Basic Valuation Theory

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1 Absolute Values

1.1 Absolute Values - Completions

Let K be a field. An **absolute value** on K is a map

$$| \ | : K \to \mathbb{R}$$

satisfying the following axioms for all $x, y \in K$

1. $|x| > 0$ for all $x \neq 0$, and $|0| = 0$
2. $|xy| = |x||y|$
3. $|x + y| \leq |x| + |y|$

The absolute value sending all $x \neq 0$ to 1 is called the **trivial** absolute value on K.

Observation: $|1|^2 = |1^2| = |1|$, $|1| = 1 = |-1|$, $|x| = |-x|$ for all $x \in K$, $|x^{-1}| = |x|^{-1}$ for $x \neq 0$.

Proposition 1.1. *The set $\{|n \cdot 1| \mid n \in \mathbb{Z}\}$ is bounded iff $| \ |$ satisfies the "ultra-metric" inequality*

$$|x + y| \leq \max\{|x|, |y|\} \tag{1}$$

for all $x, y \in K$

Proof. $\Leftarrow$: Easy, bounded by 1

$\Rightarrow$: let $|n \cdot 1| \leq C$, then

$$|x + y|^n = |(x + y)^n| \leq \sum_{\nu} \left| \binom{n}{\nu} x^\nu y^{n-\nu} \right| \leq (n + 1)C \max(|x|, |y|)^n$$

□

If an absolute value satisfies (1), it is called **non-archimedean**; otherwise it is called **archimedean**. Clearly, if $\operatorname{char} K \neq 0$, K cannot carry any archimedean absolute value.

Example 1.1. Let

$$|x|_0 = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

for all $x \in \mathbb{R}$; we call $|\ |_0$ the **usual** absolute value on $\mathbb{R}$. This is an archimedean absolute value.

Example 1.2. For every prime p, the p**-adic** absolute value $|\ |_p$ on $\mathbb{Q}$ is defined by $|0|_p = 0$ and

$$\left|p^\nu \frac{m}{n}\right|_p = \frac{1}{e^\nu}$$

where $\nu \in \mathbb{Z}$, and $n, m \in \mathbb{Z} \setminus \{0\}$ are not divisible by p. In this case

$$\{|n \cdot 1|_p \mid n \in \mathbb{Z}\} = \{e^{-\nu} \mid \nu \in \mathbb{N}\}$$

is bounded in $\mathbb{R}$.

Example 1.3. Let F be a field and let $F[[T]] = \{\sum_{i=0}^{\infty} a_i T^i \mid a_i \in F\}$, which is called the **formal power series over** F. We can define the absolute value $|\ |$ as

$$|f| = e^{-m}$$

when $f = \sum_{i=m}^{\infty} a_i T^i$ where $a_m \neq 0$.

Example 1.4. We define for every irreducible polynomial $p \in k[X]$, k a field, the following absolute value $|\ |_p$ on the rational function field $K = k(X)$: Let $|0|_p = 0$ and

$$\left|p^\nu \frac{f}{g}\right|_p = \frac{1}{e^\nu}$$

where $\nu \in \mathbb{Z}$ and $f, g \in k[X] \setminus \{0\}$ are not divisible by p. Hence the set $\{|n \cdot 1|_p \mid n \in \mathbb{Z}\}$ is bounded in $\mathbb{R}$.

Proposition 1.2. *If A is a domain, K is the fraction field of A and $|\ |$ is an absolute value, then we can uniquely extend $|\ |$ to K*

Proof. For any $a, b \in A$ and $b \neq 0$,

$$|a| = \left|b \cdot \frac{a}{b}\right| = |b|\left|\frac{a}{b}\right|$$

□

An absolute value $|\ |$ on K defines a metric by taking $|x-y|$ as distance, for $x, y \in K$. In particular, $|\ |$ induces a topology on K by taking basic open balls $B_\epsilon(a) = \{x : |x-a| < \epsilon\}$.

Since a non-trivial absolute value $|\ |$ defines a metric on K, we may consider the completion of K w.r.t. $|\ |$. Fix a $|\ |$.

