

Strict Multiscale Frangi Prefiltering for Segmentation

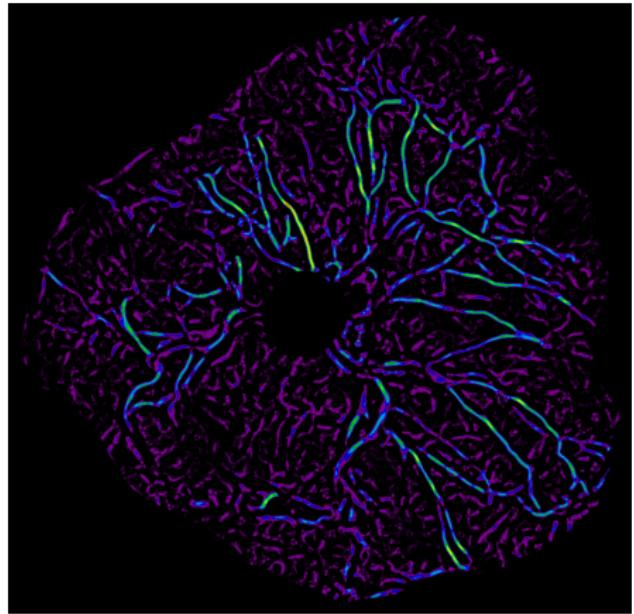
Towards an automated PCSVN extraction

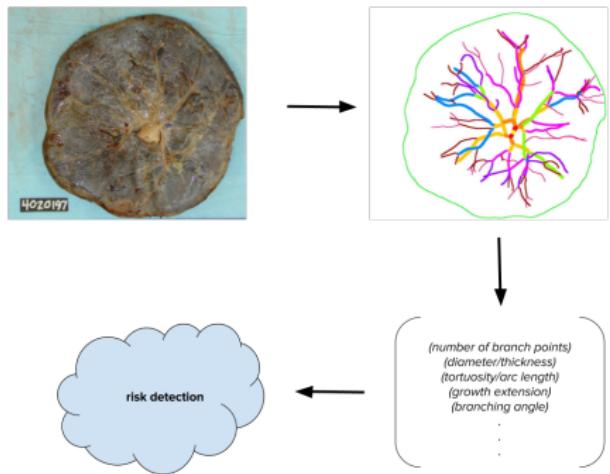
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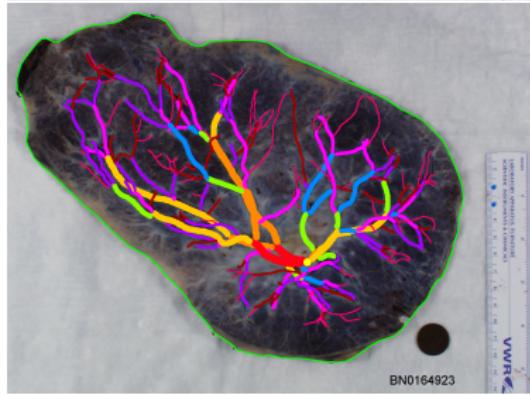
Vascular Network Extraction in Placentas

- **Motivation:** Accurate measurement of the vascular structure of a placental sample can be used to predict neonatal risk factors, specifically ASD.
- **Challenge:** Currently no automated method of obtaining traces of PCSVN. Manual tracing is labor intensive but necessary for feature analysis.
- **Research Goal:** Provide a fully automated method of extraction.

The Image Processing Problem

Our image domain

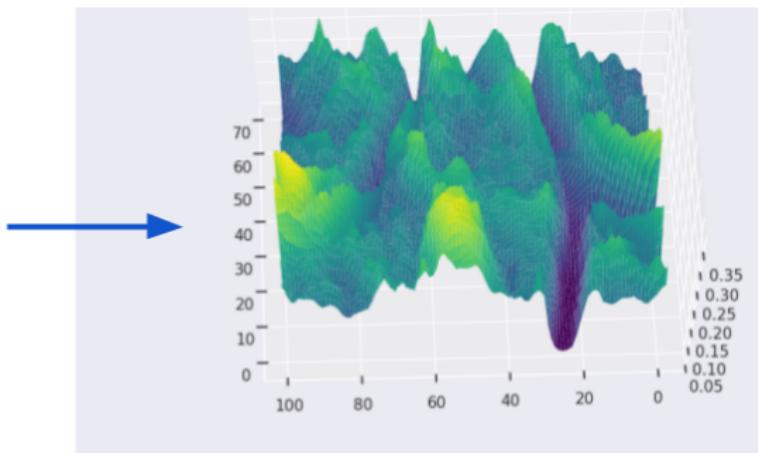
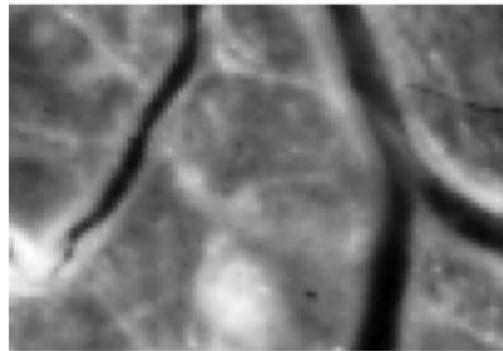
- The PCSVN is a connected network of veins and arteries on the surface of the placenta
- Ground truth for 201 samples from private NCS dataset
- Placentas have been formalin-fixed, so arteries are more prominent.
- Placental images are comparatively noisy

