Ch 3 Determinants (cont.)

Properties of Determinants (from Section 3.2)

P1. If A is an $n \times n$ upper triangular or lower triangular matrix then

$$\det A = a_{11} a_{22} \cdots a_{nn}$$

P2. If B is the matrix obtained by permuting two rows of A then

$$\det(B) = -\det(A)$$

P3. If B is the matrix obtained by multiplying one row of A by any scalar then

$$\det(B) = k\det(A)$$

P4. If B is the matrix obtained by adding a multiple of any row of A to a different row of A then

$$\det(B) = \det(A)$$

We may use these properties to calculate determinants

Ex/ Suppose $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = 7$

Suppose $B = \begin{bmatrix} 4g & 4h & 4i \\ a+d & b+e & c+f \\ a-2g & b-2h & c-2i \end{bmatrix}$. Find $\det(B)$.

$$\det(B) = \det(A) \cdot -1 \cdot 4$$

$$= -28$$

P5. $\det(A^T) = \det(A)$

P6. Let $a_1, a_2, \ldots, a_n$ denote the row vectors of A. If the ith row vector of A is the sum of two row vectors, say $a_i = b_i + c_i$ then $\det(A) = \det(B) + \det(C)$ where

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_i = b_i + c_i \\ \vdots \\ a_n \end{bmatrix} \quad B = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ b_i \\ \vdots \\ a_n \end{bmatrix} \quad C = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ c_i \\ \vdots \\ a_n \end{bmatrix}$$

The corresponding property is also true for columns

P7. If A has a row (or column) of zeros then $\det A = 0$

P8. If two rows (or two columns) are scalar multiples of each other then $\det(A) = 0$

P9. $\det(AB) = \det(A)\det(B)$

P10 If A is an invertible matrix then $\det(A) \neq 0$ and $\det(A^{-1}) = \frac{1}{\det(A)}$

Ex/ Let A and B be 4×4 matrices with $\det(A) = 3$ and $\det(B) = 4$. If $C = 5A \cdot 2B$, find $\det(C)$.

$$\det(5A) = 5^4 \cdot 3$$

$$\det(2B) = 2^4 \cdot 4$$

$$\det(C) = \det(5A) \cdot \det(2B)$$

$\boxed{\text{HW11}}$

Calculate
$$\begin{vmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{vmatrix}$$

$\boxed{\text{HW12}}$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \det(A) = 6$$

Find $\det(B)$ where

$$B = \begin{bmatrix} 2a & 2d & 2g \\ b-c & e-f & h-i \\ c-a & f-d & i-g \end{bmatrix}$$

$\boxed{\text{HW13}}$ Let A and B be 4×4 matrices such that $\det(A) = 5$ and $\det(B) = 3$

Find a) $\det\,(2B)^{-1}(AB)^{T}$

b) $\det\,(4B)^{3}$

c) $\det\,(A^{-1}B^{2})^{2}$

$\boxed{\text{HW14}}$ Determine all values of the constant k for which

$$x_1 + kx_2 = b_1$$
$$kx_1 + 4x_2 = b_2$$

has a unique solution