Incentives for Present-Biased Preferences

Kaidi Wu

Abstract

This paper studies the self-control problem caused by time inconsistent preference and investigates the existence of the incentive scheme that would achieve the efficient outcome for present-biased agents. For naive agents who have misbelief about their future preference, they intend to procrastinate current effort into the future to enjoy immediate gratification. For sophisticated agents who are aware of future time inconsistency, without commitment devices, they postpone costly effort into the future even more. In view of such problems, I find there exist optimal incentive schemes for benevolent principles to induce best performance of such agents. This could be an important implication for policy makers to create incentives for better performance and higher social welfare.

Keywords: Present bias; Self-control; Self-awareness; Incentive design

Contents

1 Introduction

Ambitious people have some goals to realize in the long future. However, from a student who wants to pass an exam to an athlete who wants to obtain the gold medal in the olympic game, realization of the goal needs a long time of diligence along the way. Ex-ante they all want to overcome the cost of efforts in every day to approach their final dreams. But if people have present bias, when they face the pain in front of them, will they still bear the planed cost of efforts. If not, what will happen? If there is inefficient outcome due to the present bias, can we design some incentive scheme to help them to achieve the goal?

In reality, we observe that coaches reward and punish athletes according to their performances in the training before the actual game. Teachers put positive weights of mid-term exam to the overall grades if the semester is very long. This paper tries to provide a justification of these phenomena.

In the standard paradigm of economic modeling of human preferences and behavior, the assumption of time consistent discounting is widely adopted. However, from a variety of behaviors documented by psychologists, such received economic paradigm are confronted with lots of challenges. Evidence shows that the time consistency assumption is only a rough approximation of time preference. In fact, people tend to have preference for immediate gratification against future satisfaction. For example, when faced with an unpleasant task, people would procrastinate to avoid immediate cost. When making up the mind to give up smoking, present-biased agents tend to put off the action to future periods to enjoy current consumption of cigarettes.

With the development of the exciting research field of psychological economics, economists have developed formal models of time inconsistent preferences over the past years. Robert Strotz (1955) proposed a typical discount function applied to a future utility depending on the time-distance from the present date rather than the calendar date at which it occurs to model inconsistency in dynamic utility maximization. Phelps and Pollak (1968) incorporated time inconsistent preference, which is called "imperfect altruism", into intergenerational utility maximization problem under Ramsey's settings. In their study, present bias or imperfect altruism caused the under-saving problem which lowers social welfare. Empirical findings of research on human behavior suggested discount functions are approximately hyperbolic (Ainslie, 1992).[1] Based on such findings, Laibson (1994) employed Phelps and Pollak's model to analyze self control problem in individuals' intertemporal choice, which formalized the time-inconsistent preferences.

By self cognition of the self control problem, two extreme cases of present-biased agents are characterized by *Naivete* and *Sophistication* respectively. Naive agents refer to those who are present-biased but are not aware of their time inconsistent preference in future periods. Sophisticated agents have perfect foresight of their future preferences and thus have perfect control of future behaviors.

Most early economic literatures assumed sophisticated beliefs in modeling time-inconsistent preference.[2] There are mainly two methods adopted to deal with the optimal intertemporal choice by sophisticated agents. Many economists use dynamic programming to de-

[1]Recently there are other empirical evidences that conflict with hyperbolic discounting. For example, Benhabib, Basin and Schotter (2010) carried out a lab experiment that provides little evidence for quasi-hyperbolic discounting and favors present bias in a form of "fixed cost". Anderson et al (2014) find no support of quasi-hyperbolic discounting or "fixed cost" discounting. However, in this paper we focus on the traditional well-accepted assumption of quasi-hyperbolic discounting.

[2]For instance, see Pollak (1968), Phelps and Pollak (1968) and Laibson (1994,1995,1997).

termine optimal consumption-saving path under infinite horizon present bias preference setting (Phelps and Pollak [1968] , etc). However, in the finite period consumption problem, it's standard practice to treat a T-period consumption decision as a series of temporal selves playing a dynamic game (Pollak [1968], Peleg and Yaari [1973], Goldman [1980] and Laibson [1997]). Other techniques applied in solving present bias problem in-clude a dual-self model that categorizing multiple selves into "myopic short-run self" and "patient long-run self" (Fudenberg and Levine, 2006) and more complicated method in-corporating graphic theorem in dealing with present bias in multi-stage tasks (Kleinberg, Oren and Raghavan, 2016). However in our simplified case dealing with finite-period continuous effort choice without intertemporal constraints, backward induction will be sufficient to solve the optima for sophisticated agents.

Although it's natural for economists to assume rational expectations, restricting attention to complete sophistication could be a methodological and empirical mistake even if people are mostly sophisticated because "any degree of naivete can yield different predictions than complete sophistication" (O'Donoghue and Rabin, 2000). Some other researchers places focus on naive belief[3] since naive behavior is far more tractable than sophisticated behavior (O'Donoghue and Rabin, 1999b). However, behavioral evidences indicate that people act partial sophistication (or partial naivete) where agents are partially aware of their changing tastes in many cases.[4]

No matter which type of agents analysed, present-biased preferences cause problems like over-consumption or under-investment on a macro level and lead agents to procrastinate completion of tasks with immediate costs and preproperate recreational activities with immediate rewards on a micro level (O'Donoghue and Rabin, 1999). The self-control problem thus lowers long-run welfare. Empirical literatures have documented evidences of the demand for incentives to mitigate self-control problem. Kaur, Kremer and Mullainathan (2015) analyse the importance of self-control at work with data from a field experiment and suggest high-powered incentives or effort monitoring may provide self-control benefits. In education, Paula and Scoppa (2015) find remedial program that induces students to provide effort on a regular basis is effective to overcome procrastination caused by self-control problems. Observing the problems present-bias would bring, e-conomists sought design of incentives to mitigate such problems. O'Donoghue and Rabin (2005) classified incentives into four categories – paternalistic incentives from benevolent third-parties, commitment devices from agents themselves, exploitation of interest from profit-seeking firms and inducement of efficient exchange from profit-seeking firms.[5] However, most of these study either focus on infinite horizon intertemporal consumption-saving decision or procrastination in one-activity case. In the first case, infinite horizon is a very strong assumption which cannot be applied to analyse present bias in short period context, which is a more realistic way of modeling human behavior[6]. The second case is limited, too. Though there is study analysing multiple activity choices, it can't

[3]For instance, see Akerlof (1991), Rabin (1999) and O'Donoghue and Rabin (2000).

[4]See experimental evidences in Ariely and Wertenbroch (2002) who find excessive preference for membership contracts in health clubs and DellaVigna and Malmendier (2004) proving positive effect of deadlines on homework grades and preference for deadlines. And also see a series of empirical work summarized in DellaVigna (2009).

[5]For the first type, see O'Donoghue and Rabin (2006). For the second, Augenblick et al (2015) validated the demand of binding commitment device through a longitudinal experiment. For the third type, see Gilpatric (2008), Heidhues and *Kõszegi* (2010).

[6]i.e. Infinite horizon doesn't fit the case where people are faced with tasks asked to be completed at a given date.

be generalized to investigate continuous case where agents can arbitrarily allocate time or energy in completing the tasks.

This paper contributes to existing literature in the following aspects. First, I analyse the case where agents are faced with tasks with fixed time-to-completion. The agents make continuous choices on efforts exerted at each period. This make sense since such setting applies to many realistic cases. For example, for students who have term papers to submit at certain date, they choose time spent on this paper everyday and manage to submit it by the deadline. For athletes who registered for important games, they choose everyday efforts exerted when preparing for the game. Second, I incorporate immediate incentive scheme from paternalistic principle to deal with self-control problem and check the existence of such incentive scheme, which provide reference for policy makers to design incentives for present-biased agents to reach the efficient outcome.

We start our analysis from the benchmark model where there is no time-consistent discounting ($\delta = 1$) for simplicity. In Section 2, we put forward basic assumptions about the benefit and cost functions and begin with a two-period model to get a brief understanding of the self-control problem. In Section 3, we extend the time period into more than 2, where naive agents and sophisticated agents will behave differently. We proved existence of effective incentive scheme in both cases and provide examples to get an intuitive understanding of how the incentive will solve the self-control problem. We further analyse a model with time-consistent impatience ($\delta < 1$) in Section 4 and prove the robustness of the main results obtained in Section 3. In section 5, we make comparison of sophistication and naivete and find the impact of time-consistent impatience and degree of present bias on the performance of present-biased agents. We draw the conclusion in Section 6 and discuss future research topics.

2 A Two-period Model

We start with a simple two-period model with $T = 2$ to compare the behaviors under time consistent and inconsistent preferences.

At $t = 1$, the agent A makes an effort e_1. $C(e_1)$ is A's cost of effort at this period.

At $t = 2$, A makes an effort e_2. The cost function $C(\cdot)$ is time invariant. $C(e_2)$ is A's cost of effort. The aggregate efforts create a benefit $B(e_1 + e_2)$ at $t = 2$.

We make the following assumptions:

A1. $B'(\cdot) > 0$, $B''(\cdot) < 0$ and $C''(\cdot) > 0$.

$B'(\cdot) > 0$ means the final benefit of A is higher when A chooses a higher aggregate effort. $B''(\cdot) < 0$ reflects the fact that the impact of the effort in the earlier stage is more significant. For instance, in many sport training situations, the speed of learning is very fast in the beginning, but it is difficult to improve his situation later on although the sporter spends the same effort every day. $C''(\cdot) > 0$ reflects the increasing marginal cost of the effort. But we don't assume $C'(\cdot) > 0$ since A might enjoy the procedure of exerting effort when e is not so large.

A2. $B'(e) \to \infty$ as $e \to 0$ and $C'(e) \to \infty$ as $e \to \infty$.

$B'(e) \to \infty$ as $e \to 0$ reflects that it is not wise for A to give up his goal where giving up means exerting no efforts. $C'(e) \to \infty$ as $e \to \infty$ says A suffers infinite pain when choosing the infinite level of efforts. These assumptions also guarantee the interior solutions of many problems.

2.1 Time Consistent Agents

If A's preference is time consistent, then at $t = 1$, A solves the following problem:

$$\text{Max}_{e_1,e_2}\ B(e_1 + e_2) - C(e_1) - C(e_2) \tag{1}$$

By assumptions, it is easy to obtain the unique optimal effort choice $e_1 = e_2 = e_*$ such that

$$B'(2e) = C'(e^*) \tag{2}$$

We call it 1st best outcome.

2.2 Time Inconsistent Agents

When $T = 2$ and A has present bias only at $t = 1$, there is no difference between sophisticated agents and naive agents. If A's preference is time inconsistent, then at $t = 1$, A solves the following problem:

$$\text{Max}_{e_1,e_2}\ \beta(B(e_1 + e_2) - C(e_2)) - C(e_1) \tag{3}$$

Assume that $0 < \beta < 1$. β being smaller than 1 represents A's present bias. It is easy to see the optimal effort choices are e_1^* and e_2^* such that

$$\beta B'(e_1^* + e_2^*) = C'(e_1^*) \tag{4}$$

$$B'(e_1^* + e_2^*) = C'(e_2^*) \tag{5}$$

Then we obtain the following propositions.

Proposition 1. *$e_1^* < e^* < e_2^*$.*

Proof. (4) and (5) imply $e_1^* < e_1^*$.

Suppose $e^* \leq e_1^*$, then $2e^* < e_1^* + e_2^*$. (2) and (5) imply $e^* > e_2^*$. Thus $e^* > e_1^*$, which is a contradiction. Thus we have $e^* > e_1^*$.

Suppose $e^* \geq e_2^*$, then $2e^* > e_1^* + e_2^*$. (2) and (5) imply $e^* < e_2^*$ which is a contradiction. Thus we have $e^* < e_2^*$.

Therefore $e_1^* < e^* < e_2^*$. ∎

Proposition 1 says that, compared with the 1st best effort levels, in period 1, A exerts less effort. But in period 2, A overworks.

Proposition 2. *$B(e_1^* + e_2^*) < B(2e^*)$.*

Proof. By Proposition 1, we have $e^* < e_2^*$. Thus (2) and (5) imply $B(e_1^* + e_2^*) < B(2e^*)$. ∎

Proposition 2 means if A has self-control problem, the explicit performance of A is worse than that of time consistent A.

2.3 Incentives for Time Inconsistent Agents

After knowing that A is worse off due to his self control problem, we now check if we could do something to help A to improve his situation. Suppose there is a benevolent P who designs a paternalistic incentive scheme to help A to approach the 1st best outcome.

