Using Cross Table to Solve an Interview Question

Jingyuan Wu

Question

- Suppose there are 5 coins. Four of the coins are fair with P(heads)=0.5, and one of the coins is biased with P(heads)=0.75. Suppose a coin is randomly selected and flipped 4 times resulting in 4 heads.

- What is the probability that the selected coin was biased? That is, what is P(biased coin selected | HHHH)?

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob	P (HHHH&Fair)	P (HHHH&Biased)	P (HHHH)
Row prob	P (Fair|HHHH)	P (Biased|HHHH)	
Col prob	P (HHHH|Fair)	P (HHHH|Biased)	
NOT HHHH			
Cell prob	P (NOT HHHH&Fair)	P (NOT HHHH&Biased)	P (NOT HHHH)
Row prob	P (Fair|NOT HHHH)	P (Biased|NOT HHHH)	
Col prob	P (NOT HHHH|Fair)	P (NOT HHHH|Biased)	
Column Total	P (Fair)	P (Biased)	1

Solution

- 2 kinds of coins, Fair | Biased
- Fair: 4, Biased: 1, Total: 5
- The probability of the selected coin was fair/biased
- P(Fair) = 4/5 = 0.8
- P(Biased) = 1/5 = 0.2

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob			
Row prob			
Col prob			
NOT HHHH			
Cell prob			
Row prob			
Col prob			
Column Total	0.8	0.2	1

Solution

- According to the conditions given in the question:
- Flipped once
- P(H|Fair) = 0.5
- P(H|Biased) = 0.75

- Flipped 4 times
- 2 kinds of outcomes, HHHH | NOT HHHH
- P(HHHH|Fair) = 0.5*0.5*0.5*0.5
- P(HHHH|Biased) = 0.75*0.75*0.75*0.75
- P(NOT HHHH) = 1 - P(HHHH)
- P(NOT HHHH|Fair) = (1 - 0.5*0.5*0.5*0.5)
- P(NOT HHHH|Biased) = (1 - 0.75*0.75*0.75*0.75)

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob			
Row prob			
Col prob	0.5^4	0.75^4	
NOT HHHH			
Cell prob			
Row prob			
Col prob	$(1-0.5^4)$	$(1-0.75^4)$	
Column Total	0.8	0.2	1

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob	0.8×0.5^4	0.2×0.75^4	$0.8 \times 0.5^4 + 0.2 \times 0.75^4$
Row prob			
Col prob	0.5^4	0.75^4	
NOT HHHH			
Cell prob	$0.8 \times (1 - 0.5^4)$	$0.2 \times (1 - 0.75^4)$	$0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)$
Row prob			
Col prob	$(1 - 0.5^4)$	$(1 - 0.75^4)$	
Column Total	0.8	0.2	1

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob	0.8×0.5^4	0.2×0.75^4	$0.8 \times 0.5^4 + 0.2 \times 0.75^4$
Row prob	$\frac{0.8 \times 0.5^4}{0.8 \times 0.5^4 + 0.2 \times 0.75^4}$	$\frac{0.2 \times 0.75^4}{0.8 \times 0.5^4 + 0.2 \times 0.75^4}$	
Col prob	0.5^4	0.75^4	
NOT HHHH			
Cell prob	$0.8 \times (1 - 0.5^4)$	$0.2 \times (1 - 0.75^4)$	$0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)$
Row prob	$\frac{0.8 \times (1 - 0.5^4)}{0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)}$	$\frac{0.2 \times (1 - 0.75^4)}{0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)}$	
Col prob	$(1 - 0.5^4)$	$(1 - 0.75^4)$	
Column Total	0.8	0.2	1

Outcome	Fair	Biased	Row Total
HHHH			
Cell prob	0.8×0.5^4	0.2×0.75^4	$0.8 \times 0.5^4 + 0.2 \times 0.75^4$
Row prob	$\frac{0.8 \times 0.5^4}{0.8 \times 0.5^4 + 0.2 \times 0.75^4}$	$\frac{0.2 \times 0.75^4}{0.8 \times 0.5^4 + 0.2 \times 0.75^4}$ 0.56	
Col prob	0.5^4	0.75^4	
NOT HHHH			
Cell prob	$0.8 \times (1 - 0.5^4)$	$0.2 \times (1 - 0.75^4)$	$0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)$
Row prob	$\frac{0.8 \times (1 - 0.5^4)}{0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)}$	$\frac{0.2 \times (1 - 0.75^4)}{0.8 \times (1 - 0.5^4) + 0.2 \times (1 - 0.75^4)}$	
Col prob	$(1 - 0.5^4)$	$(1 - 0.75^4)$	
Column Total	0.8	0.2	1