Heat Eqn, 2D

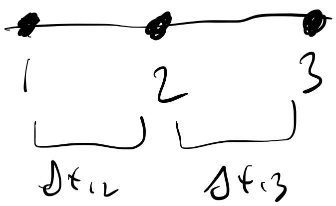
$$\frac{\partial u}{\partial t} = \Delta u \Rightarrow \frac{\partial u}{\partial t} = \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

→ α: Thermal diffusivity

↳ $\alpha = \frac{k}{\rho C_p}$ → k: thermal conductivity
ρ: density
C_p: heat capacity, constant pressure

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 \dots$$

Taylor series discretization → for a given property "ϕ": $\phi = \phi_0 + \Delta x\, \phi'_0 + \frac{\Delta x^2}{2}\phi''_0$



$$\phi_1 = \phi_2 + (-\Delta x_{12})\phi'_2 + \frac{(-\Delta x_{12})^2}{2}\phi''_2 = \phi_2 - \Delta x_{12}\phi'_2 + \frac{\Delta x_{12}^2}{2}\phi''_2$$

$$\phi_3 = \phi_2 + \Delta x_{23}\phi'_2 + \frac{\Delta x_{23}^2}{2}\phi''_2$$

assuming $\Delta x_{12} = \Delta x_{13}$: $\phi_1 + \phi_3 = 2\phi_2 + \Delta x_{23}^2 \phi''_2$

↳ $$\boxed{\frac{\phi_1 + \phi_3 - 2\phi_2}{\Delta x^2} = \phi''_2}$$

$$\phi_3 - \phi_1 = 2\Delta x\, \phi'_2 \rightarrow \boxed{\frac{\phi_3 - \phi_1}{2\Delta x} = \phi'_2}$$

let $u(x,y,t) = u^k_{i,j}$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \rightarrow \frac{u^{k+1}_{ij} - u^k_{ij}}{\Delta t} = \alpha\left(\frac{u^k_{i+1,j} + u^k_{i-1,j} - 2u^k_{i,j}}{\Delta x^2} + \frac{u^k_{i,j+1} + u^k_{i,j-1} - 2u^k_{i,j}}{\Delta y^2}\right)$$

$$\boxed{u^{k+1}_{i,j} - u^k_{i,j} = \alpha(\Delta t)\left(\frac{u^k_{i+1|j} + u^k_{i-1|j} - 2u^k_{i|j}}{\Delta x^2} + \frac{u^k_{i|j+1} + u^k_{i|j-1} - 2u^k_{i|j}}{\Delta y^2}\right)}$$