4.75 out of 6 points.

Week3

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Exercise 1. Show that the collection of formulas $x > a$ for $a \in M$ generates a complete type $\tau_M(x) \in S_1(M)$. In other words, show that the partial type $\{(x > a) : a \in M\}$ has a unique completion

+0.5

Proof. Let $\Sigma(x) = \{x > a : a \in M\}$, $p, q \in S_1(M)$, $p, q \supseteq \Sigma(x)$, $p \neq q$. Since DLO has quantifier elimination, there is a quantifier free formula $\varphi(x) \in p \setminus q$. φ has the form

φ could also be a disjunction of these formulas

$$\bigwedge_{a \in A} x > a \wedge \bigwedge_{b \in B} x \leq b \wedge \bigwedge_{c \in C} x \neq c \wedge \bigwedge_{d \in D} x = d$$

If φ is $(x>1) \wedge (x>2) \wedge (x \neq 3)$ then $a' = 2$ but $\varphi \not\equiv x > a'$...

where A, B, C, D are finite. But since $p, q \supseteq \Sigma(x)$, $B = D = \emptyset$. Also $x \neq c$ is implied by $\Sigma(x)$. Thus if we choose $a' = \max\{a : a \in A\}$, then

$$M \vdash \varphi \leftrightarrow x > a'$$

this doesn't hold for every φ in p, because of disjunctions, as well as the problem with $x \neq c$ terms.

The important thing is $M \vdash x > a' \to \varphi$ ... for $a' := \max(A \cup C)$.

Thus $\varphi \in q$, a contradiction. Hence $p = q$ □

+2

Exercise 2. Show that τ_M is definable

Proof. By exercise 3 for each $N \succeq M$, τ_M has a unique heir and thus τ_M is definable □

Exercise 3. Suppose $N \succeq M$. Show that τ_N is an heir of τ_M

Proof. Let $q \in S_1(N)$ be an heir of τ_M and suppose $x \leq a \in q(x)$ for some $a \in N$. Then there is $a' \in M$ s.t. $x \leq a' \in \tau_M$, which is impossible. Thus $\{x > a : a \in N\} \subseteq q$ and $q = \tau_N$. Hence τ_N is the unique heir of τ_M by Exercise 1 □

Exercise 4. Suppose $N \succeq M$ and N is $|M|^+$-saturated. Show that τ_N is not a coheir of τ_M

+1

Proof. Since N is $|M|^+$-saturated, there is $c \in N$ s.t. $N \models \tau_M(c)$. Then there is no $a \in M$ satisfying $x > c$. $\square$

Exercise 5. If $N \succeq M$, show that τ_M has a unique ~~coheirs~~ coheir over N

+0.5
the logic was a little unclear

Proof. Suppose τ_M has two different coheirs $p, q \in S_1(N)$. Because p and q are the same thing as cuts, we may assume that there is $c \in N$ s.t. $x < c \in p$ and $x > c \in q$. But for any x realizing ? realizing what?, $x > m$ for all $m \in M$. Thus there is no $m \in M$ satisfying $x > c$. Hence $\cancel{p}$ q is not a coheir of τ_M, a contradiction $\square$

technically this step doesn't work for general p, q in $S_1(N)$. (consider p a constant type, q the type infinitesimally higher than p)

I think this shows $c \notin M$, but there are $c \in N \setminus M$ such that $(x > c)$ is satisfiable in M.

Wait, is $c > M$? How do we know that?

$x>c$ $\in q$
$x>c$ $\notin p$

Exercise 6. Give an example of models $M \preceq N$ of DLO where τ_N is a coheir of τ_M

+0.75

Proof. $M = \mathbb{Q}$, $N = \mathbb{R}$ $\square$

Need some explanation here. (The coheir wasn't explicitly calculated in Exercise 5, so it's not 100% obvious.)

Or it could be using the fact that τ_M has a unique extension in this case, plus the existence of coheirs.

(But need to explain.)