Big DataBase

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1 Query Optimization

1.1 Introduction

Compile time system:

1. parsing: parsing, AST production
2. semantic analysis: schema lookup, variable binding, type inference
3. normalization, factorization, constant folding
4. rewrite 1: view resolution, unnesting, deriving predicates
5. plan generation: constructing the execution plan
6. rewrite 2: refining the plan, pushing group

7. code generation: producing the imperative plan

Different optimization goals:

- minimize response time
- minimize resource consumption
- minimize time to first tuple
- maximize throughput

Notation:

- $\mathcal{A}(e)$: attributes of the tuples produces by e
- $\mathcal{F}(e)$ free variable of the expression e
- binary operators $e_1 \theta e_2$ usually require $\mathcal{A}(e_1) = \mathcal{A}(e_2)$
- $\rho_{a \to b(e)}$, rename
- $\Pi_A(e)$, projection
- $\sigma_p(e)$, selection, $\{x \mid x \in e \wedge p(x)\}$
- $e_1 \bowtie_p e_2$, join, $\{x \circ y \mid x \in e_1 \wedge y \in e_2 \wedge p(x \circ y)\}$

Different join implementations have different characteristics:

- $e_1 \bowtie^{NL} e_2$ Nested Loop Join:
- $e_1 \bowtie^{BNL} e_2$ Blockwise Nested Loop Join: Read chunks of e_1 into memory and read e_2 once for each chunk. Further improvement: Use hashing for equi-joins
- $e_1 \bowtie^{SM} e_2$ Sort Merge Join: Equi-joins only
- $e_1 \bowtie^{HH} e_2$ Hybrid-Hash Join: Partitions e_1 and e_2 into partitions that can be joined in memory. Equi-joins only

1.2 Query Optimization

steps

1. translate the query into its canonical algebraic expression
2. logical query optimization
3. physical query optimization

1.2.1 Algebra Revisited

Tuple is a (unordered) mapping from attribute names to values of a domain
Schema is a set of attributes with domain, written $\mathcal{A}(t)$
concatenation of tuple:

- $t_1 \circ t_2$, note $t_1 \circ t_2 = t_2 \circ t_1$
- $\mathcal{A}(t_1) \cap \mathcal{A}(t_2) = \emptyset$
- $\mathcal{A}(t_1 \circ t_2) = \mathcal{A}(t_1) \cup \mathcal{A}(t_2)$

tuple projection:

- $t.a$, $t|_A$
- $a \in \mathcal{A}(t), A \subseteq \mathcal{A}(t)$
- $\mathcal{A}(t|_A) = A$
- $t.a$ produces a value, $t|_A$ produces a tuple

Relation is a set of tuples with the same schema. Schema of the contained tuples, written $\mathcal{A}(R)$

Real data is usually a multi set (bag). The optimizar must consider three different semantics:

- logical algebra operates on bags
- physical algebra operates on streams
- explicit duplicate elimination $\Rightarrow$ sets

Set operations are part of the algebra:

- union, intersection, difference
- but have schema constraints
- $\mathcal{A}(L) = \mathcal{A}(R)$
- $\mathcal{A}(L \cup R) = \mathcal{A}(L) = \mathcal{A}(R)$, $\mathcal{A}(L \cup R) = \mathcal{A}(L) = \mathcal{A}(R)$, $\mathcal{A}(L \setminus R) = \mathcal{A}(L) = \mathcal{A}(R)$

$\mathcal{F}(e)$ are the free variables of e
Selection:

- $\sigma_p(R)$
- $\mathcal{F}(p) \subseteq \mathcal{A}(R)$
- $\mathcal{A}(\sigma_p(R)) = \mathcal{A}(R)$

Projection:

- $\Pi_A(R)$
- eliminates duplicates for set semantic, keeps them for bag semantic
- $A \subseteq \mathcal{A}(R)$
- $\mathcal{A}(\Pi_A(R)) = A$

Rename:

- $\rho_{a \to b}(R)$
- $a \in \mathcal{A}(R), b \notin \mathcal{A}(R)$
- $\mathcal{A}(\rho_{a \to b}(R)) = \mathcal{A}(R) \setminus \{a\} \cup \{b\}$

