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CBE 140A: Pipe Networks

1 Introduction

The industrial applications of transport phenomena are ubiquitous: fluids must be transported, often in a system of interconnected pipes, or a "pipe network." These networks might consist of a single pipe or a complex system comprised of pipes of different diameters and lengths, multiple inlet and outlet points, and looping sections. As an example, consider the water system supplying Engineering Tower—it was (hopefully) designed to ensure faucets on all ten floors have access to water delivered at similar pressures and flow rates. Consider what characteristics this pipe network might have to meet the desired design criteria. Using this apparatus, flow properties of an incompressible fluid in pipe networks will be investigated.

2 Objective

In this experiment, the flow characteristics of water are to be explored in a number of different diameter pipes. Subsequently, flow properties of multiples pipes in parallel or in series will be documented. A cyclical piping network, or ring main, with one inlet and up to three outlets can be characterized. The effect on flow rate of doubling a pipeline can also be investigated.

3 Theory

Osborne Reynolds was the first to investigate the transition between laminar and turbulent flow in cylindrical pipes in his paper from 1883. [1] Defining a dimensionless group, the Reynolds number, in the following way,

$$Re = \frac{\rho v D}{\mu} \tag{1}$$

$$\begin{aligned} \text{where } \rho &= \text{density}, \\ v &= \text{average velocity}, \\ D &= \text{pipe inner diameter, and} \\ \mu &= \text{dynamic viscosity}, \end{aligned}$$

he discovered that flow was laminar below a Reynolds number of approximately 2,100, underwent a transition period up to values of 4,000, and exhibited turbulent flow at higher values of Reynolds number. Using this dimensionless group, flow can be classified into one of these three regimes based on the fluid physical properties and the geometry and dimensions of the conduit.

The Bernoulli equation [2, 310], applied between two points in space, takes the following form when there are major frictional losses but no pump work:

$$\frac{\Delta v^2}{2g} + \frac{\Delta p}{\rho g} + \Delta z = f_D \left(\frac{L}{D} \right) \frac{v^2}{2g} \tag{2}$$

where $g =$ gravitational acceleration,
$\Delta p =$ the difference in fluid pressure between the two points,
$\Delta z =$ the difference in height between the two points,
$L =$ length of pipe, and
$f_D =$ the Darcy friction factor.

The right-hand side's first term, f_D or the Darcy friction factor, is not to be confused with the Fanning friction factor, f_F. Different fields of study and textbooks use one form more often than the other, so be careful not to mix definitions! The Darcy friction factor is equal to four times the Fanning friction factor.

Laminar flow will likely not be achieved in this experiment—nevertheless, in the laminar regime, the Darcy friction factor is analytically shown to be: [2, 173]

$$f_D = \frac{64}{Re} \tag{3}$$

Substituting this into the Bernoulli equation, ignoring changes in velocity or height, and replacing the Reynolds number with its definition results in the following,

$$\Delta p = \left(\frac{32\mu L}{D^2} \right) v \tag{4}$$

which shows a linear relationship between velocity and pressure drop in a pipe.

The friction factor for completely turbulent flow (referred to here as $f_{C.T.}$) is independent of Reynolds number, but is influenced by the relative roughness of the pipe wall. Empirical correlations exist describing the friction factor as a function of relative roughness. [2, 299] Evaluating for pressure drop from the Bernoulli equation under these conditions,

$$\Delta p = \left(\frac{f_{C.T.} \rho L}{2D} \right) v^2 \tag{5}$$

a quadratic relationship between pressure drop and average fluid velocity emerges.