If the effort of A is observable by P, P could achieve the 1st best outcome. P could, at the extremely high levels, punish A if $e_1 \neq e^*$ or rewarding if $e_1 = e^*$. Then the solution is trivial.

We assume that the effort of A is not observable by P, but at $t = 1$, P could observe a stochastic signal x which is correlated with e_1. Let $f(x, e_1)$ be the density function of x if the effort level at $t = 1$ is e_1. Based on the observed x, P gives A the reward or punishment $R(x)$. $R(\cdot)$ is in term of A's utility. The reward or punishment is not necessarily a monetary incentive. For example, the rewards could be medals, honors and so on.

Suppose the population of A is very large and P's budget for each A in term of A's utility is W. By law of large number, the incentive scheme is constrained by $E[R(x)] \leq W$.

In a nutshell, the timing is as following:

At $t = 1$, the agent A makes an effort e_1. e_1 produces a signal x at this period. P observes x and gives A the reward $R(x)$.

At $t = 2$, A makes an effort e_2. The aggregate benefit $B(e_1 + e_2)$ is produced at $t = 2$.

Suppose both P and A are risk neutral. Thus P solves the following problem:

$$\begin{aligned}
&\text{Max}_{e_1,e_2,R(\cdot)} B(e_1 + e_2) - C(e_1) - C(e_2) + E[R(x)] && (6)\\
&s.t. E[R(x)] \leq W && (7)\\
&e_1, e_2 \in \text{argmax } \beta(B(e_1 + e_2) - C(e_2)) + E[R(x)] - C(e_1) && (8)
\end{aligned}$$

(6) reflects P maximize the benefit net the cost of A plus the reward which A gets at $t = 1$. (7) is the budget constraint of P. (8) says the agent solves his problem with an additional reward-punishment acheme when A faces self control at $t = 1$.

It is equivalent to solving the following problem:

$$\begin{aligned}
&\text{Max}_{e_1,e_2,R(\cdot)} B(e_1 + e_2) - C(e_1) - C(e_2) && (9)\\
&s.t.\ E[R(x)] = W && (10)\\
&e_1, e_2 \in \text{argmax } \beta(B(e_1 + e_2) - C(e_2)) + E[R(x)] - C(e_1)
\end{aligned}$$

We know the optimal solution of (9) without any constraint is characterized by (2) in Section 2.1. Thus the only work left is to check if we could still realize it by designing $R(\cdot)$. Suppose $E[R(x)]$ is increasing and concave in e_1, then the 1st best outcome could be gained by a $R(\cdot)$ such that

$$\beta B'(2e^*) + \left.\frac{d\ E[R(x)]}{d\ e_1}\right|_{e_1=e^*} = C'(e^*) \tag{11}$$

Thus we obtain the following proposition:

Proposition 3. *The 1st best outcome $e_1 = e_2 = e^*$ could be gained by an increasing and concave function $E[R(x)]$ where (11) and (10) are satisfied.*

Example 1. *Let $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$ and $\beta = \frac{1}{2}$. P has $W = 0$. Suppose x is normally distributed with $f(x, e_1) = \frac{1}{\sqrt{\pi}} exp(-(x - e_1)^2)$. Thus the mean of x is e_1 and the variance of x is $\frac{1}{2}$.*

The optimal effort level in each period is $e^* = \frac{1}{\sqrt{2}} \approx 0.7$.

The real effort choices are $e_1^* \approx 0.4$ *and* $e_2^* \approx 0.8$.

Now we consider the following simple linear incentive scheme: $R(x) = ax + b$. *Thus* $E[R(x)] = ae_1 + b$ *is indeed increasing and concave in* e_1.

By (11), we have $a = \frac{E[R(x)]}{e_1}\Big|_{e_1 = e^*} = \frac{\sqrt{2}}{4}$.

By (10), we have $b = -\frac{1}{4}$.

Hence the optimal incentive scheme is $R(x) = \frac{\sqrt{2}}{4}x - \frac{1}{4}$.

3 A T-period Model

Now we consider a T-period model with $T > 2$.

At each period t, A makes an effort e_t. $C(e_t)$ is A's cost of effort at t. The aggregate efforts create a benefit $B(\sum_{t=1}^{T} e_t)$ at terminal period T. Assume also that $B'(\cdot) > 0$, $B''(\cdot) < 0$ and $C''(\cdot) > 0$.

3.1 Time Consistent Agents

If A's preference is time consistent, then at $t = 1$ A solves the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^{T}} \; B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t) \tag{12}$$

It is easy to see that the optimal effort choice is $e_t = e^*$ for all t such that

$$B'(Te^*) = C'(e^*) \tag{13}$$

3.2 Time Inconsistent Naive Agents

If A's preference is time inconsistent, then at $t = 1$, A has a plan to solve the following problem.

$$\text{Max}_{\{e_t\}_{t=1}^{T}} \; \beta(B(\sum_{t=1}^{T} e_t) - C(\sum_{t=2}^{T} e_t)) - C(e_1) \tag{14}$$

It is easy to see that the optimal plan is $e_1 = e_1^*$ and $e_t = e_1'$ for $t \neq 1$ such that

$$\beta B'(e_1^* + (T-1)e_1') = C'(e_1^*) \tag{15}$$
$$B'(e_1^* + (T-1)e_1') = C'(e_1') \tag{16}$$

Thus at $t = 1$, A makes a real effort at level e_1^* and plans to make equal effort e_1' in all future periods. The equal future efforts reflect that A is naive. e_1' is only an imaginary future effort level of A expected at $t = 1$. At $t = 2$, A faces the self control problem again. Given e_1^*, A solves the following problem.

$$\text{Max}_{\{e_t\}_{t=2}^{T}} \; \beta(B(e_1^* + \sum_{t=2}^{T} e_t) - C(\sum_{t=3}^{T} e_t)) - C(e_2) \tag{17}$$

It's easy to see that the optimal plan is $e_2 = e_2^*$ and $e_t = e_2'$ for $t > 2$ such that

$$\beta B'(e_1^* + e_2^* + (T-2)e_2') = C'(e_2^*) \tag{18}$$
$$B'(e_1^* + e_2^* + (T-2)e_2') = C'(e_2') \tag{19}$$

e_2^* is real and e_2' in future is imaginary.

In general, at $t = k$ wit $k = 2, 3, ..., T-1$, A solves the following problem:

$$\text{Max}_{\{e_t\}_{t=k}^T} \ \beta(B(\sum_{t=1}^{k-1} e_t^* + \sum_{t=k}^{T} e_t) - C(\sum_{t=k+1}^{T} e_t)) - C(e_k) \tag{20}$$

The optimal plan is $e_k = e_k^*$ and $e_t = e_k'$ for $t > k$ such that

$$\beta B'(\sum_{t=1}^{k} e_t^* + (T-k)e_k') = C'(e_k^*) \tag{21}$$

$$B'(\sum_{t=1}^{k} e_t^* + (T-k)e_k') = C'(e_k') \tag{22}$$

e_k^* at $t = k$ is real and e_k' in future is imaginary.

Note that, at $t = T-1$, $e_{T-1}' = e_T^*$. It reflects that the effort level at T which is imaginary at $T-1$ is nothing but the real effort level at T, since there is no self control problem at the terminal stage.

Formally, at $t = T-1$, the optimal plan is $e_{T-1} = e_{T-1}^*$ and $e_T = e_T'$ such that

$$\beta B'(\sum_{t=1}^{T-1} e_t^* + e_{T-1}') = C'(e_{T-1}^*) \tag{23}$$

$$B'(\sum_{t=1}^{T-1} e_t^* + e_{T-1}') = C'(e_{T-1}') \tag{24}$$

At $t = T$, the optimal effort is $e_T = e_T^*$ such that

$$B'(\sum_{t=1}^{T-1} e_t^* + e_T^*) = C'(e_T^*) \tag{25}$$

Obviously, the optimal effort at $t = T$ which is characterized by (25) is the same as e_{T-1}' which is characterized in (24).

Proposition 4. *e_t^* is strictly increasing in t.*

Proof. We firstly show that $e_1^* < e_2^*$.

(15) and (16) imply $e_1^* < e_1'$ $\quad(*)$. (18) and (19) imply $e_2^* < e_2'$ $\quad(**)$.

Suppose $e_1^* \geq e_2^*$, then (16) and (19) imply that if $e_1' > e_2'$ then $e_1^* + (T-1)e_1' > e_1^* + e_2^* + (T-2)e_2'$, then $e_1' < e_2'$ which is a contradiction with $e_1^* \geq e_2^*$.

Thus, there must be $e_1' \leq e_2'$. However, if equality holds, (16) and (19) imply $e_1^* + (T-1)e_1' = e_1^* + e_2^* + (T-2)e_2'$ which implies $e_2^* = e_1' = e_2'$ which contradicts with $(**)$.

Thus it must be that $e_1' < e_2'$. (16) and (19) imply $e_1^* + (T-1)e_1' > e_1^* + e_2^* + (T-2)e_2'$. Then (15) and (18) imply $e_1^* < e_2^*$ which contradicts with $e_1^* \geq e_2^*$.

Therefore, we have $e_1^* < e_2^*$.

By the same reasoning, we could show $e_t^* < e_{t+1}^*$ for all $t < T-1$.

For $t = T-1$, $e_{T-1}^* < e_T^*$ could be gained by (23), (24) and (25).

Thus e_t^* is strictly increasing in t. ∎

Proposition 4 says the agent will work harder and harder when moving to the terminal stage.

Proposition 5. *e'_t is strictly increasing in t.*

Proof. Since $e_1^* < e_2^*$ by Proposition 4, (15) and (18) imply $\beta B'(e_1^*+(T-1)e_1') < \beta B'(e_1^*+e_2^*+(T-2)e_1')$. Thus $B'(e_1^*+(T-1)e_1') < B'(e_1^*+e_2^*+(T-2)e_1')$ implies $e_1' < e_2'$.

By the same reasoning and Proposition 4, we could show $e'_t < e'_{t+1}$ for $\forall t$. Thus e'_t is strictly increasing in t. ■

Proposition 5 says the agent's belief of future effort level will be increasing when moving to the terminal stage.

Proposition 6. *$e'_t > e^*$ for all t.*

Proof. Suppose $e_1' \le e^*$, $(*)$ in the proof of Proposition 4 implies $e_1^* < e_1' \le e^*$. Then $e_1^* + (T-1)e_1' < Te^*$. Thus (16) and (13) imply $e_1' > e^*$ which contradicts with $e_1' \le e^*$.

Thus $e_1' > e^*$. By Proposition 5, we have $e'_t > e^*$ for all t. ■

In words, Proposition 6 says that the effort level the agent believes he will exert in future will be always higher than the most efficient level of effort e^*.

Proposition 7. $B(\sum_{t=1}^T e_t^*) < B(Te^*)$.

Proof. By Proposition 6, $e'_{T-1} > e^*$. Since $e'_{T-1} = e_T^*$, we have $e_T^* > e^*$. By (13) and (25), we have $B(\sum_{t=1}^T e_t^*) < B(Te^*)$. ■

Proposition 7 is an extension of Proposition 2. It says that if A has problem of self-control, the explicit performance of A is worse than time consistent A for all T.

3.3 Incentives for Time Inconsistent Naive Agents

In a T-period model, we have proved that present bias causes a decrease in the benefit of naive agents. Again we check if there still exists a incentive scheme for P to resume the 1st best outcome.

Following the assumptions in the two-period model, P solves the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(\cdot)} B(\sum_{t=1}^T e_t) - \sum_{t=2}^T C(e_t) + \sum_{t=2}^{T-1} E[R_t(x_t)] \tag{26}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] \le W \tag{27}$$

$$e_1 \in \text{argmax } \beta(B(\sum_{t=1}^T e_t) - \sum_{t=1}^T C(e_t) + \sum_{t=1}^{T-1} E[R_t(x_t)]) + E[R_1(x_1)] - C(e_1) \tag{28}$$

$$e_k \in \text{argmax } \beta(B(\sum_{t=1}^{k-1} e_t^* + \sum_{t=k}^T e_t) - \sum_{t=k+1}^T C(e_t) + \sum_{t=k+1}^{T-1} E[R_t(x_t)]) + E[R_k(x_k)] - C(e_k)$$
$$k = 2, 3, ..., T-2 \tag{29}$$

$$e_{T-1}, e_T \in \text{argmax } \beta(B(\sum_{t=1}^{T-2} e_t^* + e_{T-1} + e_T) - C(e_T)) + E[R_{T-1}(x_{T-1})] - C(e_{T-1}) \tag{30}$$

(26) says P maximizes A's net benefit plus total reward at the end of T. (27) is the budget constraint of P. (28)-(30) are the maximization problems the naive agent faces from period 1 to $T-1$ with the incentive schedule. Since there is no present bias at the last period, the agent chooses optimal effort level e^*_{T-1} and e^*_T both at $T-1$.