$$
\begin{aligned}
\sigma_{p_1 \wedge p_2} &\equiv \sigma_{p_1}(\sigma_{p_2}(e)) && (1)\\
\sigma_{p_1}(\sigma_{p_2}(e)) &\equiv \sigma_{p_2}(\sigma_{p_1}(e)) && (2)\\
\Pi_{A_1}(\Pi_{A_2}(e)) &\equiv \Pi_{A_1}(e) && (3)\\
&\equiv \text{if } A_1 \subseteq A_2\\
\sigma_p(\Pi_A(e)) &\equiv \Pi_A(\sigma_p(e)) && (4)\\
&\equiv \text{if } \mathcal{F}(p) \subseteq A\\
\sigma_p(e_1 \cup e_2) &\equiv \sigma_p(e_1) \cup \sigma_p(e_2) && (5)\\
\sigma_p(e_1 \cap e_2) &\equiv \sigma_p(e_1) \cap \sigma_p(e_2) && (6)\\
\sigma_p(e_1 \setminus e_2) &\equiv \sigma_p(e_1) \setminus \sigma_p(e_2) && (7)\\
\Pi_A(e_1 \cup e_2) &\equiv \Pi_A(e_1) \cup \Pi_A(e_2) && (8)\\
e_1 \times e_2 &\equiv e_2 \times e_1 && (9)\\
e_1 \bowtie_p e_2 &\equiv e_2 \bowtie_p e_1 && (10)\\
(e_1 \times e_2) \times e_3 &\equiv e_1 \times (e_2 \times e_3) && (11)\\
(e_1 \bowtie_{p_1} e_2) \bowtie_{p_2} e_3 &\equiv e_1 \bowtie_{p_1} (e_2 \bowtie_{p_2} e_3) && (12)\\
\sigma_p(e_1 \times e_2) &\equiv e_1 \bowtie_p e_2 && (13)\\
\sigma_p(e_1 \times e_2) &\equiv \sigma_p(e_1) \times e_2 && (14)\\
&\equiv \text{if } \mathcal{F}(e) \subseteq \mathcal{A}(e_1)\\
\sigma_{p_1}(e_1 \bowtie_{p_2} e_2) &\equiv \sigma_{p_1}(e_1) \bowtie_{p_2} e_2 && (15)\\
&\equiv \text{if } \mathcal{F}(p_1) \subseteq \mathcal{A}(e_1)\\
\Pi_A(e_1 \times e_2) &\equiv \Pi_{A_1}(e_1) \times \Pi_{A_2}(e_2) && (16)\\
&\equiv \text{if } A = A_1 \cup A_2, A_1 \subseteq \mathcal{A}(e_1), A_2 \subseteq \mathcal{A}(e_2)
\end{aligned}
$$

1.2.2 Canonical Query Translation

Restrictions:

- only **select distinct**
- no **group by, order by, union, intersect, except**
- only attributes in **select** clause
- no nested queries
- not discussed here: `NULL` values

1.2.3 Logical Query Optimization

- foundation: algebraic equivalence

Which plans are better?

- plans can only be compared if there is a cost function
- cost functions need details that are not available when only considering logical algebra
- consequence: logical query optimization remains a heuristic

Phases

1. break up conjunctive selection predicates, $(1) \rightarrow$
2. push selections down, $(2) \rightarrow, (14) \rightarrow$
3. introduce joins, $(13) \rightarrow$
4. determine join order $(9), (10), (11), (12)$
5. introduce and push down projections $(3) \leftarrow, (4) \leftarrow, (16) \rightarrow$
    - eliminate redundant attributes

This kind of phases has limitation: different join order would allow further push down. The phases are interdependent

1.2.4 Physical Query Optimization

- add more execution information to the plan
- allow for cost calculations
- select index structures/access paths
    - scan+selection could be done by an index lookup
    - multiple indices to choose from
    - table scan might be the best, even if an index is available
    - depends on selectivity, rule of thumb: 10%
    - detailed statistics and costs required
    - related problem: materialized view

  - even more complex, as more than one operator could be substitued

- choose operator implementations
  - replace a logical operator (e.g. $\bowtie$) with a physical one (e.g. $\bowtie^{HH}$)
  - semantic restrictions: e.g., most join operators require equi-conditions
  - $\bowtie^{BNL}$ is better than $\bowtie^{NL}$
  - $\bowtie^{SM}$ and $\bowtie^{HH}$ are usually better than both
  - $\bowtie^{HH}$ is often the best if not reusing sorts
  - decision must be cost-based
  - even $\bowtie^{NL}$ can be optimal
  - not only joins, has to be done for all operators
- add property enforcer
  - certain physical operators need certain properties
  - example: sort for $\bowtie^{SM}$
  - example: in a distributed database, operators need the data locally to operate
  - many operator requirements can be modeled as properties
- choose when to materialize
  - temp operator stores input on disk
  - essential for multiple consumers (factorization, DAGs)
  - also relevant for $\bowtie^{NL}$

1.3 Join Ordering

1.3.1 Basics

Concentrate on join ordering, that is:

- conjunctive queries
- simple predicates
- predicates have the form $a_1 = a_2$ where a_1 is an attribute and a_2 is either an attribute or a constant

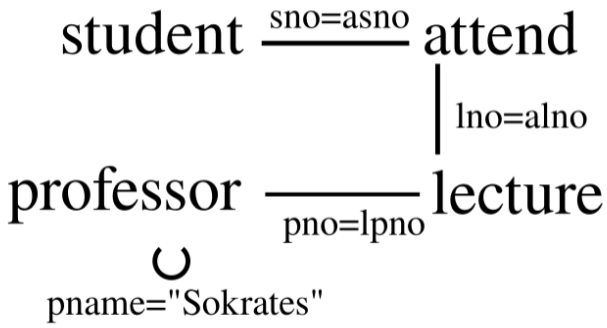
- even ignore constants in some algorithms

We join relations $R_1, \ldots, R_n$ where R_i can be

- a base relation
- a base relation including selections
- a more complex building block or access path

Queries of this type can be characterized by their query graph:

- the query graph is an undirected graph with $R_1, \ldots, R_n$ as nodes
- a predicate of the form $a_1 = a_2$ where $a_1 \in R_i$ and $a_2 \in R_j$ forms an edge between R_i and R_j labeled with the predicate
- a predicate of the form $a_1 = a_2$ where $a_1 \in R_i$ and a_2 is a constant forms a self-edge on R_i labeled with the predicate



