Supplementary Material

1 Clique Construction

The constructing procedure (GetClique, Algorithm 4) generates the maximal weight cliques with diversity, which is integrated into population initialization, local optimization, and population updating of HEA-D. It starts from a random vertex v from the vertex set V which includes all vertices in the graph G. Add the selected vertex v into a partial clique c. Let $Candset$ denote a candidate vertex set, which is a set of the neighbourhoods of vertex v and each vertex in it is adjacent to all vertices already in c. Then, GetClique repeats a series of iterations until $CandSet$ becomes empty (lines 4–13). At each iteration, if $Candset$ is not empty, the search procedure continues to find another vertex which can be added into the partial clique c from $Candset$. The *best from multiple selection* (BMS) heuristic strategy is applied to improve the efficiency of constructing procedure, which has been used to solving maximum weight clique problem and diversified top-k cohesive subgraph problem [Cai, 2015; Wu *et al.*, 2020; Wu and Yin, 2021a]. The benefit of a vertex in $CandSet$ is calculated by the formula as follows.

$$\hat{b}[v] = \sum_{u \in (N(v) \cap Candset)} w(u). \tag{1}$$

Moreover, the BMS strategy can adaptively adjust the number of vertices selected from the candidate set in the process of constructing the maximal weight cliques based on the parameters m, m_0, and m_{max}. Among them, m is the maximum number of vertices picked from the candidate set each time and is initialized as m_0. And in the next search round of HEA-D, the value of m is adjusted to $2m$ ($m_0 < m < m_{max}$). If $m > m_{max}$, m is reset to $(m_0 + 1)$ and $m_0 = m_0 + 1$. The BMS has been theoretically shown that it can approximate the best-picking strategy very well in O(1). The tuning of the parameters m_0 and m_{max} is discussed in Section 2.

Therefore, we use the BMS strategy to select the remaining vertices which can be added into c after the first vertex is confirmed (lines 2–3). If $|CandSet| < m$, i.e., there are less than m vertices in $CandSet$, the vertex with the greatest benefit is selected to be added into c (lines 5–6). Otherwise, the vertex with the greatest benefit among m vertices which are randomly selected from $CandSet$ is added into c (lines 7–12). Then $CandSet$ is updated to keep all vertices in $CandSet$ are adjacent to all vertices already in c (line 13).

Algorithm 4: GetClique(G, m)

Input: a weighted graph $G(V, E, w)$, parameter m of BMS strategy
Output: a maximal weight clique c

```
1  c ← ∅;
2  v ← randomly select a vertex from V;
3  c ← {v}, CandSet ← {u|u ∈ N(v)};
4  while CandSet ≠ ∅ do
5      if |CandSet| < m then
6          pick the vertex v from CandSet with the
             greatest b̂, breaking ties in favour of the older
             one;
7      else
8          v ← randomly select a vertex from CandSet;
9          for iter = 1 to m − 1 do
10             v′ ← randomly select a vertex from
                 CandSet;
11             if b̂[v′] > b̂[v] then
12                 v ← v′, b̂[v] ← b̂[v′];
13     c ← c ∪ {v}; CandSet ← CandSet ∩ N(v);
14 return c;
```

2 Parameter Tuning

HEA-D requires another 6 parameters (p, θ_{size}, θ_{cool}, θ_{reduce}, l and L except m_0 and m_{max}) (see Table A1). Among them, p is the size of the population, θ_{size} and θ_{cool} are required by the simulated annealing procedure, while l and L are the maximum unimproved rounds of local optimization and the maximum unimproved iterations of population, respectively. R is the set of the individuals which are removed from the population in the updating procedure and $\theta_{reduce} = |R|/|Pop|$ is defined as the removal rate. To tune these parameters, the automatic configuration tool irace [López-Ibáñez *et al.*, 2016] is used. In addition, for this tuning experiment, a set of 18 graphs in which CPLEX solver can obtain at least one solution with k varying in $\{10, 20, 30, 40, 50\}$ is chosen from the weighted large real-world graphs. The tuning budget is set to 3000 runs and the

Parameter	Description	Range	Value
p	size of population	{10, 15, 20, 25, 30}	20
θ_{size}	max iteration coefficient per temperature of LocalOptimization	{2, 4, 8, 16, 32}	8
θ_{cool}	cool ratio of LocalOptimization	{0.90, 0.91, . . . , 0.99}	0.96
θ_{reduce}	remove ratio of population	{0.1,0.2,. . . ,0.9}	0.5
l	max unimproved rounds of local optimization	{1,2,. . . ,10}	5
L	max unimproved iterations of population	{50,100,150,200}	100
m_0	min iteration steps of BMS strategy	{1,2,4,8}	4
m_{max}	max iteration steps of BMS strategy	{16,32,64,128}	64

Table A1: Parameter setting of HEA-D and TOPKWCLQ

time budget is set to 600 seconds per run. The range of possible parameter values and final values suggested by irace are represented in Table A1. The parameters employed in TOPKWCLQ are used the default values reported by the authors from the corresponding literature [Wu and Yin, 2021b].

3 Computational Results

This section presents computational results and comparisons on a broad range of real-world graphs. Table A2 shows the results of comparison on advertisement putting. Columns 1 to 2 indicate the basic information of each graph (name, number of vertices, and edges) and the value of the parameter "k". Columns "w_{best}" and "w_{HEA-D}" provide the optimal solutions and the best solution obtained by HEA-D, respectively. Since these instances which transformed from advertisement putting are so easy that all algorithms (exact WPMS solver, heuristic WPMS solver, and HEA-D) can find the optimal solutions, we only show the results of the optimal ("w_{best}") reported in the literature in Table A2 for each k value. Compared with the existing algorithms, HEA-D can find the optimal solutions in almost the same time (less than 10^{-3} second) on all advertisement putting instances. And the dominating values are marked in bold.

The solutions of HEA-D and its competitors (CPLEX solver and TOPKWCLQ) on large real-world instances are presented in Tables A3-A7. "LB" and "UB" mean the *lower bound* and the *upper bound* found by CPLEX, respectively. w_b denotes the best value found and w_a indicates the average value obtained over 10 runs. If an algorithm fails to solve an instance, the corresponding column is marked as "N/A". "-" indicates that the reference algorithm fails to reach a solution within a limited time. The average time over 10 runs of the TOPKWCLQ and HEA-D are reported in $time$ columns. If the run time is less than 10^{-3} second, the entry is filled with "0". The bold values indicate the best solution value among HEA-D and the reference algorithms. To verify whether there exists a statistically significant difference between HEA-D and its competitors in terms of the best (average) objective values, the p-values from the non-parametric Wilcoxon signed ranks test [Derrac *et al.*, 2011; Carrasco *et al.*, 2020] with a significance level of 0.05 are provided in Table A8. Column R^+_{best} (R^+_{avg}) denotes the sum of ranks for the instances in which HEA-D outperforms the compared algorithms, in terms of the best (average) objective value, while R^-_{best} (R^-_{avg}) refers to the sum of ranks for the opposite.

Instance	k	w_{best}	w_{HEA-D}
G10 (10,45)	1	**1628**	**1628**
	2	**1628**	**1628**
	3	**1628**	**1628**
	4	**1628**	**1628**
	5	**1628**	**1628**
G20 (20,133)	1	**2341**	**2341**
	2	**2617**	**2617**
	3	**2845**	**2845**
	4	**2970**	**2970**
	5	**3051**	**3051**
G30 (30,213)	1	**2572**	**2572**
	2	**3109**	**3109**
	3	**3556**	**3556**
	4	**3783**	**3783**
	5	**3983**	**3983**
G40 (40,473)	1	**2341**	**2341**
	2	**3674**	**3674**
	3	**5045**	**5045**
	4	**5272**	**5272**
	5	**5472**	**5472**
G50 (50,689)	1	**4133**	**4133**
	2	**5828**	**5828**
	3	**6422**	**6422**
	4	**6814**	**6814**
	5	**7128**	**7128**
G60 (60,952)	1	**4575**	**4575**
	2	**6562**	**6562**
	3	**7656**	**7656**
	4	**8319**	**8319**
	5	**8766**	**8766**
G70 (70,1356)	1	**5525**	**5525**
	2	**8085**	**8085**
	3	**9223**	**9223**
	4	**9886**	**9886**
	5	**10333**	**10333**
G87 (87,1968)	1	**5926**	**5926**
	2	**9519**	**9519**
	3	**11040**	**11040**
	4	**11990**	**11990**
	5	**12570**	**12570**

Table A2: Comparison of solution quality on advertisement putting.