A sequence $(x_n)_{n\in\mathbb{N}}$ of elements of k is called a **Cauchy sequence** if for every $\epsilon > 0$ there exists $N \in \mathbb{N}$ s.t. for all $n, m > N$ we have

$$|x_n - x_m| < \epsilon$$

We say a sequence $(x_n)_{n\in\mathbb{N}}$ **converges** to $x \in K$ and write $\lim_{n\to\infty} x_n = x$ if for every $\epsilon > 0$ there is an $N \in \mathbb{N}$ s.t. for all $n > N$ we have

$$|x_n - x| < \epsilon$$

K is **complete** if every Cauchy sequence from K converges to some element of K.

The next theorem will show that every field K with a non-trivial absolute value can be densely embedded into a field complete with respect to an ab- solute value extending the given one on K.

Theorem 1.3. *There exists a field $\hat{K}$, complete under an absolute value $|\hat{\ }|$, and an embedding $\iota : K \to \hat{K}$, s.t. $|x| = |\widehat{\iota x}|$ for all $x \in K$. The image $\iota(K)$ is dense in $\hat{K}$. If $(\hat{K}', \iota')$ is another such pair, then there exists a unique continuous isomorphism $\varphi : \hat{K} \to \hat{K}'$ preserving the absolute value and making the diagram*

$$\begin{tikzcd}
\hat{K} \arrow[rr, "\varphi"] & & \hat{K}' \\
& K \arrow[ul, "\iota"] \arrow[ur, "\iota'"'] &
\end{tikzcd}$$

*Such a pair is called a **completion** of the valued field $K, |\ |$*

Proof. Sketch of completion:

Let $\mathcal{C}$ be the set of all Cauchy sequences $(x_n)_{n\in\mathbb{N}}$ of elements of K. $\mathcal{C}$ is a ring with componentwise addition and multiplication. $\mathcal{N} = \{(x_n)_{n\in\mathbb{N}} \mid \lim_{n\to\infty} x_n = 0\}$ is an ideal of $\mathcal{C}$.

Each $(a_n)_{n\in\mathbb{N}} \in \mathcal{C} \setminus \mathcal{N}$ has a positive lower bound, and therefore there is $M \in \mathbb{N}$ and $\eta > 0$ s.t. $|a_n| > \eta$ for every $n > M$.

Setting $c_n = 1$ for every $n = 1, \ldots, M$ and $c_n = a_n^{-1}$ for every $n > M$. Then $(c_n)_{n\in\mathbb{N}}$ is a Cauchy sequence, and $(a_n)_{n\in\mathbb{N}}(c_n)_{n\in\mathbb{N}} - (1)_{n\in\mathbb{N}} \in \mathcal{N}$. Thus the ideal $\mathcal{N}$ is a maximal ideal of $\mathcal{C}$, and the quotient ring $\hat{K}$ is a field.

The map $\iota : K \to \hat{K}$ defined by $\iota(x) = (x_n)_{n\in\mathbb{N}} + \mathcal{N}$, where $x_n = x$ for every n, embeds K in $\hat{K}$.

For $(a_n)_{n\in\mathbb{N}} \in \mathcal{C}$ the sequence $(|a_n|)_{n\in\mathbb{N}}$ is a Cauchy sequence of real numbers, since $\left||a_p| - |a_q|\right|_0 \leq |a_p - a_q|$ for all p, q. Moreover, for every sequence $(a_n)_{n\in\mathbb{N}} \in \mathcal{N}$ the sequence of real numbers $(|a_n|)_{n\in\mathbb{N}}$ has limit 0. Consequently for $\xi = (a_n)_{n\in\mathbb{N}} + \mathcal{N}$ the value

$$\left|\hat{\xi}\right| = \lim_{n\to\infty} |a_n|$$

does not depend on the representative $(b_n)_{n\in\mathbb{N}}$ of ξ.And it's an absolute value of $\hat{K}$ that induces $|\ |$ on K. $\square$

Definition 1.4. Let $\mathbb{Q}_p$ be the completion of $\mathbb{Q}$ w.r.t. the p-adic absolute value $|\ |_p$, called p**-adic numbers**. The ring of p**-adic integers** is $\mathbb{Z}_p = \{x \in \mathbb{Q}_p \mid |x|_p \leq 1\}$

Fact 1.5. *1. $\mathbb{Z}_p$ is the completion of $\mathbb{Z}$ w.r.t. the p-adic absolute value.*

2. $\mathbb{Q}_p = \mathbb{Z}_p[1/p]$.