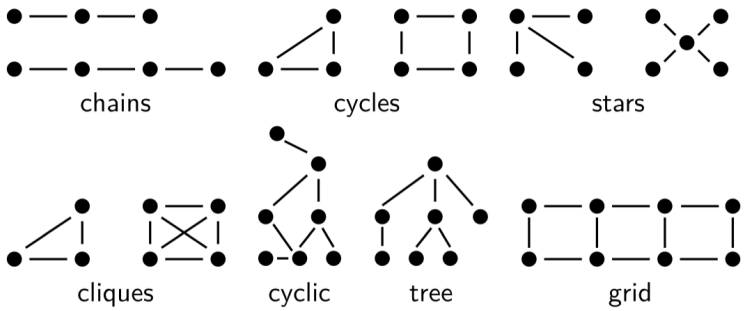


Figure 1: Shapes of Query Graphs

A join tree is a binary tree with

- join operators as inner nodes
- relations as leaf nodes

Commonly used classes of join trees:

- left-deep tree
- right-deep tree
- zigzag tree
- bushy tree

The first three are summariezed as **linear trees**

1.3.2 Search Space

1.3.3 Greedy Heuristics

1.3.4 IKKBZ

1.3.5 MVP

1.3.6 Dynamic Programming

1.3.7 Simplifying the Query Graph

1.3.8 Adaptive Optimization

1.3.9 Generating Permutations

1.3.10 Transformative Approaches

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1.3.13 Iterative Dynamic Programming

1.3.14 Order Preserving Joins

1.3.15 Complexity of Join Processing

1.4 Accessing the Data

1.5 Physical Properties

1.6 Query Rewriting

1.7 Self Tuning

2 Transaction System

2.1 Computational Models

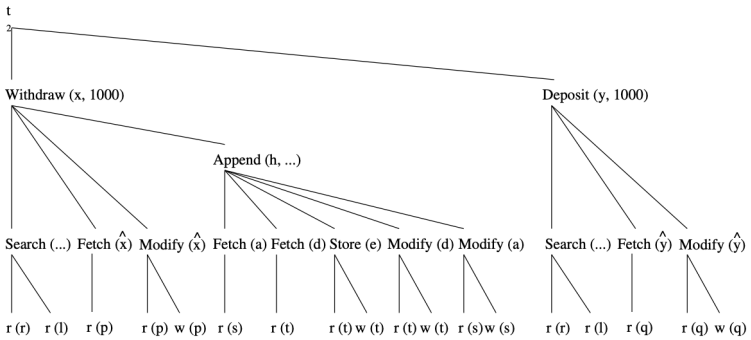
2.1.1 Page Model

Definition 2.1 (Page Model Transaction)**.** A **transaction** t is a partial order of steps of the form $r(x)$ or $w(x)$ where $x \in D$ and reads and writes as well as multiple writes applied to the same object are ordered. We write $t = (op, <)$ for transaction t with step set op and partial order $<$

2.1.2 Object Model

Definition 2.2 (Object Model Transaction)**.** A **transaction** t is a (finite) tree of labeled nodes with

- the transaction identifier as the label of the root node,
- the names and parameters of invoked operations as labels of inner nodes, and
- page-model read/write operations as labels of leafs nodes, along with a partial order $<$ on the leaf nodes s.t. for all leaf-node operations p and q with p of the form $w(x)$ and q of the form $r(x)$ or $w(x)$ or vice versa, we have $p < q \vee q < p$.



2.2 Notions of Correctness for the Page Model

2.2.1 Canonical Synchronization Problems

Lost Update Problem:

P1	Time	P2
	/* x = 100 */	
r (x)	1	
	2	r (x)
x := x+100	4	x := x+200
w (x)	5	
	/* x = 200 */	
	6	w (x)
	/* x = 300 */	

update "lost"

Observation: problem is the interleaving $r_1(x)$ $r_2(x)$ $w_1(x)$ $w_2(x)$

Inconsistent Read Problem

P1	Time	P2
	1	r (x)
	2	x := x – 10
	3	w (x)
sum := 0	4	
r (x)	5	
r (y)	6	
sum := sum +x	7	
sum := sum + y	8	
	9	r (y)
	10	y := y + 10
	11	w (y)

"sees" wrong sum

Observations:

problem is the interleaving $r_2(x)\ w_2(x)\ r_1(x)\ r_1(y)\ r_2(y)\ w_2(y)$

no problem with sequential execution

Dirty Read Problem

P1	Time	P2
r (x)	1	
x := x + 100	2	
w (x)	3	
	4	r (x)
	5	x := x - 100
failure & rollback	6	
	7	w (x)

cannot rely on validity
of previously read data

Observation: transaction rollbacks could affect concurrent transactions

2.2.2 Syntax of Histories and Schedules

Definition 2.3 (Schedules and histories)**.** Let $T = \{t_1, \ldots, t_n\}$ be a set of transactions, where each $t_i \in T$ has the form $t_i = (op_i, <_i)$

1. A **history** for T is a pair $s = (op(s), <_s)$ s.t.
   (a) $op(s) \subseteq \bigcup_{i=1}^{n} op_i \cup \bigcup_{i=1}^{n} \{a_i, c_i\}$
   (b) for all $1 \leq i \leq n$, $c_i \in op(s) \Leftrightarrow a_i \notin op(s)$
   (c) $\bigcup_{i=1}^{n} <_i \subseteq <_s$

(d) for all $1 \leq i \leq n$ and all $p \in op_i$, $p <_s c_i \vee p <_s a_i$

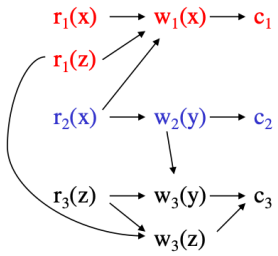
(e) for all $p, q \in op(s)$ s.t. at least one of them is a write and both access the same data item: $p <_s q \vee q <_s p$

2. A **schedule** is a prefix of a history

Definition 2.4. A history s is **serial** if for any two transactions t_i and t_j in s, where $i \neq j$, all operations from t_i are ordered in s before all operations from t_j or vice versa

Definition 2.5.

