Homework 4

Introductory Model Theory

Autumn 2021

Due 2021-10-21

If you don't submit on time, the score you get will multiply $(1 - \frac{n}{5})$ if you delay n days for $n < 5$ and 0 for $n \geq 5$.

Problem 1. *Let $\mathcal{L} = \{P\}$, a language with only one unary relation symbol. Classify complete theoies with $\mathcal{L}$, i.e. determine all complete theories with only one unary symbol.*

Problem 2. *Show that there is a structure $(M, +, \cdot, <, 0, 1)$ elementarily equivalent to $(\mathbb{R}, +, \cdot, <, 0, 1)$ such that the order on M is not complete: there is a bounded set with no supremum.*

Hint: use Löwenheim's theorem and the classification of countable dense linear orders.

Problem 3. *Show that the open interval $((0, 1), <)$ is an elementary substructure of $(\mathbb{R}, <)$.*

Hint: Don't use Löwenheim's theorem, use Fraïssé's theorem and the fact from Chapter 1 that local isomorphisms are p-isomorphisms for all p.

Problem 4. *Show that every formula is equivalent to a "nice" formula. "Nice" is defined in compactness.pdf.*

Problem 5. *Let T be the set of $\mathcal{L}_{ring}$-sentences true in $(\mathbb{R}, +, \cdot, 0, 1)$. Show that T is finitely satisfiable and complete, but does not have the witness property.*