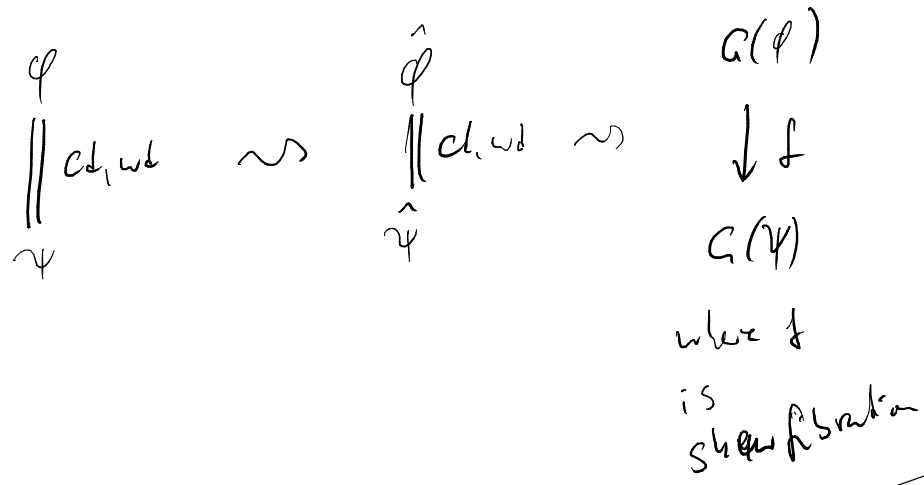


φ, ψ ... FO-formulas α, β, γ : prop. formula

$\hat{\varphi}, \hat{\psi}$ - propositional encodings
 $G(\varphi) = G(\hat{\varphi})$

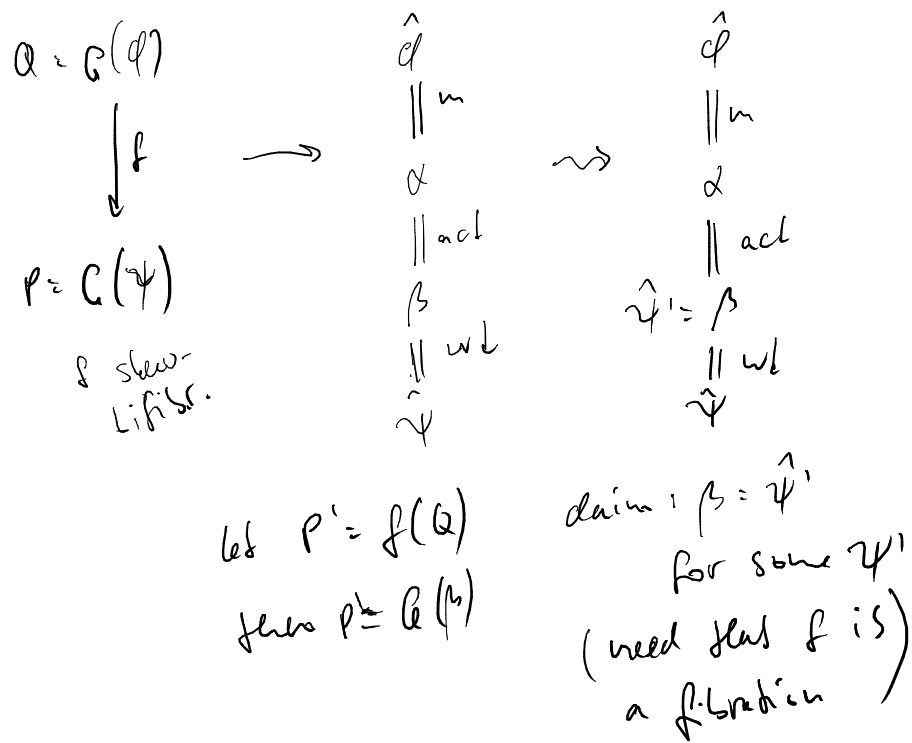


need to show:
 f is also a filtration
 pf: each cl and ul
 is a filtration
 • Relations compose
 $\Rightarrow f$ is a filtration

COMPLETENESS

f is a shrink filtration

SOUNDNESS



let x be a prop. formula

Define x^* as follows

$$\begin{aligned} \alpha^* &= \alpha \quad \text{if } \alpha \text{ is an ordinary atom} \\ (u_x \vee \dots \vee u_y)^* &= u_x \quad (u \vee v)^* = u^* \vee v^* \\ (E_x \vee \dots \vee E_y)^* &= E_x \quad \wedge \psi)^* = P^* \wedge \psi^* \end{aligned}$$

this way get γ s.

$$\begin{array}{c} \alpha \\ \parallel_{act} \\ \gamma \\ \parallel_{act} \\ \vdots \end{array}$$

and

$$\begin{array}{c} \beta^* \\ \parallel_{act} \\ \gamma^* \\ \parallel_{act} \\ \vdots \end{array}$$

$$\beta = \beta^*$$

$$\hat{\varphi}^*$$

$$\parallel_m^*$$

$$\alpha^*$$

$$\parallel_{act}$$

$$\beta^* = \gamma^* = \beta = \hat{\psi}'$$

$$\parallel_{ul}$$

$$\hat{\psi}$$

← is propositional
 in Γ formula

next: we show that each line in

$$\begin{array}{c} \hat{\varphi}^* \\ \parallel_m^* \\ \alpha^* \end{array}$$

is a prop encoding
 of a FO formula

• $\alpha^* = \hat{\theta}$ for some FO formula θ

• Finally, each m^* is either an $m \frac{(A \wedge C) \vee (B \wedge D)}{(A \vee B) \wedge (C \vee D)}$
 or $\frac{(E_x \wedge C) \vee (E_x \wedge D)}{E_x \wedge (C \vee D)}$

which is the propos. encoding of $m \frac{\exists x C \vee \exists x D}{\exists x (C \vee D)}$

Hence, we can go at each step from conclusion to
 premise, to conclude that $\hat{\varphi}^*$ is a prop. encoding
 \Rightarrow But not necessarily $\hat{\varphi} = \hat{\varphi}^*$ because the number of u_x
 might be messed up.
 But this can be fixed with $m \frac{\forall x C \vee \forall x D}{\forall x (C \vee D)}$

$$\begin{array}{c} \Rightarrow \\ \parallel_{m, m', m''} \\ \varphi \\ \parallel_{act} \\ \psi' \\ \parallel_{ul} \\ \psi \end{array}$$

Attention: we need to take care of variables:

In an ordinary instance of contraction

$$\frac{(\forall x. D) \vee (\forall x. D)}{\forall x. D} \quad \text{or} \quad \frac{(\exists x. D) \vee (\exists x. D)}{\exists x. D}$$

the premise is not redified. we should
 in fact write

$$\frac{\forall y. D \vee \forall z. D}{\forall x. D} \quad \text{and} \quad \frac{\exists y. D \vee \exists z. D}{\exists x. D}$$