Peter Shire - Olympic Disco , USC 1984

Math 225: Linear Algebra + Differential Equations

Instructor: Spencer Gerhardt

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office hours: MW 1-2 pm, F 12-1 pm

KAP 406 J

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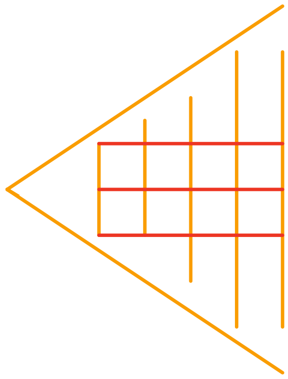
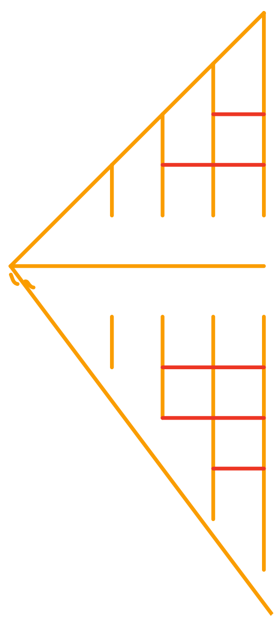
Course components



lectures MWF

discussion sections T

(Open discussion Th 10-12pm
in KAP 265 starting Week Two)



HW

Quizzes

Midterms / final

Lecture format

Subject to current university policies, lectures will be held in person.

It is possible some students will need to enter the USC health and safety protocols throughout the semester.

To best accomodate this eventuality,
I will post lecture notes on
Brightspace.

Also you don't need to rush to write
everything down (unless you like to).

It's also possible due to health and safety protocols that an individual class section may be held online via Zoom.

If this is the case, I will notify you in advance via Brightspace.

Syllabus highlights

Submitted work

Homework

$\boxed{75\%}$

There will be a few HW problems assigned each class. All HW problems for the week will [be due] the following Monday and submitted through Gradescope.

In case you need to miss lecture, the problems can be viewed on the posted notes in Brightspace.

Quizzes

17.5 %

There will be weekly quizzes (except exam weeks, and the first week)

They will be held in the discussion sections.

The lowest quiz score will be dropped.

Midterms

40 %

Held in class on

Monday October 6th
Friday November 7th

Final Exam

$\boxed{35\%}$

Wednesday December 10th 11-1pm

Cumulative exam same format as midterms

Course Overview

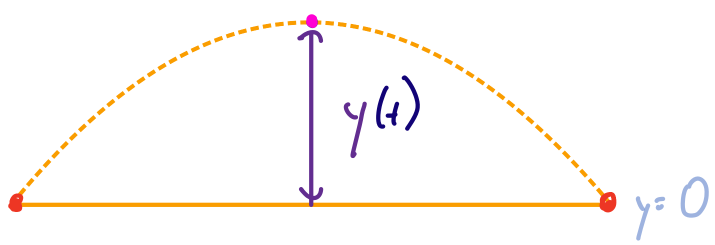
- This is an introduction to linear algebra and differential equations with applications to engineering, physics considered.

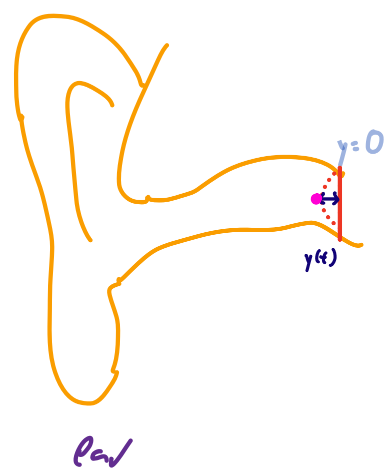
These are some of the most important tools for describing the physical world, and are basic tools for computing, machine learning, etc.

Ex (Harmonic Motion)

point on a cello string

point on the basilar membrane of the cochlea





Consider a particle of mass m subject to a force F towards an equilibrium position $y=0$ whose magnitude is proportional to the distance $y(t)$ from the equilibrium position. Then

1) $$\boxed{F = -ky}$$

constant of proportionality

Now Newton's second law of motion says:

2) $$F = ma$$

where $a = \dfrac{d^2y}{dt^2}$

Combining these two equations yields:

$$\boxed{\frac{d^2y}{dt^2} + \frac{ky}{m} = 0}$$

differential equation governing simple harmonic motion

This is a differential equation

(A differential equation is an equation involving one or more derivatives of a function).

(We'll see)

Solutions to this equation are the functions of the form:

$$\boxed{y = A\cos\left(\sqrt{k/m}\,t\right) + B\sin\left(\sqrt{k/m}\,t\right)}$$

The fact that these are the solutions of this differential equation explains why sine waves are the basis for the harmonic analysis of stringed instruments

(and Western music theory in general).