- $trans(s) := \{t_i \mid s \text{ contains step of } t_i\}$
- $commit(s) := \{t_i \in trans(s) \mid c_i \in s\}$
- $abort(s) := \{t_i \in trans(s) \mid a_i \in s\}$
- $active(s) := trans(s) - (commit(s) \cup abort(s))$



trans(s):=
{t_i | s contains step of t_i}
commit(s):=
{$t_i \in$ trans(s) | $c_i \in$ s}
abort(s):=
{$t_i \in$ trans(s) | $a_i \in$ s}
active(s):=
trans(s) – (commit(s) ∪ abort(s))

$r_1(x)\ r_2(z)\ r_3(x)\ w_2(x)\ w_1(x)\ r_3(y)\ r_1(y)\ w_1(y)\ w_2(z)\ w_3(z)\ c_1\ a_3$

2.2.3 Herbrand Semantics of Schedules

Definition 2.6 (Herbrand Semantics of Steps)**.** For schedule s the **Herbrand semantics** H_s of steps $r_i(x), w_i(x) \in op(s)$ is :

1. $H_s[r_i(x)] := H_s[w_j(x)]$ where $w_j(x)$ is the last write on x in s before $r_i(x)$
2. $H_s[w_i(x)] := f_{ix}(H_x[r_i(y_1)], \ldots, H_s[r_i(y_m)])$ where the $r_i(y_j)$, $1 \leq j \leq m$, are all read operations of t_i that occur in s before $w_i(x)$ and f_{ix} is an uninterpreted m-ary function symbol.

Definition 2.7 (Herbrand Universe)**.** For data items $D = \{x, y, z, \dots\}$ and transactions t_i, $1 \leq i \leq n$, the **Herbrand universe HU** is the smallest set of symbols s.t.

1. $f_{0x}() \in HU$ for each $x \in D$ where f_{0x} is a constant, and
2. if $w_i(x) \in op_i$ for some t_i, there are m read operations $r_i(y_1), \dots, r_i(y_m)$ that precede $w_i(x)$ in t_i, and $v_1, \dots, v_m \in HU$, then $f_{ix}(v_1, \dots, v_m) \in HU$

Definition 2.8 (Schedule Semantics)**.** The **Herbrand semantics of a schedule** s is the mapping $H[s] : D \to HU$ defined by $H[s](x) := H_s[w_i(x)]$ where $w_i(x)$ is the last operation from s writing x, for each $x \in D$

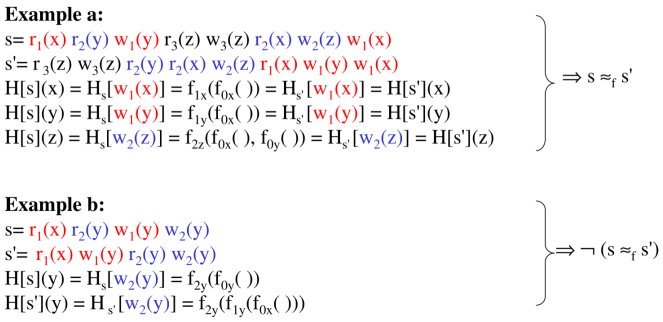
$$s = w_0(x)\ w_0(y)\ c_0\ r_1(x)\ r_2(y)\ w_2(x)\ w_1(y)\ c_2\ c_1$$

$$\begin{aligned}
&H_s[w_0(x)] = f_{0x}() \\
&H_s[w_0(y)] = f_{0y}() \\
&H_s[r_1(x)] = H_s[w_0(x)] = f_{0x}() \\
&H_s[r_2(y)] = H_s[w_0(y)] = f_{0y}() \\
&H_s[w_2(x)] = f_{2x}(H_s[r_2(y)]) = f_{2x}(f_{0y}()) \\
&H_s[w_1(y)] = f_{1y}(H_s[r_1(x)]) = f_{1y}(f_{0x}())
\end{aligned}$$

$$\begin{aligned}
&H[s](x) = H_s[w_2(x)] = f_{2x}(f_{0y}()) \\
&H[s](y) = H_s[w_1(y)] = f_{1y}(f_{0x}())
\end{aligned}$$

2.2.4 Final-State Serializability

Definition 2.9. Schedules s and s' are called **final state equivalent**, denoted $s \approx_f s'$ if $op(s) = op(s')$ and $H[s] = H[s']$



Definition 2.10 (Reads-from Relation)**.** Given a schedule s, extended with an initial and a final transaction, t_0 and t_∞

1. $r_j(x)$ **reads** x **in** s **from** $w_i(x)$ if $w_i(x)$ is the last write on x s.t. $w_i(x) <_s r_j(x)$
2. The **reads-from relation** of x is

$$RF(s) := \{(t_i, x, t_j) \mid \text{an } r_j(x) \text{ reads } x \text{ from a } w_i(x)\}$$