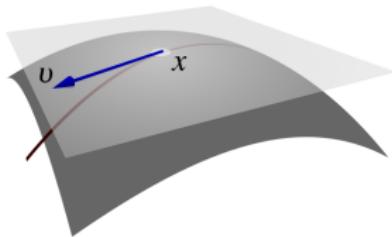


Appealing to Differential Geometry

Idealize image as a 3D surface (a graph) with (x, y) spatial coordinates and intensity as the height function $h(x, y)$.

$$\mathbf{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ where } (x, y) \mapsto (x, y, h(x, y))$$





Meusnier's Theorem

If you look at a point on the surface and fix a tangent vector, then all surface curves through that point with that velocity will have the same curvature there. So the curvature is intrinsic to the surface. Call it **normal curvature**.

Definition

Extremal values of normal curvature are called **principal curvatures** of the surface at that point. The extremizing tangent vectors are **principal directions**.

Theorem of Olinde Rodrigues

These principal curvatures/directions are the eigenvalues/eigenvectors of a particular map called the **Weingarten map**.

Weingarten Map for Graphs

Given the graph $f : U \rightarrow \mathbb{R}^3$ where $(x, y) \mapsto (x, y, h(x, y))$, the matrix representation of its Weingarten map is given by

$$\widehat{\mathbf{L}} = \text{Hess}(h)\tilde{\mathbf{G}}$$

where $\tilde{\mathbf{G}} := \frac{1}{(1+h_x^2+h_y^2)^{3/2}} \begin{bmatrix} 1+h_y^2 & -h_x h_y \\ -h_x h_y & 1+h_x^2 \end{bmatrix}$ and $\text{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix}$

Approximating

- In particular, given a point where $h_x \approx h_y \approx 0$, we have $\tilde{\mathbf{G}} \approx \text{Id}$, and thus $\widehat{\mathbf{L}} \approx \text{Hess}(h)$.
- For ease of use, we can simply find eigenvalues of the Hessian instead.
- This gives rise to a class of filters, the so-called Hessian-based filters.

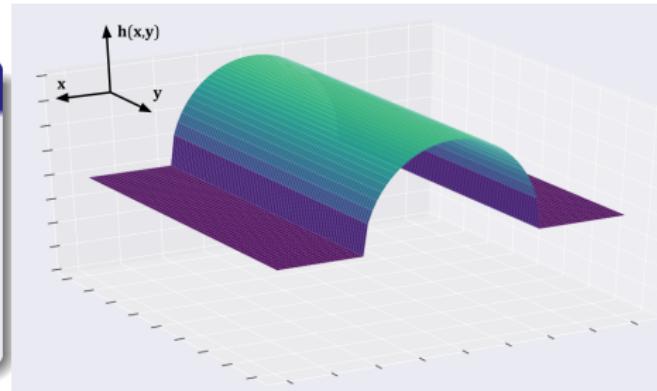
Example: Finding Weingarten map and Principal Curvatures

Cylindrical Ridge of radius r

Let f be the graph given by

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ by } f(x, y) = (x, y, h(x, y)),$$

$$\text{with } h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$$



$$h_x = \frac{-x}{\sqrt{r^2 - x^2}} \quad , \quad h_{xx} = \frac{-r^2}{(r^2 - x^2)^{3/2}}$$
$$h_y = 0 \quad , \quad h_{yy} = 0 \quad , \quad h_{xy} = 0$$

Example: Finding Weingarten map and Principal Curvatures

Weingarten matrix representation (for graphs)

$$\widehat{\mathbf{L}} = \text{Hess}(h)\tilde{\mathbf{G}}, \quad \text{where} \quad \tilde{\mathbf{G}} := \frac{1}{(1 + h_x^2 + h_y^2)^{3/2}} \begin{bmatrix} 1 + h_y^2 & -h_x h_y \\ -h_x h_y & 1 + h_x^2 \end{bmatrix}$$

$$\text{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{\mathbf{G}} = \frac{1}{\left(1 + \frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 + \frac{x^2}{r^2-x^2} \end{bmatrix}$$

$$\begin{aligned} \widehat{\mathbf{L}} &= \text{Hess}(h)\tilde{\mathbf{G}} = \frac{1}{\left(1 + \frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} \frac{r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \\ &= \dots = \begin{bmatrix} \frac{-1}{r} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