Finally, considering flow that is neither laminar nor completely turbulent, no analytical solution exists. A number of empirical equations have been suggested relating the friction factor to the Reynolds number and the pipe roughness, such as the Blasius equation, the Prandtl equation, and the Colebrook equation. [2, 300]

The friction factor can be more easily determined (for a given Reynolds number and relative pipe roughness) using the Moody chart, which is found in most fluid dynamics textbooks, and is displayed below in Figure 1. [3]

When multiple pipes are in parallel, all pipes must have the same pressure differentials, as their starting and ending pressures are fixed at the branching points. Flow rate, denoted here with Q, need not be evenly distributed, however. By conservation of mass, the total flow rate into a junction is identical to the sum of outgoing branch's flow rates:

$$\sum Q_{\text{in}} = \sum Q_{\text{out}} \tag{6}$$

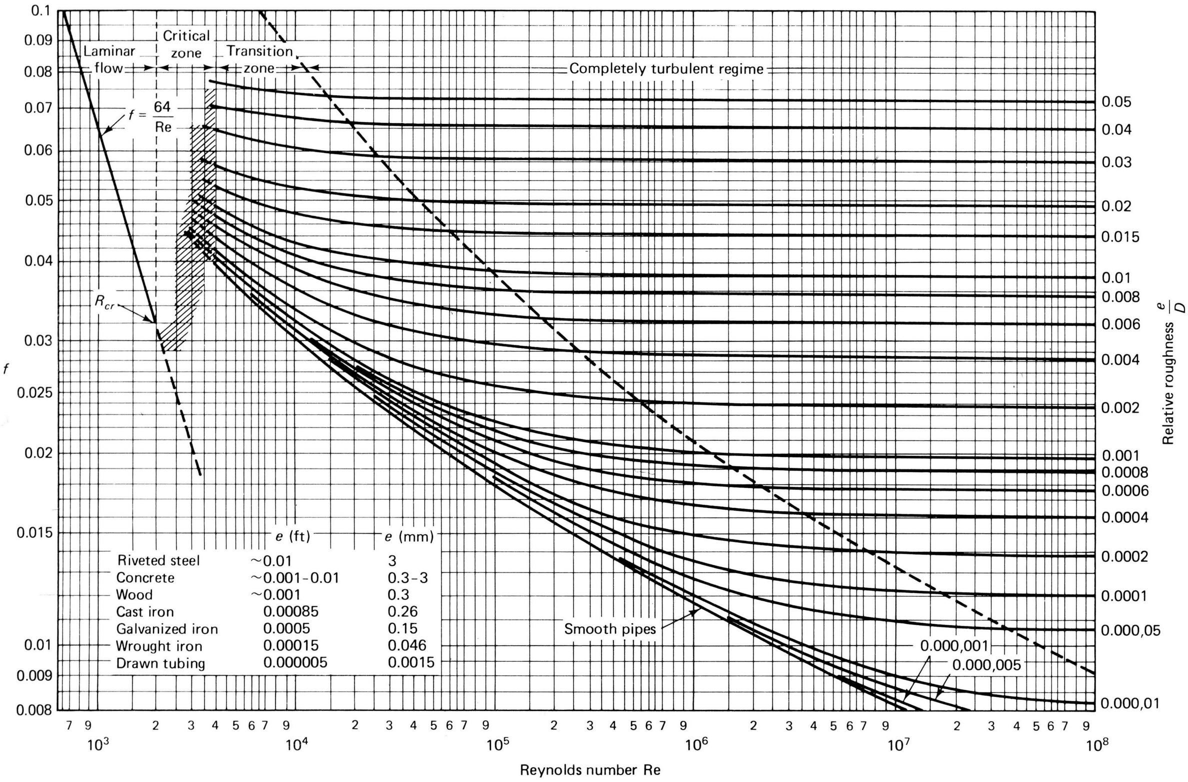

Figure 1: Friction factors for commercial pipes, from [3].

A comparison to electrical circuits is a common metaphor: At a branching point, the sum of currents into the junction is equal to the sum of currents exiting the junction, known as Kirchhoff's first law. When considering a closed loop, Kirchhoff's second law states the sum of voltage differences around the circuit is equal to zero. For fluid flow, the equivalent statement would be that the directed sum of pressure drops around a circuit is equal to zero. For parallel pipes in this apparatus, we can conclude each pathway of piping has an identical pressure drop.

