# Homework 4: Morley sequences and the order property

Advanced Model Theory

Due March 24, 2022

1. Find a set $A$ and a relation $R \subseteq A \times A$ such that

$$\exists^\infty x \in A \; \exists^\infty y \in A : (x,y) \in R$$
$$\neg \exists^\infty y \in A \; \exists^\infty x \in A : (x,y) \in R.$$

2. Consider the structure $(\mathbb{R}, +, -, \cdot, 0, 1, \leq)$. Let $\varphi(x,y)$ be the formula $y - 1 \leq x \wedge x \leq y + 1$. Show that $\varphi(x,y)$ has the order property (in a monster model $\mathbb{M} \succeq \mathbb{R}$). *Hint:* you don't really need to go to the elementary extension.

3. Let $\mathbb{M}$ be a monster model of DLO. Let $\tau \in S_1(\mathbb{M})$ be the type at $+\infty$ as in last week's homework. Consider the Morley product $\tau \otimes \tau \in S_2(\mathbb{M})$. Show that $(\tau \otimes \tau)(x,y)$ is the unique completion of $\tau(x) \cup \tau(y) \cup \{x < y\}$. *Hint:* in one approach to this problem, you first show that $\tau(x) \cup \tau(y) \cup \{x < y\}$ has a unique completion, which is similar to the proof that $\tau$ is well-defined on last week's homework. Then you only need to show that $(\tau \otimes \tau)(x,y) \vdash \tau(x) \cup \tau(y) \cup \{x < y\}$, which isn't *that* hard.

4. Let $\mathbb{M}$ be a monster model of a complete theory $T$. Suppose $\mathbb{M}$ is an expansion of a linear order. (This means that there is a binary relation symbol $\leq$ in the language, and $(\mathbb{M}, \leq)$ is a linear order.) Let $p \in S_1(\mathbb{M})$ be a global $A$-invariant 1-type. Suppose that $p$ commutes with itself. Show that $p$ is a constant/realized type, meaning that $p = \mathrm{tp}(c/\mathbb{M})$ for some $c \in \mathbb{M}$. *Hint:* take $(a,b)$ realizing $p$ over a small set and compare $a$ with $b$.