Advanced Model Theory

PHIL130307/PHIL630142

Last updated: February 18, 2022

Basic information: The class meets at H2106B on Thursdays in periods 3–5 (9:55-12:30) during weeks 1–16 (February 24 to June 9). The entire course is in English. This class is for both undergraduate and graduate students. Auditors are welcome to attend. The course is 3 credits.
Textbook: *A Course in Model Theory*, by Bruno Poizat. You can access it electronically in the Fudan Library system, and I will post it on eLearning. We will cover chapters 11–13 and 15–18.
Prerequisites: This course assumes you have taken classes in mathematical logic and introductory model theory, and have basic mathematical literacy. *This is a mathematics class.*
Exams: The midterm exam is in class on April 14, 2022. The final exam is in class on June 9, 2022. Both exams are open book.
Teachers: The main teacher is Will Johnson, willjohnson@fudan.edu.cn, office HGW2503, office hours to be determined. The TA is to be determined.
Homework assignments: There will be weekly homework assignments posted in the WeChat group and on eLearning. If homework is turned in n days late, the score will be multiplied by $(1 - n/5)$. Your solutions must be written in English. I will post solutions on eLearning.
Grading: 50% of the grade comes from homework, and 25% comes from each exam. After combining these values, the final grades will be curved. Undergraduate and graduate students will be curved separately. Attendance will not affect your grade.

1 Course content

This class is about stability theory, a subject within model theory, a subject within mathematical logic.

- *Mathematical logic* is the formal mathematical study of logical reasoning.
- *Model theory* is a brach of mathematical logic which analyzes a theory by studying its class of models—the mathematical structures where the axioms of the theory are true.

- *Stability theory* is the study of theories satisfying a mysterious condition called "stability," which arises naturally in many model-theoretic problems.

Stability has been a central topic in model theory since the 1970's. Our goal for the course will be to prove *Morley's Theorem*, the earliest major theorem in stability theory. It says:

> Let T be a theory in a countable language and let κ, λ be uncountable cardinals.
> If T has exactly one model of size κ, then T has exactly one model of size λ.

The statement of Morley's theorem is not as interesting as the proof, which will take us on a tour of the basic notions of stability theory, such as heirs, coheirs, stability, indiscernibility, forking, ranks, and prime models.

2 Schedule

Week	Date	Main topic	Reading
1	2/24	Heirs and definable types	§11.1–11.2
2	3/3	Stability and invariant types	§11.4, 12.1
3	3/10	Coheirs and Morley sequences	§12.2–12.3
4	3/17	NIP	§12.4–12.5
5	3/24	The order property and Ramsey's theorem	§12.8–12.9
6	3/31	The fundamental order and the stability spectrum	§13.1–13.2
7	4/7	Forking	§15.1–15.2
8	4/14	**MIDTERM EXAM**	
9	4/21	More forking	§15.3, 16.1
10	4/28	Imaginaries	§16.4–16.5
11	5/5	(Holiday)	
12	5/12	Lascar and Shelah ranks	§17.1–17.2
13	5/19	Morley and local ranks	§17.3–17.4
14	5/26	Prime models	§18.1–18.2
15	6/2	Morley's theorem	§18.5–18.6
16	6/9	**FINAL EXAM**	

The readings are from the textbook *A Course in Model Theory*, by Bruno Poizat.

Note: The date of the midterm and final exams are fixed. The rest of the schedule will be adjusted for holidays.