A similar outcome is obtained for multiple pipes in series: By conservation of mass, the flow rate in each section of piping must be equal. However, the pressure drop can vary from one section of piping to another. The total pressure drop is related to the differential in each section as follows:

$$\Delta p_{\text{Total}} = \sum \Delta p_{\text{section}} \tag{7}$$

A ring main can be considered fully understood if the flow rate for each section is known. Consider a junction in a ring main, as displayed in Figure 2: by conservation of mass, the flow rates entering the junction must equal flow rates exiting, or

$$Q_4 = Q_5 + Q_7 \tag{8}$$

Combining all material balance equations and the relations between pressure drop and flow rate for each section, the flow rates in each segment can be determined. Note the directionality given to flows Q_4 and Q_6 in Figure 2 are arbitrary. If negative flow rates are calculated, this suggests the opposite direction of flow for those segments.

When a pipeline's capacity is increased by installing another pipe in parallel, the pressure drop across each pipe must be equal, as described earlier. If the pipes are identical except for their

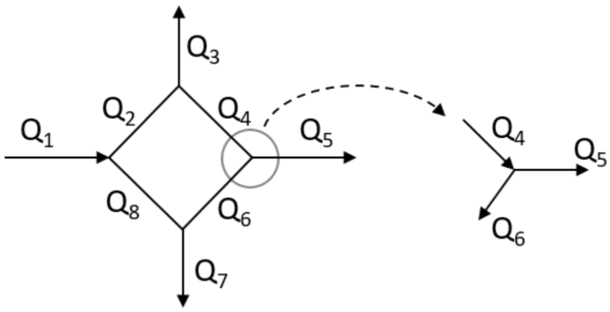

Figure 2: A ring main with one inlet and three outlets, with focus on one junction.

diameter, the following equation can be derived by equating two Bernoulli equations, one for each pipe:

$$\frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5} \quad \text{and} \quad Q_{\text{Total}} = Q_1 + Q_2 \tag{9}$$

Returning to the Bernoulli equation to consider minor, as opposed to major, frictional losses (that is, those associated with fittings and valves instead of friction in straight pipe) are commonly modeled with one of two common methods: fitting factors or equivalent length. As a note, when needing to consider major and minor frictional losses, their terms can be summed on the right-hand side of the Bernoulli equation.

Modeling minor frictional loss using a fitting factor, K, results in the following Bernoulli equation (when changes in velocity and height are negligible):

$$\frac{\Delta p}{\rho g} = \sum K \frac{v^2}{2g} \tag{10}$$

The summation denotes inclusion of all appropriate fittings for the system under study, and where each fitting has a unique fitting factor, K. Tables of empirical fitting factors can be found in Perry's handbook [4] and many fluid dynamics textbooks. [2, 316]

The equivalent length method models [4, 6-16] the pressure drop across various fittings as the amount of straight pipe required to affect the same decrease in pressure. This effective length to diameter ratio, denoted as $L_e \cdot D^{-1}$, depends on the type and geometry of a fitting and, for laminar flow, the Reynolds number. As before, the summation symbol denotes the inclusion of all fittings in the system being studied.

$$\frac{\Delta p}{\rho g} = f_D \left(\sum \frac{L_e}{D} \right) \frac{v^2}{2g} \tag{11}$$

Manometers allow for the measurement of a pressure differential between two points. For this apparatus, an inverted differential U tube manometer is employed, where the system fluid (water) contacts air, as shown later in Figure 5. The primary equation governing a manometer is:

$$\Delta p = \rho g \Delta z \tag{12}$$

describing difference in pressure between two heights, $\Delta z = z_2 - z_1$, within one fluid phase of a density denoted by ρ. Given the manometer size and air's density, there are negligible changes in pressure of the gas phase filling the upper section of the manometer. As such, pressure at the water's surface on both sides of the manometer are identical. The pressure differential between the two points connected to the manometer can, therefore, be calculated from height differential between the surfaces of water on either side of the manometer.