3. *Every $x \in \mathbb{Z}_p$ can be written in the form*

$$x = b_0 + b_1 p + b_2 p^2 + \cdots + b_n p^n + \ldots$$

where $0 \leq b_i \leq p-1$, and this representation is unique.

4. *Every $x \in \mathbb{Q}_p$ can be written in the form*

$$x = \sum_{n\geq -n_0} b_n p^n$$

where $0 \leq b_n \leq p-1$ and $|x|_p = e^{n_0}$. This representation is unique.

1.2 Archimedean Complete Fields

Let K be a field complete w.r.t. an archimedean absolute value $|\ |$. Since the set $\{|n \cdot 1| \mid n \in \mathbb{Z}\}$ is not bounded, char $K = 0$. Thus K contains the field $\mathbb{Q}$ of rationals.

$|\ |$ restricted to $\mathbb{Q}$ induces the same topology as the usual absolute value of $\mathbb{Q}$. Thus the complete field K contains the completion of $\mathbb{Q}$ w.r.t. the ordinary absolute value, i.e., K contains $\mathbb{R}$ as a closed subfield.

Then K must be equal to $\mathbb{R}$ or to $\mathbb{C}$. Consequently, every field K admitting an archimedean absolute value may be considered as a subfield of $\mathbb{C}$ or even $\mathbb{R}$ with the absolute value dependent on the induced one from $\mathbb{C}$ (or from $\mathbb{R}$)

1.3 Non-Archimedean Complete Fields

Assume $|\ |$ is a non-trivial, non-archimedean absolute value on the field K, we can define an "additive" presentation of the absolute value $|\ |$:

$$v(x) := -\ln|x|$$

In the case of the p-adic absolute value $|\ |_q$ on $\mathbb{Q}$, we obtain

$$v_p(p^\nu \frac{m}{n}) = \nu$$

v_p is called the **p-adic valuation** on $\mathbb{Q}$.

Using the additive notion, the axioms of a non-archimedean absolute value

$$v : K \to \mathbb{R} \cup \{\infty\}$$

now reads for all $x, y \in K$

1. $v(x) \in \mathbb{R}$ for $x \neq 0$, $v(0) = \infty$
2. $v(xy) = v(x) + v(y)$
3. $v(x + y) \geq \min\{v(x), v(y)\}$

First we note that only the additive structure of $\mathbb{R}$ together with the ordering on $\mathbb{R}$ is used, we will generalize this later. Secondly, ∞ is a symbol that satisfies, for all $\gamma \in \mathbb{R}$, the following axiom:

$$\infty = \infty + \infty = \gamma + \infty = \infty + \gamma$$

By an **ordered abelian group** we mean an abelian group $(\Gamma, +, 0)$ together with a binary relation $\leq$ on Γ, where $\leq$ is a linear order on Γ and for any $\gamma, \delta, \lambda \in \Gamma$,

$$\gamma \leq \delta \Rightarrow \gamma + \lambda \leq \delta + \lambda$$

Let Γ be an ordered abelian group, and ∞ a symbol satisfying for all $\gamma \in \Gamma$,

$$\infty = \infty + \infty = \gamma + \infty = \infty + \gamma.$$

We then define a **valuation** v on a field K to be a surjective map

$$v : K \twoheadrightarrow \Gamma \cup \{\infty\}$$

satisfying the following axioms: for all $x, y \in K$,

1. $v(x) = \infty \Rightarrow x = 0$
2. $v(xy) = v(x) + v(y)$
3. $v(x + y) \geq \min\{v(x), v(y)\}$

If $\Gamma = \{0\}$, we call v the **trivial valuation**; for all $x, y \in K$:

$$v(1) = 0, \qquad v(x^{-1}) = -v(x), \qquad (-x) = v(x),$$
$$v(x) < v(y) \Rightarrow v(x + y) = v(x)$$

Definition 1.6. Let $v : K^\times \to \Gamma$ be a valuation on a field. We set

1. $\mathcal{O}_v := \{x \in K : v(x) \geq 0\}$
2. $\mathfrak{m}_v := \{x \in K : v(x) > 0\}$
3. $\mathbf{k}_v := \mathcal{O}_v / \mathfrak{m}_v$.