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	**6867**	**6867**	**6867**	6860.8	121.529	**6867**	**6867**	0.178
bio-diseasome	**8672**	**8672**	**8672**	**8672**	0	**8672**	**8672**	0
bio-dmela	N/A	N/A	6261	6176.5	172.733	**6274**	**6274**	2.422
bio-yeast	**4991**	5020	**4991**	**4991**	0.005	**4991**	**4991**	0.008
ca-AstroPh	N/A	N/A	**48124**	48107.9	114.107	**48124**	**48124**	25.748
ca-citeseer	N/A	N/A	**76978**	76864.6	3.944	**76978**	**76978**	42.466
ca-coauthors-dblp	N/A	N/A	**306617**	**306617**	18.654	**306617**	**306617**	14.694
ca-CondMat	N/A	N/A	**20325**	**20325**	0.231	**20325**	**20325**	0.15
ca-CSphd	**4059**	**4059**	**4059**	**4059**	0	**4059**	**4059**	0
ca-dblp-2010	N/A	N/A	**65041**	**65041**	3.073	**65041**	**65041**	2.746
ca-dblp-2012	N/A	N/A	**64257**	**64257**	1.1	**64257**	**64257**	1.778
ca-Erdos992	N/A	N/A	6013	5991.1	130.412	**6048**	6037.6	224.546
ca-GrQc	N/A	N/A	**25844**	**25844**	0.048	**25844**	**25844**	0.042
ca-HepPh	N/A	N/A	**79624**	**79624**	2.5	**79624**	**79624**	0.256
ca-hollywood-2009	N/A	N/A	934087	933840.5	286.157	**934287**	934141.2	387.604
ca-MathSciNet	N/A	N/A	**24017**	**24017**	0.606	**24017**	**24017**	0.618
ca-netscience	**7588**	**7588**	**7588**	**7588**	0	**7588**	**7588**	0
ia-email-EU	N/A	N/A	10086	9976.2	339.157	**10762**	10709.4	292.5
ia-email-univ	**9570**	9718	**9570**	9567.8	107.826	**9570**	**9570**	12.526
ia-enron-large	N/A	N/A	17020	16615.7	438.43	**18142**	18006	490.156
ia-enron-only	**4331**	4338	**4331**	**4331**	11.478	**4331**	**4331**	0.692
ia-fb-messages	5738	7441	**5961**	5936.3	280.465	**5961**	**5961**	14.08
ia-infect-dublin	**10946**	11001	**10946**	10933.6	239.643	**10946**	**10946**	4.83
ia-infect-hyper	4446	5292	4376	4298	261.069	**4581**	4554.2	192.25
ia-reality	N/A	N/A	3015	3011.9	151.681	**3017**	**3017**	0.626
ia-wiki-Talk	N/A	N/A	11921	11697.2	367.257	**13550**	13520.2	474.074
inf-power	N/A	N/A	**6613**	**6613**	0.077	**6613**	**6613**	0.09
inf-roadNet-CA	N/A	N/A	**6515**	**6515**	1.731	**6515**	**6515**	1.472
inf-roadNet-PA	N/A	N/A	**6086**	**6086**	0.7	**6086**	**6086**	0.82
inf-road-usa	N/A	N/A	**6719**	6692	173.532	**6719**	**6719**	113.452
rec-amazon	N/A	N/A	**8931**	8928.2	0.645	**8931**	**8931**	0.446
rt-retweet	**1575**	1578	**1575**	**1575**	0	**1575**	**1575**	0
rt-retweet-crawl	N/A	N/A	9612	9477.7	80.458	**9728**	9673.8	259.284
rt-twitter-copen	**4661**	**4661**	**4661**	**4661**	0.009	**4661**	**4661**	0.016
sc-ldoor	N/A	N/A	**40747**	**40747**	34.021	**40747**	**40747**	24.278
sc-msdoor	N/A	N/A	**40691**	**40691**	12.584	**40691**	**40691**	11.072
sc-nasasrb	N/A	N/A	**43792**	43786.4	7.769	**43792**	**43792**	69.296
sc-pkustk11	N/A	N/A	**47548**	47545.6	70.576	**47548**	**47548**	2.802
sc-pkustk13	N/A	N/A	**57960**	57903.9	196.501	**57960**	**57960**	3.654
sc-pwtk	N/A	N/A	**45528**	**45528**	5.523	**45528**	**45528**	4.74
sc-shipsec1	N/A	N/A	**31900**	31898.9	6.013	**31900**	**31900**	4.286
sc-shipsec5	N/A	N/A	**43260**	**43260**	0.532	**43260**	**43260**	0.392
soc-BlogCatalog	N/A	N/A	19608	19285.3	321.322	**21317**	20751.2	501.56
soc-brightkite	N/A	N/A	19398	18687.3	210.193	**20323**	20110	431.52
soc-buzznet	N/A	N/A	18268	17904.6	312.259	**19585**	19130.2	539.012
soc-delicious	N/A	N/A	12779	12645.6	109.774	**13165**	13121.8	482.56
soc-digg	N/A	N/A	27957	26954.4	359.908	**28748**	28224.4	453.37
soc-dolphins	**1226**	**1226**	**1226**	**1226**	0.009	**1226**	**1226**	0
soc-douban	N/A	N/A	9758	9547	171.104	**9956**	**9956**	104.456
soc-epinions	N/A	N/A	13189	12828.8	194.049	**13359**	13356.6	259.346
socfb-A-anon	N/A	N/A	23927	23611.8	462.452	**24832**	24385.2	435.346
socfb-B-anon	N/A	N/A	22741	22339.5	446.51	**23176**	22867.4	583.964
socfb-Berkeley13	N/A	N/A	37836	37398.6	199.215	**39905**	39857.4	520.028
socfb-CMU	N/A	N/A	31686	31265.1	289.008	**34100**	34060.4	411.812

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Table A3 – continued from previous page

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	N/A	N/A	29635	29182.7	253.73	**32359**	32187.8	401.866
socfb-Indiana	N/A	N/A	42713	42274.1	262.268	**45814**	45624.6	508.782
socfb-MIT	N/A	N/A	30755	30317.3	351.148	**32614**	32572.8	481.508
socfb-OR	N/A	N/A	26666	26337	255.168	**28839**	28765.8	514.692
socfb-Penn94	N/A	N/A	37827	37443.1	254.848	**39724**	39608.6	452.312
socfb-Stanford3	N/A	N/A	37528	36412.8	200.555	**40370**	40244.6	437.192
socfb-Texas84	N/A	N/A	40982	40483	255.799	**45051**	44801.8	526.558
socfb-uci-uni	N/A	N/A	8777	8636.7	425.483	**8859**	8784.4	337.472
socfb-UCLA	N/A	N/A	41293	40387.8	286.558	**44246**	44103	541.246
socfb-UConn	N/A	N/A	36941	36384.1	295.565	**39556**	39486.4	461.888
socfb-UCSB37	N/A	N/A	40672	39904.7	171.366	**44129**	44071.4	514.072
socfb-UF	N/A	N/A	49857	49351.3	276.168	**53584**	53461.6	556.5
socfb-UIllinois	N/A	N/A	44447	43776.4	271.594	**47611**	47336.6	556.782
socfb-Wisconsin87	N/A	N/A	32533	32156.2	308.538	**35245**	35119.6	493.238
soc-flickr	N/A	N/A	24435	23992.5	209.199	**26232**	25718.6	492.004
soc-flixster	N/A	N/A	27651	26991	239.732	**29064**	28436.8	477.002
soc-FourSquare	N/A	N/A	12582	12386.4	295.201	**12631**	12519.4	401.6
soc-gowalla	N/A	N/A	20138	19869.6	145.985	**20624**	20533.6	421.45
soc-karate	**472**	**472**	**472**	**472**	0.003	**472**	**472**	0
soc-lastfm	N/A	N/A	14765	14471.3	171.909	**15092**	14970.2	567.816
soc-livejournal	N/A	N/A	137836	137052.9	414.093	**139006**	137767	432.142
soc-LiveMocha	N/A	N/A	10458	10305.1	211.407	**11413**	11278.4	525.678
soc-orkut	N/A	N/A	**40071**	39739.5	390.926	**40071**	39672.4	353.496
soc-pokec	N/A	N/A	22402	22132.9	444.383	**23111**	22647.6	522.222
soc-slashdot	N/A	N/A	13595	13365	342.717	**14745**	14475.6	558.832
soc-twitter-follows	N/A	N/A	6457	6319.5	40.989	**6475**	6457.8	125.8
soc-wiki-Vote	6376	6768	**6432**	6419.5	275.404	**6432**	6431.8	276.836
soc-youtube	N/A	N/A	13805	13519	77.156	**14642**	14477	514.838
soc-youtube-snap	N/A	N/A	14092	13706.7	183.177	**14588**	14337	453.232
tech-as-caida2007	N/A	N/A	7383	7276.6	242.271	**8363**	8080.2	224.864
tech-as-skitter	N/A	N/A	32313	31589	441.04	**33713**	33289.4	524.008
tech-internet-as	N/A	N/A	8025	7632.2	159.953	**8556**	8283	397.372
tech-p2p-gnutella	N/A	N/A	**5883**	**5883**	1.186	**5883**	**5883**	0.604
tech-RL-caida	N/A	N/A	12577	12276.8	216.554	**13417**	13264.2	332.212
tech-routers-rf	3914	207554	10062	9963.7	255.858	**10094**	**10094**	0.71
tech-WHOIS	N/A	N/A	30373	29557.1	314.206	**32550**	32368.4	463.59
web-arabic-2005	N/A	N/A	**92108**	**92108**	0.139	**92108**	**92108**	0.142
web-BerkStan	N/A	N/A	**11200**	**11200**	0.052	**11200**	**11200**	0.02
web-edu	N/A	N/A	**11250**	**11250**	0	**11250**	**11250**	0
web-google	**13126**	**13126**	**13126**	**13126**	0	**13126**	**13126**	0
web-indochina-2004	N/A	N/A	**44052**	**44052**	0.15	**44052**	**44052**	0.1
web-it-2004	N/A	N/A	**415850**	**415850**	2.553	**415850**	**415850**	3.168
web-polblogs	5926	6428	6030	6000	303.5	**6034**	**6034**	30.418
web-sk-2005	N/A	N/A	**63958**	**63958**	1.97	**63958**	**63958**	2.426
web-spam	N/A	N/A	14512	14361.2	277.037	**15226**	15106.2	341.904
web-uk-2005	N/A	N/A	**441613**	**441613**	0.27	**441613**	**441613**	0.286
web-webbase-2001	N/A	N/A	**20648**	**20648**	0.183	**20648**	**20648**	0.138
web-wikipedia2009	N/A	N/A	**31340**	**31340**	5.616	**31340**	**31340**	4.014

Table A3: Comparison of solution quality on real-world large graphs with $k = 10$.