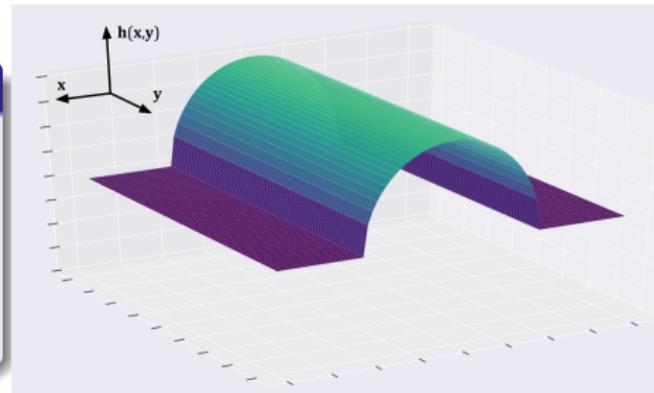
Example: Finding Weingarten Map and Principal Curvatures

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Let f be the graph given by

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ by } f(x, y) = (x, y, h(x, y)),$$

$$\text{with } h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$$



$$\hat{L} = \begin{bmatrix} -\frac{1}{r} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{Matrix of Weingarten map}$$

$u_2 = (1, 0)$, $u_1 = (0, 1)$ principal directions (eigenvectors of \hat{L})

$\lambda_2 = -\frac{1}{r}$, $\lambda_1 = 0$ principal curvatures (eigenvalues of \hat{L})

The (Uniscale) Frangi Filter

$$\mathcal{V}(x, y) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) & \text{otherwise} \end{cases}$$

where

$$A := |\lambda_1 / \lambda_2| \quad (\text{Anisotropy Measure})$$

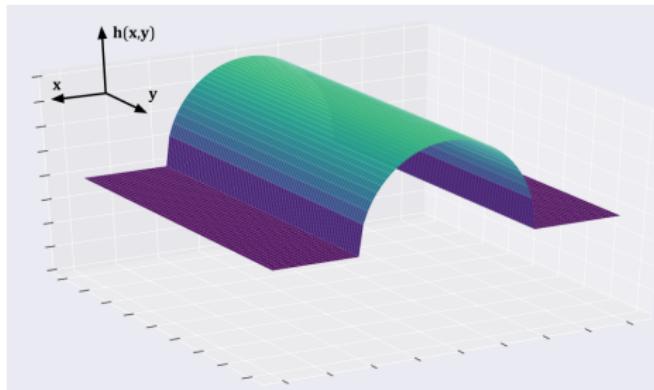
$$\text{and } S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad (\text{Structureness Measure})$$

for eigenvalues λ_1, λ_2 of the Hessian (at point (x,y)), $|\lambda_1| \leq |\lambda_2|$
and β and c are parameters

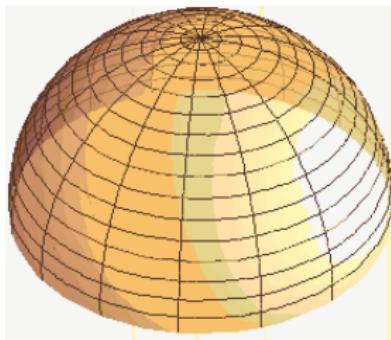
Anisotropy Factor

$$\exp\left(-\frac{A^2}{2\beta^2}\right) \quad , \quad A := \left| \frac{\lambda_1}{\lambda_2} \right| \quad (1)$$

- For selecting anisotropic content (lines not blobs)
- When A is very close to 0, $|\lambda_2| \gg |\lambda_1|$, and the factor is ≈ 1 .
- Choosing parameter: Frangi suggested $\beta = \frac{1}{2}$ as a reasonable default.



(a) Anisotropic



(b) Isotropic

Structureness Factor

$$\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \quad , \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad (2)$$

- Purpose: Filter out numerically insignificant values.
- Important to pick a reasonable value for the image at hand / image domain.
- Frangi suggested “half the maximum value of the Hessian norm”. We will define the parameter γ and define $c(\gamma) = \gamma S_{\max}$, since S_{\max} is the Frobenius norm of the Hessian.

