The Function problems

Coding the Matrix, 2015

For auto-graded problems, edit the file The_Function_problems.py to include your solution.

Problem 1: `tuple_sum(A, B)`
input: lists A and B of the same length, where each element in each list is a pair (x, y) of numbers
output: list of pairs (x, y) in which the first element of the ith pair is the sum of the first element of the ith pair in A and the first element of the ith pair in B
example: given lists $[(1, 2), (10, 20)]$ and $[(3, 4), (30, 40)]$, return $[(4, 6), (40, 60)]$.

Problem 2: `inv_dict(d)`
input: dictionary d representing an invertible function f
output: dictionary representing the inverse of f, the returned dictionary's keys are the values of d and its values are the keys of d
example: given an English-French dictionary
`{'thank you': 'merci', 'goodbye':  'au revoir'}`
return a French-English dictionary
`{'merci':'thank you', 'au revoir':'goodbye'}`

**Problem 3:** First write a procedure `row(p, n)` with the following spec:

- *input:* integer p, integer n

- *output:* n-element list such that element i is $p + i$

- *example:* given $p = 10$ and $n = 4$, return $[10, 11, 12, 13]$

Next write a comprehension whose value is a 15-element list of 20-element lists such that the jth element of the ith list is $i + j$. You can use `row(p, n)` in your comprehension.
Finally, write the same comprehension but without using `row(p, n)`. Hint: replace the call to `row(p, n)` with the comprehension that forms the body of `row(p, n)`.

Functional Inverses

Ungraded problem: Is the following function invertible? If yes, explain why. If not, can you change domain and/or codomain of the function to make it invertible?
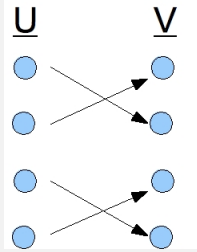
Ungraded problem: Is the following function invertible? If yes, explain why. If not, can you change domain and/or codomain of the function to make it invertible?
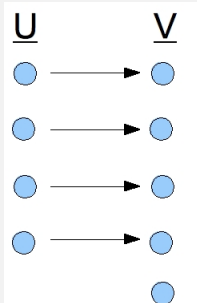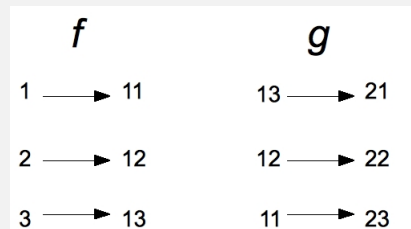

Functional composition

Ungraded problem: Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = abs(x)$. Is there a choice of domain and co-domain for the function $g(x)$ with rule $g(x) = \sqrt{x}$ such that $g \circ f$ is defined? If so, specify it. If not, explain why not. Could you change domain and/or codomain of f or g so that $g \circ f$ will be defined?

Ungraded problem: Consider functions f and g in the following figure:

Is $f \circ g$ defined? If so, draw it, otherwise explain why not.

Problem 4: A function $f(x) = x + 1$ with domain $\{1, 2, 3, 5, 6\}$ and codomain $\{2, 3, 4, 6, 7\}$ has the following probability function on its domain: $\Pr(1) = 0.5$, $\Pr(2) = 0.2$ and $\Pr(3) = \Pr(5) = \Pr(6) = 0.1$. What is the probability of getting an even number as an output of $f(x)$? An odd number?

Problem 5: A function $g(x) = x \bmod 3$ with domain $\{1, 2, 3, 4, 5, 6, 7\}$ and codomain $\{0, 1, 2\}$ has the following probability function on its domain: $\Pr(1) = \Pr(2) = \Pr(3) = 0.2$ and $\Pr(4) = \Pr(5) = \Pr(6) = \Pr(7) = 0.1$. What is the probability of getting 1 as an output of $g(x)$? What is the probability of getting 0 or 2?