Homework 1

Introductory Model Theory

Due September 15, 2022

General comments: Please try to write answers in complete sentences in English. You don't need to include random scratch work. The main part of your solution should be an explanation of why your answer is correct (an informal proof).

Turn in your solutions to me or the TA by the end of the day next Thursday. You will lose points if you turn in the homework late, as detailed in the course information document. If you have mathematical questions about the homework problems, you can ask them via WeChat or email. These homework problems are based on Sections 1.1–1.3 of the course notes, which are posted on eLearning. It is good to type up your solutions using LaTeX, but it is also okay to write them by hand.

If x and y are real numbers, let $x \oplus y$ be $x + y + xy$.

1. Show that $(\mathbb{R}, \oplus)$ is a semigroup.
2. Show that there is an identity element $e \in \mathbb{R}$ for the semigroup $(\mathbb{R}, \oplus)$.
3. Find the inverse of 5.
4. Show that not every element in $\mathbb{R}$ has an inverse, so that $(\mathbb{R}, \oplus)$ is not a group.

If x and y are real numbers, let $x \odot y$ denote $2x - y$.

5. What is the interpretation of the term $t(x, y) = (x \star y) \star x$ in the magma $(\mathbb{R}, \odot)$? Try to write the answer using normal mathematical operations, not $\odot$.
6. Find an equation that is satisfied by the magma $(\mathbb{R}, \odot)$ and not satisfied by some other magma. *Hint:* write out a few simple terms and see if you can find any coincidences or things that cancel out.