Frangi filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{s^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \quad (3)$$

$$\text{where } A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2}, \quad |\lambda_2| \geq |\lambda_1| \quad (4)$$

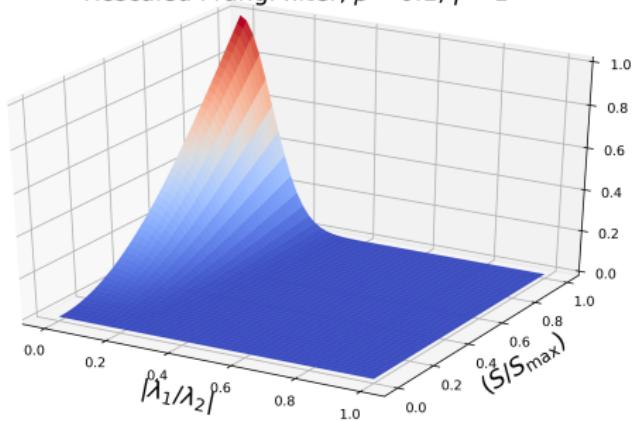
- $\lambda_2 > 0$ means the point would be a local minimum (at a critical point). We are looking for a local maximum by looking for bright curvilinear objects. You can switch this ($\lambda_2 < 0$ as the case) if you are looking for dark curvilinear.
- Point of $\exp(\dots)$ and $1 - \exp(\dots)$ structure is that the filter decays rapidly as anisotropy or structureness decrease.

Frangi filter anatomy: Choosing Parameters

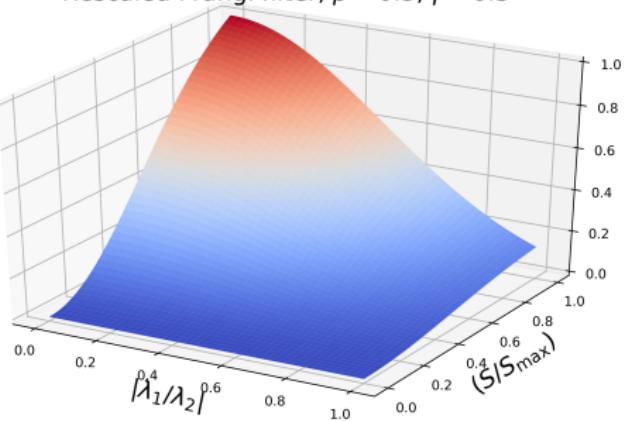
$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{s^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

where $A := |\lambda_1/\lambda_2|$ and $S := \sqrt{\lambda_1^2 + \lambda_2^2}$, $|\lambda_2| \geq |\lambda_1|$

Rescaled Frangi filter, $\beta = 0.1, \gamma = 1$



Rescaled Frangi filter, $\beta = 0.5, \gamma = 0.5$



Linear Scale Space Theory for Kids

- Obviously the image is not actually a continuous surface. It is a particular sampling I of the surface.
- We want to create a “family of derived images” with a “resolution” parameter $\sigma \geq 0$, ideally from some operator T_σ acting on the image I .

$$K(x, y; \sigma) = T_\sigma$$

Some Axioms

- Linear shift and rotational invariance
- Semigroup property $T_{\sigma_1 + \sigma_2} I = T_{\sigma_2} \circ T_{\sigma_1} I$
- Continuity of scale parameter σ
- Causality condition

Moral

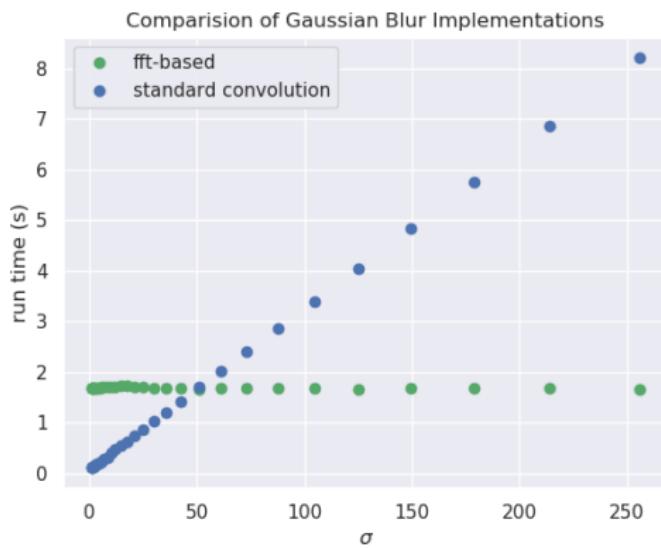
Convolution by Gaussian solves these problems.

