

# Homework 7: acl and imaginaries

## Advanced Model Theory

Due April 21, 2022

Note this homework is due in *two* weeks, on April 21.

Let  $T$  be a complete theory of infinite fields, such as RCF or  $\text{ACF}_0$ . (Make sure to not assume  $T = \text{ACF}_0$ .) Let  $\mathbb{M}$  be a monster model of  $T$ . Suppose  $a, b, c \in \mathbb{M}$ .

1. Suppose  $\sqrt{a}$  and  $\sqrt{b}$  both exist (in  $\mathbb{M}$ ). Show that  $\sqrt{a} + \sqrt{b} \in \text{acl}(\{a, b\})$ .
2. Suppose  $c \neq 0$ . Let  $D = \{(x, y, z) \in \mathbb{M}^3 : ax + by + cz = 0\}$ . Show that  $(a/c, b/c)$  is a “code” for  $D$ , in the sense defined in class.
3. Let  $D = \mathbb{M}^3 \setminus \{(0, 0, 0)\}$ . Let  $E$  be the equivalence relation on  $D$  where  $(a_1, a_2, a_3)E(b_1, b_2, b_3)$  iff the two vectors are parallel, that is, there is  $\lambda \in \mathbb{M}$  such that

$$\lambda a_1 = b_1, \lambda a_2 = b_2, \lambda a_3 = b_3.$$

Find a definable function  $f : D \rightarrow \mathbb{M}^n$  such that

$$f(a_1, a_2, a_3) = f(b_1, b_2, b_3) \iff (a_1, a_2, a_3)E(b_1, b_2, b_3).$$

4. Suppose  $\leq$  is a definable linear order on  $\mathbb{M}$ . Show that  $\text{dcl}(A) = \text{acl}(A)$  for any  $A \subseteq \mathbb{M}$ .