Obviously, budget constraint (27) must be binding because more reward is always better for A. Then it is equivalent to solving the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(.)} B(\sum_{t=1}^T e_t) - \sum_{t=1}^T C(e_t) \tag{31}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] = W \tag{32}$$

$$e_1 \in \text{argmax } \beta(B(\sum_{t=1}^T e_t) - \sum_{t=1}^T C(e_t) + \sum_{t=1}^{T-1} E[R_t(x_t)]) + E[R_1(x_1)] - C(e_1)$$

$$e_k \in \text{argmax } \beta(B(\sum_{t=1}^{k-1} e^*_t + \sum_{t=k}^T e_t) - \sum_{t=k+1}^T C(e_t) + \sum_{t=k+1}^{T-1} E[R_t(x_t)]) + E[R_k(x_k)] - C(e_k)$$

$$k = 2, 3, ..., T-2$$

$$e_{T-1}, e_T \in \text{argmax } \beta(B(\sum_{t=1}^{T-2} e^*_t + e_{T-1} + e_T) - C(e_T)) + E[R_{T-1}(x_{T-1})] - C(e_{T-1})$$

The optimal solution of (31) without constraint is characterized by (13) in Section 3.1. Then we check if there exist an incentive scheme $\{R_t(.)\}_{t=1}^{T-1}$ to reach the 1st best outcome. Suppose $E[R_t(.)]$ is increasing and concave in e_t, then the 1st best outcome could be gained by $\{R_t(.)\}_{t=1}^{T-1}$ such that

$$\beta B'(Te^*) + \left.\frac{dE[R_t(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) \qquad t = 1, 2, ..., T-1 \tag{33}$$

Then we obtain the following proposition:

Proposition 8. *The 1st best outcome $e_t = e^*$ ($t = 1, 2, ..., T$) could be obtained by a series of increasing and concave function $\{E[R_t(x)]\}_{t=1}^{T-1}$ where (32) and (33) are satisfied.*

Example 2. *Let $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$, $T = 4$, $\beta = \frac{1}{2}$ and $W = 0$. As in Example 1, assume x_t is normally distributed with $f(x_t, e_t) = \frac{1}{\sqrt{\pi}} exp(-(x_t - e_t)^2)$. Thus the mean of x_t equals e_t and the variance of x_t is $\frac{1}{2}$ for $t = 1, 2, ..., T-1$.*

The optimal effort level in each period is $e^* = 0.5$.

The real effort choice at each time period is $e^*_1 = 0.2673$ $e^*_2 = 0.2906$ $e^*_3 = 0.3257$ $e^*_4 = 0.6514$.

The imaginary future effort choice at each time period is $e_1 = 0.5345$ $e_2 = 0.5813$ $e_3 = 0.6514$.

Now consider the simple linear incentive scheme: $R_t(x) = a_t x + b_t$. Thus $E[R_t(x_t)] = a_t e_t + b_t$ is increasing and concave in e_t.

By (33), we have $a_t = \frac{dE[R_t(x_t)]}{de_t}|_{e_t=e^*} = C'(e^*) - \beta B'(Te^*) = 0.25$ for t=1,2,3.

By (32), $\sum_{t=1}^3 b_t = -\sum_{t=1}^3 a_t e^* = -0.375$. If the incentive scheme is time consistent, we have $b_t = -0.125$ for t=1,2,3.

Hence the time invariant optimal incentive scheme is $R(x) = 0.25x - 0.125$.

3.4 Time Inconsistent Sophisticated Agents

For sophisticated agents, preferences now over future behavior are different from the preferences they will have when the future arrives. Compared with naive agents who have misconception about their time preferences, sophisticated agents are aware of future self-control problems and can correctly predict how their future selves will behave. We start with the simple case where there is no time-consistent discounting for the sophisticated agent ($\delta = 1$).

Since sophisticated agents are aware of their changing preferences and can correctly predict how future selves will behave, they could do backward induction to determine the optimal effort path. Let U_t denote overall utility self-t receives at t and u_t denote instant utility self-t receives at t. Let h_t denote history from first period to period $t-1$.

At $t = T$, the agent maximizes his overall utility given choices made in previous periods and there is no present-bias problem :

$$\text{Max}_{\{e_T|h_T\}} \quad U_T = u_T = B(\sum_{t=1}^{T} e_t) - C(e_T) \tag{34}$$

At $t = T-1$, the agent maximizes his overall utility discounted at $T-1$ and solves the following problem given efforts made in previous periods :

$$\begin{aligned} \text{Max}_{\{e_{T-1},e_T|h_{T-1}\}} \quad U_{T-1} &= u_{T-1} + \beta U_T \\ &= -C(e_{T-1}) + \beta B(\sum_{t=1}^{T} e_t) - \beta C(e_T) \end{aligned} \tag{35}$$

At $t = k$, the agent solves the following problem :

$$\begin{aligned} \text{Max}_{\{e_t|h_t\}_{t=k}^T} \quad U_k &= u_k + \beta U_{k-1} \\ &= \beta^{T-k}[B(\sum_{t=1}^{T} e_t) - C(e_T)] - \sum_{\tau=k}^{T-1} \beta^{\tau-k} C(e_\tau) \end{aligned} \tag{36}$$

At $t = 1$, the agent solves the following problem :

$$\begin{aligned} \text{Max}_{\{e_t\}_{t=1}^T} \quad U_1 &= u_1 + \beta U_2 \\ &= \beta^{T-1} B(\sum_{t=1}^{T} e_t) - \sum_{\tau=1}^{T} \beta^{\tau-1} C(e_\tau) \end{aligned} \tag{37}$$

Then the optimal effort plan $\{e_t\}_{t=1}^T$ must satisfy :

$$\beta^{T-k} B'(\sum_{t=1}^{T} e_t^\star) = C'(e_k^\star) \qquad \forall k = 1, 2, ..., T \tag{38}$$

Proposition 9. *$e_t^\star$ is strictly increasing in t.*

Proof. From the first order conditions, we have $C'(e_1^\star) < C'(e_2^\star) < ... < C'(e_T^\star)$. By monotonicity of $C'(\cdot)$, we have $e_1^\star < e_2^\star < ... < e_{T-1}^\star < e_T^\star$. ■

Proposition 9 says that the sophisticated agents will exert more and more effort when the deadline approaches to maximize their overall utility. We can view these agents as procrastinators in this sense.

Proposition 10. $e_T^\star > e^*$.

Proof. Suppose $e_T^\star \leq e^\star$, then $e_t^\star < e_T^\star \leq e^\star$ for $\forall\ t < T$. Then we must have $\sum_{t=1}^T e_t^\star < Te^\star$ and from diminishing marginal benefit, there is $B'(\sum_{t=1}^T e_t^\star) > B'(Te^\star)$. From (13) and (38) we have $C'(e_T^\star) > C'(e^*)$ implicating $e_T^\star > e^*$ which contradicts with the assumption. Hence, there must be $e_T^\star > e^*$. ∎

Proposition 11. $B(\sum_{t=1}^T e_t^\star) < B(Te^*)$.

Proof. Since $e_T^\star > e^*$, from increasing marginal cost, we have $C'(e_T^\star) > C'(e^*)$. Then from (13) and (38) we have $B'(\sum_{t=1}^T e_t^\star) > B'(Te^\star)$. From decreasing marginal benefit, we have $\sum_{t=1}^T e_t^\star < Te^\star$. Then $B(\sum_{t=1}^T e_t^\star) < B(Te^\star)$. ∎

Like the case of naive agents, the self control problem causes sophisticated agents to perform worse than the 1st best outcome.

3.5 Incentives for Time Inconsistent Sophisticated Agents

We know the present bias can lead to worse performance even to agents who are aware of the self control problem. In this section, we check if benevolent P can design an incentive scheme to help sophisticated agents reach the 1st best outcome.

Under the assumptions made in previous sections, P solves the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(\cdot)} B(\sum_{t=1}^T e_t) - \sum_{t=1}^T C(e_t) + \sum_{t=1}^{T-1} E[R_t(x_t)] \tag{39}$$

$$\text{s.t.} \sum_{t=1}^{T-1} E[R_t(x_t)] \leq W \tag{40}$$

$$\{e_t\}_{t=1}^T \in \text{argmax}\ \beta^{T-1}[B(\sum_{t=1}^T e_t) - C(e_T)] + \sum_{\tau=1}^{T-1} \beta^{\tau-1}[E[R_\tau(x_\tau)] - C(e_\tau)] \tag{41}$$

(39) and (40) state the maximization problem the principle faces under the budget constraint of total reward. (41) represents the backward induction the agent's multiple selves play under the incentive scheme. The budget constraint must be binding since more reward always add to total payoff A gets. Then the problem is equivalent to:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(\cdot)}\ B(\sum_{t=1}^T e_t) - \sum_{t=1}^T C(e_t) \tag{42}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] = W \tag{43}$$

$$\{e_t\}_{t=1}^T \in \text{argmax}\ \beta^{T-1}[B(\sum_{t=1}^T e_t) - C(e_T)] + \sum_{\tau=1}^{T-1} \beta^{\tau-1}[E[R_\tau(x_\tau)] - C(e_\tau)]$$

The optimal solution of (42) without constraint is characterized by (13) in Section 3.1. We check if we could design an incentive scheme $\{R_t(\cdot)\}_{t=1}^{T-1}$ to reach the 1st best outcome.

Suppose $E[R_t(\cdot)]$ is increasing and concave in e_t, then the 1st best outcome could be gained by $\{R_t(\cdot)\}_{t=1}^{T-1}$ such that

$$\beta^{T-t}B'(Te^*) + \left.\frac{dE[R_t(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) \tag{44}$$

Then we obtain the following proposition:

Proposition 12. *The 1st best outcome $e_t = e^*$ $(t = 1, 2, ..., T)$ could be obtained by a series of increasing and concave function $\{E[R_t(x)]\}_{t=1}^{T-1}$ where (43) and (44) are satisfied.*

Follow the settings of Example 2, we now calculate the optimal effort path for sophisticated agents without incentives and find the corresponding incentive scheme.

Example 3. *Let $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$, $T = 4$, $\beta = \frac{1}{2}$ and $W = 0$. As in Example 1, assume x_t is normally distributed with $f(x_t, e_t) = \frac{1}{\sqrt{\pi}}exp(-(x_t - e_t)^2)$. Thus the mean of x_t equals e_t and the variance of x_t is $\frac{1}{2}$ for $t = 1, 2, ..., T-1$.*

The optimal effort level in each period is $e^* = 0.5$.

The real effort choice at each time period is $e_1^* = 0.0913$ $e_2^* = 0.1826$ $e_3^* = 0.3651$ $e_4^* = 0.7303$.

Consider the simple linear incentive scheme: $R_t(x) = a_t x + b_t$. Thus $E[R_t(x_t)] = a_t e_t + b_t$ is increasing and concave in e_t.

By (44), we have $a_1 = 0.4375, a_2 = 0.375, a_3 = 0.25$.

By (54), $\sum_{t=1}^{3} b_t = -\sum_{t=1}^{3} a_t e^* = -0.53125$. In the case where efficient behavior will not be rewarded nor punished, we have $b_1 = -0.21875, b_2 = -0.1875, b_3 = -0.125$.

4 Incorporating Time Consistent Impatience

In this section, we incorporate time-consistent discount factor δ into the model to check the more general case where people have consistent impatience over future period utility.

4.1 Naive Agents

At t=1, A plans to solve the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^{T}} \beta(\delta^{T-1}B(\sum_{t=1}^{T} e_t) - \sum_{t=2}^{T} \delta^{t-1}C(e_t)) - C(e_1) \tag{45}$$

Due to the impatience over future periods, there is no symmetric solution to anticipated future efforts e_1^t $t = 2, 3, ..., T$. The optimal real effort e_1^{**} and expected future efforts $\{e_1^t\}_{t=2}^{T}$ must satisfy the following first order conditions:

$$\beta\delta^{T-1}B'(e_1^{**} + \sum_{t=2}^{T} e_1^t) = C'(e_1^{**}) \tag{46}$$

$$\delta^T B'(e_1^{**} + \sum_{t=2}^{T} e_1^t) = \delta^t C'(e_1^t) \quad t = 2, 3, ..., T \tag{47}$$

By increasing marginal cost of effort and $0 < \delta < 1$, we can easily prove that the agent plans to make increasing effort over future periods. The increasing imaginary efforts reflect that the agent is sophisticated since he not only cares about present payoff but also is time-inconsistent among payoffs that occur at future periods.