Implementation Detail: Calculating Discrete Hessian

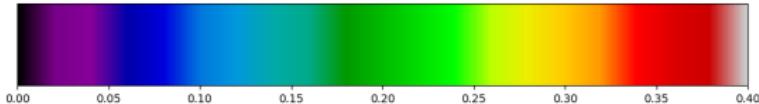
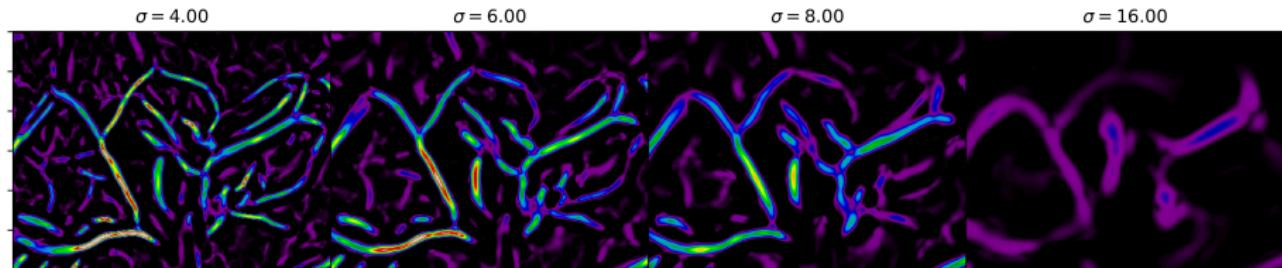
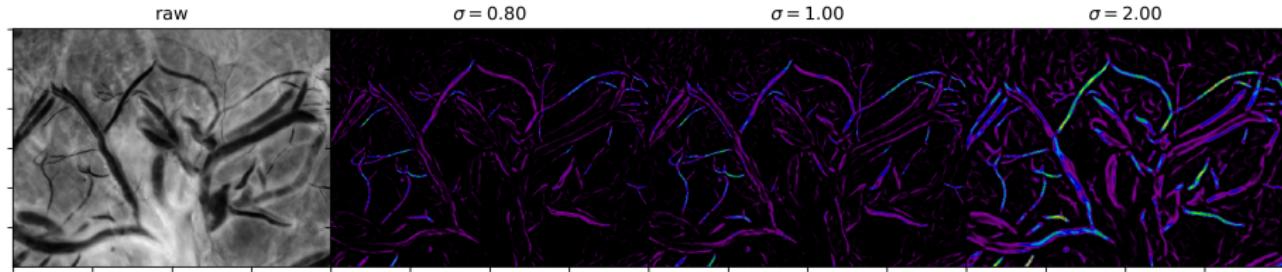
$$K(x, y; \sigma) = T_\sigma u_0 = G_\sigma \star u_0$$

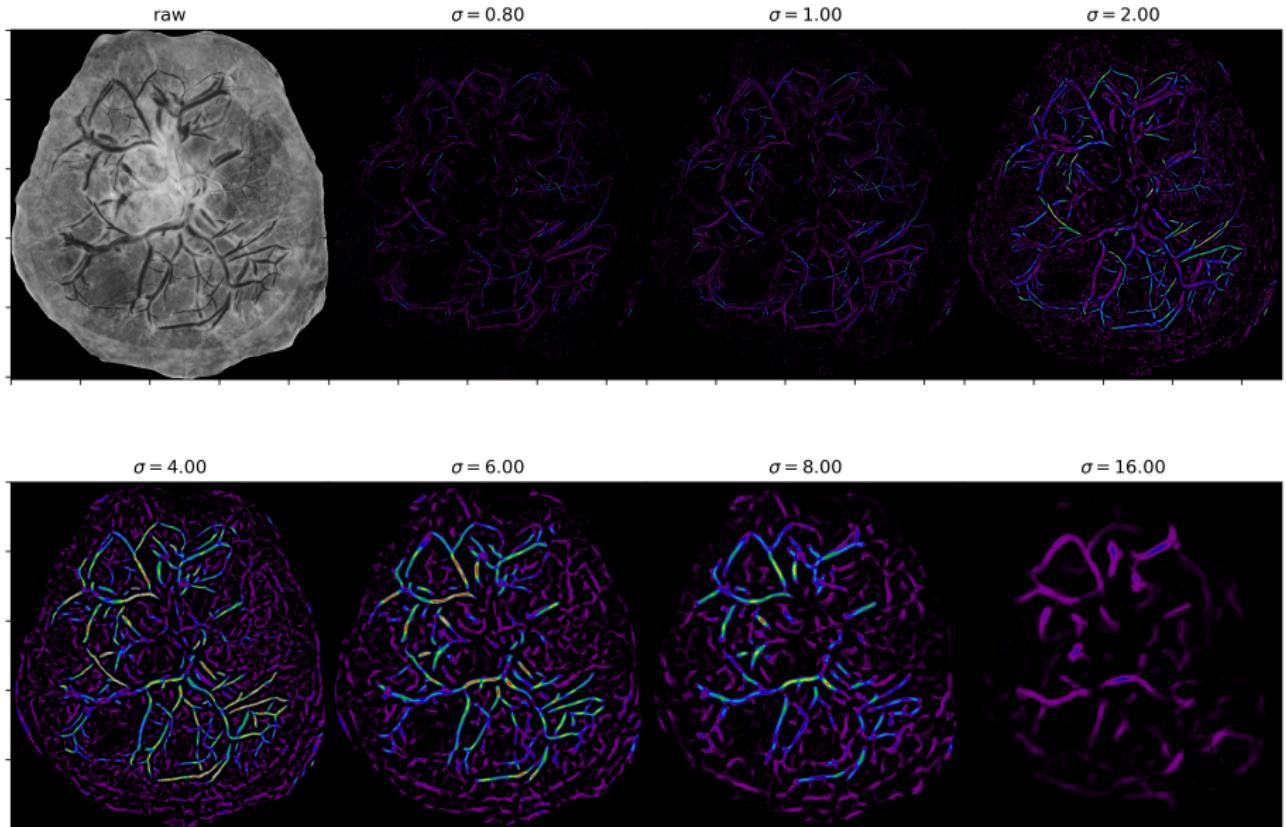
where $G_\sigma(x, y) := \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$