3. Step p is **directly useful** for step q, denoted $p \to q$, if q reads from p, or p is a read step and q is a subsequent write step of the same transaction. $\to^*$, the **useful relation**, denotes the reflexive and transitive closure of $\to$.
4. Step p is **alive** in s if it is useful for some step from t_∞, i.e.,

$$(\exists q \in t_\infty) p \xrightarrow{*} q$$

   and **dead** otherwise
5. The **live-reads-from relation** of s is

$$LRF(s) := \{(t_i, x, t_j) \mid \text{an alive } r_j(x) \text{ reads } x \text{ from } w_i(x)\}$$

Theorem 2.11. *For schedules s and s' the following statements hold:*

1. *$s \approx_f s'$ iff $op(s) = op(s')$ and $LRF(s) = LRF(s')$*
2. *For s let the step graph $D(s) = (V, E)$ be a directed graph with vertices $V := op(s)$ and edges $E := \{(p, q) \mid p \to q\}$, and the reduced step graph $D_1(s)$ be derived from $D(s)$ by removing all vertices that correspond to dead steps. Then $LRF(s) = LRF(s')$ iff $D_1(s) = D_1(s')$*

Proof. For a given schedule s, we can construct a "step graph" $D(s) = (V, E)$ as follows

$$V := op(s)$$
$$E := \{(p, q) \mid p, q \in V, p \to q\}$$

From a step graph $D(s)$, a reduced step graph $D_l(s)$ can be derived by dropping all vertices (and thier incident edges) that represent dead steps. Then the following can be proven:

1. $LRF(s) = LRF(s') \Leftrightarrow D_l(s) = D_l(s')$

   If $D_l(s) \neq D_l(s')$, if there is $r(x) \in D_l(s) \setminus D_l(s')$, then clearly $LRF(s) \neq LRF(s')$; if there is $w_i(x) \in D_l(s) \setminus D_l(s')$, then $(t_i, x, t_\infty) \in LRF(s) \setminus LRF(s')$.

   If $LRF(s) \neq LRF(s')$, suppose $(t_i, x, t_j) \in LRF(s) \setminus LRF(s')$, then clearly $D_l(s) \neq D_l(s')$

2. $s \approx_f s'$ iff $op(s) = op(s')$ and $D_l(s) = D_l(s')$

□

Corollary 2.12. *Final-state equivalence of two schedules s and s' can be decided in time that is polynomial in the length of the two schedules.*

2.2.5 View Serializability

As we have seen, FSR emphasizes steps that are alive in a schedule. However, since the semantics of a schedule and of the transactions occurring in a schedule are unknown, it is reasonable to require that in two equivalent schedules, each transaction reads the same values, independent of its liveliness.

Lost update anomaly: $L = r_1(x)r_2(x)w_1(x)w_2(x)c_1c_2$. History is not FSR, $LRF(L) = \{(t_0, x, t_2), (t_2, x, t_\infty)\}$, $LRF(t_1t_2) = \{(t_0, x, t_1), (t_1, x, t_2), (t_2, x, t_\infty)\}$ and $LRF(t_2t_1) = \{(t_0, x, t_2), (t_2, x, t_1), (t_1, x, t_\infty)\}$

Inconsistent read anomaly: $I = r_2(x)w_2(x)r_1(x)r_1(y)r_2(y)w_2(y)c_1c_2$, history is FSR $LFR(I) = LFR(t_1t_2) = LFR(t_2t_1) = \{(t_0, x, t_2), (t_0, y, t_2), (t_2, x, t_\infty), (t_2, y, t_\infty)\}$

Definition 2.13 (View Equivalence)**.** Schedules s and s' are **view equivalent**, denoted $s \approx_v s'$, if the following hold:

1. $op(s) = op(s')$

2. $H[s] = H[s']$

3. $H_s[p] = H_{s'}[p]$ for all (read or write) steps

Theorem 2.14. *For schedules s and s' the following statements hold.*

1. $s \approx_v s'$ *iff* $op(s) = op(s')$ *and* $RF(s) = RF(s')$

2. $s \approx_v s'$ *iff* $D(s) = D(s')$

Proof. 1. $\Rightarrow$: Consider a read step $r_i(x)$ from s. Then $H_s[r_i(x)] = H_{s'}[r_i(x)]$ implies that if $r_i(x)$ reads from some step $w_j(x)$ in s, the same holds in s', and vice versa.

$\Leftarrow$: If $RF(s) = RF(s')$, this in particular applies to t_∞; hence $H[s] = H[s']$. Similarly, for all other reads $r_i(x)$ in s, we have $H_s[r_i(x)] = H_{s'}[r_i(x)]$.

Suppose for some $w_i(x)$, $H_s[w_i(x)] \neq H_{s'}[w_i(x)]$. Thus the set of values read by t_i prior to step w_i is different in s and s', a contradiction to our assumption that $RF(s) = RF(s')$.

