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CBE 140A: Heat Conduction

1 Introduction

Relevant applications of conductive heat transfer can be found in a myriad of unit operations: shell-and-tube heat exchangers, air and spacecraft surface materials, development of higher temperature resilient materials for use in turbines or jets, and more. Predicting and characterizing the behavior of these types of systems requires an understanding of the fundamentals of conductive heat transfer, which is introduced in this experiment.

2 Objective

This apparatus will make use of insulated metal cylinders, electrically heated on one end and cooled with water at the other. The impact of metal alloy composition, heat sink paste, inclusion of insulator, cross-sectional area, and geometry on the temperature profile throughout the materials and the effectiveness of heat transfer will be explored.

3 Theory

The rate of one-dimensional heat transfer through and between solids is well-described by Fourier's law:

$$q = -k\frac{dT}{dx} \tag{1}$$

where q = heat flux in units of energy per area per time,
k = thermal conductivity,
T = temperature, and
x = position in the solid in the direction of heat transfer.

Note that this equation does not assume the type of geometry—so long as the transfer is one-dimensional, x could be a direction in rectangular space or a radial direction in cylindrical or spherical coordinates, more often denoted as r.

To predict the temperature profile in a solid, the differential equation must be solved for $T(x)$. For heat flux in the axial direction through a constant cross-sectional area cylinder, assuming temperature is known at one position for use as a boundary condition ($T(x_0) = T_0$), the resulting equation is:

$$T(x) = T_0 - \frac{q}{k}(x - x_0) \tag{2}$$

Note that q and $x - x_0$ are vectors, so their product can change sign depending on their direction. Empirical thermal conductivities, under these conditions, can be determined with a slope of the best-fit line generated from temperature and position data. An estimation of k can still be made if the temperatures are only known at the cylinder endpoints but will be less accurate, with no ability to quantify the uncertainty.

When heat is conducted in the axial direction through a composite cylinder of different materials and/or cross-sectional areas, an overall heat transfer coefficient can be utilized. Let us assume there are three such sections. To begin, each sequential section will have equal heat powers, but not fluxes, at steady state.

$$Q_i = q_i A_i = -k_i A_i \left(\frac{dT}{dx}\right)_i \tag{3}$$

Where Q represents heat transfer in units of power, A is cross-sectional area, and subscript i can take values of 1, 2, or 3, denoting the section of the composite cylinder. Writing a separate equation for heat transfer across the entire system:

$$Q = UA\Delta T \tag{4}$$

Where UA represents the overall heat conductance and ΔT is the difference in temperature between the two ends of the composite cylinder. By combining equations 3 and 4, one can derive the following:

$$UA = \left(\frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2} + \frac{L_3}{k_3 A_3}\right)^{-1} \tag{5}$$

Where the length of each section is denoted with L. Using this approach, the thermal conductivity for one section of the composite can be estimated, so long as the other two materials are well understood. To reiterate, this method works for variations in material and/or cross-sectional area. If the composite is composed of more than three sections, simply include additional terms in Equation 5 to account for the other resistances to heat transfer.

Later in the experimental procedure, the importance of using a highly conductive heat paste between sections will be stressed. From a theoretical point of view, when heat is desired to transfer between adjacent solids, the entirety of their contact area will not be fully flush—one can imagine that, at a microscopic scale, surface roughness will lead to small air pockets between the two materials. Given air's physical properties, this can appear as an additional resistance to heat transfer. This contact resistance between two solids, in rectangular coordinates, is modeled in the following way:

$$UA = \left(\frac{L_1}{k_1 A_1} + \frac{1}{h_C A_C} + \frac{L_2}{k_2 A_2}\right)^{-1} \tag{6}$$

Where A_C is the contact surface area and h_C is the contact heat conductance, its inverse being the contact resistance. With a thorough literature search, it may be possible to find prior work correlating material type, surface roughness, and interfacial pressure with thermal contact resistance.

Predicting the steady-state temperature profile for one-dimensional heat conduction for a cylinder in the radial direction requires beginning again with Fourier's law and applying to a differential energy balance. The solution, easily derived in combination with a boundary condition of the form $T(r_0) = T_0$, is as follows:

$$T(r) = T_0 - \frac{Q}{2\pi k L} \log\left(\frac{r}{r_0}\right) \tag{7}$$

Where L is the axial length of the cylinder. This assumes r and Q have identical vector orientation—if not, the second term on the right-hand side should be negative. Estimation of a material's thermal conductivity through this technique requires linearizing Equation 7, performing a linear regression, and subsequent calculation of k from the slope.

