Math 578 HW#4

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Problem 1. The entirety of this code is contained in the included single file `hw4.py`. In the main executing loop (`if __name__ == "__main__":`), various sections have been commented out. Uncomment as desired and run.

(a) Cholesky decomposition is performed by the function `hw4.cholesky`. The particular implementation handles sparse matrices as well, and most of the code is simply dedicated to choosing the right method based on the data object provided:

```
def cholesky(A):
    """
    computes the cholesky decomposition for symmetric, positive definite
    matrices. returns a lower-triangular matril L with positive diagonal
    entries so that A=LL^T.
    also returns an integer nzl that gives the number of
    nonzero entries in the Cholesky factor L.

    INPUT:

    A - a positive definite matrix nxn
        (may be np.array or scipy.sparse.spmatrix)

    OUTPUT:

    L - the cholesky factor L s.t. A = LL^T
    nzl - number of nonzero entries in L i.e. where |L_{ij}| > 0

    """

    if sparse.issparse(A):
        G = sparse.tril(A)
        #a sparse matrix that still allows elementwise access
        G = G.tocsc()
    else:
        G = np.tril(A)

    n = A.shape[0]

    for k in range(n):
        G[k:,k] -= G[k:,:k] @ G[k,:k].T
        G[k:,k] /= np.sqrt(G[k,k])

    if sparse.issparse(G):
        nzl = G.count_nonzero()
    else:
        nzl = np.count_nonzero(G)

    return G, nzl
```

The following suggested sanity check was performed, which yielded the desired result:

```
In [1]: cholesky(np.array([[2,1,0],[1,2,1],[0,1,2]],dtype='f'))
Out[1]:
(array([[ 1.41421354,  0.        ,  0.        ],
        [ 0.70710677,  1.2247448 ,  0.        ],
        [ 0.        ,  0.81649655,  1.15470064]], dtype=float32), 5)
```

(b) Output of spy(L) for $n = 2000$:

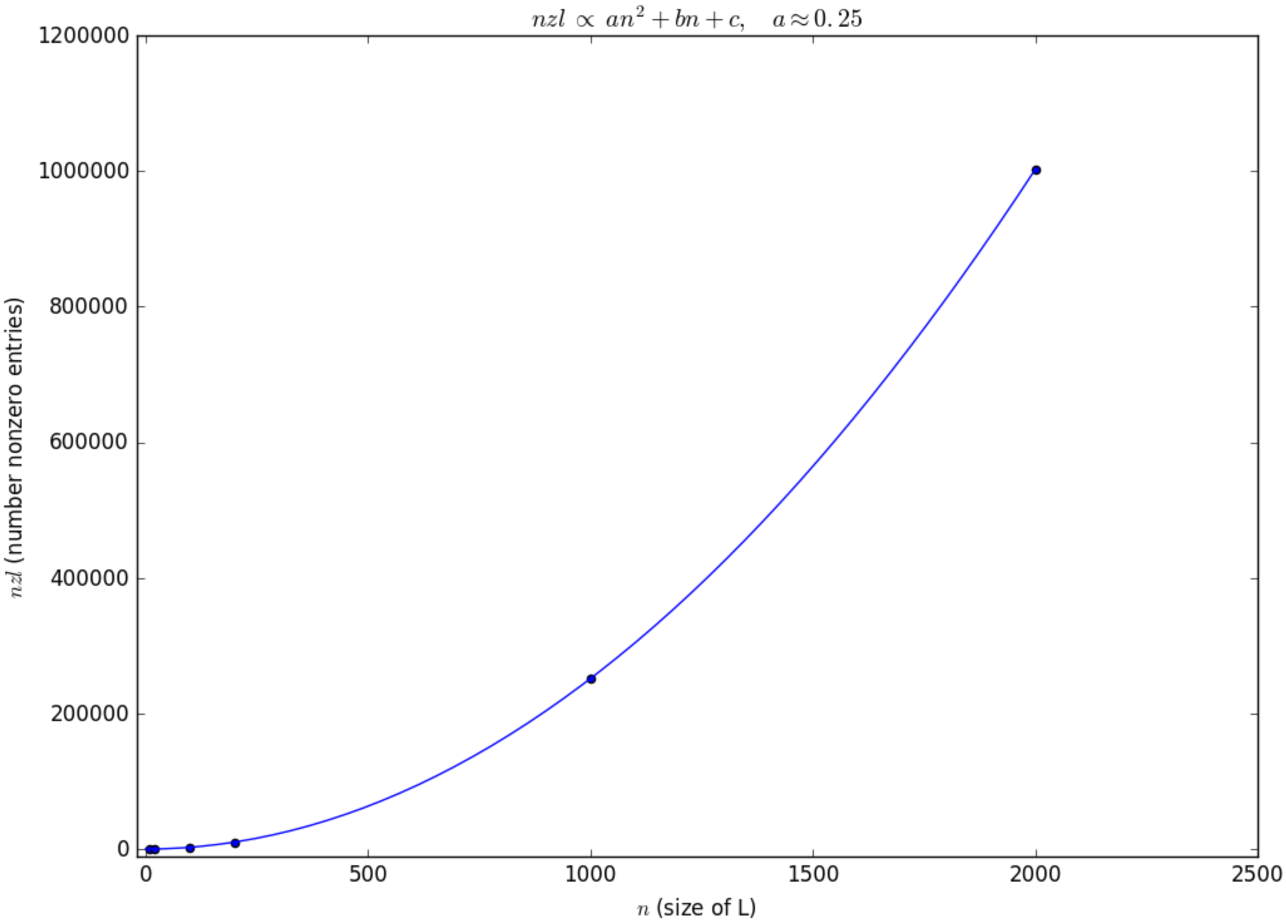

Figure 1: Relationship between size of system n and number of nonzero elements in Cholesky factor. The relationship was found to be quadratic with leading coefficient $a \approx 0.25$.

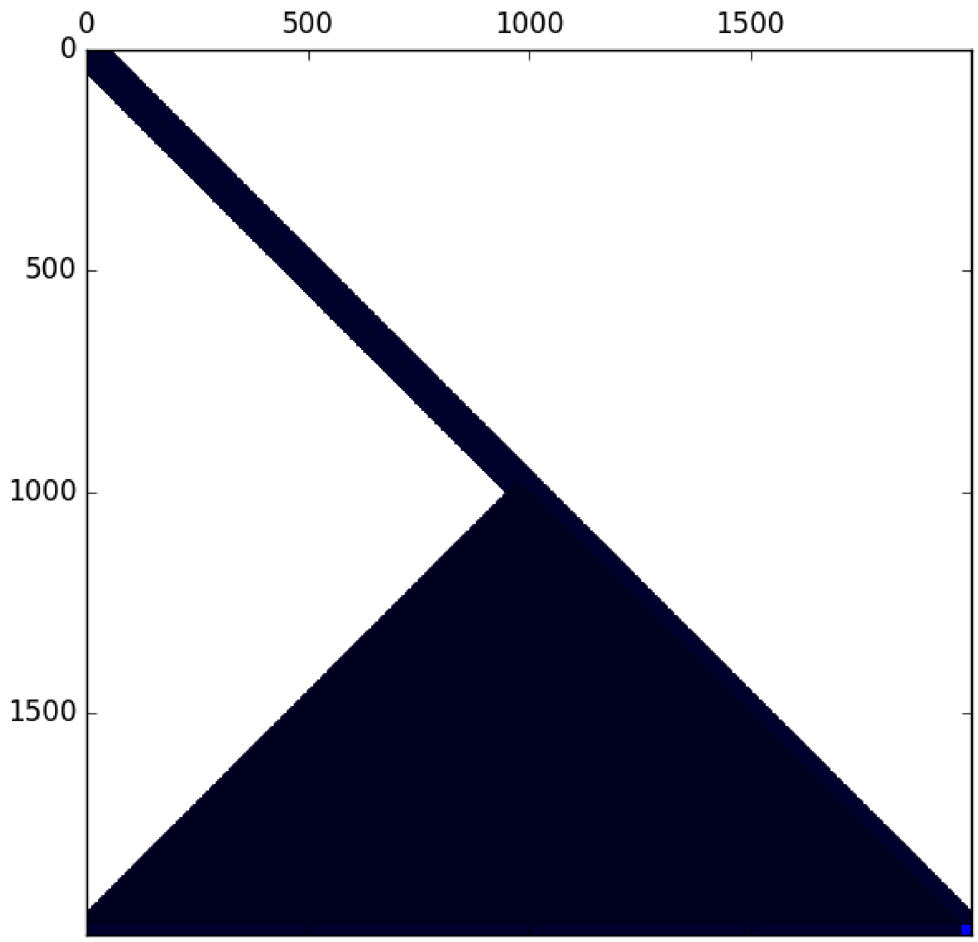

Figure 2: A visualization of nonzero elements of the cholesky factor L when $n = 1000$ of the matrix A