The performance of a pump, often characterized in what is referred to as a pump characteristic curve, demonstrates the increase in fluid pressure at a given flow rate. A pump's general behavior is to produce the highest pressure fluid at low flow rates, effecting smaller increases in pressure as the flow increases, eventually imparting no increase in pressure at a critical maximum flow rate. See Figure 10-28 in [4] for an illustration. Pump vendors will often provide the pump curves associated with their products and are generally easily accessible.

4 Experimental Apparatus

To begin, Figure 3 illustrates a side view of the apparatus. A water reservoir in the station's bottom holds a pump (2) which when turned on (3) supplies flow through an inlet valve (1) to a horizontal manifold which can be customized with additional piping from four potential branches to run across the workstation's top surface. (These configurations will be shown later.) Regardless of the configuration, exiting water should return to the basin (5). This basin can be plugged, allowing accumulated volume to be observed via the connected sight gauge (4).

Straight pipes of roughly 13, 17.5, and 22 mm inner diameter available. One at a time, each size of piping can be connected to the closest opening on the manifold, with an outlet piece (consisting of three 90° turns, a quick-connect valve, and a ball valve) secured after the pipe. Pipes are sealed with rubber o-rings and secured together with male-female threaded connections—do not torque beyond hand-tight, as the plastic might crack and subsequently leak.

Using up to four sections of straight pipes and various additional fittings, several networks can be configured. An illustration of these is provided in Figure 4. Insert A shows how individual pipes can be installed, as described in the previous paragraph. Insert B shows a configuration of four pipes in parallel. Insert C shows how three pipes can be connected together in series. Insert D illustrates how a ring main can be constructed, with one inlet and three outlets. Finally, insert E has a doubled pipe arrangement, where they join together downstream. For all configurations, experimentalists are recommended to explore the effect of pipe diameter by alternating the placement of different straight pipes within the network.

Many quick-connection valve ports are installed throughout the apparatus. (Not shown in Figures 3 or 4). Connecting tubing between these valves and the manometer allows for measurement

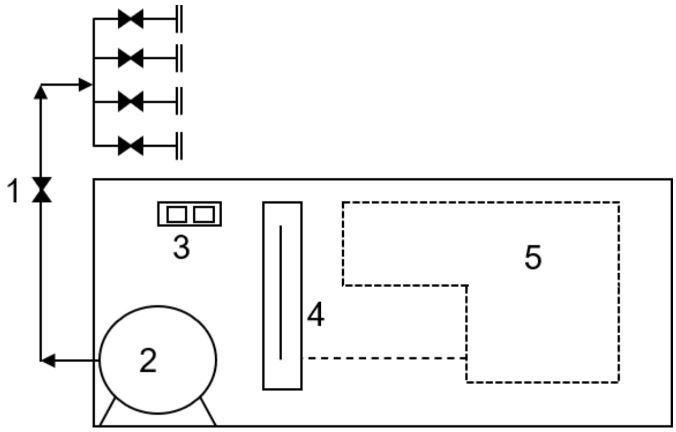

Figure 3: Experiment 2: Pipe networks apparatus.

