Topology Notes

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August 13, 2021

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Definition 0.1. A **topology** on a set is a collection $\mathcal{T}$ of subsets of X having the following properties

1. $\emptyset$ and X are in $\mathcal{T}$
2. The union of the elements of any subcollection of $\mathcal{T}$ is in T
3. The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$

A set X for which a topology $\mathcal{T}$ has been specified is called a **topological space**

Definition 0.2. If X is a set, a **basis** for a topology on X is a collection $\mathcal{B}$ of subsets of X (called **basis element**) s.t.

1. for each $x \in X$, there is at least one basis element B s.t. $x \in B$
2. if $x \in B_1 \cap B_2$, then there is a basis element B_3 s.t. $x \in B_3 \subset B_1 \cap B_2$

If $\mathcal{B}$ satisfies these conditions, then we define the **topology** $\mathcal{T}$ **generated by** $\mathcal{B}$ as follows: A subset U of X is said to be open in X if for each $x \in U$, there is a basis $B \in \mathcal{B}$ s.t. $x \in B \subset U$.

Definition 0.3. A collection $\mathcal{A}$ of subsets of a space X is said to **cover** X, or to be a **covering** of X, if $\bigcup \mathcal{A} = X$. It is called an **open covering** of X if its elements are open subsets of X

Definition 0.4. A space X is said to be **compact** if every open covering $\mathcal{A}$ of X contains a finite subcollection that also covers X.

Definition 0.5. A collection $\mathcal{A}$ of subsets of the space X is said to have **order** $m+1$ if some point of X lies in $m+1$ elements of $\mathcal{A}$, and no point of X lies in more than $m+1$ elements of $\mathcal{A}$.

Given a collection $\mathcal{A}$ of subsets of X, a collection $\mathcal{B}$ is said to **refine** $\mathcal{A}$, or to be a **refinement** of $\mathcal{A}$, if for each element B of $\mathcal{B}$ there is an element A of $\mathcal{A}$ s.t. $B \subset A$

Definition 0.6. A space X is said to be **finite dimensional** if there is some integer m s.t. for every open covering $\mathcal{A}$ of X, there is an open covering $\mathcal{B}$ of X that refines $\mathcal{A}$ and has order at most $m+1$. The **topological dimension** of X is defined to be the smallest value of m for which this statement holds; we denote it by $\dim X$.