Math 225 11 AM (Spencer) Quiz Solutions

Quiz 1

1. To properly define matrix multiplication we require the number of columns of the previous matrix to match the number of rows of the following matrix, that is, the entry is the dot product of a row in the previous matrix and a column in the following matrix. Thus the dimensions of B must be $n \times r$ and dimensions of ABC is $m \times s$

2. The constraint $AB=0$ is "harder" to realize than $BA \neq 0$, thus let us look at what A, B give $AB=0$: Recall that

$$\mathrm{Row}_i(AB) = \sum_{j=1}^{2} A_{ij} \cdot \mathrm{Row}_j B = a_{i1}\,\mathrm{Row}_1 B + a_{i2}\,\mathrm{Row}_2 B$$

we want $AB=0$, thus $\mathrm{Row}_i(AB)=0$ for all i, thus we can let $\mathrm{Row}_1 B = k\,\mathrm{Row}_2 B$ and $a_{i2} = -k\,a_{i1}$, which can be

$$A = \begin{pmatrix} a & -ka \\ a & -ka \end{pmatrix} \quad B = \begin{pmatrix} kb_1 & kb_2 \\ \cancel{k}b_1 & \cancel{k}b_2 \end{pmatrix} = \begin{pmatrix} kb_1 & kb_2 \\ b_1 & b_2 \end{pmatrix} \text{ where } \left.\begin{matrix} k \\ a \\ b_1 \text{ or } b_2 \end{matrix}\right\} \neq 0.$$

Let us compute
$$BA = \begin{pmatrix} ak(b_1+b_2) & -ak^2(b_1+b_2) \\ a(b_1+b_2) & -ak(b_1+b_2) \end{pmatrix}$$

Since we limit that $\left.\begin{matrix} k \\ a \\ b_1 \text{ or } b_2 \end{matrix}\right\} \neq 0$, thus $BA \neq 0$ iff $b_1+b_2 \neq 0$

Therefore, one such example can be $A = a\begin{pmatrix} 1 & -k \\ 1 & -k \end{pmatrix}$

$B = \begin{pmatrix} kb_1 & kb_2 \\ b_1 & b_2 \end{pmatrix}$ where $k \neq 0$, $a \neq 0$, b_1 or $b_2 \neq 0$ and $b_1 \neq -b_2$

3. We compute $A^2 = \begin{pmatrix} x^2-2 & x+y \\ -2x-2y & -2+y^2 \end{pmatrix} = A = \begin{pmatrix} x & 1 \\ -2 & y \end{pmatrix}$, thus

$$\begin{cases} x^2-2 = x & \text{i)} \\ x+y = 1 & \text{ii)} \\ -2(x+y) = -2 & \text{iii)} \\ -2+y^2 = y & \text{iv)} \end{cases}$$

we see ii) and iii) repeats thus we only need $x+y=1$, now we solve i) and iv) we get $x = \begin{cases} -1 \\ 2 \end{cases}$, $y = \begin{cases} -1 \\ 2 \end{cases}$, but sum of x & y is 1,

... solution ... $\{x=-1$ or $\{x=2$

In Problem 3, 4 matrix A is arbitrary dimension $m \times n$

Quiz 2.

1. We compute $A^2 = \begin{pmatrix} 2\times 2 + (-5)\times 6 & 2\times(-5) + (-5)\times(-6) \\ 6\times 2 + (-6)\times 6 & 6\times(-5) + (-6)\times(-6) \end{pmatrix}$

$4A = \begin{pmatrix} 4\times 2 & 4\times(-5) \\ 4\times 6 & 4\times(-6) \end{pmatrix}$ $\quad 18 I_2 = \begin{pmatrix} 18 & 0 \\ 0 & 18 \end{pmatrix}$ thus

$$A^2 + 4A + 18 I_2 = \begin{pmatrix} \underbrace{2\times 2 + 4\times 2}_{12} + \underbrace{(-5)\times 6}_{-30} + 18 & (-5)\times(\underbrace{2-6+4}_{=0}) \\ 6\times(\underbrace{2+(-6)+4}_{=0}) & \underbrace{(-6)\times(-6+4) + 6\times(-5)}_{-6\times(-6+4+5) = -18} + 18 \end{pmatrix}$$

$$= \begin{pmatrix} 12+18-30 & 0 \\ 0 & -18+18 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2$$

2. $A^{\#} = \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & -1 & 1 \end{array}\right) \overset{R_2 - 2R_1}{\sim} \underbrace{\left(\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -3 & -9 \end{array}\right)}_{\text{"REF}(A^{\#})\text{"}}$ "Echelon" (without normal pivot in 2^{nd} row

Now we read "REF$(A^{\#})$" as

$$\begin{cases} -3x_2 = -9 \\ x_1 + x_2 = 5 \end{cases} \Rightarrow \begin{cases} x_2 = 3 \\ x_1 = 5 - x_2 = 5 - 3 = 2 \end{cases}$$

3. matrix A is $m\times n$, thus A^T is $n \times m$ matrix, we previously learned if #(columns in A) = #(rows in A^T) in our multiplication AA^T, then such multiplication is defined and result AA^T is an m^2 square matrix

4. AA^T is symmetric, use definition of symmetric matrix S: $S^T = S$, we investigate $(AA^T)^T = (A^T)^T \cdot A^T = A \cdot A^T$, thus AAT is symmetric