For all $x, y \in \mathcal{O}_v$ we have

$$v(x \pm y) \geq \min\{v(x), v(\pm y)\} \geq 0$$
$$v(xy) = v(x) + v(y) \geq 0$$

Hence $x \pm y, xy \in \mathcal{O}$. From $v(x^{-1}) = -v(x)$, we deduce that x is a unit in $\mathcal{O}_v$ iff $v(x) = 0$ and for every $x \in K$, either x or x^{-1} or both lie in $\mathcal{O}_v$. A subring $\mathcal{O}$ of K satisfying

$$x \in \mathcal{O} \quad \text{or} \quad x^{-1} \in \mathcal{O}$$

for all $x \in K^\times$ is called a **valuation ring** of K. Thus $\mathcal{O}_v$ is a valuation ring. Moreover, $\mathfrak{m}_v$ is an ideal of $\mathcal{O}_v$. Since $\mathfrak{m}_v$ consists exactly of the non-units of $\mathcal{O}_v$, $\mathfrak{m}_v$ is a maximal ideal, and in fact the only maximal ideal of $\mathcal{O}_v$. Thus $\mathcal{O}_v$ is a local ring(ring with only one maximal ideal) and $\mathbf{k}_v$ is a field, called the **residue class field** of v. The residue class of $a \in \mathcal{O}_v$ is denoted by $\bar{a}$. Note that v is trivial iff $\mathcal{O}_v = K$ iff $\mathbf{k}_v = K$. The group $v(K^\times)$ will be called the **value group** of v.

Proposition 1.7. *Let $\mathcal{O} \subseteq K$ be a valuation ring of K. Then there exists a valuation v on K s.t. $\mathcal{O} = \mathcal{O}_v$.*

Proof. Denote by $\mathcal{O}^\times$ the group of units of $\mathcal{O}$. The group $\Gamma = K^\times / \mathcal{O}^\times$ is an abelian group and we can define a binary relation on Γ by

$$x\mathcal{O}^\times \leq y\mathcal{O}^\times \Leftrightarrow \frac{y}{x} \in \mathcal{O}$$

We can check that Γ is an ordered abelian group. The valuation is defined by

$$v(x) = x\mathcal{O}^\times \in \Gamma$$

for $x \in K^\times$, and $v(0) = \infty$. If $v(x) \leq v(y)$, then $y/x \in \mathcal{O}$. Therefore $(x + y)/x = 1 + y/x \in \mathcal{O}$ and $v(x+y) \geq v(x) = \min\{v(x), v(y)\}$. Now

$$\mathcal{O}_v = \{x \in K \mid v(x) \geq 0\} = \{x \in K \mid x \in \mathcal{O}\} = \mathcal{O}$$

$\square$

Example 1.5. Consider $K = \mathbb{Q}$, $v = v_p$, then

$$\mathcal{O}_{v_p} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \text{ is not divisible by } p\}$$
$$\mathfrak{m}_{v_p} = \{\frac{pa}{b} \mid a, b \in \mathbb{Z}, b \text{ is not divisible by } p\}$$

$\mathcal{O}_{v_p}$ is the localization $\mathbb{Z}_{(p)} = (\mathbb{Z} - (p))^{-1}\mathbb{Z})$ of the ring $\mathbb{Z}$ at the prime ideal $(p) = p\mathbb{Z}$, and $\mathfrak{m}_{v_p}$ is $p\mathbb{Z}_{(p)}$. Thus the residue class field $\mathbf{k}_{v_p}$ is isomorphic to the finite field $\mathbb{F}_p$.

Example 1.6. Consider $K = F((T)) = \{\sum_{n=m}^{\infty} a_n T^n \mid m \in \mathbb{Z}, a_n \in F\}$, field of formal Laurent series with valuation $v(f) = m$ where $f = \sum_{n=m}^{\infty} a_n T^n$ and $a_m \neq 0$, then $\mathcal{O}_v = F[[T]]$, $\mathfrak{m}_v$ is all series $\sum_{n=m}^{\infty} a_n T^n$ where $m > 0$ and the residue field $\mathbf{k}_v$ is F.