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	11489	11747	11509	11417.5	328.368	**11544**	**11544**	1.734
bio-diseasome	**14313**	14345	**14313**	**14313**	0.032	**14313**	**14313**	0.002
bio-dmela	N/A	N/A	11224	11152.4	288.738	**11354**	**11354**	51.414
bio-yeast	8874	9730	**9067**	**9067**	0.027	**9067**	**9067**	0.02
ca-AstroPh	N/A	N/A	**88438**	88227.9	19.705	**88438**	88406.6	86.468
ca-citeseer	N/A	N/A	**133919**	133886.4	23.886	**133919**	**133919**	41.464
ca-coauthors-dblp	N/A	N/A	**544001**	543974.6	53.299	**544001**	**544001**	22.538
ca-CondMat	N/A	N/A	**36370**	**36370**	34.656	**36370**	**36370**	0.4
ca-CSphd	-	-	7943	7943	0	7943	7943	0
ca-dblp-2010	N/A	N/A	**110035**	**110035**	8.845	**110035**	**110035**	5.332
ca-dblp-2012	N/A	N/A	**104685**	**104685**	8.842	**104685**	**104685**	7.53
ca-Erdos992	N/A	N/A	10615	10512	295.322	**10629**	**10629**	71.558
ca-GrQc	N/A	N/A	**37559**	**37559**	0.287	**37559**	**37559**	0.086
ca-HepPh	N/A	N/A	**114481**	114454.1	146.47	**114481**	**114481**	0.434
ca-hollywood-2009	N/A	N/A	1367714	1366829	280.652	**1368903**	1368500	462.924
ca-MathSciNet	N/A	N/A	**42409**	**42409**	2.154	**42409**	**42409**	1.758
ca-netscience	13178	13196	**13189**	**13189**	0.001	**13189**	**13189**	0
ia-email-EU	N/A	N/A	17253	17054.8	188.851	**18898**	18820.4	432.532
ia-email-univ	11024	18426	15880	15797.5	307.977	**15930**	15929.2	127.506
ia-enron-large	N/A	N/A	30552	30043.3	319.851	**32447**	32358.8	448.996
ia-enron-only	6922	7237	6917	6885	218.089	**6938**	6925.2	243.002
ia-fb-messages	6204	15510	10834	10792.5	235.978	**10901**	10890.6	252.236
ia-infect-dublin	15009	18694	17563	17466.3	306.297	**17588**	17588	259.35
ia-infect-hyper	5972	6554	5943	5874	130.892	**6261**	6204.2	158.824
ia-reality	N/A	N/A	5734	5714.6	134.038	**5742**	5736.2	145.934
ia-wiki-Talk	N/A	N/A	21146	20961.8	417.833	**23946**	23618.2	527.852
inf-power	N/A	N/A	**11801**	**11801**	56.104	**11801**	**11801**	1.088
inf-roadNet-CA	N/A	N/A	**12488**	**12488**	1.692	**12488**	**12488**	1.656
inf-roadNet-PA	N/A	N/A	**12056**	**12056**	0.637	**12056**	**12056**	0.726
inf-road-usa	N/A	N/A	**12740**	12678.4	170.794	**12740**	**12740**	117.518
rec-amazon	N/A	N/A	**17414**	**17414**	1.741	**17414**	**17414**	0.94
rt-retweet	2732	2758	**2754**	**2754**	0	**2754**	**2754**	0.002
rt-retweet-crawl	N/A	N/A	17130	17022.5	102.011	**17485**	17395	448.48
rt-twitter-copen	8364	8525	**8366**	**8366**	0.521	**8366**	**8366**	0.012
sc-ldoor	N/A	N/A	**81375**	81372.9	52.368	**81375**	**81375**	25.424
sc-msdoor	N/A	N/A	**81053**	81046	127.563	**81053**	**81053**	10.894
sc-nasasrb	N/A	N/A	85152	85055.6	169.151	**85168**	**85168**	1.478
sc-pkustk11	N/A	N/A	**91735**	91686.4	106.545	**91735**	**91735**	3.104
sc-pkustk13	N/A	N/A	111649	111314.5	312.215	**112515**	**112515**	8.802
sc-pwtk	N/A	N/A	**90144**	**90144**	253.998	**90144**	**90144**	5.08
sc-shipsec1	N/A	N/A	61914	61875.4	107.947	**61929**	**61929**	8.008
sc-shipsec5	N/A	N/A	**83202**	**83202**	4.094	**83202**	**83202**	3.396
soc-BlogCatalog	N/A	N/A	32250	31945.7	358.614	**35143**	34631.6	488.588
soc-brightkite	N/A	N/A	31856	31255.8	255.972	**33426**	33230.6	505.974
soc-buzznet	N/A	N/A	31214	30589.3	341.3	**33596**	32949.8	530.632
soc-delicious	N/A	N/A	23337	23067.4	166.775	**24313**	24195.2	492.914
soc-digg	N/A	N/A	44417	43431.6	396.56	**46508**	45478.6	512.014
soc-dolphins	**1861**	1871	**1861**	1858.8	292.925	**1861**	**1861**	1.114
soc-douban	N/A	N/A	16673	16486.4	182.181	**17316**	**17316**	132.902
soc-epinions	N/A	N/A	22912	22718	184.69	**23924**	23876.2	174.824
socfb-A-anon	N/A	N/A	44125	43833.1	469.613	**45992**	45322.2	414.072
socfb-B-anon	N/A	N/A	42421	42162	466.9	**43663**	43357	471.516
socfb-Berkeley13	N/A	N/A	65979	64980.9	383.114	**72508**	72361.8	534.596
socfb-CMU	N/A	N/A	53623	52629.3	216.637	**60006**	59757.8	524.202

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Table A4 – continued from previous page

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	N/A	N/A	52716	52024.4	336	**59580**	59323	572.25
socfb-Indiana	N/A	N/A	76872	75792.1	230.515	**84348**	84161.8	553.382
socfb-MIT	N/A	N/A	53108	52605	318.317	**59167**	59105.4	564.864
socfb-OR	N/A	N/A	47768	47342.5	283.672	**52597**	52384.8	522.028
socfb-Penn94	N/A	N/A	69138	68324.7	309.005	**74932**	74654	548.494
socfb-Stanford3	N/A	N/A	63550	63046.3	261.696	**71196**	71022.8	573.312
socfb-Texas84	N/A	N/A	72640	71501.1	201.496	**79857**	79450.2	583.112
socfb-uci-uni	N/A	N/A	16667	16430.9	413.77	**16969**	16850.6	361.4
socfb-UCLA	N/A	N/A	68039	67474.8	286.517	**75996**	75732	517.7
socfb-UConn	N/A	N/A	61789	60885.8	277.107	**68176**	67795.4	567.876
socfb-UCSB37	N/A	N/A	64937	64364.9	424.212	**72476**	72211.6	555.62
socfb-UF	N/A	N/A	83601	82180.8	319.916	**94399**	93888.2	556.238
socfb-UIllinois	N/A	N/A	77626	76264.6	274.991	**83836**	83630.6	564.712
socfb-Wisconsin87	N/A	N/A	58168	57628	363.847	**64133**	64007.8	561.59
soc-flickr	N/A	N/A	43338	42825.6	252.001	**45916**	45635	550.828
soc-flixster	N/A	N/A	50125	49001.8	237.433	**51482**	50934.4	477.226
soc-FourSquare	N/A	N/A	22156	21937	381.32	**22530**	22362.6	404.926
soc-gowalla	N/A	N/A	37324	36576.2	305.692	**38715**	38566.6	514.624
soc-karate	**629**	**629**	**629**	**629**	0	**629**	**629**	0
soc-lastfm	N/A	N/A	26753	26582.8	190.194	**28008**	27853.6	525.576
soc-livejournal	N/A	N/A	226339	224347.3	475.404	**232264**	229330.6	509.484
soc-LiveMocha	N/A	N/A	18906	18567.4	283.677	**21111**	20681.2	543.55
soc-orkut	N/A	N/A	**75929**	74769.9	275.86	**75929**	74921.4	263.258
soc-pokec	N/A	N/A	41260	40724	448.337	**42401**	41993.4	518.286
soc-slashdot	N/A	N/A	22478	22268.5	321.575	**23879**	23731.2	562.364
soc-twitter-follows	N/A	N/A	12033	11937.9	148.231	**12223**	12170.4	277.694
soc-wiki-Vote	10649	12613	11265	11178.7	236.995	**11269**	11262.2	219.24
soc-youtube	N/A	N/A	24875	24611	107.244	**26772**	26590	541.714
soc-youtube-snap	N/A	N/A	25295	24989.3	185.307	**26374**	26114.8	523.994
tech-as-caida2007	N/A	N/A	12594	12397.2	291.076	**13287**	13177	348.246
tech-as-skitter	N/A	N/A	52352	50979.3	456.218	**54517**	52839.8	457.614
tech-internet-as	N/A	N/A	13222	13145.1	232.823	**14125**	13969.8	466.596
tech-p2p-gnutella	N/A	N/A	**11424**	**11424**	2.444	**11424**	**11424**	2.018
tech-RL-caida	N/A	N/A	22825	22361.6	327.455	**23799**	23722.6	474.6
tech-routers-rf	N/A	N/A	15827	15690.2	243.73	**15946**	15941.8	405.022
tech-WHOIS	N/A	N/A	42439	42086.9	237.877	**46539**	46291.8	552.046
web-arabic-2005	N/A	N/A	**178434**	**178434**	0.228	**178434**	**178434**	0.156
web-BerkStan	N/A	N/A	**18758**	**18758**	2.373	**18758**	**18758**	0.414
web-edu	N/A	N/A	**16615**	16611.8	126.966	**16615**	**16615**	0.084
web-google	14174	22943	**21479**	**21479**	0.013	**21479**	**21479**	0.014
web-indochina-2004	N/A	N/A	**74320**	74315.6	55.269	**74320**	**74320**	1.714
web-it-2004	N/A	N/A	**797123**	**797123**	14.356	**797123**	**797123**	14.028
web-polblogs	9808	11906	10349	10297.7	287.062	**10464**	10429.6	125.088
web-sk-2005	N/A	N/A	**97490**	97476.4	3.701	**97490**	**97490**	5.036
‘web-spam	N/A	N/A	24480	24164.3	177.823	**25680**	25597.2	347.54
web-uk-2005	N/A	N/A	**789896**	**789896**	0.344	**789896**	**789896**	0.374
web-webbase-2001	N/A	N/A	**33815**	**33815**	30.703	**33815**	**33815**	0.576
web-wikipedia2009	N/A	N/A	**52555**	**52555**	52.305	**52555**	**52555**	47.682

Table A4: Comparison of solution quality on real-world large graphs with $k = 20$.