□

Corollary 2.15. *View equivalence of two schedules s and s' can be decided in time that is polynomial in the length of the two schedules*

Definition 2.16. A schedule s is **view serializable** if there exists a serial schedule s' s.t. $s \approx_v s'$. VSR denotes the class of all view-serializable histories

Theorem 2.17. $VSR \subset FSR$

Theorem 2.18. *Let s be a history without dead steps. Then $s \in VSR$ iff $s \in FSR$*

Theorem 2.19. *The problem of deciding for a given schedule s whether $s \in VSR$ holds is NP-complete*

Definition 2.20 (Monotone Classes of Histories)**.** Let s be a schedule and $T \subseteq trans(s)$. $\pi_T(s)$ denotes the projection of s onto T. A class of histories is called **monotone** if the following holds:

If s is in E, then $\Pi_T(s)$ is in E for each $T \subseteq trans(s)$

VSR is not monotone

2.2.6 Conflict Serializability

Definition 2.21 (Conflicts and Conflict Relations)**.** Let s be a schedule, $t, t' \in trans(s)$, $t \neq t'$

1. Two data operations $p \in t$ and $q \in t'$ are in **conflict** in s if they access the same data item and at least one of them is a write
2. $conf(s) := \{(p, q) \mid p, q$ are in conflict and $p <_s q\}$ is the **conflict relation** of s

Definition 2.22. Schedules s and s' are **conflict equivalent**, denoted $s \approx_c s'$, if $op(s) = op(s')$ and $conf(s) = conf(s')$

Definition 2.23. Schedule s is **conflict serializable** if there is a serial schedule s' s.t. $s \approx_c s'$. CSR denotes the class of all conflict serializable schedules.

Theorem 2.24. $CSR \subset VSR$

Definition 2.25. Let s be a schedule. The **conflict graph** $G(s) = (V, E)$ is a directed graph with vertices $V := commit(s)$ and edges $E := \{(t, t') \mid t \neq t' \wedge \exists p \in t, q \in t' : (p, q) \in conf(s)\}$

Theorem 2.26. *Let s be a schedule. Then $s \in CSR$ iff $G(s)$ is acyclic.*

Proof. $\Rightarrow$: There is a serial history s' s.t. $op(s) = op(s')$ and $conf(s) = conf(s')$. Consider $t, t' \in V$, $t \neq t'$ with $(t, t') \in E$. Then we have

$$(\exists p \in t)(\exists q \in t')p <_s q \wedge (p, q) \in conf(s)$$

Then $p <_{s'} q$. Also all of t occur before all of t' in s'.

Suppose $G(s)$ were cyclic. Then we have a cycle $t_1 \to t_2 \to ... \to t_k \to t_1$. The same cycle also exists in $G(s')$, a contradiction

$\Leftarrow$: □

Corollary 2.27. *Testing if a schedule is in CSR can be done in time polynomial to the schedule's number of transactions*

Commutativity rules:

1. $C_1 : r_i(x)r_j(y) \sim r_j(y)r_i(x)$ if $i \neq j$
2. $C_2 : r_1(x)w_j(y) \sim w_j(y)r_i(x)$ if $i \neq j$ and $x \neq y$
3. $C_3 : w_i(x)w_j(y) \sim w_j(y)w_i(x)$ if $i \neq j$ and $x \neq y$

Ordering rule:

4. C_4: $o_i(x)$, $p_j(y)$ unordered $\Rightarrow$ $o_i(x)p_j(y)$ if $x \neq y$ or both o and p are reads

Definition 2.28. Schedules s and s' s.t. $op(s) = op(s')$ are **commutativity based equivalent**, denoted $s \sim^* s'$, if s can be transformed into s' by applying rules C1, C2, C3, C4 finitely.

Theorem 2.29. *Let s and s' be schedules s.t. $op(s) = op(s')$. Then $s \approx_c s'$ iff $s \sim^* s'$*

Definition 2.30. Schedule s is **commutativity-based reducible** if there is a serial schedule s' s.t. $s \sim^* s'$

Corollary 2.31. *Schedule s is commutativity-based reducible iff $s \in CSR$*

Definition 2.32. Schedule s is **order preserving conflict serializable** if it is conflict equivalent to a serial schedule s' and for all $t, t' \in trans(s)$, if t completely precedes t' in s, then the same holds in s'. OSCR denotes the class of all schedules with this property.

Theorem 2.33. $OCSR \subset CSR$

$s = w_1(x)r_2(x)c_2w_c(y)c_3w_1(y)c_1 \in CSR \setminus OCSR$

Definition 2.34. Schedules s is **commit order preserving conflict serializable** if for all $t_i, t_j \in trans(s)$, if there are $p \in t_i$, $q \in t_j$ with $(p, q) \in conf(s)$, then $c_i <_s c_j$.

COCSR denotes the class of all schedules with this property

Theorem 2.35. $COCSR \subset CSR$

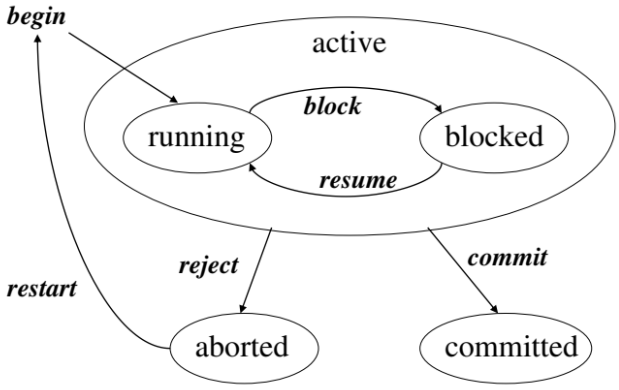
Theorem 2.36. *Schedule s is in COCSR iff there is a serial schedule s' s.t. $s \approx_c s'$ and for all $t_i, t_j \in trans(s)$: $t_i <_{s'} t_j \Leftarrow c_i <_s c_j$*