2 Hensel's Lemma

Definition 2.1. A local domain A with maximal ideal $\mathfrak{m}$ is **henselian** if whenever $f(x) \in A[X]$ and there is $a \in A$ s.t. $f(a) \in \mathfrak{m}$ and $f'(a) \notin \mathfrak{m}$, then there is $\alpha \in A$ s.t. $f(\alpha) = 0$ and $\alpha - a \in \mathfrak{m}$.

A **valued field** is a pair $(K, \mathcal{O})$ where K is a field and A is a valuation ring. A valued field is **henselian** if its valuation ring is henselian.

Remark. A ring is local iff all non-units form an ideal, therefore henselianity is a first-order property.

Theorem 2.2 (Hensel's Lemma)**.** *Suppose K is a complete field with non-archimedean absolute value $|\ |$ and valuation ring $\mathcal{O} = \{x \in K : |x| \leq 1\}$. Then $\mathcal{O}$ is henselian*

Proof. Suppose $a \in \mathcal{O}_v$, $|f(a)| = \epsilon < 1$ and $|f'(a)| = 1$. We think of a as our first approximation to a zero of f and use Newton's method to find a better approximation.

Let $\delta = \frac{-f(a)}{f'(a)}$. Note that $|\delta| = \left|\frac{f(a)}{f'(a)}\right| = \epsilon$. Consider the Taylor expansion

$$f(a+x) = f(a) + f'(a)x + \text{terms of degree at least 2 in } x$$

Thus

$$f(a+\delta) = f(a) + f'(a)\frac{-f(a)}{f'(a)} + \text{terms of degree at least 2 in } \delta$$

Thus $|f(a+\delta)| \leq \epsilon^2$. Similarly

$$f'(a+\delta) = f'(a) + \text{terms of degree at least 2 in } \delta$$

and $|f'(a+\delta)| = |f'(a)| = 1$.

Thus starting with an approximation where $|f(a)| = \epsilon < 1$ and $|f'(a)| = 1$, we get a better approximation b where $|f(b)| \leq \epsilon^2$ and $|f'(b)| = 1$. We now iterate this procedure to build $a = a_0, a_1, a_2, \ldots$ where

$$a_{n+1} = a_n - \frac{a_n}{f'(a_n)}$$

It follows, by induction, that for all n:

1. $|a_{n+1} - a_n| \leq \epsilon^{2^{n+1}}$
2. $|f(a_n)| \leq \epsilon^{2^n}$
3. $|f'(a_n)| = 1$

Thus $(a_n)_{n\in\mathbb{N}}$ is a Cauchy sequence and converges to α, $|\alpha - a| \leq \epsilon$, and $f(\alpha) = \lim_{n\to\infty} f(a_n) = 0$ □

Therefore we have henselian field $(\mathbb{Q}_p, \mathbb{Z}_p)$ and $(F((T)), F[[T]])$.

Fact 2.3 (Chevalley)**.** *For a field K, let $A \subseteq K$ be a subring and let $P \subseteq A$ be a prime ideal of A. Then there exists a valuation ring $\mathcal{O}$ of K s.t.*

$$R \subseteq \mathcal{O} \quad \textit{and} \quad M \cap R = P$$

where M is the maximal ideal of $\mathcal{O}$.

Lemma 2.4. *Let K_2/K_1 be a field extension and let $\mathcal{O}_1 \subseteq K_1$ be a valuation ring. Then there is a valuation ring $\mathcal{O}_2 \subseteq K_2$ with $\mathcal{O}_2 \cap K_1 = \mathcal{O}_1$.*

Proof. Since $\mathcal{O}_1$ is a subring of K_2, according to Chevalley's Theorem there exists a valuation ring $\mathcal{O}_2$ of K_2 with $\mathcal{O}_1 \subseteq \mathcal{O}_2$ and $\mathfrak{m}_2 \cap \mathcal{O}_1 = \mathfrak{m}_1$ for maximal ideals. Since $\mathcal{O}_2 \cap K_1$ and $\mathcal{O}_1$ are valuation rings with the same maximal ideal they must coincide. $\square$

Fact 2.5. *Let $(K, \mathcal{O})$ be a valued field. T.F.A.E.:*

1. *$(K, \mathcal{O})$ is henselian.*
2. *For any separable extension L/K there is a unique extension of $\mathcal{O}$ to a valuation ring of L.*
3. *For any algebraic extension L/K there is a unique extension of $\mathcal{O}$ to a valuation ring of L.*

3 Hahn Series

For each group Γ and field k, there is a field $K = k((t^\Gamma))$ with valuation v having Γ as the value group and k as the residue field.