At the beginning of t=2, A faces the self control problem again given realized effort level e_1^{**} and solves the following problem:

$$\text{Max}_{\{e_t\}_{t=2}^T} \beta(\delta^{T-2} B(e_1^{**} + \sum_{t=2}^{T} e_t) - \sum_{t=3}^{T} \delta^{t-2} C(e_t)) - C(e_2) \tag{48}$$

The optimal plan $\{e_2^{**}, e_2^3, e_2^4, ..., e_2^T\}$ must satisfy the following conditions:

$$\beta\delta^{T-2} B'(e_1^{**} + e_2^{**} + \sum_{t=3}^{T} e_2^t) = C'(e_2^{**}) \tag{49}$$

$$\delta^T B'(e_1^{**} + e_2^{**} + \sum_{t=3}^{T} e_2^t) = \delta^t C'(e_2^t) \quad t = 3, 4, ..., T \tag{50}$$

e_2^{**} is real effort made at $t = 2$ and $\{e_2^t\}_{t=3}^T$ is imaginary.

In general, at $t = k$ with $k = 2, 3, ..., T-1$, A solves the following problem:

$$\text{Max}_{\{e_t\}_{t=k}^T} \beta(\delta^{T-k} B(\sum_{t=1}^{k-1} e_t^{**} + \sum_{t=k}^{T} e_t) - \sum_{t=k+1}^{T} \delta^{t-k} C(e_t)) - C(e_k) \tag{51}$$

The optimal plan is $e_k = e_k^{**}$ and $e_t = e_k^t$ for $t > k$ such that:

$$\beta\delta^{T-k} B'(\sum_{t=1}^{k} e_t^{**} + \sum_{t=k+1}^{T} e_k^t) = C'(e_k^{**}) \tag{52}$$

$$\delta^T B'(\sum_{t=1}^{k} e_t^{**} + \sum_{t=k+1}^{T} e_k^t) = \delta^t C'(e_k^t) \quad t = k+1, k+2, ..., T \tag{53}$$

e_k^{**} is real effort made at $t = k$ and $\{e_k^t\}_{t=k+1}^T$ is imaginary.

Note that at $t = T-1$, there is no self control problem at the last period, so the imaginary future effort level at T is real effort made at T. Hence, $e_{T-1}^T = e_T^{**}$. We can formally justify the deduction following the steps below:

At $t = T-1$, the optimal plan is $e_{T-1} = e_{T-1}^{**}$ and $e_T = e_{T-1}^T$ such that:

$$\beta\delta B'(\sum_{t=1}^{T-1} e_t^{**} + e_{T-1}^T) = C'(e_{T-1}^{**}) \tag{54}$$

$$B'(\sum_{t=1}^{T-1} e_t^{**} + e_{T-1}^T) = C'(e_{T-1}^T) \tag{55}$$

At $t = T$, the optimal effort is $e_T = e_T^{**}$ such that

$$B'(\sum_{t=1}^{T-1} e_t^{**} + e_T^{**}) = C'(e_T^{**}) \quad (56)$$

Obviously, the optimal effort made at $t = T$ from (56) solves (55) as well.

Proposition 13. *e_t^{**} is strictly increasing in t.*

Proof. Follow the steps in the proof of Proposition 5, we first show that $e_1^{**} < e_2^{**}$.

From (46) and (47), we know at $t = 1$,

$$\beta\delta^{t-1}C'(e_1^t) = C'(e_1^{**}) \quad (57)$$

$$C'(e_1^t) = \delta C'(e_1^{t+1}) \quad (58)$$

Since $0 < \beta < 1$, $0 < \delta < 1$ and $C''(.) > 0$, we have

$$e_1^{**} < e_1^2 < ... < e_1^t < e_1^{t+1} < ... < e_1^T \quad (59)$$

From (49) and (50), we know at $t = 2$,

$$\beta\delta^{t-2}C'(e_2^t) = C'(e_2^{**}) \quad (60)$$

$$C'(e_2^t) = \delta C'(e_2^{t+1}) \quad (61)$$

Then we have

$$e_2^{**} < e_2^3 < ... < e_2^t < e_2^{t+1} < ... < e_2^T \quad (62)$$

Suppose $e_1^{**} \geq e_2^{**}$, then (47) and (50) imply if $e_1^3 \geq e_2^3$ then $e_1^t \geq e_2^t$ for $t \geq 3$.

By (36) and (39), $e_1^{**} + \sum_{t=2}^T e_1^t \leq e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t$. Then we have $e_1^2 \leq e_2^{**}$. With the assumption that $e_1^{**} \geq e_2^{**}$, we get $e_1^{**} \geq e_1^2$ which contradicts with (48).

Then it must be $e_1^3 < e_2^3$. By (58) and (61), $e_1^t < e_2^t$ for $t \geq 3$. (47) and (50) imply $e_1^{**} + \sum_{t=2}^T e_1^t > e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t$. Since $B'(\cdot) < 0$, then $B'(e_1^{**} + \sum_{t=2}^T e_1^t) < B'(e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t)$. By (48) and (51), $e_1^{**} < e_2^{**}$, which contradicts with $e_1^{**} \geq e_2^{**}$.

Hence, we have $e_1^{**} < e_2^{**}$.

Following the same reasoning, we can show that $e_t^{**} < e_{t+1}^{**}$ for $t = 1, 2, ..., T-2$.

For $t = T-1$, $e_{T-1}^{**} < e_T^{**}$ could be gained by (54) (55) and (56).

Thus, e_t^{**} is strictly increasing in t. ■

Proposition 13 indicates that the naive agents will exert more and more effort at each period as when there is time-consistent discounting over future periods.

Proposition 14. *$e_t^{t+1} > e_{t+1}^{**}$ for $t = 1, 2, ..., T-2$.*

Proof. We first show that $e_1^2 > e_2^{**}$.

Suppose $e_1^2 \leq e_2^{**}$, then $C'(e_1^2) \leq C'(e_2^{**})$. By (47) and (49), we have $B'(e_1^{**} + \sum_{t=2}^T e_1^t) \leq \beta B'(e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t)$. With $0 < \beta < 1$ and decreasing marginal benefit, there must be $B'(e_1^{**} + \sum_{t=2}^T e_1^t) < B'(e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t)(*)$ and $e_1^2 + \sum_{t=3}^T e_1^t > e_2^{**} + \sum_{t=3}^T e_2^t$, thus $\sum_{t=3}^T e_1^t > \sum_{t=3}^T e_2^t$.

Now we prove for each $t = 3, 4, ..., T-1$ there must be $e_1^t > e_2^t$. If there exist some k such that $e_1^k \leq e_2^k$, from (58) (61) and $C''(\cdot) > 0$, we have $e_1^n \leq e_2^n$ for all $n \neq k$. Then $\sum_{t=3}^T e_1^t \leq \sum_{t=3}^T e_2^t$ is the contradiction. Hence, we must have $e_1^t > e_2^t$ $(t = 3, 4, ..., T)$.

Then from (47) and (50), we have $B'(e_1^{**} + \sum_{t=2}^T e_1^t) > B'(e_1^{**} + e_2^{**} + \sum_{t=3}^T e_2^t)$ which contradicts with $(*)$. Therefore we have $e_1^2 > e_2^{**}$.

By the same reasoning, we could show $e_t^{t+1} > e_{t+1}^{**}$ for $t = 1, 2, ..., T-2$. At $t = T-1$, we have shown $e_{T-1}^T = e_T^{**}$. ■

Proposition 14 says that the real effort made at next period will be less than the planned level for sophisticated agents.

Proposition 15. *$B(\sum_{t=1}^{T} e_t^{**}) < B(\sum_{t=1}^{T} e_t^{*}) < B(Te^*)$.*

Proof. The second inequality has been proved in Proposition 8. The only work to do is to check if $\sum_{t=1}^{T} e_t^{**} < \sum_{t=1}^{T} e_t^{*}$ holds.

At t=1, assume $e_1^{**} \geq e_1^*$. Then $C'(e_1^{**}) \geq C'(e_1^*)$.

From (15) and (46), we have

$$B'(e_1^* + (T-1)e_1') = \beta^{-1}C'(e_1^*) \leq \beta^{-1}C'(e_1^{**}) = \delta^{T-1}B'(e_1^{**} + \sum_{t=2}^{T} e_1^t) < B'(e_1^{**} + \sum_{t=2}^{T} e_1^t)$$

Since $B''(.) < 0$, this implies $\sum_{t=2}^{T} e_1^t < (T-1)e_1' \quad (1')$.

From (16) and (57), we have

$$C'(e_1') = \beta^{-1}C'(e_1^*) \leq \beta^{-1}C'(e_1^{**}) = \delta^{t-1}C'(e_1^t) < C'(e_1^t) \quad t = 2, 3, ..., T$$

Since $C''(.) > 0$, we get $e_1^t > e_1'$ for each $t \geq 2$. Then $\sum_{t=2}^{T} e_1^t > (T-1)e_1'$, which contradicts with $(1')$. Hence, $e_1^{**} < e_1^*$.

At t=2, assume $e_1^{**} + e_2^{**} \geq e_1^* + e_2^*$. We have already shown $e_1^{**} < e_1^*$, then $e_2^{**} > e_2^*$ and $C'(e_2^{**}) > C'(e_2^*)$.

From (18) and (49), we have

$$B'(e_1^*+e_2^*+(T-2)e_2') = \beta^{-1}C'(e_2^*) < \beta^{-1}C'(e_2^{**}) = \delta^{T-2}B'(e_1^{**}+e_2^{**}+\sum_{t=3}^{T} e_2^t) < B'(e_1^{**}+e_2^{**}+\sum_{t=3}^{T} e_2^t)$$

Since $B''(.) < 0$, this implies $e_1^{**} + \sum_{t=3}^{T} e_2^t < e_1^* + (T-2)e_2' \quad (2')$.

From (19) and (49), we have

$$C'(e_2') = \beta^{-1}C'(e_2^*) < \beta^{-1}C'(e_2^{**}) = \delta^{t-2}C'(e_2^t) < C'(e_2^t) \quad t = 3, 4, ..., T$$

Since $C''(.) > 0$, we get $e_2^t > e_2'$ for each $t \geq 3$. Then $\sum_{t=3}^{T} e_2^t > (T-2)e_2'$, which contradicts with $(2')$. Hence, $e_1^{**} + e_2^{**} < e_1^* + e_2^*$.

Following the above reasoning, it can be shown that for $k = 1, 2, ..., T-1$, we have $\sum_{t=1}^{k} e_t^{**} < \sum_{t=1}^{k} e_t^{*}$.

At t=T, suppose $\sum_{t=1}^{T} e_t^{**} \geq \sum_{t=1}^{T} e_t^{*}$. Since $\sum_{t=1}^{T-1} e_t^{**} < \sum_{t=1}^{T-1} e_t^{*}$, then it must be $e_T^{**} > e_T^*$. Then by (25) and (56), we have

$$B'(\sum_{t=1}^{T} e_t^*) = C'(e_T^*) < C'(e_T^{**}) = B'(\sum_{t=1}^{T} e_t^{**})$$

Since $B''(.) < 0$, this implies $\sum_{t=1}^{T} e_t^{**} < \sum_{t=1}^{T} e_t^{*}$ which leads to a contradiction with the assumption.

Hence, for any t we always have $\sum_{t=1}^{T} e_t^{**} < \sum_{t=1}^{T} e_t^{*}$ and $B(\sum_{t=1}^{T} e_t^{**}) < B(\sum_{t=1}^{T} e_t^{*})$. ∎

From Proposition 15 we know that naive agents make the effort during the whole process and gain less benefit when they have time-consistent impatience. If a present-biased agent has time inconsistent preference, the explicit performance of A will be even worse for time-inconsistent naive agent.

4.2 Incentives for Naive Agents

We know the performance of sophisticated agents is the worst among three types of agent. Again we check if P can design a incentive scheme to help A achieve the 1st best outcome.