- Convolution by gaussian matrix is slow
- Calculate in frequency space as a multiplication according to the convolution theorem.
- While you're at it, use a FFT.



Frangi Filter Anatomy: Scale parameter





Multiscale Frangi filter

For a set of scales to probe, $\Sigma := \{\sigma_0, \sigma_1, \dots, \sigma_n\}$, the set of n scales at which to probe,

$$\mathcal{V}_{\max}(x_0, y_0) := \max_{\sigma \in \Sigma} \{\mathcal{V}_\sigma(x_0, y_0)\} \quad (5)$$

where \mathcal{V}_σ is the Frangi vesselness measure at scale σ for the pixel (x_0, y_0)

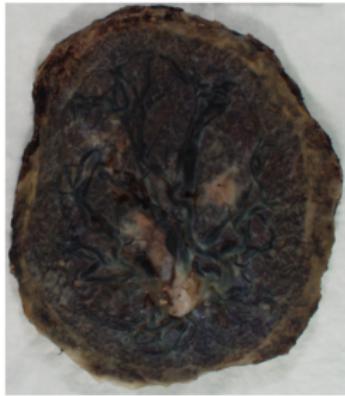
Notes

- The default “merging” strategy suggested by Frangi.
- Alternatively, we can process each scale by itself too if we want to. (Keep track of $\mathcal{V}_{\arg\max}$, \mathcal{V}_Σ , etc.)

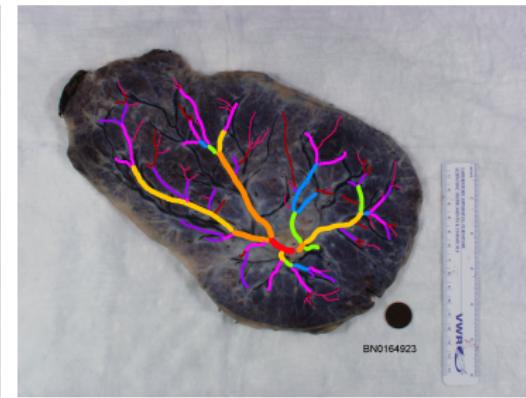
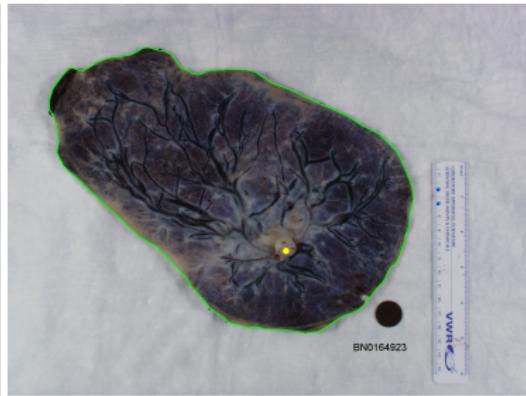
What scales to use?

- Logarithmic spacing makes some intuitive sense.
- Experiment to determine what is large / small enough.
- Smaller scales for smaller features, larger scales for larger features.

