Math 225: Linear Algebra — Fall 22

Exam 1 — 7 October 2022

First Name: (as in student record)

Last Name: (as in student record)

USC ID: Signature:

- This exam has 5 problems, and will last 50 minutes.
- You may use one page of notes, but no calculator.
- Show all of your work and justify every answer to receive full credit.
- Feel free to continue answers on other pages as long as you clearly indicate to the grader where they can find your solution.
- Work quickly, but carefully. Good luck!

Do not write in the box below:

Q01	Q02	Q03	Q04	Q05
/10	/10	/10	/10	/10

/50

Question 1 (10 points)**.** Determine whether the following statements are **true** or **false**. You do not need to justify your answer.

(a) Assume A and B are $n \times n$ matrices. Then $det(AB) = det(BA)$.

(b) The vector equation $Ax = b$ has infinitely many solutions if A has more columns than rows.

(c) If $det(A) = 2det(B)$ then AB is not invertible.

(d) Let A and B be $n \times n$ matrices where $AB + BA = 0$. Then either A or B is not invertible.

(e) Assume S and T are subspaces of a vector space V. Then

$$S \cup T = \{x \mid x \in S \text{ or } x \in T\}$$

is a subspace of V.

Question 2 (10 points).

(a) Suppose $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} a-d & b-e & c-f \\ 2g & 2h & 2i \\ -d & -e & -f \end{bmatrix}$ and $det(A) = 4$.

Find $det(ABA^T B^{-2})$.

(b) Let $A = \begin{bmatrix} 2 & k & k \\ 2 & 1 & 4 \\ 1 & k & 0 \end{bmatrix}$. Find all values k for which the matrix fails to be invertible.

Question 3 (10 points). Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. Can A be written as a product of elementary matrices? Justify your answer.

If your answer is yes, write A as a product of elementary matrices.

Question 4 (10 points)**.** Let S be the solution set to the system of linear equations

$$\begin{aligned} x - y + 3z - 2w &= 0 \\ 2x + y + 3z - 4w &= 0 \\ 3x + 4y + 2z - 6w &= 0 \end{aligned}$$

(a) Find S.

(b) Show that S is a subspace of $\mathbb{R}^4$.

Question 5 (10 points)**.** Let S be the subspace of $M_2(\mathbb{R})$ consisting of all 2×2 matrices whose four elements sum to zero.

Find three vectors v_1, v_2, v_3 in S where $span\{v_1, v_2, v_3\} = S$. Justify your answer.