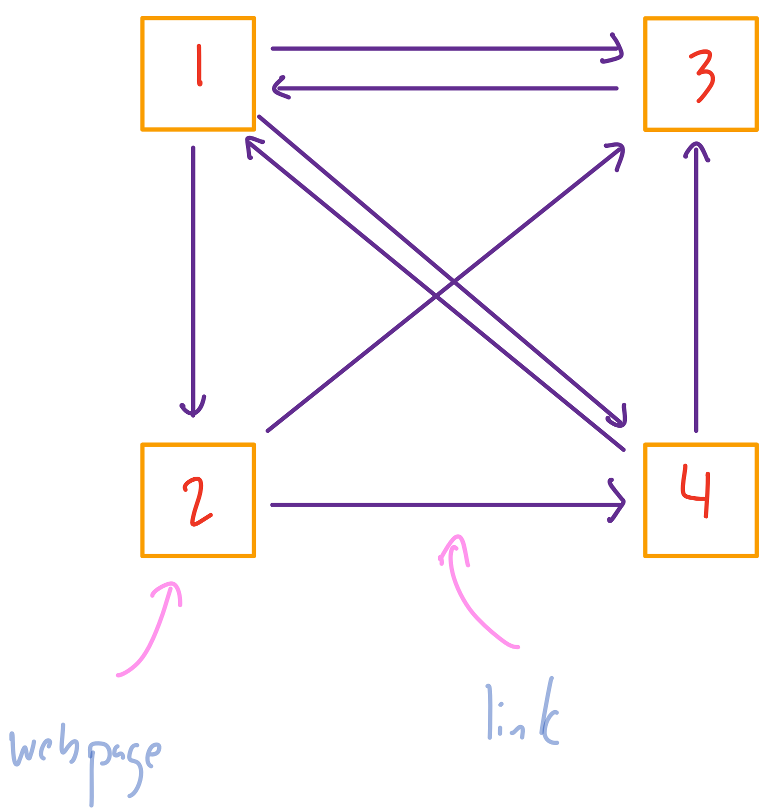
Furthermore, this differential equation governs the movement of any point on the basilar membrane and hence governs the human perception of sound.

A single differential equation can yield many far ranging applications.

Now let's look at linear algebra.

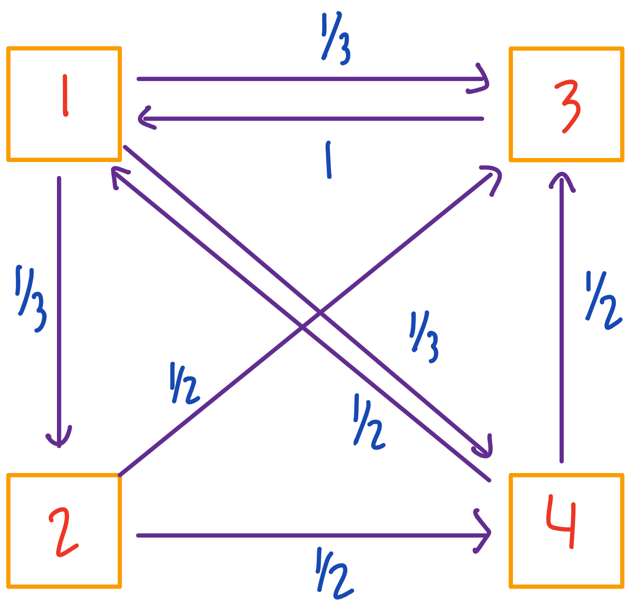
Ex (Google pagerank algorithm)

Imagine the internet as a directed graph with nodes represented by web pages and edges by links between them



We want to rank the importance of each page (for our search engine)

Each page transfers evenly its importance to the pages it links to. This yields the picture:



Q: Which page is the most important?

Using columns to represent each webpage this information can be encoded in a matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

webpage 2 links to pages 3 and 4

Let x_1, x_2, x_3, x_4 represent the importance of the four webpages.

This gives us a system of linear equations:

$$\begin{cases} x_1 = 1 \cdot x_3 + \frac{1}{2} \cdot x_4 \\ x_2 = \frac{1}{3} x_1 \\ x_3 = \frac{1}{3} \cdot x_1 + \frac{1}{2} \cdot x_2 + \frac{1}{2} \cdot x_4 \\ x_4 = \frac{1}{3} \cdot x_1 + \frac{1}{2} x_2 \end{cases}$$

Solving this system tells us the relative importance of each web page.

(We'll see)

This is equivalent to solving the matrix equation

$$A \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

or to finding an eigenvector corresponding to the eigenvalue 1.

Finding eigenvectors and eigenvalues of matrices is one of the main topics of this course.

So some computer scientists were able to monetize undergraduate linear algebra... Maybe you can do the same...

Solving the system (and scaling appropriately) you find:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{bmatrix}$$

A: So website 1 is the most important, even though more links go to website 3.

Course overview

In this course, we will cover most of Ch 2-4, 6-7 in the book, and parts of Ch 8 and Ch 9.

We will start with the linear algebra part of the course and end with differential equations.

See the syllabus for further details