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	14528	16193	15082	14981	233.54	**15199**	**15199**	49.19
bio-diseasome	12610	20103	**18431**	**18431**	6.529	**18431**	**18431**	0.022
bio-dmela	N/A	N/A	15978	15834	309.367	**16189**	16173.8	120.012
bio-yeast	-	-	**12845**	**12845**	0.04	**12845**	**12845**	0.048
ca-AstroPh	N/A	N/A	120785	120472.2	248.297	**121237**	**121237**	13.646
ca-citeseer	N/A	N/A	**182155**	182050.8	110.636	**182155**	**182155**	45.514
ca-coauthors-dblp	N/A	N/A	**760014**	759929.3	92.724	**760014**	**760014**	25.606
ca-CondMat	N/A	N/A	**50658**	50562.8	168.691	**50658**	**50658**	1.476
ca-CSphd	N/A	N/A	**11742**	**11742**	0.008	**11742**	**11742**	0.004
ca-dblp-2010	N/A	N/A	**149809**	149795	11.204	**149809**	**149809**	6.246
ca-dblp-2012	N/A	N/A	**137088**	**137088**	13.086	**137088**	**137088**	11.594
ca-Erdos992	N/A	N/A	14656	14569.1	147.755	**14887**	**14887**	7.094
ca-GrQc	N/A	N/A	**46325**	**46325**	0.117	**46325**	**46325**	0.094
ca-HepPh	N/A	N/A	141521	141385.4	226.765	**141529**	**141529**	0.432
ca-hollywood-2009	N/A	N/A	1686859	1685740	232.406	**1688898**	1688708	459.116
ca-MathSciNet	N/A	N/A	**58496**	**58496**	8.31	**58496**	**58496**	4.394
ca-netscience	17341	17997	**17781**	**17781**	0.025	**17781**	**17781**	0.006
ia-email-EU	N/A	N/A	23143	22882.1	257.099	**25410**	25207.4	390.852
ia-email-univ	8733	26220	21297	21128.6	343.832	**21452**	**21452**	289.332
ia-enron-large	N/A	N/A	42777	42556.7	194.357	**45845**	45760.6	476.744
ia-enron-only	8030	9226	8543	8494	222.645	**8649**	**8649**	53.258
ia-fb-messages	7135	23605	15307	15213	198.032	**15411**	15398	159.224
ia-infect-dublin	12608	26004	22693	22569.2	304.605	**22977**	**22977**	7.08
ia-infect-hyper	**6554**	**6554**	6538	6510.2	105.637	**6554**	**6554**	23.866
ia-reality	N/A	N/A	8239	8208.9	241.731	**8256**	8240.4	150.002
ia-wiki-Talk	N/A	N/A	29885	29557.4	261.334	**33074**	32935.8	564.198
inf-power	N/A	N/A	**16625**	16623.3	12.29	**16625**	**16625**	4.394
inf-roadNet-CA	N/A	N/A	**18458**	**18458**	1.863	**18458**	**18458**	1.75
inf-roadNet-PA	N/A	N/A	**18026**	**18026**	0.649	**18026**	**18026**	0.676
inf-road-usa	N/A	N/A	**18710**	18668.6	209.722	**18710**	**18710**	158.318
rec-amazon	N/A	N/A	**25468**	**25468**	1.844	**25468**	**25468**	1.504
rt-retweet	**3609**	**3609**	**3609**	**3609**	0.007	**3609**	**3609**	0.004
rt-retweet-crawl	N/A	N/A	24379	24222.1	92.049	**24931**	24821.2	432.924
rt-twitter-copen	11777	12474	**11804**	11792.9	213.571	**11804**	**11804**	11.034
sc-ldoor	N/A	N/A	**121807**	121779	122.017	**121807**	**121807**	27.758
sc-msdoor	N/A	N/A	121156	121082.5	205.22	**121163**	**121163**	17.97
sc-nasasrb	N/A	N/A	124728	124453	99.139	**124840**	124825.6	136.164
sc-pkustk11	N/A	N/A	135034	134873.5	215.591	**135085**	**135085**	4.158
sc-pkustk13	N/A	N/A	163616	162812.6	221.862	**165079**	**165079**	8.862
sc-pwtk	N/A	N/A	133872	133827.2	101.677	**133920**	**133920**	20.882
sc-shipsec1	N/A	N/A	90538	90353.1	98.576	**90607**	**90607**	28.762
sc-shipsec5	N/A	N/A	**120702**	**120702**	6.668	**120702**	**120702**	5.898
soc-BlogCatalog	N/A	N/A	43493	42989.9	228.977	**48000**	47307.2	568.608
soc-brightkite	N/A	N/A	42785	42279.1	278.536	**45209**	45002.2	561.12
soc-buzznet	N/A	N/A	42262	41653.1	333.262	**46710**	45850	570.386
soc-delicious	N/A	N/A	32604	32193.4	172.806	**34277**	34171.2	505.646
soc-digg	N/A	N/A	59440	58381.1	381.475	**62215**	61074	482.234
soc-dolphins	**2015**	**2015**	**2015**	**2015**	0.064	**2015**	**2015**	0.532
soc-douban	N/A	N/A	23281	22996.9	155.889	**24047**	**24047**	238.954
soc-epinions	N/A	N/A	31359	31122.5	180.529	**33192**	33111.4	388.114
socfb-A-anon	N/A	N/A	63456	62909.4	481.843	**65917**	65477.8	490.442
socfb-B-anon	N/A	N/A	61754	60873.3	466.443	**63422**	63212.4	505.75
socfb-Berkeley13	N/A	N/A	89569	87664.4	375.264	**99420**	98819.6	563.996
socfb-CMU	N/A	N/A	69979	68798.1	129.992	**79662**	79599	539.158

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Table A5 – continued from previous page

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	N/A	N/A	71990	71474.2	323.987	**83644**	82906.4	572.748
socfb-Indiana	N/A	N/A	105371	104101.7	284.459	**117639**	117158	576.468
socfb-MIT	N/A	N/A	71406	70551.1	206.005	**81405**	81226.8	544.06
socfb-OR	N/A	N/A	67381	66256.4	347.074	**74569**	74237	565.498
socfb-Penn94	N/A	N/A	95496	93933.4	390.619	**105466**	105163.2	592.11
socfb-Stanford3	N/A	N/A	86452	85854.6	194.788	**98176**	97947.6	574.492
socfb-Texas84	N/A	N/A	99665	99083.4	305.918	**110839**	110238.4	585.22
socfb-uci-uni	N/A	N/A	24261	24025.2	403.562	**24737**	24596.6	427.29
socfb-UCLA	N/A	N/A	92858	91524.2	233.788	**103751**	103129	569.456
socfb-UConn	N/A	N/A	83300	81790.6	240.332	**91861**	91370.6	566.276
socfb-UCSB37	N/A	N/A	87074	85755.2	270.471	**95495**	94964.4	579.062
socfb-UF	N/A	N/A	110179	109293.9	293.966	**125827**	124482.8	581.518
socfb-UIllinois	N/A	N/A	104988	104200.9	343.514	**116521**	116048.4	571.412
socfb-Wisconsin87	N/A	N/A	81327	80430.8	336.952	**90379**	89943.4	547.454
soc-flickr	N/A	N/A	61246	60462	294.321	**64809**	64010.2	552.73
soc-flixster	N/A	N/A	68666	67883.6	289.368	**72471**	71924.2	552.112
soc-FourSquare	N/A	N/A	31078	30787.6	278.917	**31927**	31705.6	545.176
soc-gowalla	N/A	N/A	51669	51370.2	321.814	**54931**	54811	575.674
soc-karate	**629**	**629**	**629**	**629**	0	**629**	**629**	0
soc-lastfm	N/A	N/A	38093	37861.6	178.233	**39880**	39665.8	521.628
soc-livejournal	N/A	N/A	295449	292498.8	487.355	**301000**	298494.2	436.96
soc-LiveMocha	N/A	N/A	26715	26350	313.599	**29435**	29016.2	563.074
soc-orkut	N/A	N/A	**108504**	108034	280.25	**108504**	108097.8	309.696
soc-pokec	N/A	N/A	58794	58246.1	469.77	**60606**	60437	543.064
soc-slashdot	N/A	N/A	30393	30026.2	364.077	**32201**	31815	475.578
soc-twitter-follows	N/A	N/A	17575	17462.4	211.386	**17838**	17803.6	410.95
soc-wiki-Vote	9944	18426	15219	15076.4	212.469	**15472**	15472	63.678
soc-youtube	N/A	N/A	35174	34939	261.12	**38038**	37533.6	509.814
soc-youtube-snap	N/A	N/A	36160	35517.9	188.562	**37640**	37323	517.918
tech-as-caida2007	N/A	N/A	17374	17189.3	175.477	**18139**	18101.8	391.954
tech-as-skitter	N/A	N/A	68630	66825.9	530.071	**72343**	70997	526.506
tech-internet-as	N/A	N/A	18415	18268.8	320.048	**19410**	19364.8	292.188
tech-p2p-gnutella	N/A	N/A	**16836**	**16836**	3.424	**16836**	**16836**	2.21
tech-RL-caida	N/A	N/A	31609	31460.2	246.159	**33725**	33618.2	486.29
tech-routers-rf	N/A	N/A	21008	20718.6	302.506	**21115**	21110.4	89.538
tech-WHOIS	N/A	N/A	53093	52112.7	427.894	**57050**	56743	525.766
web-arabic-2005	N/A	N/A	**263602**	**263602**	3.791	**263602**	**263602**	2.748
web-BerkStan	N/A	N/A	**25959**	25958.7	0.415	**25959**	**25959**	0.406
web-edu	N/A	N/A	21049	21041.8	256.64	**21061**	**21061**	0.196
web-google	10767	28599	**26866**	**26866**	0.051	**26866**	**26866**	0.034
web-indochina-2004	N/A	N/A	**99285**	99168.8	101.244	**99285**	**99285**	2.182
web-it-2004	N/A	N/A	**1115823**	**1115823**	21.326	**1115823**	**1115823**	20.734
web-polblogs	8579	17312	13996	13932.7	151.76	**14190**	14188	66.604
web-sk-2005	N/A	N/A	**127612**	127590.2	3.794	**127612**	**127612**	3.354
web-spam	N/A	N/A	32740	32297.6	197.122	**34409**	34288.4	371.26
web-uk-2005	N/A	N/A	**1050477**	**1050477**	1.029	**1050477**	**1050477**	0.98
web-webbase-2001	N/A	N/A	**44669**	44664.9	6.973	**44669**	**44669**	79.156
web-wikipedia2009	N/A	N/A	**70080**	**70080**	95.313	**70080**	**70080**	66.592

Table A5: Comparison of solution quality on real-world large graphs with $k = 30$.