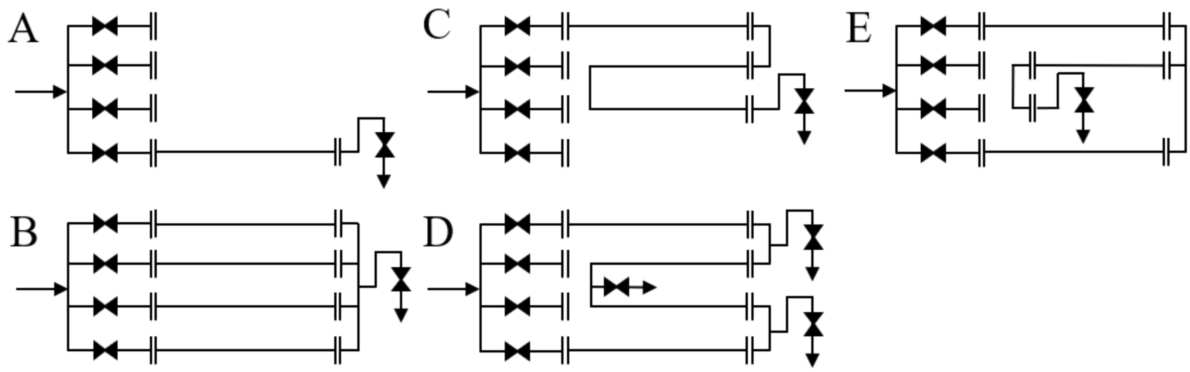

Figure 4: Possible pipe network configurations.

of the pressure differential between two points, as detailed in Figure 5. To connect, simply push the test probe directly into the connection valve. To disconnect, push in the metal clip on its side while pulling out the probe. The four valves above the water reservoir (5) are either closed, allowing water to flow back to the reservoir, or open, connecting the analog manometer to the system. The range of measurable pressure differentials is no more than one meter of water. If the manometer becomes flooded, or needs to be filled, air can be reintroduced into, or bled from, the manometer via the bleed valve at its top, respectively. When the manometer is in operation, the bleed valve should be closed.

5 Safety

The major hazards associated with this experiment are the following: The pump, powered from an electrical outlet, introduces the risk of injury from electric shock. Avoid by ensuring the pump power cord does not become submerged and is not touched when in operation. Through accumulation of biological contaminants in the water, there is also a risk of infection. Contact the instructional staff if you believe the tubing needs cleaned or replaced. If water splashes out of the apparatus, there are risks to clothing damage, slips, and falls. Avoid this by carefully considering the impact of opening or closing of valves prior to acting, particularly the valves at the inlet manifold, and clean spills immediately if they do occur. This list is only a starting point and not intended to be fully comprehensive—please keep the safety of yourself and others in mind throughout the laboratory sessions.

6 Experimental Procedure

To begin, assemble the pipe network illustrated in Figure 4.A using the smallest diameter pipe, roughly 13 mm. Fully open valve 1 in Figure 3, the closest inlet manifold valve, and the exit valve. Close all three of the other inlet manifold valves. Start the pump and wait until all air has been cleared from the piping and steady state has been reached. Measure the volumetric flow rate and pressure differential across the pipe for several flow rates. As this data will be used to predict results for subsequent parts of this experiment, it is recommended that at least six trials be conducted per pipe.

Flow rates can be measured using the volumetric tank (4, 5) and timing with a stopwatch. Record both the volume and timespan in which the measured amount of water fills the tank. Flow

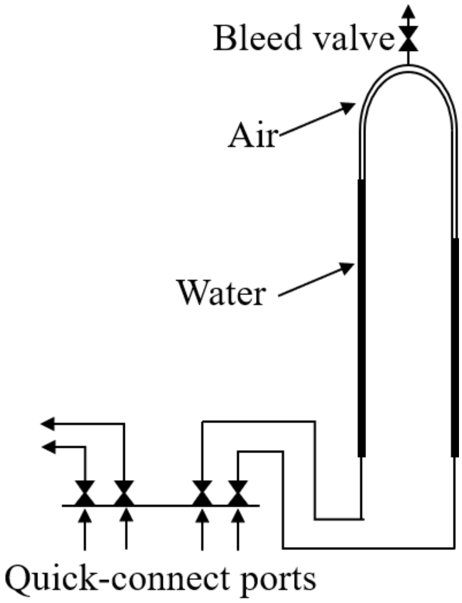

Figure 5: Connection diagram for analog manometer.

rates are best controlled with the exit valve(s), keeping the relevant inlet valves fully open. Pressure differentials across sections of interest can be measured using the manometer. A caliper is available for measuring the diameter of different piping pieces and confirming the pipes are smooth—if substantial roughness is observed, the caliper can be used to measure it. Record the locations of differently sized straight pipes in the networks assembled.

