Homework 5: Ramsey's theorem and indiscernible sequences

Advanced Model Theory

Due March 31, 2022

Let $\mathbb{M}$ be a monster model elementary extension of $(\mathbb{R}, +, \cdot, -, 0, 1, \leq)$. Let $a_1, a_2, a_3, \ldots$ be a non-constant indiscernible sequence of singletons/elements in $\mathbb{M}$.

1. Show that $a_1, a_2, \ldots$ is not totally indiscernible.
2. Show that $a_1 a_2 > 0$. *Hint:* more generally, $a_i a_j > 0$. Think about what this means, concretely.
3. Suppose $a_2 - a_1 \geq 1$. Show that $a_2 - a_1 \geq 7$. *Hint:* (a_1, a_8) has the same type as (a_1, a_2).
4. Show that at least one of the following is true: $a_2 < (1.01) \cdot a_1$ or $a_2 > 200 \cdot a_1$. *Hint:* proof by contradiction, compound interest.
5. Show that $a_i + a_j \neq a_k$ for any i, j, k (possibly equal). *Hint:* there are probably clever ways to prove this, but the most routine way to do this is to break into cases depending on the relative order of i, j, k.
6. Show that there is an indiscernible sequence $b_1, b_2, b_3, \ldots$ such that $b_2 > 200 \cdot b_1$. *Hint:* extract an indiscernible sequence from a well-chosen sequence in $\mathbb{R}$.

For these problems, you don't need to explain that properties of $\mathbb{R}$ extend to properties of $\mathbb{M}$ because it's an elementary extension.