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	10037	22077	18349	18205.7	170.363	**18459**	**18459**	142.486
bio-diseasome	10450	24138	**21945**	21938.8	88.679	**21945**	**21945**	18.318
bio-dmela	N/A	N/A	20387	20239.9	270.694	**20771**	20732.6	245.596
bio-yeast	-	-	**16468**	**16468**	0.066	**16468**	**16468**	0.05
ca-AstroPh	N/A	N/A	149165	148935.4	225.88	**150317**	**150317**	22.79
ca-citeseer	N/A	N/A	225069	224803.5	13.025	**225076**	**225076**	67.898
ca-coauthors-dblp	N/A	N/A	963124	962802.7	74.075	**963240**	**963240**	36.806
ca-CondMat	N/A	N/A	64118	63917.3	244.806	**64175**	**64175**	22.146
ca-CSphd	N/A	N/A	**15440**	**15440**	0.029	**15440**	**15440**	0.028
ca-dblp-2010	N/A	N/A	**186676**	186661.7	48.57	**186676**	**186676**	9.396
ca-dblp-2012	N/A	N/A	**166039**	166017	42.977	**166039**	**166039**	12.512
ca-Erdos992	N/A	N/A	18582	18444.8	239.364	**18890**	18838.2	42.798
ca-GrQc	N/A	N/A	**54003**	**54003**	0.321	**54003**	**54003**	0.116
ca-HepPh	N/A	N/A	162656	162506.2	321.608	**162825**	**162825**	1.232
ca-hollywood-2009	N/A	N/A	1943869	1942635	287.481	**1949728**	1948901	469.514
ca-MathSciNet	N/A	N/A	**73431**	**73431**	9.715	**73431**	**73431**	5.882
ca-netscience	10519	22011	**21076**	**21076**	5.299	**21076**	**21076**	1.952
ia-email-EU	N/A	N/A	28430	28174.9	243.833	**31182**	30989.2	473.91
ia-email-univ	10977	32641	26034	25828	262.21	**26523**	26510	157.75
ia-enron-large	N/A	N/A	54611	54363.5	265.945	**58300**	58032.8	422.074
ia-enron-only	8159	10121	9520	9478.8	129.207	**9695**	9688.8	207.008
ia-fb-messages	-	-	19409	19289.5	199.138	**19703**	19685.4	103.034
ia-infect-dublin	17535	30618	26586	26456.4	217.779	**27309**	27299.4	85.282
ia-infect-hyper	**6554**	**6554**	**6554**	**6554**	0	**6554**	**6554**	0
ia-reality	N/A	N/A	10584	10563.7	198.451	**10641**	10615.4	2.188
ia-wiki-Talk	N/A	N/A	37697	37402.5	289.388	**41963**	41748.2	585.212
inf-power	N/A	N/A	**21185**	21182.7	50.491	**21185**	**21185**	2.504
inf-roadNet-CA	N/A	N/A	**24428**	**24428**	2.548	**24428**	**24428**	2.16
inf-roadNet-PA	N/A	N/A	**23996**	**23996**	0.789	**23996**	**23996**	0.766
inf-road-usa	N/A	N/A	**24680**	24646	241.127	**24680**	**24680**	95.122
rec-amazon	N/A	N/A	**33364**	**33364**	2.658	**33364**	**33364**	2.256
rt-retweet	**4161**	4169	**4161**	**4161**	0.036	**4161**	**4161**	0.04
rt-retweet-crawl	N/A	N/A	31489	31343.1	119.941	**32207**	32112.6	430.664
rt-twitter-copen	14772	16130	15000	14970.7	244.286	**15010**	15008	191.47
sc-ldoor	N/A	N/A	162050	162015.7	125.608	**162071**	**162071**	28.566
sc-msdoor	N/A	N/A	160963	160888.9	340.852	**161049**	**161049**	15.694
sc-nasasrb	N/A	N/A	163384	162800.6	194.151	**163664**	163491.2	22.774
sc-pkustk11	N/A	N/A	177661	177488.8	216.837	**177892**	**177892**	4.478
sc-pkustk13	N/A	N/A	213908	213258	238.247	**216651**	**216651**	14.01
sc-pwtk	N/A	N/A	177256	177130.4	237.606	**177312**	177305.6	176.892
sc-shipsec1	N/A	N/A	118039	117681.6	243.848	**118262**	**118262**	51.318
sc-shipsec5	N/A	N/A	**157040**	157023.2	9.103	**157040**	**157040**	6.454
soc-BlogCatalog	N/A	N/A	53891	53185.7	263.863	**59644**	59029.8	562.364
soc-brightkite	N/A	N/A	52972	52426.6	248.134	**56094**	55909.8	565.11
soc-buzznet	N/A	N/A	52170	51783.2	323.303	**57865**	56988.2	572.424
soc-delicious	N/A	N/A	40914	40779.1	205.53	**43683**	43582.2	502.614
soc-digg	N/A	N/A	73644	72460.4	421.056	**79072**	76366.8	477.212
soc-dolphins	**2015**	**2015**	**2015**	**2015**	0	**2015**	**2015**	0
soc-douban	N/A	N/A	29342	29190.6	203.633	**30466**	30443.2	292.656
soc-epinions	N/A	N/A	39217	39073.3	332.845	**41764**	41531.2	419.768
socfb-A-anon	N/A	N/A	82078	81036.2	493.926	**84982**	84463.2	452.278
socfb-B-anon	N/A	N/A	79859	78853.8	438.337	**82637**	82019	491.85
socfb-Berkeley13	N/A	N/A	109294	108345.6	168.298	**123283**	122702.8	581.908
socfb-CMU	N/A	N/A	83878	82830.8	241.084	**96273**	95727.4	576.614

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Table A6 – continued from previous page

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	N/A	N/A	90512	89272.5	267.513	**103786**	103053.8	569.8
socfb-Indiana	N/A	N/A	130154	128728.3	324.013	**145974**	145230	585
socfb-MIT	N/A	N/A	86659	85831.9	332.073	**99338**	98896.2	570.882
socfb-OR	N/A	N/A	85283	84298.7	314.907	**95007**	94381	570.918
socfb-Penn94	N/A	N/A	118993	116455.4	303.589	**131155**	130566.4	573.478
socfb-Stanford3	N/A	N/A	107097	105933.3	183.05	**122458**	121894.2	589.094
socfb-Texas84	N/A	N/A	124978	123827.3	355.353	**140522**	139562	580.49
socfb-uci-uni	N/A	N/A	31690	31566.3	476.695	**32409**	32218.6	401.278
socfb-UCLA	N/A	N/A	114727	113778.1	250.139	**129878**	128848.4	566.898
socfb-UConn	N/A	N/A	100918	99325.7	213.631	**112536**	112276.2	583.528
socfb-UCSB37	N/A	N/A	106783	104280.2	309.675	**116591**	116351.6	550.142
socfb-UF	N/A	N/A	135141	133713.2	295.26	**153363**	152681.2	578.25
socfb-UIllinois	N/A	N/A	131159	129295.4	225.922	**146436**	145346.8	593.914
socfb-Wisconsin87	N/A	N/A	103200	101561.7	363.285	**113964**	113624	577.87
soc-flickr	N/A	N/A	77788	76712.6	345.138	**82602**	82059.2	557.432
soc-flixster	N/A	N/A	86269	85645.2	259.232	**91463**	90766.2	550.564
soc-FourSquare	N/A	N/A	39863	39617.4	298.756	**40615**	40507.6	472.99
soc-gowalla	N/A	N/A	65828	65355.9	308.548	**70139**	70035.2	566.354
soc-karate	**629**	**629**	**629**	**629**	0	**629**	**629**	0
soc-lastfm	N/A	N/A	48913	48386.2	240.7	**50933**	50792.6	557.926
soc-livejournal	N/A	N/A	356963	352603.3	480.09	**365429**	361325.4	430.674
soc-LiveMocha	N/A	N/A	34118	33748	290.217	**37293**	37004	587.94
soc-orkut	N/A	N/A	**141168**	139796	315.83	**141168**	139791.4	276.472
soc-pokec	N/A	N/A	75753	75368.7	450.124	**78463**	77846.2	511.556
soc-slashdot	N/A	N/A	37629	37239.4	244.211	**39668**	39407.4	510.47
soc-twitter-follows	N/A	N/A	22941	22864.7	361.006	**23372**	23302	255.026
soc-wiki-Vote	8107	23864	18798	18652.9	281.426	**19179**	19167.6	408.754
soc-youtube	N/A	N/A	45323	44814.5	188.897	**47810**	47551.4	532.986
soc-youtube-snap	N/A	N/A	46260	45668.3	253.685	**48296**	47918.2	573.03
tech-as-caida2007	N/A	N/A	22023	21851.5	215.712	**22955**	22857	312.65
tech-as-skitter	N/A	N/A	84773	82688.5	496.076	**88791**	88080.4	521.164
tech-internet-as	N/A	N/A	23450	23240.7	159.153	**24493**	24465.6	337.598
tech-p2p-gnutella	N/A	N/A	**22127**	22126.6	17.534	**22127**	**22127**	2.452
tech-RL-caida	N/A	N/A	40487	40165.7	232.533	**42744**	42533.8	505.192
tech-routers-rf	N/A	N/A	25414	25232.7	234.97	**25845**	25837.4	149.632
tech-WHOIS	N/A	N/A	60640	60273	256.301	**66069**	65989.2	471.62
web-arabic-2005	N/A	N/A	**348027**	347983.8	1.861	**348027**	**348027**	19.164
web-BerkStan	N/A	N/A	**32833**	32831.7	158.527	**32833**	**32833**	2.608
web-edu	N/A	N/A	25094	25076.2	219.301	**25114**	**25114**	0.376
web-google	-	-	**31277**	**31277**	47.631	**31277**	**31277**	0.084
web-indochina-2004	N/A	N/A	**120622**	120602.7	0.634	**120622**	**120622**	0.364
web-it-2004	N/A	N/A	**1314144**	**1314144**	25.787	**1314144**	**1314144**	23.966
web-polblogs	9606	21840	17417	17292.9	312.21	**17660**	17643.6	172.618
web-sk-2005	N/A	N/A	**155177**	155162.6	3.453	**155177**	**155177**	4.914
web-spam	N/A	N/A	40180	39836.4	247.928	**42564**	42383.4	316.524
web-uk-2005	N/A	N/A	**1277887**	**1277887**	1.808	**1277887**	**1277887**	2.582
web-webbase-2001	N/A	N/A	54026	53887	189.172	**54040**	**54040**	10.09
web-wikipedia2009	N/A	N/A	**86055**	86001	141.163	**86055**	86036.2	172.032