To find well-established empirical values of thermal conductivity for various materials, many reference texts should provide helpful, for example, Tables 2-326 to 2-335 in [1]. To predict the thermal conductivity of a solid *ab initio*, "there is no reliable method for estimating solid thermal conductivity at this time." [1, 2-513]

4 Experimental Apparatus

Figure 1 illustrates the heat conduction apparatus. A control console, which can be powered on using the switch (1), provides variable electrical power to a resistive heater embedded in the top of the cylindrical composite rod (9), which is surrounded by plastic insulation. Heat travels through the brass rod and is removed from water flowing through the bottom of the cylinder (10). The power supplied to the heater is controlled by the knob (2) and metered by display (3) with selector knob (5). Temperatures are detected with thermocouples through ports in the heated section (11), the insert (12), and in the cooled section (13). These thermocouples are wired to control console ports (7), although the wires are not shown here for simplicity. A USB hub (8) allows the data from the console to be streamed to and recorded by software on a nearby computer. The middle insert section has several replacement pieces of different materials (stainless steel, aluminum, or brass) and cross-sectional areas. A top-down view of an additional apparatus, displayed on the right-hand side of Figure 1, allows for study of radial heat dissipation. In this module, the resistive heater is in the center (14) while the cooling water travels around the perimeter (16). Like the first system, several thermocouple ports (15) are installed throughout the device.

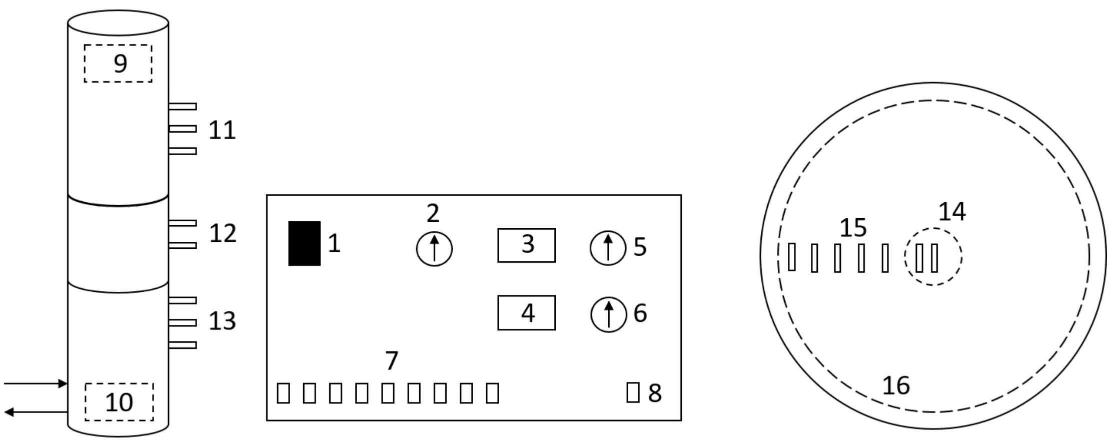

Figure 1: Experiment 8: heat conduction apparatus.

5 Safety

The major hazards associated with this experiment are the following: The control console, powered from an electrical outlet, introduces the risk of injury from electric shock. Avoid with tidy cable management and by ensuring the power cord is not touched when in operation. As this system can reach nearly 100 °C, burns are also a risk—mitigate by avoiding direct contact with the heater sections of the apparatus or using heat-resistant gloves. This list is only a starting point and not intended to be fully comprehensive—please keep the safety of yourself and others in mind throughout the laboratory sessions.

6 Experimental Procedure

Assemble the axial heat conduction apparatus using the brass insert with the additional two thermocouples. Apply thermal heat paste on both sides of the brass insert to minimize contact resistance, but be careful to not apply the paste on the insulation. Manipulate the voltage and current supplied to the heater until 20 watts of power are provided. After ensuring the water tubing is securely connected to the apparatus and drains to the sink, open the water supply valve. Set up the computer software to periodically record temperatures at all positions as the system heats up. Measure the thermocouple positions in the metal inserts relative to bar's open surfaces. Once steady state is reached, save the temperature dataset with a detailed file name and record the type of insert, geometry, and power setting.

Conduct additional trials investigating the effects of: geometry (axial or radial), swapping for a different metal insert material (stainless steel or aluminum), swapping for a different cross-sectional area insert, inclusion of insulators such as paper or cork, use or absence of thermal heat paste, and heat power supplied (setting the power to zero allows study of cooling time). Note the radial heat conduction module cannot be disassembled and does not have swappable sections. Be sure to record the thickness of insulators used.

Notes: The time to reach steady state might be shortened if the system is throttled by increasing heating power beyond the set point until the expected steady state temperature profile is reached and subsequently reducing the power back to the set point. Do not throttle the system for all trials, as the transient behavior of the system will also be studied. The control console has an automatic shut-off feature if the system overheats. It will restore control to the user once it sufficiently cools to a safe temperature. When using insulators, avoid the automatic shut-off by using lower power settings.

7 Data Analysis

For each trial, characterize heat conduction by: plotting the temperature profile as a function of time and position; calculate experimental values of k, h_C, and UA. Compare these charts and estimates to their expected shapes and values—if there is disagreement, what might be the cause, and is there evidence for it?

Based on the trials you were able to complete, investigate the effect of geometry, material type, cross-sectional area, and heat power on k, h_C, and UA. Note this is combinatorial, allowing several methods of analysis. Comment on significant results. Do they agree with the predicted relationships between these parameters? If not, what might be the cause, and is there evidence for it?

References

[1] Don W. Green, editor. *Perry's chemical engineers' handbook*. McGraw-Hill, eighth edition, 2008.