2.2.7 Commit Serializability

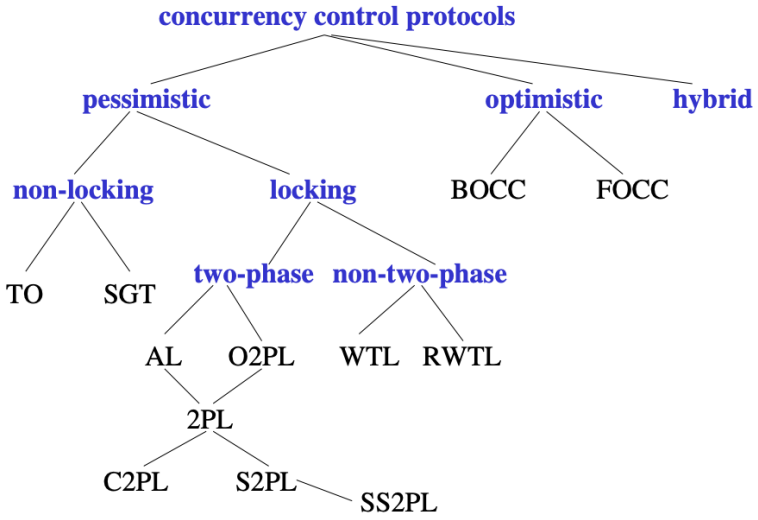
2.2.8 An Alternative Criterion: Interleaving Specifications

2.3 Concurrency Control Algorithms

2.3.1 General Scheduler Design



Definition 2.37 (CSR Safety)**.** For a scheduler S, $Gen(S)$ denotes the set of all schedules that S can generate. A scheduler is called **CSR safe** if $Gen(S) \subseteq CSR$



2.3.2 Locking Schedulers

1. Introduction General locking rules:

   (a) Each data operation $o_i(x)$ must be preceded by $ol_i(x)$ and followed by $ou_i(x)$

   (b) For each x and t_i there is at most one $ol_i(x)$ and at most one $ou_i(x)$

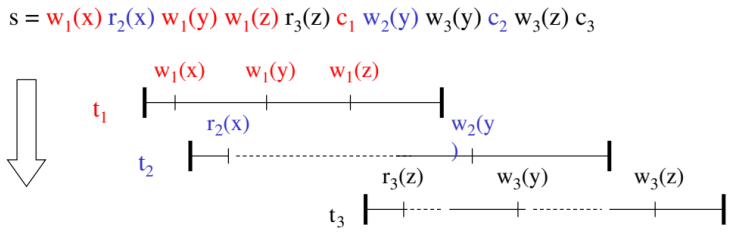
   (c) No $ol_i(x)$ or $ou_i(x)$ is redundant

   (d) If x is locked by both t_i and t_j, then these locks are compatible

   Let DT(s) denote the projection of s onto the steps of type r, w, a, c. CP(s) denotes the committed projection of s.

2. Two-Phase Locking

   Definition 2.38. A locking protocol is **two-phase** if for every output schedule s and every transaction $t_i \in trans(s)$ no ql_i step follows the first ou_i step $(q, 0 \in \{r, w\})$



Lemma 2.39. *Let s be the output of a 2PL scheduler. Then for each transaction $t_i \in commit(DT(s))$, the following holds:*

(a) if $o_i(x)$, $o \in \{r, w\}$, occurs in CP(DT(s)), then so do $ol_i(x)$ and $ou_i(x)$ with the sequencing $ol_i(x) < o_i(x) < ou_i(x)$.

(b) If $t_j \in commit(DT(s))$, $i \neq j$, is another transaction s.t. some steps $p_i(x)$ and $q_j(x)$ from CP(DT(s)) are in conflict, then either $pu_i(x) < ql_j$ or $qu_j(x) < pl_i(x)$ holds.

(c) If $p_i(x)$ and $q_j(y)$ are in CP(DT(s)), then $pl_i(x) < qu_i(y)$, i.e., every lock operation occurs before every unlock operation of the same transaction.

Lemma 2.40. *Let s be the output of a 2PL scheduler, and let $G := G(CP(DT(s)))$ be the conflict graph of $CP(DT(s))$, then the following holds:*

(a) *If (t_i, t_j) is an edge in G, then $pu_i(x) < ql_j(x)$ for some data item x and two operations $p_i(x)$, $q_j(x)$ in conflict.*

(b) *If $(t_1, \ldots, t_n)$ is a path in G, $n \geq 1$, then $pu_1(x) < ql_n(y)$ for two data items x and y as well as operations $p_1(x)$ and $q_n(y)$.*

(c) *G is acyclic.*

Since the conflict graph of an output produced by a 2PL scheduler is acyclic, we have

Theorem 2.41. $Gen(2PL) \subset CSR$

Example 2.1 (Strict inclusion)**.** Let $s = w_1(x)r_2(x)c_2r_3(y)c_3w_1(y)c_1$. $s \in$ CSR as $s \approx_c t_3t_1t_2$. And s cannot be produced by a 2PL scheduler

Theorem 2.42. $Gen(2PL) \subset OCSR$

3. Deadlock Handling Deadlock detection:

   (a) maintain dynamic **waits-for graph** (WFG) with active transactions as nodes and an edge from t_i to t_j if t_j waits for a lock held by t_i

   (b) Test WFG for cycles

   Deadlock resolution: Choose a transaction on a WFG cycles as a **deadlock victim** and abort this transaction, and repeat until no more cycles.

   Possible victim selection strategies:

   (a) Last blocked

   (b) Random

   (c) Youngest

   (d) Minimum locks

   (e) Minimum work

   (f) Most cycles

   (g) Most edges

   Deadlock Prevention: Restrict lock waits to ensure acyclic WFG at all times. Reasonable deadlock prevention strategies when t_i is blocked by t_j:

(a) **wait-die**: if t_i started before t_j then wait else abort t_i.