Table A6: Comparison of solution quality on real-world large graphs with $k = 40$.

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	14811	25659	21277	21159.4	260.986	**21427**	21422.4	224.26
bio-diseasome	13041	27569	**25175**	25171.3	191.286	**25175**	**25175**	87.348
bio-dmela	N/A	N/A	24628	24471.3	247.951	**25123**	25105.2	267.298
bio-yeast	N/A	N/A	**19980**	19976.8	95.747	**19980**	**19980**	0.1
ca-AstroPh	N/A	N/A	175325	174729	272.983	**176816**	**176816**	218.348
ca-citeseer	N/A	N/A	265104	264691.1	12.125	**265128**	**265128**	42.982
ca-coauthors-dblp	N/A	N/A	1152503	1152223	148.394	**1153307**	**1153307**	64.852
ca-CondMat	N/A	N/A	77062	76970.2	278.551	**77259**	**77259**	4
ca-CSphd	N/A	N/A	**19006**	**19006**	0.055	**19006**	**19006**	0.058
ca-dblp-2010	N/A	N/A	**220402**	220317.5	19.203	**220402**	**220402**	26.268
ca-dblp-2012	N/A	N/A	**192742**	192691.5	165.589	**192742**	192740.4	50.196
ca-Erdos992	N/A	N/A	22362	22179	229.123	**22621**	22550.2	140.588
ca-GrQc	N/A	N/A	**60986**	60979.2	39.632	**60986**	**60986**	0.124
ca-HepPh	N/A	N/A	179533	179301.1	274.322	**179704**	**179704**	2.338
ca-hollywood-2009	N/A	N/A	2164148	2161979	344.216	**2169116**	2167979	436.752
ca-MathSciNet	N/A	N/A	**87418**	87409.6	58.524	**87418**	**87418**	16.008
ca-netscience	14917	25103	**23697**	23695.8	160.899	**23697**	**23697**	0.082
ia-email-EU	N/A	N/A	33131	32888.2	337.461	**36627**	36400.2	504.676
ia-email-univ	-	-	30405	30163.6	103.367	**31066**	31050.6	395.366
ia-enron-large	N/A	N/A	65907	65439.1	382.204	**70023**	69782.8	547.32
ia-enron-only	9157	10428	10207	10162.7	210.451	**10338**	10311	82.072
ia-fb-messages	-	-	23267	23093.1	146.372	**23666**	23633.6	241.13
ia-infect-dublin	21426	34156	29911	29622.9	190.206	**30761**	30730.6	124.342
ia-infect-hyper	**6554**	**6554**	**6554**	**6554**	0	**6554**	**6554**	0
ia-reality	N/A	N/A	12861	12842.7	100.485	**12894**	12892	156.246
ia-wiki-Talk	N/A	N/A	45072	44699.3	266.007	**50050**	49688.6	552.678
inf-power	N/A	N/A	**25586**	25579.7	138.864	**25586**	**25586**	0.552
inf-roadNet-CA	N/A	N/A	**30398**	**30398**	2.277	**30398**	**30398**	2.168
inf-roadNet-PA	N/A	N/A	**29966**	**29966**	1.09	**29966**	**29966**	1.068
inf-road-usa	N/A	N/A	**30650**	30585.6	288.99	**30650**	30649.8	134.672
rec-amazon	N/A	N/A	**41152**	**41152**	2.347	**41152**	**41152**	1.626
rt-retweet	**4526**	4620	**4526**	**4526**	0	**4526**	**4526**	0
rt-retweet-crawl	N/A	N/A	38356	38188.3	151.032	**39262**	39200.8	532.96
rt-twitter-copen	17314	19812	17984	17939.7	246.817	**17998**	17989	206.426
sc-ldoor	N/A	N/A	202230	202171.9	158.898	**202251**	**202251**	37.096
sc-msdoor	N/A	N/A	200607	200513.7	205.267	**200765**	**200765**	21.74
sc-nasasrb	N/A	N/A	201040	200714	220.41	**201940**	201647.2	292.758
sc-pkustk11	N/A	N/A	220072	219540.9	177.563	**220324**	**220324**	43.474
sc-pkustk13	N/A	N/A	263746	263016.5	238.227	**267484**	**267484**	32.036
sc-pwtk	N/A	N/A	220304	220216.8	239.229	**220536**	220531.2	95.622
sc-shipsec1	N/A	N/A	144570	144172.2	183.521	**145341**	**145341**	32.852
sc-shipsec5	N/A	N/A	**191848**	191674	39.231	**191848**	**191848**	7.088
soc-BlogCatalog	N/A	N/A	63601	62615.2	243.916	**70642**	69668.6	573.022
soc-brightkite	N/A	N/A	62263	61747.6	306.293	**66325**	66205.8	545.202
soc-buzznet	N/A	N/A	62077	61477.2	334.557	**69088**	68309.2	546.702
soc-delicious	N/A	N/A	49253	48927.1	201.375	**52416**	52332.8	540.346
soc-digg	N/A	N/A	87283	85799.6	411.41	**92027**	90762.2	499.092
soc-dolphins	**2015**	**2015**	**2015**	**2015**	0	**2015**	**2015**	0
soc-douban	N/A	N/A	35411	35049.5	267.641	**36908**	36849	419.944
soc-epinions	N/A	N/A	47182	46672.8	268.237	**49855**	49622.2	478.624
socfb-A-anon	N/A	N/A	99795	99062.2	526.275	**104160**	103510.6	417.442
socfb-B-anon	N/A	N/A	97336	96725.1	470.755	**100916**	100408.6	410.344
socfb-Berkeley13	N/A	N/A	130233	128991.7	362.908	**145329**	144952.6	559.756
socfb-CMU	N/A	N/A	97086	96067.4	308.901	**111519**	111060.8	578.104

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Table A7 – continued from previous page

Instance	CPLEX		TOPKWCLQ			HEA-D		
	LB	UP	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	N/A	N/A	106515	105435.3	166.969	**123265**	122597.2	583.92
socfb-Indiana	N/A	N/A	150760	149759.6	273.878	**172222**	171774.2	587.448
socfb-MIT	N/A	N/A	100380	99438.5	226.717	**115271**	115027.6	539.728
socfb-OR	N/A	N/A	102266	101600	206.273	**113874**	113469	582.142
socfb-Penn94	N/A	N/A	138545	137025	246.735	**155112**	154536	574.066
socfb-Stanford3	N/A	N/A	125388	123998	213.711	**143516**	142497.8	594.92
socfb-Texas84	N/A	N/A	149260	147447.7	293.685	**166938**	165831.6	588.95
socfb-uci-uni	N/A	N/A	39180	38947.5	459.381	**39903**	39744	453.08
socfb-UCLA	N/A	N/A	136076	134339.1	304.762	**153515**	152530.6	589.122
socfb-UConn	N/A	N/A	117014	115386.6	336.784	**131021**	130587.4	585.356
socfb-UCSB37	N/A	N/A	124172	121216.7	294.994	**136277**	135631.4	585.358
socfb-UF	N/A	N/A	160471	156844.6	195.45	**180566**	178331.4	584.228
socfb-UIllinois	N/A	N/A	154314	152892.1	132.627	**173086**	172279.2	584.522
socfb-Wisconsin87	N/A	N/A	121805	120845	263.188	**136496**	135882.8	575.958
soc-flickr	N/A	N/A	92954	92055.8	337.054	**98758**	98421.8	553.978
soc-flixster	N/A	N/A	103929	102865.7	317.511	**110735**	109761.6	545.362
soc-FourSquare	N/A	N/A	48223	48053	381.055	**49026**	48828.6	484.132
soc-gowalla	N/A	N/A	79941	78546.1	278.674	**84483**	84348.2	554.672
soc-karate	**629**	**629**	**629**	**629**	0	**629**	**629**	0
soc-lastfm	N/A	N/A	59002	58558.9	232.192	**62139**	61472	527.558
soc-livejournal	N/A	N/A	413377	407286.2	524.225	**421326**	414355	487.162
soc-LiveMocha	N/A	N/A	41187	40718.2	269.24	**45683**	45239.2	575.358
soc-orkut	N/A	N/A	**172032**	171004.5	333.038	**172032**	171086.8	446.518
soc-pokec	N/A	N/A	92258	91793.1	476.993	**95899**	95383.6	542.81
soc-slashdot	N/A	N/A	44276	43853.9	275.335	**46980**	46678.8	473.352
soc-twitter-follows	N/A	N/A	28136	28051.7	256.186	**28792**	28720.8	229.022
soc-wiki-Vote	10097	28039	21996	21879.7	177.814	**22666**	22589.8	186.972
soc-youtube	N/A	N/A	54558	54264.5	170.117	**57997**	57719	547.828
soc-youtube-snap	N/A	N/A	55979	55505.6	235.688	**58512**	58142.8	545.096
tech-as-caida2007	N/A	N/A	26677	26487.1	404.648	**27578**	27523.4	392.594
tech-as-skitter	N/A	N/A	99537	97886.4	482.007	**105397**	103651.6	544.344
tech-internet-as	N/A	N/A	28207	28094.5	261.259	**29543**	29481.4	453.88
tech-p2p-gnutella	N/A	N/A	**27278**	27271.2	122.195	**27278**	**27278**	2.532
tech-RL-caida	N/A	N/A	48738	48501.8	336.885	**51603**	51478.6	504.144
tech-routers-rf	N/A	N/A	29737	29466.8	242.254	**30297**	30283.8	203.702
tech-WHOIS	N/A	N/A	67871	67180.2	262.593	**73568**	73415.2	538.83
web-arabic-2005	N/A	N/A	**430893**	430839	1.771	**430893**	**430893**	14.346
web-BerkStan	N/A	N/A	**39523**	39520.9	102.249	**39523**	**39523**	10.342
web-edu	N/A	N/A	29028	29017.7	236.265	**29056**	**29056**	0.546
web-google	-	-	**35204**	35192.6	206.659	**35204**	35200.8	157.632
web-indochina-2004	N/A	N/A	**139541**	139496.3	228.965	**139541**	**139541**	0.782
web-it-2004	N/A	N/A	**1502580**	**1502580**	32.053	**1502580**	**1502580**	26.086
web-polblogs	9793	25434	20428	20347.1	219.857	**20901**	**20901**	93.494
web-sk-2005	N/A	N/A	**181571**	181561.4	5.458	**181571**	**181571**	4.34
web-spam	N/A	N/A	47230	46794.1	180.398	**50071**	49957.2	411.774
web-uk-2005	N/A	N/A	**1497314**	**1497314**	3.577	**1497314**	**1497314**	3.668
web-webbase-2001	N/A	N/A	61219	61144.9	300.616	**61310**	61295	204.278
web-wikipedia2009	N/A	N/A	**100908**	100841.1	147.676	**100908**	100906.8	364.038