Under the settings of the two-period model, P solves the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(\cdot)} B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t) + \sum_{t=1}^{T-1} E[R_t(x_t)] \tag{63}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] \leq W \tag{64}$$

$$e_1 \in \text{argmax } \beta(\delta^{T-1} B(\sum_{t=1}^{T} e_t) - \sum_{t=2}^{T} \delta^{t-1} C(e_t) + \sum_{t=2}^{T-1} \delta^{t-1} E[R_t(x_t)]) + E[R_1(x_1)] - C(e_1) \tag{65}$$

$$\begin{aligned} e_k \in \text{argmax } & \beta(\delta^{T-k} B(\sum_{t=1}^{k-1} e_t^{**} + \sum_{t=k}^{T} e_t) - \sum_{t=k+1}^{T} \delta^{t-k} C(e_t) + \sum_{t=k+1}^{T-1} \delta^{t-k} E[R_t(x_t)]) \\ & + E[R_k(x_k)] - C(e_k) \quad k = 2, 3, ..., T-2 \end{aligned} \tag{66}$$

$$e_{T-1}, e_T \in \text{argmax } \beta(\delta B(\sum_{t=1}^{T-2} e_t^{**} + e_{T-1} + e_T) - \delta C(e_T)) + E[R_{T-1}(x_{T-1})] - C(e_T) \tag{67}$$

By (63), P maximize net benefit of A plus total reward A gets during the time periods. (64) represents P's budget constraint. (65)-(67) are the optimal choices, including real effort to make at current period and planned effort to make in future periods, the agent makes at each t with the incentive scheme.

Budget constraint (64) must be binding because more reward for A is always better. It's equivalent to solving the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^T, R(.)} B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t) \tag{68}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] = W \tag{69}$$

$$e_1 \in \text{argmax } \beta(\delta^{T-1} B(\sum_{t=1}^{T} e_t) - \sum_{t=2}^{T} \delta^{t-1} C(e_t) + \sum_{t=2}^{T-1} \delta^{t-1} E[R_t(x_t)]) + E[R_1(x_1)] - C(e_1)$$

$$\begin{aligned} e_k \in \text{argmax } & \beta(\delta^{T-k} B(\sum_{t=1}^{k-1} e_t^{**} + \sum_{t=k}^{T} e_t) - \sum_{t=k+1}^{T} \delta^{t-k} C(e_t) + \sum_{t=k+1}^{T-1} \delta^{t-k} E[R_t(x_t)]) \\ & + E[R_k(x_k)] - C(e_k) \quad k = 2, 3, ..., T-2 \end{aligned}$$

$$e_{T-1}, e_T \in \text{argmax } \beta(\delta B(\sum_{t=1}^{T-2} e_t^{**} + e_{T-1} + e_T) - \delta C(e_T)) + E[R_{T-1}(x_{T-1})] - C(e_T)$$

The solution to (68) without any constraint is characterized by (13) in Section 3.1. Suppose there exists an incentive scheme $\{R_t(\cdot)\}_{t=1}^{T-1}$ where $E[R_t(x)]$ is increasing and concave in e_t such that the 1st best outcome is reached. Then,

$$\beta\delta^{T-t}B'(Te^*) + \left.\frac{dE[R_t(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) \qquad t = 1, 2, ..., T-1 \tag{70}$$

Then we obtain the following proposition:

Proposition 16. *The 1st best outcome $e_t = e^*$ $(t = 1, 2, ..., T)$ could be obtained by a series of increasing and concave function $\{E[R_t(x)]\}_{t=1}^{T-1}$ where (69) and (70) are satisfied.*

Example 4. *Let $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$, $T = 4$, $\beta = \frac{1}{2}$, $\delta = 0.96$ and $W = 0$. As in Example 4, assume x_t is normally distributed with $f(x_t, e_t) = \frac{1}{\sqrt{\pi}}exp(-(x_t - e_t)^2)$. Thus the mean of x_t equals e_t and the variance of x_t is $\frac{1}{2}$ for $t = 1, 2, ..., T-1$.*

The optimal effort level in each period is $e^* = 0.5$.

The effort plan at t=1 is $e_1^{**} = 0.24264$, $e_1^2 = 0.50549$, $e_1^3 = 0.52655$, $e_1^4 = 0.54849$.

The effort plan at t=2 is $e_2^{**} = 0.26981$, $e_2^3 = 0.5621$, $e_2^4 = 0.58552$.

The effort plan at t=3 is $e_3^{**} = 0.31307$, $e_3^4 = e_4^{**} = 0.65222$.

The real effort choices at each period are $e_1^{**} = 0.24264$, $e_2^{**} = 0.26981$, $e_3^{**} = 0.31307$, $e_4^{**} = 0.65222$.

Now consider the simple linear incentive scheme: $R_t(x) = a_t x + b_t$. Thus $E[R_t(x_t)] = a_t e_t + b_t$ is increasing and concave in e_t.

From (70), we have $a_t = \frac{dE[R_t(x_t)]}{de_t}|_{e_t=e^*} = C'(e^*) - \beta\delta^{T-t}B'(Te^*)$. Then $a_1 = 0.7788$, $a_2 = 0.7696$, $a_3 = 0.76$.

From (69), $\sum_{t=1}^{3} b_t = -\sum_{t=1}^{3} a_t e^*$, i.e. $b_t = -a_t e^*$. Then we have $b_1 = -0.3894$, $b_2 = -0.3848$, $b_3 = -0.38$.

Hence the optimal incentive scheme is $\{R_1(x) = 0.7788x - 0.3894$, $R_2(x) = 0.7696x - 0.3848$, $R_3(x) = 0.76x - 0.38\}$.

4.3 Sophisticated Agents

In this section, we incorporate time-consistent discount factor δ into the model to see if time-consistent impatience would cause larger deviation from the 1st best result.

Since sophisticated agents are aware of their changing preferences and can correctly predict how future selves will behave, they could do backward induction to determine the optimal effort path. Let U_t denote overall utility self-t receives at t and u_t denote instant utility self-t receives at t.

At $t = T$, the agent maximizes his overall utility given choices made in previous periods and there is no present-bias problem :

$$\text{Max}_{\{e_T|h_T\}} \quad U_T = u_T = B(\sum_{t=1}^{T} e_t) - C(e_T) \tag{71}$$

At $t = T-1$, the agent maximizes his overall utility discounted at $T-1$ and solves the following problem given efforts made in previous periods :

$$\begin{aligned} \text{Max}_{\{e_{T-1},e_T|h_{T-1}\}} \quad U_{T-1} &= u_{T-1} + \beta\delta U_T \\ &= -C(e_{T-1}) + \beta\delta B(\sum_{t=1}^{T} e_t) - \beta\delta C(e_T) \end{aligned} \tag{72}$$

At $t = k$, the agent solves the following problem :

$$\begin{aligned}\text{Max}_{\{e_t|h_t\}_{t=k}^T} \quad U_k &= u_k + \beta\delta U_{k-1} \\ &= (\beta\delta)^{T-k}[B(\sum_{t=1}^{T} e_t) - C(e_T)] - \sum_{\tau=k}^{T-1}(\beta\delta)^{\tau-k}C(e_\tau)\end{aligned} \tag{73}$$

At $t = 1$, the agent solves the following problem :

$$\begin{aligned}\text{Max}_{\{e_t\}_{t=1}^T} \quad U_1 &= u_1 + \beta\delta U_2 \\ &= (\beta\delta)^{T-1}B(\sum_{t=1}^{T} e_t) - \sum_{\tau=1}^{T}(\beta\delta)^{\tau-1}C(e_\tau)\end{aligned} \tag{74}$$

Then the optimal effort plan $\{e_t\}_{t=1}^T$ should satisfy :

$$(\beta\delta)^{T-k}B'(\sum_{t=1}^{T} e_t^{\star\star}) = C'(e_k^{\star\star}) \qquad \forall k = 1, 2, ..., T \tag{75}$$

Proposition 17. *$e_t^{\star\star}$ is strictly increasing in t.*

Proof. From the first order conditions, we have $C'(e_1^{\star\star}) = (\beta\delta)C'(e_2^{\star\star}) = (\beta\delta)^2C'(e_3^{\star\star}) = ... = (\beta\delta)^{T-1}C'(e_T^{\star\star})$. Then $C'(e_1^{\star\star}) < C'(e_2^{\star\star}) < C'(e_3^{\star\star}) < ... < C'(e_T^{\star\star})$. By monotonicity of $C'(\cdot)$, we have $e_1^{\star\star} < e_2^{\star\star} < e_3^{\star\star} < ... < e_T^{\star\star}$. ■

Proposition 17 says that when there is time consistent impatience, the sophisticated agents will still exert more and more effort when the deadline approaches to maximize their overall utility.

Proposition 18. *$e_T^{\star\star} > e^*$.*

Proof. Suppose $e_T^{\star\star} \le e^*$, then $e_t^{\star\star} \le e^*$ for $\forall\ t$. Then we must have $\sum_{t=1}^T e_t^{\star\star} < Te^*$ and from diminishing marginal benefit, there is $B'(\sum_{t=1}^T e_t^{\star\star}) > B'(Te^*)$. From (13) and (75) we have $C'(e_T^{\star\star}) > C'(e^*)$, implicating $e_T^{\star\star} > e^*$, which contradicts with the assumption. Hence, there must be $e_T^{\star\star} > e^*$. ■

Proposition 19. $B(\sum_{t=1}^T e_t^{\star\star}) < B(\sum_{t=1}^T e_t^{\star}) < B(Te^*)$.

Proof. The second inequality has been proved in Prop.11 . The only work left to do is to check if $B(\sum_{t=1}^T e_t^{\star\star}) < B(\sum_{t=1}^T e_t^{\star})$ holds.

Suppose $\sum_{t=1}^T e_t^{\star\star} \ge \sum_{t=1}^T e_t^{\star}$, From diminishing marginal benefit, we have $B'(\sum_{t=1}^T e_t^{\star\star}) \le B'(\sum_{t=1}^T e_t^{\star})$. By (38) and (75), $C'(e_T^{\star\star}) = (\beta\delta)^{T-k}B'(\sum_{t=1}^T e_t^{\star\star} < \beta^{T-k}B'(\sum_{t=1}^T e_t^{\star\star} \le \beta^{T-k}B'(\sum_{t=1}^T e_t^{\star} = C'(e_T^{\star})$. So there is $e_k^{\star\star} < e_k^{\star}$ for $\forall\ k = 1, 2, ..., T$. Then, we have $\sum_{t=1}^T e_t^{\star\star} < \sum_{t=1}^T e_t^{\star}$, contradicting with the assumption.

Hence, we have $\sum_{t=1}^T e_t^{\star\star} < \sum_{t=1}^T e_t^{\star}$ and $B(\sum_{t=1}^T e_t^{\star\star}) < B(\sum_{t=1}^T e_t^{\star})$. ■

Like the case of naive agents, the self control problem causes sophisticated agents to perform worse than the 1st best outcome. What's more, Proposition 19 implies that the performance of sophisticated agents gets even worse when time-consistent impatience occurs.

4.4 Incentives for Sophisticated Agents

In this section, we check if benevolent P could design an incentive scheme to help sophisticated agents who have time-consistent impatience realize the 1st best outcome.

Under the assumptions made in previous sections with time-consistent discounting factor δ $(0 < \delta < 1)$, P solves the following problem:

$$\text{Max}_{\{e_t\}_{t=1}^{T}, R(\cdot)} B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t) + \sum_{t=1}^{T-1} E[R_t(x_t)] \tag{76}$$

$$s.t. \sum_{t=1}^{T-1} E[R_t(x_t)] \leq W \tag{77}$$

$$\{e_t\}_{t=1}^{T} \in \text{argmax } (\beta\delta)^{T-1}[B(\sum_{t=1}^{T} e_t) - C(e_T)] + \sum_{\tau=1}^{T-1} (\beta\delta)^{\tau-1}[E[R_\tau(x_\tau)] - C(e_\tau)] \tag{78}$$

(76) represents P's target of maximizing the agent's overall net benefit under the incentive schedule. (77) is budget constraint of total reward. (78) represents the intrapersonal game the agent's multiple selves play under the incentive scheme. The budget constraint must be binding since more reward always add to A's total payoff. The problem is equivalent to solving:

$$\text{Max}_{\{e_t\}_{t=1}^{T}, R(\cdot)} \ B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t) \tag{79}$$

$$s.t. \ \sum_{t=1}^{T-1} E[R_t(x_t)] = W \tag{80}$$

$$\{e_t\}_{t=1}^{T} \in \text{argmax } (\beta\delta)^{T-1}[B(\sum_{t=1}^{T} e_t) - C(e_T)] + \sum_{\tau=1}^{T-1} (\beta\delta)^{\tau-1}[E[R_\tau(x_\tau)] - C(e_\tau)]$$

The optimal solution of (79) without constraint is characterized by (13) in Section 3.1. We check if we could design an incentive scheme $\{R_t(\cdot)\}_{t=1}^{T-1}$ to reach the 1st best outcome. Suppose $E[R_t(\cdot)]$ is increasing and concave in e_t, then the 1st best outcome could be gained by $\{R_t(\cdot)\}_{t=1}^{T-1}$ such that

$$(\beta\delta)^{T-t} B'(Te^*) + \left.\frac{dE[R_t(x_t)]}{de_t}\right|_{e_t = e^*} = C'(e^*) \tag{81}$$

Then we obtain the following proposition:

Proposition 20. *The 1st best outcome $e_t = e^*$ $(t = 1, 2, ..., T)$ could be obtained by a series of increasing and concave function $\{E[R_t(x)]\}_{t=1}^{T-1}$ where (80) and (81) are satisfied.*

Follow the settings of Example 2, we now calculate the optimal effort path for sophisticated agents without incentives and find the corresponding incentive scheme.