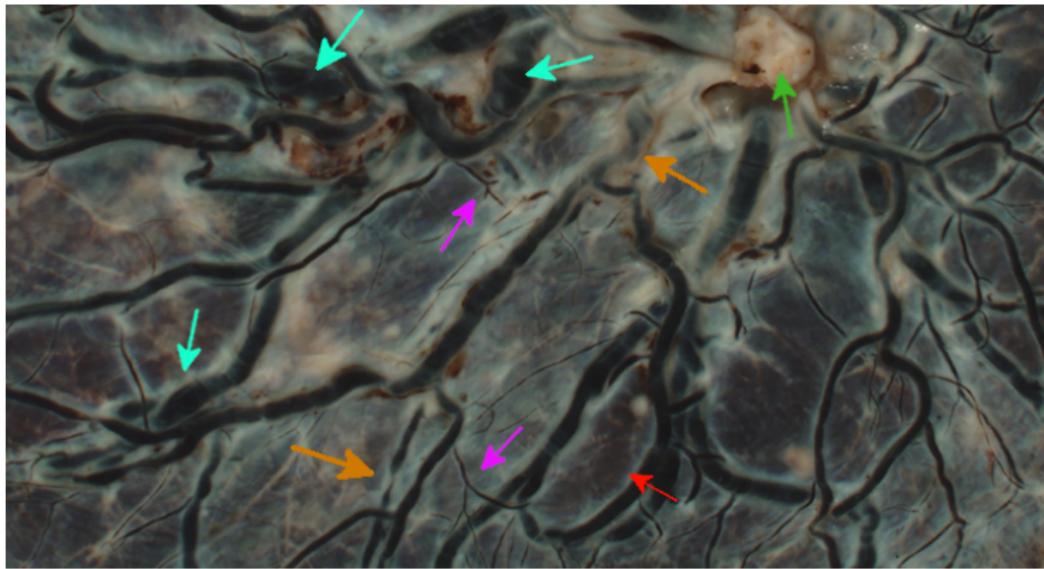
The data set



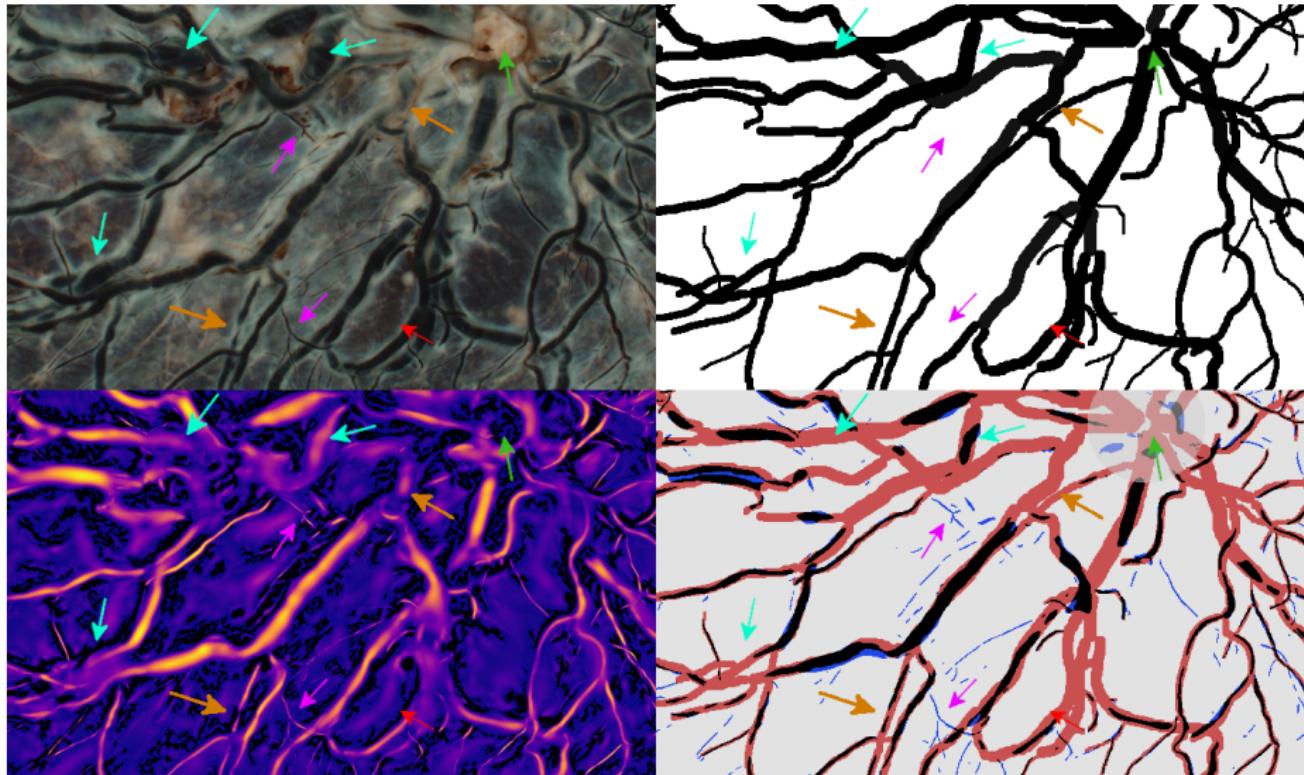
Ground truth



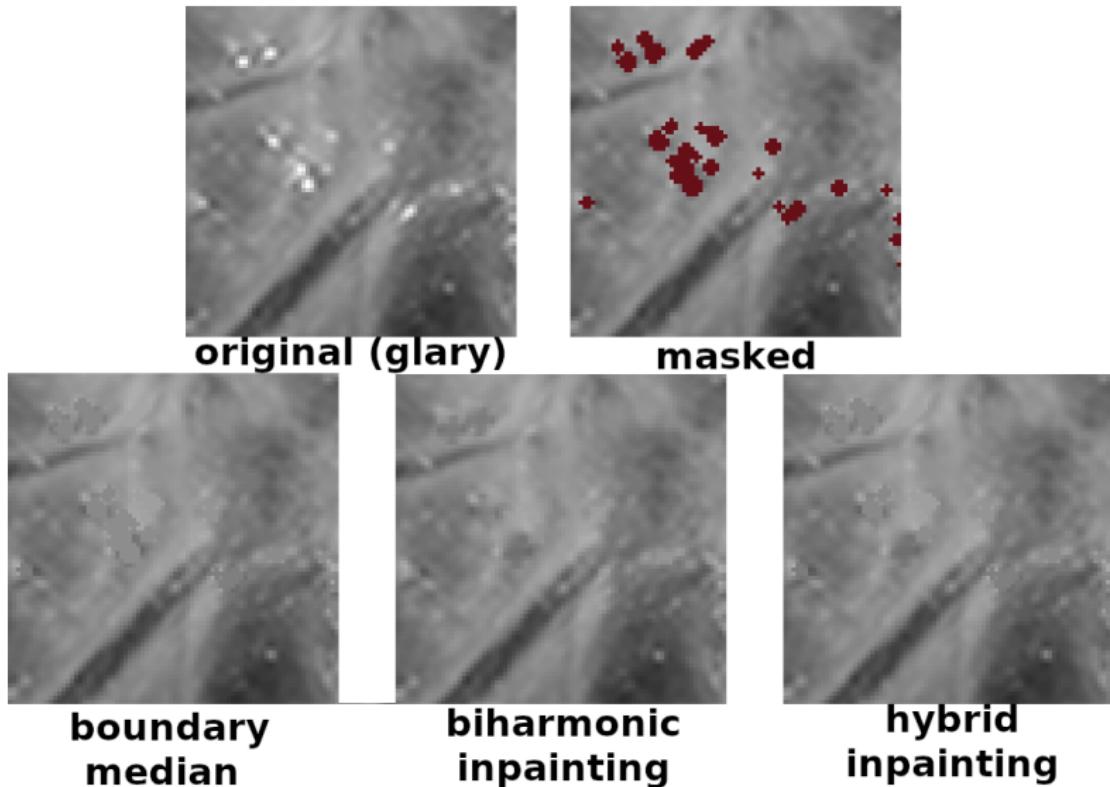
Imperfections in Data set (1/2)



Imperfections in Data set (2/2)

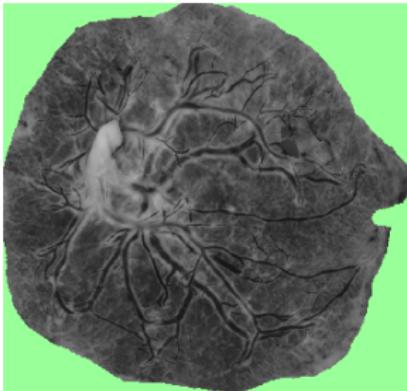


Preprocessing: Dealing with Glare