Table A7: Comparison of solution quality on real-world large graphs with $k = 50$.

Comparison	k	R^+_{best}	R^-_{best}	p-value	R^+_{avg}	R^-_{avg}	p-value
HEA-D versus CPLEX	10	89	0	2.59e-16	89	0	2.59e-16
	20	99	0	5.78e-18	99	0	5.78e-18
	30	98	0	8.46e-18	98	0	8.46e-18
	40	98	0	8.46e-18	98	0	8.46e-18
	50	98	0	8.46e-18	98	0	8.46e-18
HEA-D versus TOPKWCLQ	10	55	0	1.14e-10	67	1	1.59e-12
	20	63	0	5.29e-12	76	0	3.69e-14
	30	69	0	5.33e-13	82	0	3.73e-15
	40	74	0	7.89e-14	86	1	6.38e-16
	50	74	0	7.89e-14	92	0	8.28e-17

Table A8: Wilcoxon signed ranks test results of HEA-D and the reference algorithms in terms of both the better and the average solutions on 550 instances, with a significance level of 0.05.

4 Analysis

In this section, we perform additional experiments to analyse two key ingredients of the proposed algorithm HEA-D: the population-based memetic search framework and the simulated annealing based local optimization. The experiments were conducted on all large real-world graphs considered in this work.

Considering the methods used to solve diversified top-k cohesive subgraph problems for large-scale graphs, including diversified top-k clique [Yuan *et al.*, 2015; Yuan *et al.*, 2016; Wu *et al.*, 2020], diversified top-k weight clique [Wu and Yin, 2021b], and diversified top-k s-plex [Wu and Yin, 2021a], they are both heuristic or approximate algorithms. Therefore, HEA-D is the first evolutionary algorithm dedicated to the challenging of diversified top-k cohesive subgraph problems. To assess the effectiveness of the population-based memetic search framework and the simulated annealing based local optimization, we compared HEA-D with two HEA-D variants: HEA-D-Descent and HEA-D-Restart.

- **HEA-D-Descent**, in this variant, we replace the simulated annealing based local optimization procedure by a pure descent procedure and keep the other components unchanged. Specifically, the clique-based crossover operator is also retained such that HEA-D-Descent uses the same neighbourhood as the HEA-D does. But HEA-D-Descent always chooses the best neighbour solution from the search space each time, breaking the ties in favour of the random one.
- **HEA-D-Restart**, we disable the other components and keep only the simulated annealing based local optimization. For a fair comparison, this local optimization procedure starts with an initial population (set size of Pop to 1) and restarts this search process until the allowed cutoff time (600 seconds) is reached.

Accordingly, we can verify the importance of the simulated annealing based local optimization procedure and the population-based memetic framework by comparing HEA-D with HEA-D-Descent and HEA-D-Restart, respectively. For this experiment, the parameter k is set to 10 and the comparative results are reported in Table A10 with the same statistics as in the previous tables. As shown in Table A10 and Figure A1, we can make the following observations:

Comparison	R^+_{best}	R^-_{best}	p-value	R^+_{avg}	R^-_{avg}	p-value
HEA-D versus HEA-D-Descent	54	0	1.67e-10	64	0	3.61e-12
HEA-D versus HEA-D-Restart	81	0	5.46e-15	94	1	6.42e-17

Table A9: Wilcoxon signed ranks test results of HEA-D and its two variants HEA-D-Descent and HEA-D-Restart on 110 instances, with a significance level of 0.05.

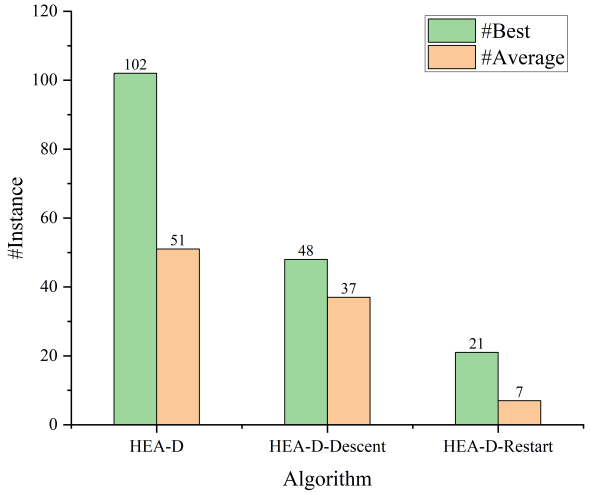

Figure A1: Number of the instances in terms of the best and average objective values for large real-world instances and $k = 10$.

1. HEA-D-Restart reports the worst results in terms of best and average objective values, which are significantly worse than the results of HEA-D, indicating that removing the memetic framework (set the population size to 1) drastically degrades the performance of the HEA-D and the memetic framework is one key components that ensuring HEA-D's high performance.
2. HEA-D-Descent reports better results than HDA-D-Restart, but its results are still significantly worse than the results of HEA-D, indicating that disabling the simulated annealing based local optimization procedure negatively impacts the performance of HEA-D, i.e., this local optimization procedure positively contributes to the high performance of the HEA-D.

In addition, the Wilcoxon signed ranks test results from Table A9 indicate that HEA-D performs better than HEA-D-Descent and HEA-D-Restart with a significance level of 0.05.