Lemma 3.1. *Let $A, B \subseteq \Gamma$ be well-ordered (by the ordering of Γ). Then $A \cup B$ is well-ordered, the set $A + B := \{\alpha + \beta : \alpha \in A, \beta \in B\}$ is well-ordered, and for each $\gamma \in \Gamma$ there are only finitely many $(\alpha, \beta) \in A \times B$ s.t. $\alpha + \beta = \gamma$.*

Proof. Suppose $(a_0, b_0), (a_1, b_1), \ldots$ are distinct s.t. $a_i + b_i > a_j + b_j$ for $i < j$. Then we can find a strictly monotone subsequence of the a_i. Since A is well-ordered, the sequence cannot be decreasing. But then there is a strictly decreasing subsequence of b_i. $\square$

Lemma 3.2 (Neumann's Lemma)**.** *Let $A \subseteq \Gamma^{>0}$ be well-ordered. Then*

$$[A] := \{\alpha_1 + \cdots + \alpha_n : \alpha_1, \ldots, \alpha_n \in A\} \qquad (\textit{allowing } n = 0)$$

is also well-ordered, and for each $\gamma \in [A]$ there are only finitely many tuples $(n, \alpha_1, \ldots, \alpha_n)$ with $\alpha_1, \ldots, \alpha_n \in A$ s.t. $\gamma = \alpha_1 + \cdots + \alpha_n$

Define $K = k((t^\Gamma))$ to be the set of all formal series $f(t) = \sum_{\gamma \in \Gamma} a_\gamma t^\gamma$ with coefficients $a_\gamma \in k$, s.t. the support of f,

$$\operatorname{supp}(f) := \{\gamma \in \Gamma : a_\gamma \neq 0\}$$

is a well-ordered subset of Γ.By the first lemma, we can define binary operations of addition and multiplication on $k((t^\Gamma))$ as

$$\sum a_\gamma t^\gamma + \sum b_\gamma t^\gamma = \sum (a_\gamma + b_\gamma) t^\gamma$$
$$\left(\sum a_\gamma t^\gamma\right)\left(\sum b_\gamma t^\gamma\right) = \sum_\gamma \left(\sum_{\alpha+\beta=\gamma} a_\alpha b_\beta\right) t^\gamma$$

Define $v : K \setminus \{0\} \to \Gamma$ by

$$v(\sum a_\gamma t^\gamma) := \min\{\gamma : a_\gamma \neq 0\}$$

Then v is a valuation on K. If $v(f) > 0$, then by the second lemma $\sum_{n=0}^\infty f^n$ makes sense as an element of K: for any $\gamma \in \Gamma$ there are only finitely many n s.t. the coefficients of t^γ in f^n is not zero. Then

$$(1-f)\sum_{n=0}^\infty f^n = 1$$

Now for any $g \in K \setminus \{0\}$, $g = ct^\gamma(1-f)$, with $c \in k^\times$ and $v(f) > 0$. Then $g^{-1} = c^{-1}t^{-\gamma}\sum_n f^n$.

For $f = \sum a_\gamma t^\gamma \in K$, call a_0 the constant term of f. The map sending sending f to its constant term sends $\mathcal{O}_v$ onto k, and this is a ring homomorphism. Its kernel is $\mathfrak{m}_v$. Therefore $\mathcal{O}_v/\mathfrak{m}_v \cong k$.

We call K the **Hahn field**.

Definition 3.3. Let K be a valued field. We say that K is **spherically complete** if whenever $(I, <)$ is a linear order and $(B_i : i \in I)$ is a family of open balls s.t. $B_i \supset B_j$ for all $i < j$, then $\bigcap_{i \in I} B_i \neq \emptyset$.

Definition 3.4. If (K, v) is a valuation field extending L as a subfield, then K is an **immediate extension** if $v(K) = v(L)$ and $\mathbf{k}_K = \mathbf{k}_L$.

Fact 3.5.

1. *Hahn field is henselian.*
2. *Hahn field is spherically complete.*
3. *Hahn field has no proper immediate extensions.*