Homework 3: coheirs and invariant types

Advanced Model Theory

Due March 17, 2022

If $n < m$, you may assume the result of Problem n in your solution to Problem m.

Let DLO be the theory of dense linear orders[1], like $(\mathbb{Q}, \leq)$ or $(\mathbb{R}, \leq)$. Fix a model $M \models \mathrm{DLO}$.

1. Show that the collection of formulas $x > a$ for $a \in M$ generates a complete type $\tau_M(x) \in S_1(M)$. In other words, show that the partial type $\{(x > a) : a \in M\}$ has a unique completion. *Hint:* you will need to use quantifier elimination or the characterization of "ω-equivalences" (partial elementary maps) or something similar.

The type τ_M of Exercise 1 is called the *type at* $+\infty$.

2. Show that τ_M is definable.

3. Suppose $N \succeq M$. Show that τ_N is an heir of τ_M. *Hint:* do Problems 2 and 3 together.

4. Suppose $N \succeq M$ and N is $|M|^+$-saturated. Show that τ_N is *not* a coheir of τ_M.

When N is a monster model, Exercise 4 gives an example of a type $p \in S_1(M)$ with more than one M-invariant global extension (the heir τ_N is one M-invariant extension, and any coheir of M is another).

5. If $N \succeq M$, show that τ_M has a unique coheir over N. *Hint:* it may help to remember that 1-types in DLO are the same thing as cuts.

6. Give an example of models $M \preceq N$ of DLO where τ_N is a coheir of τ_M (and $M \neq N$).

[1] Non-empty, without endpoints.