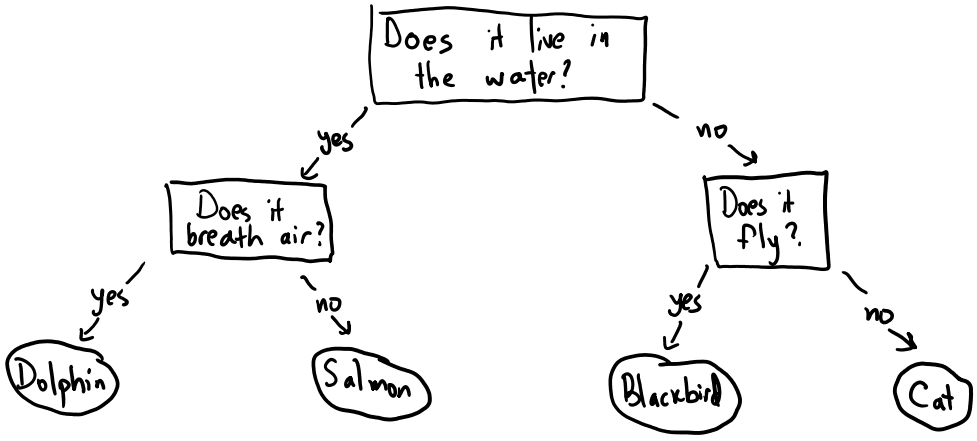
Understanding the Dichotomy Property

Wednesday, March 9, 2022 12:50 PM

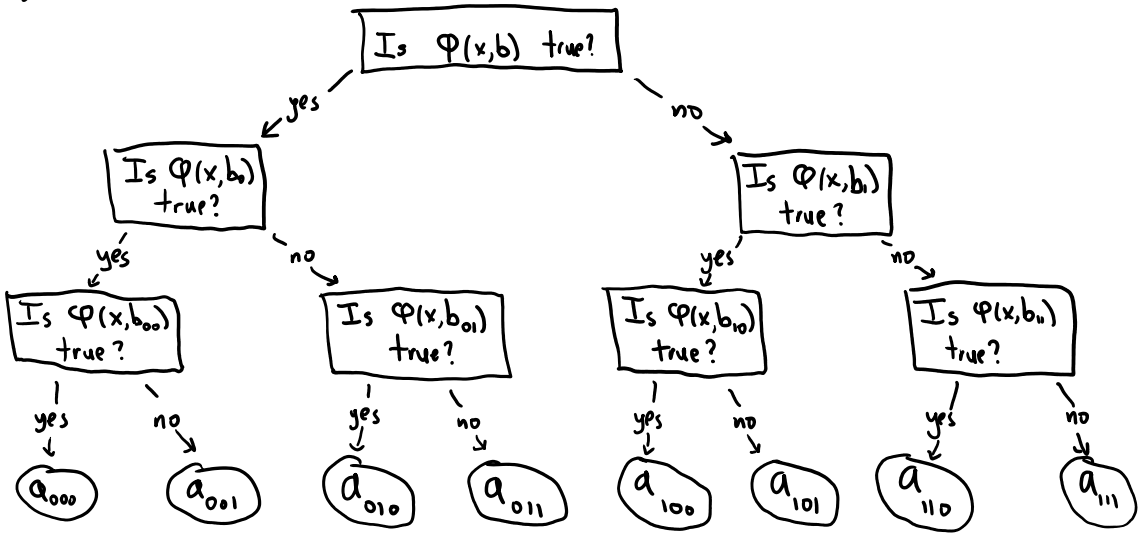
We can understand the dichotomy property in terms of **decision trees**.
A decision tree is a type of classifier in artificial intelligence.
Here is an example of a decision tree which can classify the four animals salmon, dolphin, blackbird, cat:



A formula φ has the dichotomy property if certain configurations D_n are consistent.

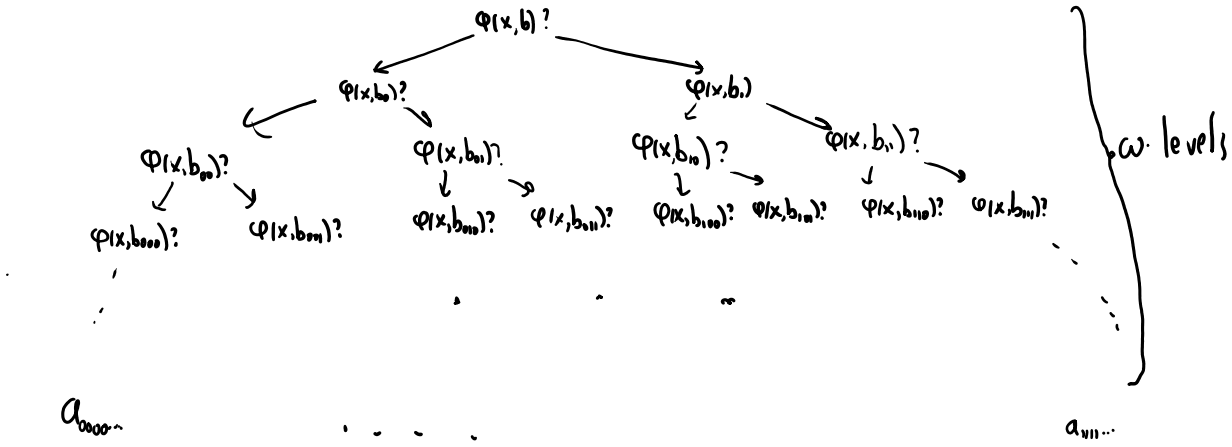
D_n can be understood as a decision tree with n levels of questions. For example,

D_3 looks like this:



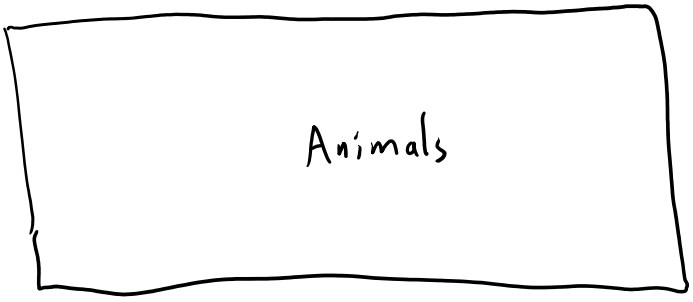
The formula phi has the dichotomy property if D_n is consistent for every n. That is, for every finite n, a tree of this sort exists with n levels of questions.