Example 5. *Let $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$, $T = 4$, $\beta = \frac{1}{2}$, $\delta = 0.96$ and $W = 0$. As in Example 4, assume x_t is normally distributed with $f(x_t, e_t) = \frac{1}{\sqrt{\pi}} exp(-(x_t - e_t)^2)$. Thus the mean of x_t equals e_t and the variance of x_t is $\frac{1}{2}$ for $t = 1, 2, ..., T-1$.*

The optimal effort level in each period is $e^* = 0.5$.

The effort choice at each time period is $e_1^{\star\star} = 0.0580$ $e_2^{\star\star} = 0.1207$ $e_3^{\star\star} = 0.2515$ $e_4^{\star\star} = 0.5240$.

Consider the simple linear incentive scheme: $R_t(x) = a_t x + b_t$. *Thus* $E[R_t(x_t)] = a_t e_t + b_t$ *is increasing and concave in* e_t.

We have $a_t = C'(e^*) - (\beta\delta)^{T-t} B'(Te^*)$. *Then,* $a_1 = 0.9447$, $a_2 = 0.8848$, $a_3 = 0.76$. *When the scheme is designed so that efficient effort won't be punished, we have* $b_t = -a_t e^*$. *Then,* $b_1 = -0.4724$, $b_2 = -0.4424$, $b_3 = -0.38$. *So the optimal incentive scheme is* $\{R_1(x) = 0.9447x - 0.4724,\ R_2(x) = 0.8848x - 0.4424,\ R_3(x) = 0.76x - 0.38\}$.

5 Discussion

5.1 Naive Agents vs. Sophisticated Agents

In previous sections we have analysed the impact of the self-control problem on agent's performance, now we focus on time inconsistent agents to see if self-awareness exacerbate the performance or welfare of agents with present-bias.

5.1.1 Explicit Performance

Follow the settings of previous examples, we graph the optimal effort path of naive agents and sophisticated agents to get a straight forward understanding of the difference between these two types.

Graphs are depicted in Appendix A.1. From Fig.1, we know both naive agents and sophisticated agents will work harder and harder to the terminal stage. Without self awareness of present bias in future periods, the sophisticated agents would postpone their current effort into the future more than naive agents and hence making a more serious procrastinator. In other words, sophistication improves the agents' propensity to procrastinate and aggravates the self control problem. Intuitively, we expect worse performance of sophisticated agents against naive agents. Below we formally justify this assumption.

For clarity, we denote real effort made by naive agents in each period by e_t^n and optimal effort made by sophisticated agents in each period by e_t^s.

Proposition 21. $B(\sum_{t=1}^T e_t^s) < B(\sum_{t=1}^T e_t^n)$ *when* β *is small enough.*

Proof. In the baseline model where $\delta = 1$, e_t^n and e_t^s are determined by (21)-(25) and (38).

Suppose $\sum_{t=1}^T e_t^s \geq \sum_{t=1}^T e_t^n$, then $B'(\sum_{t=1}^T e_t^s) \leq B'(\sum_{t=1}^T e_t^n)$. From (25) and (38), we have $C'(e_T^s) \leq C'(e_T^n)$ and $e_T^s \leq e_T^n$. Then, we must have $\sum_{t=1}^{T-1} e_t^s \geq \sum_{t=1}^{T-1} e_t^n$ $(\star)$.

For naive agents, expected effort at $T-1$ equals real effort made at terminal stage, that is $e_{T-1}^T = e_T^n$. Then we have $\sum_{t=1}^T e_t^s \geq \sum_{t=1}^{T-1} e_t^n + e_{T-1}^T$. From equation (23) and (38), $C'(e_t^s) = \beta B'(\sum_{t=1}^T e_t^s) < \beta B'(\sum_{t=1}^{T-1} e_t^n + e_{T-1}^T) = C'(e_{T-1}^n)$. Then we get $e_{T-1}^s \leq e_{T-1}^n$. In view of $(\star)$, there must be $\sum_{t=1}^{T-2} e_t^s \geq \sum_{t=1}^{T-2} e_t^n$ $(\star\star)$.

At $T-2$, with β small enough such that $\beta B'(\sum_{t=1}^T e_t^s) < B'(\sum_{t=1}^{T-2} e_t^n + e_{T-2}^{T-1} + e_{T-2}^T)$, we have $C'(e_{T-2}^s) < C'(e_{T-2}^n)$. Then, $e_{T-2}^s < e_{T-2}^n$. From $(\star\star)$, we have $\sum_{t=1}^{T-3} e_t^s > \sum_{t=1}^{T-3} e_t^n$.

Follow the above reasoning, we can easily show that $e_t^s < e_t^n$ for $\forall\ t < T-1$. Then we have $\sum_{t=1}^T e_t^s < \sum_{t=1}^T e_t^n$, which contradicts with the initial assumption.

Hence, there must be $\sum_{t=1}^T e_t^s < \sum_{t=1}^T e_t^n$. And the benefit of total effort $B(\sum_{t=1}^T e_t^n) < B(\sum_{t=1}^T e_t^s)$.

We can easily arrive at the same conclusion in the extended case with the presence of time consistent impatience. ■

Proposition 21 indicates that the performance of time inconsistent agent is even worse when he is aware of the self-control problem. The intuition is straight forward. When the agent knows that he will discount utility in distant future against utility in the near future in every period that follows, he will value costs and benefit that occurs in the future less and place greater importance on current gratification. By backward induction, he will exert less effort in every period. When the present bias is strong enough, the sophisticated agent will perform even worse than naive agents.

5.1.2 Incentive Schemes

So far we know sophisticated agents are confronted with greater decline in their benefit of performance due to serious present bias problem. Intuitively the principle should design a stronger incentive scheme to stimulate greater effort among these agents. In this section, we will provide justification for this speculation.

Proposition 22. *The incentive is stronger for sophisticated agents than naive agents to close the effort gap between present-biased agents and time consistent agents.*

Proof. Here, we directly prove the proposition in the extended model. The result holds taking $\delta = 1$ in the simplified case.

From (70) and (81) we have

$$\left.\frac{dE[R_t^n(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) - \beta\delta^{T-t}B'(Te^*) \tag{82}$$

$$\left.\frac{dE[R_t^s(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) - (\beta\delta)^{T-t}B'(Te^*) \tag{83}$$

Since $0 < \beta < 1$, we have $\frac{dE[R_t^n(x_t)]}{de_t} < \frac{dE[R_t^s(x_t)]}{de_t}$ at $e_t = e^*$.

By continuity of $R(\cdot)$, there exists a neighborhood of e^*, denoted as $N_\delta(e^*)$, such that the inequality holds in this interval. Then a small deviation from the first best level would lead to a larger change in incentives designed by the principle for sophisticated agents.

Especially in a linear case where $R_t(e) = a_t e + b_t$, we have $a_t^s > a_t^n$, indicating a stronger incentive scheme for sophisticated agents. ■

5.1.3 Social Welfare

So far we have shown that the incentive scheme can improve the explicit performance of present-biased agents. However, greater efforts not only bring about greater benefit of performance but also incur greater costs. Now we look into the problem whether the incentive scheme helps enhance social welfare, in other words, if it is worthy to design the incentive scheme to correct the deviation of effort level. Welfare comparisons for agents with present-biased preferences are inherently problematic since the individual's preferences differ at different points in time, raising the question of how to weight the utility of the different period selves relative to each other. The usual approach (advocated by O'Donoghue and Rabin, 1999a, 2001), which we adopt here, makes welfare comparisons from the perspective of the agent's prior or "long-run" perspective, reflecting the fact that present-biased preferences are seen as capturing a self-control problem that does not represent the individual's "true" preferences (this perspective is also taken less explicitly by Akerlof, 1991).

Proposition 23. *Define the welfare function* $W(e_1, e_2, ..., e_T) \triangleq B(\sum_{t=1}^{T} e_t) - \sum_{t=1}^{T} C(e_t)$*. Then* $W(e_1^s, e_2^s, ...e_T^s) < W(e_1^n, e_2^n, ..., e_T^n) < W(e^*, e^*, ..., e^*)$ *with* β *sufficiently small.*

Proof. Obviously, we have $W(e^*, e^*, ..., e^*)$ the largest among the three values since $\{e^*\}_{t=1}^{T}$ maximizes the welfare function $W(\cdot)$. Then we only need to compare the social welfare gained from naive agents and sophisticated agents without the incentive scheme.

With β sufficiently small such that $\beta^{T-k-1}B'(\sum_{t=1}^{T} e_t^s) < B'(\sum_{t=1}^{k} e_t^n + \sum_{t=k+1}^{T} e_k^t)$ for $k < T-1$, then we have $C'(e_t^s) < C'(e_t^n)$ and hence $e_t^s < e_t^n$ for $\forall t < T$. Take partial derivative of $W(\cdot)$ with respect to e_t, we have

$$\frac{\partial W}{\partial e_k} = B'(\sum_{t=1}^{T} e_t) - C'(e_k)$$
$$\frac{\partial^2 W}{\partial e_k^2} = B''(\sum_{t=1}^{T} e_t) - C''(e_k)$$

Since $B''(\cdot) < 0$, $C''(\cdot) > 0$ and $B'(Te^*) = C'(e^*)$, for present-biased agents, we have $\frac{\partial W}{\partial e_t} > 0$ for $t < T$. That is, total welfare is increasing in effort exerted in period t with all else equal when the effort level is below the efficient one. Then,

$$\begin{aligned}
&W(e_1^s, e_2^s, ..., e_T^s) - W(e_1^n, e_2^n, ..., e_T^n) && (84)\\
=&[B(\sum_{t=1}^{T} e_t^s) - \sum_{t=1}^{T} C(e_t^s)] - [B(\sum_{t=1}^{T} e_t^n) - \sum_{t=1}^{T} C(e_t^n)] && (85)\\
=&[B(\sum_{t=1}^{T} e_t^s) - B(e_1^n + e_2^s + e_3^s + ... + e_T^s)] - [C(e_1^s) - C(e_1^n)] && (86)\\
&+ [B(e_1^n + e_2^s + e_3^s + ... + e_T^s) - B(e_1^n + e_2^n + e_3^s + ... + e_T^s)] - [C(e_2^s) - C(e_2^n)] && (87)\\
&+ [B(e_1^n + e_2^n + e_3^s + ... + e_T^s) - B(e_1^n + e_2^n + e_3^n + ... + e_T^s)] - [C(e_3^s) - C(e_3^n)] && (88)\\
&+ ...\\
&+ [B(e_1^n + e_2^n + ... + e_{T-1}^n + e_T^s) - B(\sum_{t=1}^{T} e_t^n)] - [C(e_T^s) - C(e_T^n)] && (89)
\end{aligned}$$

For sophisticated agents, e_T^n maximizes $W(e_1^n, e_2^n, ..., e_T^n)$ given $\{e_t^n\}_{t=1}^{T-1}$ at the terminal stage. Then (89) is a negative term with (86)-(88) all negative. Then the first inequality holds. ∎

From Prop.23 we know present bias causes a decrease in social welfare and the situation is even worse when the agents are aware of their self-control problem. With the incentive scheme, individual's welfare is raised to the first best level. In an economy with time inconsistent naive or sophisticated agents, the incentive scheme will raise overall social welfare. However, when there are multiple types of agents with differential degree of present bias, more complicated design of incentives are required to solve the self control problem.

5.2 Partial Sophistication

A more realistic case is where people are aware of the self-control but do not have precise estimation of the degree of present bias. The real degree of present bias is β whereas the perceived degree of present bias is $\hat{\beta}$.