(b) **wound-wait**: if t_i started before t_j then abort t_i else wait

(c) **Immediate restart**: abort t_i

(d) **Running priority**: if t_j is itself blocked then abort t_j else wait

(e) **Timeout**: abort waiting transaction when a timer expires.

Abort entails later restart

4. Variants of 2PL

Definition 2.43. Under **static** or **conservative 2PL** (C2PL) each transaction acquires all its locks before the first data operation.

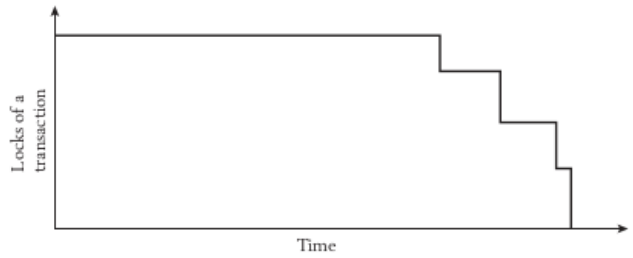


Figure 2: Conservative 2PL

Definition 2.44. Under **strict 2PL** (S2PL) each transaction holds all its write locks until the transaction terminates.

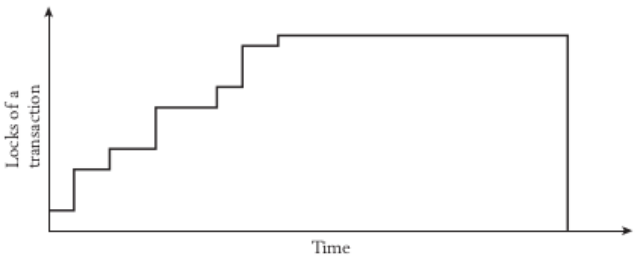


Figure 3: Strict 2PL

Definition 2.45. Under **strong 2PL** (SS2PL) each transaction holds all its locks until the transaction terminates

Theorem 2.46. $Gen(SS2PL) \subset Gen(S2PL) \subset Gen(2PL)$

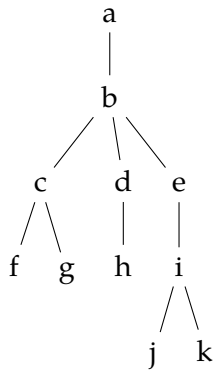
Theorem 2.47. *Gen*$(SS2PL) \subset COCSR$

5. Ordered Sharing of Locks (O2PL)

6. Altruistic Locking (AL)

7. Non-Two-Phase Locking (WTL, RWTL) Motivation: concurrent executions of transactions with access patterns that comply with organizing data items into a virtual tree

$$t_1 = w_1(a)w_1(b)w_1(d)w_1(e)w_1(i)w_1(k)$$
$$t_2 = w_2(a)w_2(b)w_2(c)w_2(d)w_2(h)$$



Definition 2.48 (Write-only Tree Locking (WTL))**.** Lock requests and releases must obey LR1 - LR4 and the following additional rules

   (a) WTL1: A lock on a node x other than the tree root can be acquired only if the transaction already holds a lock on the parent of x

   (b) WTL2: After a $wu_i(x)$ no further $wl_i(x)$ is allowed

8. Geometry of Locking

2.3.3 Non-Locking Schedulers

1. Timestamp Ordering

2.3.4 Hybrid Protocols

2.4 Multiversion Concurrency Control

2.4.1 Multiversion Schedules

Example 2.2. $s = r_1(x)w_1(x)r_2(x)w_2(y)r_1(y)w_1(z)c_1c_2 \notin$ CSR

but schedule would be tolerable if $r_1(y)$ could read the old version y_0 of y to be consistent with $r_1(x)$

Approach:

- each w step creates a new version
- each r step can choose which version it wants/needs to read
- versions are transparent to application and transient

Definition 2.49. Let s be a history with initial transaction t_0 and final transaction t_∞. A **version function** for s is a function h which associates with each read step of s a previous write step on the same data item, and the identity for writes.

Definition 2.50. A **multiversion (mv) history** for transactions $T = \{t_1, \ldots, t_n\}$ is a pair $m = (\text{op}(m), <_m)$ where $<_m$ is an order on op(m) and

1. op$(m) = \bigcup_{i=1,\ldots,n} h(\text{op}(t_i))$ for some version function h
2. for all $t \in T$ and all $p, q \in \text{op}(t_i)$: $p <_t q \Rightarrow h(p) <_m h(q)$
3. if $h(r_j(x)) = w_j(x_i)$, $i \neq j$, then c_i is in m and $c_i <_m c_j$

A **multiversion (mv) schedule** is a prefix of a multiversion history

Definition 2.51. A multiversion schedule is a **monoversion schedule** if its version function maps each read to the last preceding write on the same data item.

2.4.2 Multiversion Serializability

Definition 2.52. For mv schedule m the reads-from relation of m is RF$(m) = \{(t_i, x, t_j) \mid r_j(x_i) \in \text{op}(m)\}$

Definition 2.53. mv histories m and m' with $trans(m) = trans(m')$ are **view equivalent**, $m \equiv_v m'$, if RF(m) = RF(m')

2.4.3 Limiting the Number of Versions

2.4.4 Multiversion Concurrency Control Protocols