By a compactness argument, this is the same thing as saying that there is a tree with omega levels, and a leaf for each sequence of 0's and 1's. (There are aleph_0 -many questions in this tree, and 2^{aleph_0}-many leaves, which is why aleph_0-stability fails if you have the dichotomy property.)

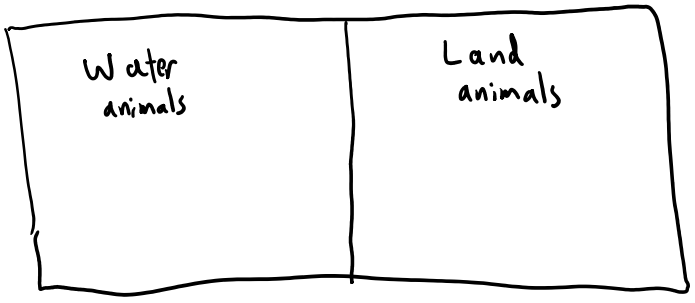


We can also understand the dichotomy property in terms of repeatedly cutting up space using formulas of the form phi(x,b).

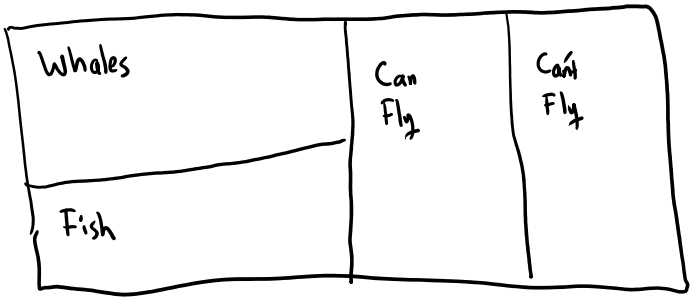
For example, the decision tree of animals corresponds to taking the set of all animals…



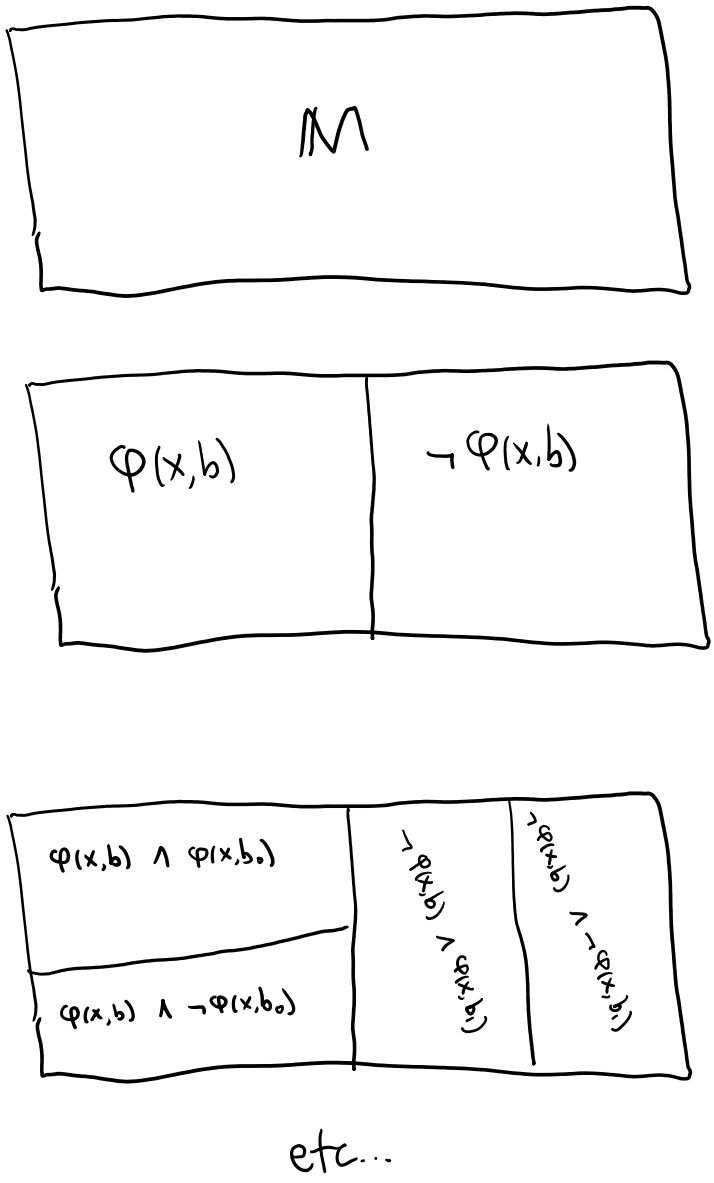
And dividing it into water animals and land animals…



And then dividing the water animals up by whether they breathe air, and the land animals by whether they fly…



The condition "D_n is consistent" means we can divide up space *n* times in this way, using a formula of the form phi(x,b) to divide things at each step.



The dichotomy property means we can do this ω times (and each region is non-empty!)