Follow the steps in analysing optimal choice of sophisticated agents, we can characterize the optimal effort path adopted by partial sophisticated agents.

In the case where agents underestimate their degree of present bias, i.e. $\hat{\beta} \in (\beta, 1)$, compared with sophisticated agents, we can expect (1) Better performance; (2) Weaker incentives; (3) Lower individual welfare and higher social welfare when $\hat{\beta}$ is sufficiently large. The reverse is true if the agents overestimate their degree of present bias, i.e. $\hat{\beta} \in (0, \beta)$.

The intuition is straightforward. Taking naivete and sophistication as two extreme cases of self awareness, partial sophistication is an intermediate case where agents are only partially aware of their present bias. When they underestimate the degree of present bias($\hat{\beta} \in (\beta, 1)$), they discount utility in distant future over near future at a discount factor larger than sophisticated agents($\beta_s = \beta$) but lower than naive agents($\beta_n = 1$) in each period. By continuity and monotonicity of benefit function and cost function, we can simply speculate that the performance and incentives for partial sophisticated agents lie between the two extreme cases. However, when the agents overestimate their degree of present bias($\hat{\beta} < \beta$), they assume a more serious present bias than intrinsic degree of present bias. Then the agents are supposed to perform worse than the sophisticated case. The proof is provided in Appendix A.2.

5.3 The Role of Time Consistent Impatience

In a general case where people have time consistent impatience over future payoff, the procrastinating problem would be more serious for present-biased agents. Fig. 2 shows optimal effort choice by naive agents and sophisticated agents with different degree of impatience.

From Fig. 2, with greater degree of time impatience (smaller δ), initial effort level would be lower since they attach less importance to future payoff. And the gap of overall performance between naive and sophisticated agents becomes larger.

When $\delta = 1$, the model is simplified to the case where there is no time consistent impatience. Sophisticated agents will exert equal effort till last but one period due to perfect foresight of their present-biased preference at each period and will choose the optimal end-of-period effort level without self-control to maximize their payoff. By contrast, the ignorance of future present-bias problem causes naive agents to adopt an increasing effort path and lowers their payoff.

Fig. 3 helps us analyse within-group difference brought by various value of δ. With larger degree of time consistent impatience, the variance of effort exerted across periods becomes bigger in both groups. This makes sense since the more impatient the agents are, the lower weights they attach to future cost. They intend to exert more effort in distant future at a lower price instead of making effort in the near future at a higher price.

Another point to which we should pay attention is that sophisticated agents are more sensitive to such change in time consistent impatience. Such difference should be due to their self awareness. With perfect foresight of future preference, effort plan is made simultaneously by multiple selves of each period rather than sequentially decided with deviation from expected preference in each period. In other words, sophisticated agents make the decision with complete information and can make more "rational" effort plan corresponding to different time consistent discounting factor.

6 Conclusion

Many cases show people have preference for immediate gratification and such time inconsistent taste may cause inefficient outcome. This paper investigates the problem of self control caused by present bias and check the existence of the incentive scheme that would achieve the first best outcome.

For agents with present bias, they intend to anticipate gratifications to the present and postpone costly effort to the future. For sophisticated agents who have perfect insight of their changing taste over future periods, they have larger propensity to procrastinate compared with naive agents who are unaware of their self-control problem. The present bias causes a deviation from the first-best effort level and leads to a decrease in agent's performance and total welfare. Hence, an incentive scheme is needed to close this gap. Under usual assumptions made for profit function and cost function, we have proved the existence of an optimal incentive scheme that would reach the 1st best outcome for present-biased agents.

There are a series of interesting topics worth investigation in the future. First, in reality, when the agents foresee their future self control problem, they will try to use commitment devices to overcome or attenuate procrastination. Then, allowing for self-commitment devices may enhance explicit performance and total welfare for sophisticated agents significantly. Second, in this paper we merely focus on an economy with only one type of agents. However, a more realistic case is where there are multiple types of agents in the economy. It is meaningful to see if there exist the optimal incentive scheme to maximize social welfare under specific distribution of agent types.

References

[1] Ainslie, George W. (1992). Picoeconomics: The Strategic Interaction of Successive Motivational States within the Person. Cambridge, UK: Cambridge University Press.

[2] Akerlof, G. A. (1991). “Procrastination and Obedience”. *American Economic Review*, 81(2), 1-19.

[3] Andersen, S., Harrison, G. W., Lau, M. I. and Rutstrom, E. E. (2014). “Discounting Behavior: A Reconsideration”. *European Economic Review*, 71, 15-33.

[4] Ariely, D. and K.Wertenbroch (2002). “Procrastination, Deadlines, and Performance: Self-Control by Precommitment”. *Psychological Science*, 13, 219-224.

[5] Augenblick, N., Niederle, M. and Sprenger, C. (2015). “Working over Time: Dynamic Inconsistency in Real Effort Tasks”. *Quarterly Journal of Economics*, 130(3), 1067-1115.

[6] Benhabib, J., Bisin, A. and Schotter, A. (2010). “Present-Bias, Quasi-Hyperbolic Discounting, and Fixed Costs”. *Games and Economic Behavior*, 69(2), 205-223.

[7] DellaVigna S. and U. Malmendier(2004). “Contract Design and Self-Control: Theory and Evidence”. *Quarterly Journal of Economics* 119, 353-402.

[8] DellaVigna, S. (2009). “Psychology and Economics: Evidence from the Field”. *Journal of Economic Literature*, 4(2), 315-372.

[9] Eliaz K. and R. Spiegler(2006). “Contracting with Diversly Naive Agents”. *Review of Economic Studies*, 73, 689-714.

[10] Fudenberg, D. and Levine, D. (2006). “A Dual-Self Model of Impulse Control”. *American Economic Review* 96(5): 1449-1476.

[11] Gilpatric, S. M. (2008). “Present-Biased Preferences, Self-Awareness and Shirking”. *Journal of Economic Behavior & Organization*, 67(3), 735-754.

[12] Heidhues, P. and $K\tilde{o}szegi, B.$ (2010). “Exploiting Naivete About Self-Control in the Credit Market”. *American Economic Review*, 100(5), 2279-2303.

[13] Kaur, S., Kremer, M. and Mullainathan, S. (2015). “Self-Control at Work”. *Journal of Political Economy*, 123(6), 1227-1277.

[14] Kleinberg, J., Oren, S. and Raghavan, M. (2016). “Planning Problems for Sophisticated Agents with Present Bias”. *ACM Conference on Economics and Computation* (pp.343-360). ACM.

[15] Laibson, D. (1997). “Golden Eggs and Hyperbolic Discounting”. *Quarterly Journal of Economics*, 62, 443-477.

[16] O’Donoghue, T. and Rabin, M. (1999a). “Doing It Now or Later”. *American Economic Review*, 89, 103-124.

[17] O’Donoghue, T. and Rabin, M. (1999b). “Incentives for Procrastinators”. *Quarterly Journal of Economics*, 114, 769-816.

[18] O'Donoghue, T. and Rabin, M. (2000). "Choice and Procrastination". *Quarterly Journal of Economics*, 116(1), 121-160.

[19] O'Donoghue, T. and Rabin, M. (2005). "Incentives and Self Control". in Advances in Economic Theory, Cambridge University Press.

[20] O'Donoghue, T. and Rabin, M. (2006). "Optimal Sin Taxes". *Journal of Public Economics*, 90(10–11), 1825-1849.

[21] Paola, M. D. and Scoppa, V. (2015). "Procrastination, Academic Success and the Effectiveness of a Remedial Program". *Journal of Economic Behavior & Organization*, 115, 217-236.

[22] Phelps, E. S. and Pollak, R. A. (1968). "On Second-Best National Saving and Game-Equilibrium Growth". *Review of Economic Studies*, 35(2), 185-199.

[23] Rabin, M. (1999). "Procrastination in Preparing For Retirement". *Behavioral Dimensions of Retirement Economics*. Washington D.C., 22(2005), 125–156.

[24] Rubinstein A. (2005). "Comments on Behavioral Economics". in Advances in Economic Theory, Cambridge University Press.

[25] Strotz R. (1955). "Myopia and Inconsistency in Dynamic Utility Maximization". *Review of Economic Studies* 23, 165-180.

[26] Sunstein, C. and R. H. Thaler(2003). "Libertarian Paternalism". *American Economic Review*, 93, 175-179.

A Appendix

A.1 Naivete vs Sophistication

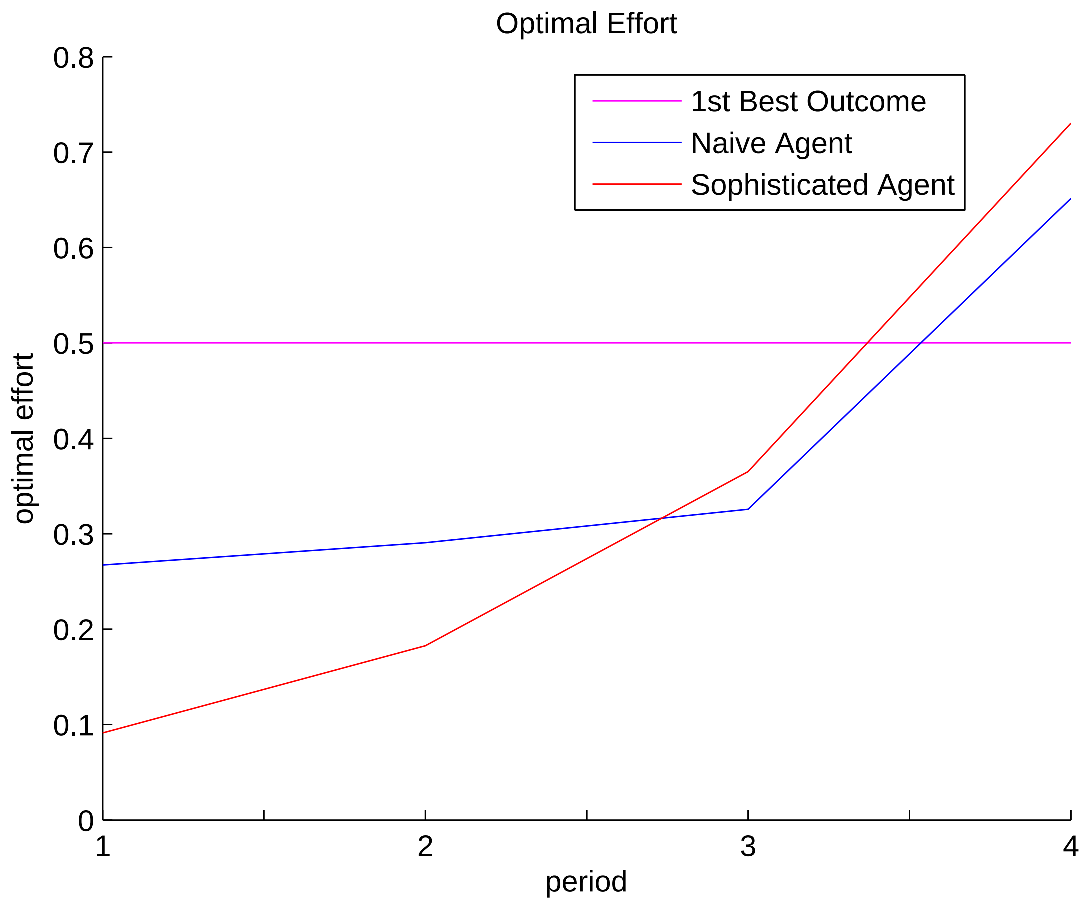

Figure 1: Optimal effort choice made by time-consistent agents, naive agents and sophisticated agents in a 4-period model. Here $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$, $\beta = \frac{1}{2}$ and $\delta = 1$.

A.2 Additional Proof

The results in Section 5.2 can be separated into the following propositions. Denote optimal effort level of sophisticated agents and partial sophisticated agents at period t as e_t^s and e_t^p respectively. We first prove the results for underestimation ($\hat{\beta} \in (\beta, 1)$).

Proposition.A1 $B(\sum_{t=1}^{T} e_t^p) > B(\sum_{t=1}^{T} e_t^s)$.

Proof. The decision rules of optimal effort are given by

$$\beta^{T-k} B'(\sum_{t=1}^{T} e_t^s) = C'(e_k^s) \tag{A.1}$$

$$\hat{\beta}^{T-k} B'(\sum_{t=1}^{T} e_t^p) = C'(e_k^p) \tag{A.2}$$

for sophisticated and partial sophisticated agents respectively when $k < T$.

