Homework 02

Problem 1

Given two chains $(C, \leq)$ and $(D, \leq)$, define the lexicographic product of C and D to be the chain defined on the Cartesian product of their universes such that $(a, b) < (c, d)$ in the sense of $C \times D$ if $b < d$ in the sense of D, or $b = d$ and $a < c$ in the sense of C.

1. Show that a discrete chains without endpoints are those that can be written in the form $\mathbb{Z} \times C$, where C is a linear order.
2. Show that if $C \sim_\omega C'$ and $D \sim_\omega D'$, then so is $C \times D \sim_\omega C' \times D'$.

Problem 2

Given two chains $(C, \leq)$ and $(D, \leq)$, by $C + D$ we mean the china

$$C \times \{0\} \cup D \times \{1\}$$

such that $C \times \{0\}$ is a copy of C, $D \times \{1\}$ is a copy of D, and each element of $C \times \{0\}$ is smaller that each element of $D \times \{1\}$.

1. Show that the linear orders $\mathbb{R}$ and $\mathbb{R} + \mathbb{Q}$ are not isomorphic;
2. Construct two discrete linear orders such that they are ∞-equivalent but not isomorphic.

Problem 3

Let (M, R) and (M', R') be structures. A local isomorphism s is in $S_{\omega+1}(M, M')$ iff it satisfis the following conditions:

- for any $a \in M$, there is $b \in M'$ such that $s \cup \{(a, b)\} \in S_\omega(M, M')$;
- for any $b \in M'$, there is $a \in M$ such that $s \cup \{(a, b)\} \in S_\omega(M, M')$;

Let $p \in \mathbb{N}$, a local isomorphism s is in $S_{\omega+p+1}(M, M')$ iff it satisfis the following conditions:

- for any $a \in M$, there is $b \in M'$ such that $s \cup \{(a, b)\} \in S_{\omega+p}(M, M')$;
- for any $b \in M'$, there is $a \in M$ such that $s \cup \{(a, b)\} \in S_{\omega+p}(M, M')$;

We say that M and M' are $\omega + p$-equivalent if $S_{\omega+p}(M, M') \neq \emptyset$, where $p = 1, 2, 3, ...$

1. Show that $\mathbb{Z} + \mathbb{Z}$ and $\mathbb{Z}$ are $\omega + 1$-equivalent but not $\omega + 2$-equivalent;
2. Construct two discrete linear orders such that they are $\omega+n$-equivalent but not $\omega+n+1$-equivalent for each $n \in \mathbb{N}$;