Instance	HEA-D			HEA-D-Descent			HEA-D-Restart		
	w_b	w_a	$time$	w_b	w_a	$time$	w_b	w_a	$time$
bio-celegans	**6867**	**6867**	0.178	**6867**	**6867**	214.863	6693	6452.8	0.062
bio-diseasome	**8672**	**8672**	0	**8672**	**8672**	0.001	**8672**	8535.6	0
bio-dmela	**6274**	**6274**	2.422	6242	6213.6	382.773	6221	6113.6	0.459
bio-yeast	**4991**	**4991**	0.008	**4991**	**4991**	0.013	**4991**	4980.4	0.001
ca-AstroPh	**48124**	**48124**	25.748	**48124**	**48124**	85.33	47997	47681.4	0.017
ca-citeseer	**76978**	**76978**	42.466	**76978**	**76978**	127.398	76412	76005	0.054
ca-coauthors-dblp	**306617**	**306617**	14.694	**306617**	**306617**	39.299	304138	303249	0.505
ca-CondMat	**20325**	**20325**	0.15	**20325**	**20325**	0.344	20249	19719.2	0.012
ca-CSphd	**4059**	**4059**	0	**4059**	**4059**	0.002	**4059**	**4059**	0.007
ca-dblp-2010	**65041**	**65041**	2.746	**65041**	**65041**	2.576	64645	63607.2	0.043
ca-dblp-2012	**64257**	**64257**	1.778	**64257**	**64257**	1.707	64198	62921.2	0.076
ca-Erdos992	**6048**	6037.6	224.546	**6048**	6036.6	455.677	5788	5729.8	0.28
ca-GrQc	**25844**	**25844**	0.042	**25844**	**25844**	0.102	25683	25161.2	0.001
ca-HepPh	**79624**	**79624**	0.256	**79624**	**79624**	7.472	78918	77853.8	0.006
ca-hollywood-2009	**934287**	934141.2	387.604	933304	932673.6	356.177	930374	913170.4	2.151
ca-MathSciNet	**24017**	**24017**	0.618	**24017**	**24017**	0.644	**24017**	23970	0.117
ca-netscience	**7588**	**7588**	0	**7588**	**7588**	0.004	**7588**	7456.8	0
ia-email-EU	**10762**	10709.4	292.5	10276	10071.8	304.461	10025	9829.6	44.043
ia-email-univ	**9570**	**9570**	12.526	**9570**	9568	149.846	9001	8838.4	0.003
ia-enron-large	**18142**	18006	490.156	16672	16642	255.464	17029	16779.2	207.854
ia-enron-only	**4331**	**4331**	0.692	**4331**	**4331**	41.575	4073	4007.4	0.002
ia-fb-messages	**5961**	**5961**	14.08	**5961**	5953.6	393.235	5904	5814	0.018
ia-infect-dublin	**10946**	**10946**	4.83	10935	10929.2	215.966	10429	10165.6	0.001
ia-infect-hyper	**4581**	4554.2	192.25	4381	4359.2	261.835	4010	3872.4	0.005
ia-reality	**3017**	**3017**	0.626	**3017**	3016.2	362.946	3007	2984.2	7.329
ia-wiki-Talk	**13550**	13520.2	474.074	11769	11641.6	296.895	13108	12784.8	260.391
inf-power	**6613**	**6613**	0.09	**6613**	**6613**	0.257	6575	6510.4	0.112
inf-roadNet-CA	**6515**	**6515**	1.472	**6515**	**6515**	1.532	6457	6448.6	366.444
inf-roadNet-PA	**6086**	**6086**	0.82	**6086**	**6086**	0.744	**6086**	**6086**	8.263
inf-road-usa	**6719**	**6719**	113.452	6695	6607.2	18.207	6462	6406.4	178.899
rec-amazon	**8931**	**8931**	0.446	**8931**	**8931**	0.484	8903	8903	0.028
rt-retweet	**1575**	**1575**	0	**1575**	**1575**	0.009	**1575**	1563.4	0
rt-retweet-crawl	**9728**	9673.8	259.284	9450	9370.2	218.575	9262	9198.2	170.665
rt-twitter-copen	**4661**	**4661**	0.016	**4661**	**4661**	0.031	**4661**	4409.8	0.001
sc-ldoor	**40747**	**40747**	24.278	**40747**	40744.2	140.44	**40747**	**40747**	109.235
sc-msdoor	**40691**	**40691**	11.072	**40691**	40681.2	118.894	**40691**	**40691**	10.974
sc-nasasrb	**43792**	**43792**	69.296	**43792**	43790.4	228.823	**43792**	43676.8	0.172
sc-pkustk11	**47548**	**47548**	2.802	**47548**	47538.4	221.126	47524	47368	0.8
sc-pkustk13	**57960**	**57960**	3.654	57915	57714.6	297.047	**57960**	57652.8	1.285
sc-pwtk	**45528**	**45528**	4.74	**45528**	**45528**	57.467	**45528**	**45528**	6.474
sc-shipsec1	**31900**	**31900**	4.286	**31900**	**31900**	145.703	31884	31799.6	0.425
sc-shipsec5	**43260**	**43260**	0.392	**43260**	**43260**	0.364	**43260**	43224	0.159
soc-BlogCatalog	**21317**	20751.2	501.56	19416	19061.6	166.402	20577	19945.4	370.122
soc-brightkite	**20323**	20110	431.52	19029	18837.4	464.457	17877	17057.4	5.77
soc-buzznet	**19585**	19130.2	539.012	17631	17490.6	319.051	18445	18092.4	460.54
soc-delicious	**13165**	13121.8	482.56	12655	12515	341.35	12616	12439.6	152.104
soc-digg	**28748**	28224.4	453.37	26551	26106.6	515.896	26282	25796	274.911
soc-dolphins	**1226**	**1226**	0	**1226**	**1226**	0.11	1202	1172.8	0
soc-douban	**9956**	**9956**	104.456	9638	9548.2	357.204	9178	9035.8	3.12
soc-epinions	**13359**	13356.6	259.346	12931	12860.6	290.462	12250	12092.2	58.794
socfb-A-anon	**24832**	24385.2	435.346	23389	23200.2	108.704	23856	23425.8	425.237
socfb-B-anon	**23176**	22867.4	583.964	22350	22077.8	209.627	22652	22495.2	491.249
socfb-Berkeley13	**39905**	39857.4	520.028	37098	36904.8	343.504	37828	37435.6	250.012
socfb-CMU	**34100**	34060.4	411.812	31761	31141.6	319.185	31916	30985.6	193.004

Continued on next page

Table A10 – continued from previous page

Instance	HEA-D			HEA-D-Descent			HEA-D-Restart		
	w_b	w_a	$time$	w_b	w_a	$time$	w_b	w_a	$time$
socfb-Duke14	**32359**	32187.8	401.866	28774	28676.2	295.856	30657	30536.6	138.929
socfb-Indiana	**45814**	45624.6	508.782	42221	41770.8	368.665	43750	43455.2	401.312
socfb-MIT	**32614**	32572.8	481.508	30221	30089.6	382.423	31229	31037	71.069
socfb-OR	**28839**	28765.8	514.692	26413	26051.8	294.819	26652	26308.8	73.466
socfb-Penn94	**39724**	39608.6	452.312	37213	37149.6	382.044	38096	37478.8	145.496
socfb-Stanford3	**40370**	40244.6	437.192	37127	36352.6	150.384	37611	37014	374.069
socfb-Texas84	**45051**	44801.8	526.558	40779	40053.8	331.965	40988	40596.4	304.9
socfb-uci-uni	**8859**	8784.4	337.472	8547	8481.2	49.554	8727	8577.6	185.028
socfb-UCLA	**44246**	44103	541.246	40538	39953.6	233.691	39998	39325.2	298.678
socfb-UConn	**39556**	39486.4	461.888	36472	36256.4	260.356	36106	34888.8	131.023
socfb-UCSB37	**44129**	44071.4	514.072	39693	39509.4	195.215	40113	38910.2	12.67
socfb-UF	**53584**	53461.6	556.5	48851	48522	247.53	52059	51712.6	406.251
socfb-UIllinois	**47611**	47336.6	556.782	43718	43321.4	314.636	43276	42331	462.064
socfb-Wisconsin87	**35245**	35119.6	493.238	31951	31678.8	132.102	32809	32509.2	173.597
soc-flickr	**26232**	25718.6	492.004	24071	23859	321.27	24935	24642.4	438.744
soc-flixster	**29064**	28436.8	477.002	26859	26357	264.604	27265	26987.2	367.032
soc-FourSquare	**12631**	12519.4	401.6	12422	12293.2	305.242	11902	11605.4	122.801
soc-gowalla	**20624**	20533.6	421.45	19855	19758.4	310.494	19959	19802.8	93.085
soc-karate	**472**	**472**	0	**472**	**472**	0.001	471	469.4	0
soc-lastfm	**15092**	14970.2	567.816	14498	14309.4	206.896	14642	14522	458.944
soc-livejournal	**139006**	137767	432.142	135209	133936.6	252.789	130688	126154.8	93.544
soc-LiveMocha	**11413**	11278.4	525.678	10265	10198.6	294.812	11195	11009	502.58
soc-orkut	**40071**	39672.4	353.496	**40071**	39672.4	335.918	39907	39901	397.552
soc-pokec	**23111**	22647.6	522.222	21571	21486	399.156	22525	22050.4	344.04
soc-slashdot	**14745**	14475.6	558.832	13612	13370.2	481.066	13343	13086.6	216.705
soc-twitter-follows	**6475**	6457.8	125.8	6416	6334.6	367.877	6251	6174.6	7.712
soc-wiki-Vote	**6432**	6431.8	276.836	**6432**	6431.2	142.255	6261	6214.4	0.02
soc-youtube	**14642**	14477	514.838	13481	13376.8	384.571	14109	13862.6	530.671
soc-youtube-snap	**14588**	14337	453.232	13657	13523.2	519.026	13983	13821.6	354.582
tech-as-caida2007	**8363**	8080.2	224.864	7428	7364.6	413.933	7192	7062.4	25.485
tech-as-skitter	**33713**	33289.4	524.008	31798	30878	320.406	32827	31270.4	253.99
tech-internet-as	**8556**	8283	397.372	7864	7728.2	366.087	7778	7502.4	79.364
tech-p2p-gnutella	**5883**	**5883**	0.604	**5883**	**5883**	0.595	**5883**	5863.2	0.195
tech-RL-caida	**13417**	13264.2	332.212	12748	12464.2	311.424	12293	12078.2	15.282
tech-routers-rf	**10094**	**10094**	0.71	10053	9965.6	264.429	9586	9301.2	0.056
tech-WHOIS	**32550**	32368.4	463.59	29431	29261.2	388.704	28926	27931.4	28.515
web-arabic-2005	**92108**	**92108**	0.142	**92108**	**92108**	0.13	**92108**	92069.8	0.115
web-BerkStan	**11200**	**11200**	0.02	**11200**	**11200**	0.02	11197	11059.6	0.008
web-edu	**11250**	**11250**	0	**11250**	**11250**	0.002	**11250**	**11250**	0.001
web-google	**13126**	**13126**	0	**13126**	**13126**	0.002	**13126**	13105.2	0
web-indochina-2004	**44052**	**44052**	0.1	**44052**	**44052**	0.132	44044	44005.2	0.004
web-it-2004	**415850**	**415850**	3.168	**415850**	**415850**	3.678	415814	415799.6	0.196
web-polblogs	**6034**	**6034**	30.418	**6034**	6025	322.205	5788	5675	0.02
web-sk-2005	**63958**	**63958**	2.426	**63958**	**63958**	6.511	63957	63677.4	0.03
web-spam	**15226**	15106.2	341.904	14405	14325	275.729	13923	13844.2	33.026
web-uk-2005	**441613**	**441613**	0.286	**441613**	**441613**	0.293	**441613**	**441613**	0.292
web-webbase-2001	**20648**	**20648**	0.138	**20648**	**20648**	0.169	**20648**	20511	0.005
web-wikipedia2009	**31340**	**31340**	4.014	**31340**	**31340**	3.209	**31340**	31255	0.564

Table A10: Comparison results of HEA-D and its two variants where the SALO is replaced by the descent (denoted by HEA-D-Descent) and a multi-start simulated annealing algorithm (denoted by HEA-D-Restart) on real-world large graphs with $k = 10$.

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