Suppose $\sum_{t=1}^{T} e_t^p \leq \sum_{t=1}^{T} e_t^s$. Then we have $B'(\sum_{t=1}^{T} e_t^p) \geq B'(\sum_{t=1}^{T} e_t^s)$. At the terminal stage, $C'(e_T^p) = B'(\sum_{t=1}^{T} e_t^p) \geq B'(\sum_{t=1}^{T} e_t^s) = C'(e_T^s)$. Hence, $e_T^p \geq e_T^s$. Under underestimation, we have $\hat{\beta} > \beta$. Then $C'(e_k^p) > C'(e_k^s)$ and thus $e_k^p > e_k^s$ for $\forall k < T$. Hence the overall effort level $\sum_{t=1}^{T} e_t^p > \sum_{t=1}^{T} e_t^s$, which is a contradiction.

So there must be $\sum_{t=1}^{T} e_t^p > \sum_{t=1}^{T} e_t^s$ and $B(\sum_{t=1}^{T} e_t^p) > B(\sum_{t=1}^{T} e_t^s)$. ■

Proposition.A2 *Weaker incentive schemes are required for partial sophisticated agents who underestimate the degree of present bias.*

Proof. For sophisticated and partial sophisticated agents the incentives are determined by

$$\left.\frac{dE[R_t^p(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) - \hat{\beta}^{T-t}B'(Te^*) \tag{A.3}$$

$$\left.\frac{dE[R_t^s(x_t)]}{de_t}\right|_{e_t=e^*} = C'(e^*) - \beta^{T-t}B'(Te^*) \tag{A.4}$$

With $\hat{\beta} > \beta$, we have $\frac{dE[R_t^p(x_t)]}{de_t} < \frac{dE[R_t^s(x_t)]}{de_t}$ at e^*. By continuity of response function, there exist some neighborhood of e^*, denoted as $N_\delta(e^*)$, such that the inequality holds in this interval. Then a small disturbance from the social optimal level would lead to smaller change in reward or punishment for partial sophisticated agents compared with sophistication case. In other words, weaker incentive schemes are required for partial sophisticated agents who underestimate the degree of present bias. ■

Proposition.A3 *Social welfare generated by partial sophisticated agents are higher if $\hat{\beta}$ and β satisfy $\hat{\beta}B'(\sum_{t=1}^{T} e_t^p) > \beta B'(\sum_{t=1}^{T} e_t^s)$.*

Proof. When $\hat{\beta}B'(\sum_{t=1}^{T} e_t^p) > \beta B'(\sum_{t=1}^{T} e_t^s)$, since $\hat{\beta} > \beta$, we have $\hat{\beta}^{T-k}B'(\sum_{t=1}^{T} e_t^p) > \beta^{T-k}B'(\sum_{t=1}^{T} e_t^s)$ for $\forall k < T$. By (A.1) and (A.2), $C'(e_k^p) > C'(e_k^s)$ for $\forall k < T$. Hence we have $e_k^p > e_k^s$ for $\forall k < T - 1$.

$$W(e_1^p, e_2^p, ..., e_T^p) - W(e_1^s, e_2^s, ..., e_T^s) \tag{A.5}$$

$$=[B(\sum_{t=1}^{T} e_t^p) - \sum_{t=1}^{T} C(e_t^p)] - [B(\sum_{t=1}^{T} e_t^s) - \sum_{t=1}^{T} C(e_t^s)] \tag{A.6}$$

$$\begin{aligned}
=&[B(\sum_{t=1}^{T} e_t^p) - B(e_1^p + e_2^p + ... + e_{T-1}^p + e_T^s)] - [C(e_T^p) - C(e_T^s)] \\
&+ [B(e_1^p + e_2^p + ... + e_{T-1}^p + e_T^s) - B(e_1^p + e_2^p + ... + e_{T-1}^s + e_T^s)] - [C(e_{T-1}^p) - C(e_{T-1}^s)] \\
&+ ... \\
&+ [B(e_1^p + e_2^s + ... + e_{T-1}^s + e_T^s) - B(\sum_{t=1}^{T} e_t^s)] - [C(e_1^p) - C(e_1^s)]
\end{aligned} \tag{A.7}$$

The first term of (A.7) is positive because e_T^p maximizes the agent's total welfare at the terminal stage given effort made in previous periods. The last $T-1$ terms of equation(A.7) are positive since welfare function $W(\cdot)$ is increasing in each element. Hence, equation (A.5) is positive, which implies that total welfare of partial sophisticated agents are higher than sophistication case. ■

As for the overestimation case, the reverse of the above propositions is true. The proof is basically the same as above.

A.3 Effect of δ

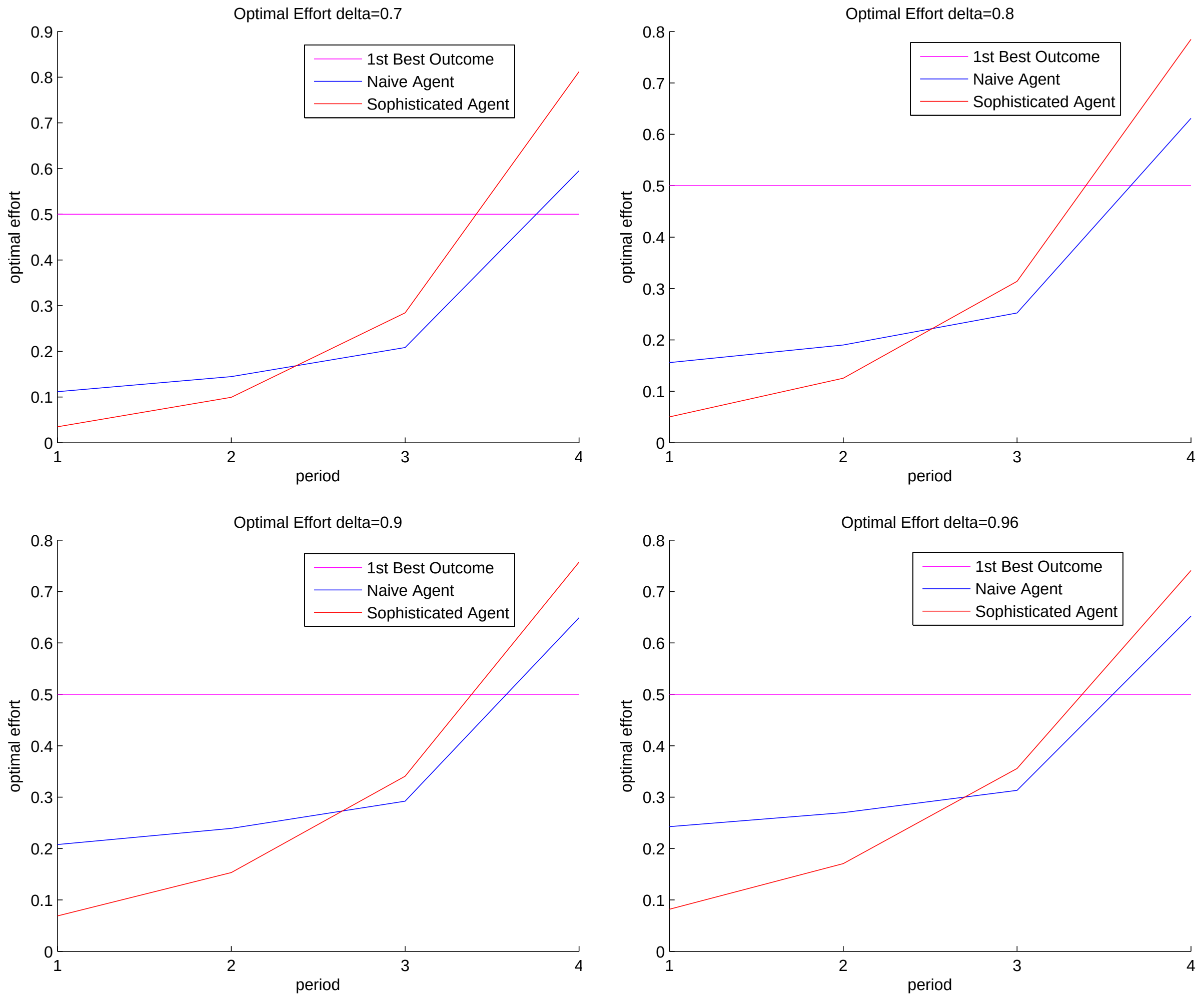

Figure 2: Optimal effort choices made by time-consistent agents, naive agents and sophisticated agents in a 4-period model with various degree of time impatience $\delta = 0.7$, $\delta = 0.8$, $\delta = 0.9$ and $\delta = 0.96$. Here $B(e) = ln(e)$, $C(e) = \frac{1}{2}e^2$ and $\beta = \frac{1}{2}$.

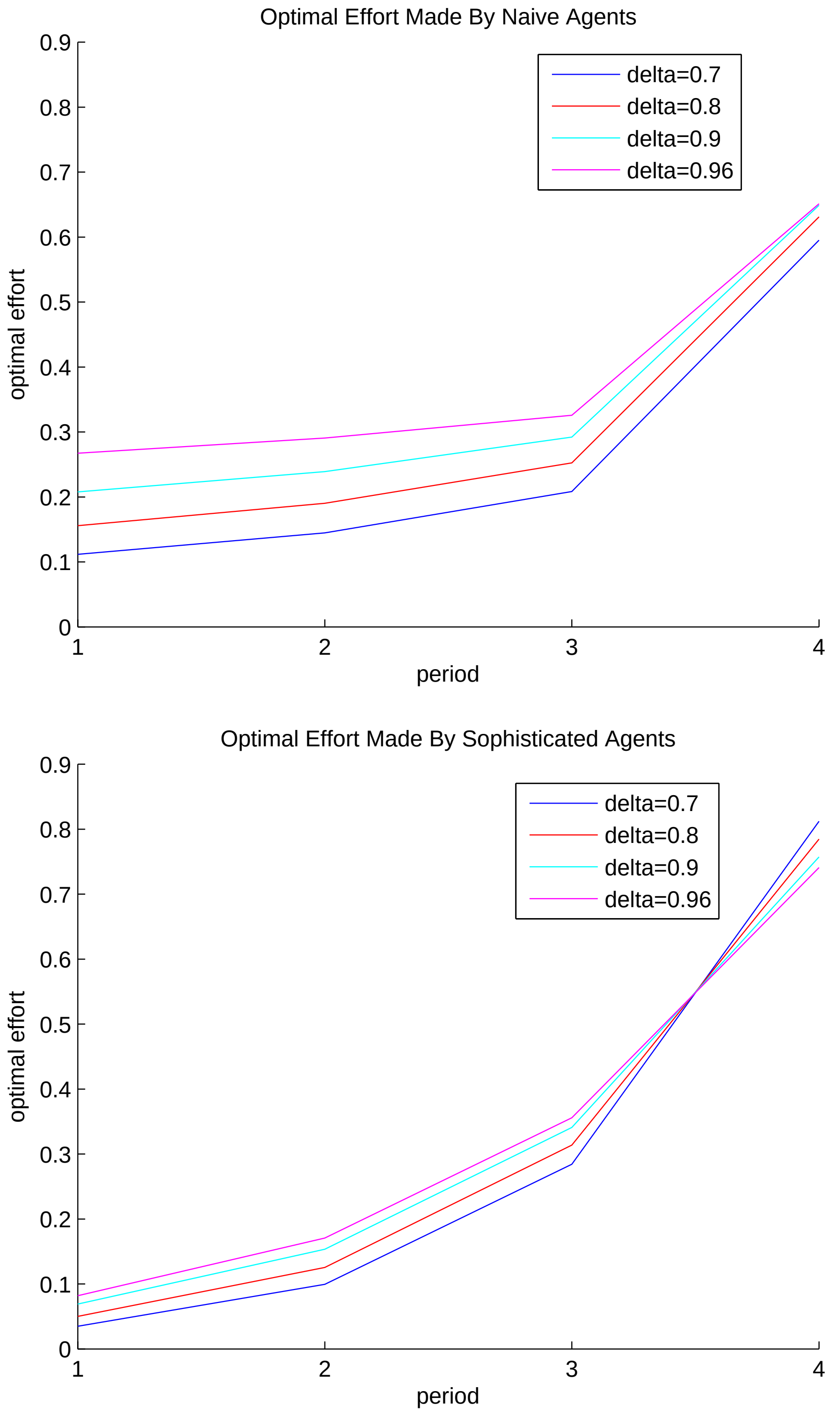

Figure 3: Within group effort change due to different level of time impatience.