As described previously, collect pressure differential and volumetric flow rate measurements across the four straight pipes for several volumetric flow rates each, as illustrated in Figure 4.A. If possible, control the flow rates to obtain results in the laminar, transitional, and turbulent regimes.

Assemble the network to resemble the parallel pipes in Figure 4.B. Collect pressure differential and volumetric flow rate measurements across the network for a range of total flow rates. Record the volumetric flow rate as well. Repeat for different arrangements of straight pipe diameters.

Assemble the network to resemble the series of pipes in Figure 4.C. Measure the pressure drop across each section of the network, as well as the volumetric flow rate, for a range of flow rates. Repeat for different arrangements of straight pipe diameters.

Assemble the pipes to resemble the ring main in Figure 4.D. Measure the pressure drop across each section of the network, as well as the total volumetric flow rate, for a range of total flow rates. The three exit valve positions for this experiment will alter the system's behavior. Be careful not to adjust until all data has been collected per trial, but it is recommended to see how pipe diameter and exit valve positions affect the ring main's behavior through multiple trials.

Assemble the network to resemble the doubled pipe in Figure 4.E. Measure the pressure drop across each section of the network and the total volumetric flow rate for a range of flow rates. Repeat for different arrangements of straight pipe diameters.

Collect the gauge pressure at the pump outlet for a variety of flow rates.

7 Data Analysis

Calculate pressure differentials and fluid volumetric flow rates for the straight pipes studied in isolation. Plot the pressure drops as a function of volumetric flow rate and pipe diameter. Are the relationships between these variables as expected? Were you able to observe a laminar or transitional regime? Estimate the upper and lower Reynolds numbers associated with the transitional regime. Does it match your expectations? Why or why not?

For subsequent sections, make calibration curves from your experimental data to relate pressure drop and flow rate for each straight pipe.

For the parallel pipe configuration, calculate the expected flow rates through each pipe as well as the expected total flow rates. Compare this to the empirically measured total flow rates. Is there agreement? Why or why not? Can you improve your predictions? If so, how, and what is the result?

For the pipes-in-series configuration, calculate the expected pressure drops across each pipe, as well as the expected total pressure drop. Compare these to the empirically measured pressure drops. Is there agreement? Why or why not? Can you improve your predictions? If so, how, and what is the result?

For the ring main configuration, calculate the flow rate and flow direction for each section of the ring. Does your system satisfy mass conservation? Do the flow rates observed at the exits agree with the calculated values? Why or why not?

For the doubled pipe configuration, calculate and plot the pressure drop across the network as a function of total flow rate. Compare this to the calibration curves constructed for the single isolated pipes—what differences are there, and what are their causes? Calculate the flow rates in each branch of the doubled pipe and comment on the reasonableness of this estimate.

Generate a pump curve, plotting pump head generated as a function of fluid flow rate. Comment on the curve shape and compare to an expected pump curve. Describe plausible causes of differences between the two curves.

References

[1] Osborne Reynolds. An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels. *Philosophical Transactions of the Royal Society of London*, 174:935–982, 1883.

[2] Stephen Whitaker. *Introduction to fluid mechanics*. Krieger Publishing Company, 1968.

[3] L. F. Moody. Friction factors for pipe flow. *Trans. ASME*, 66:671, 1944.

[4] Don W. Green, editor. *Perry's chemical engineers' handbook*. McGraw-Hill, eighth edition, 2008.