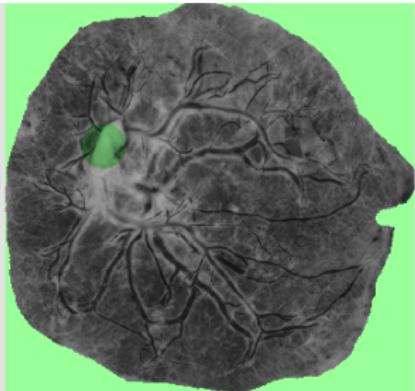
Preprocessing: Umbilical Stump

BN2432252

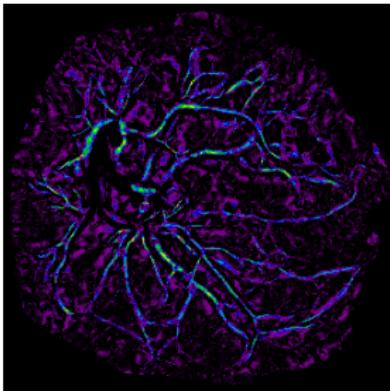


\mathcal{V}_{\max} $\beta = 0.15, \gamma = 1.0$

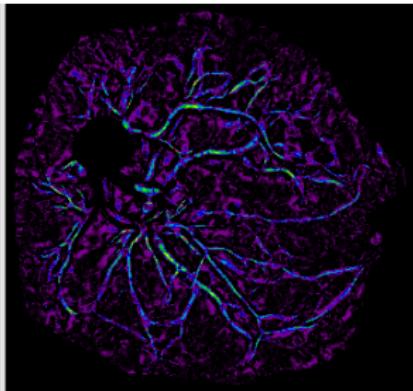
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\mathcal{V}_{\max} $\beta = 0.15, \gamma = 1.0$

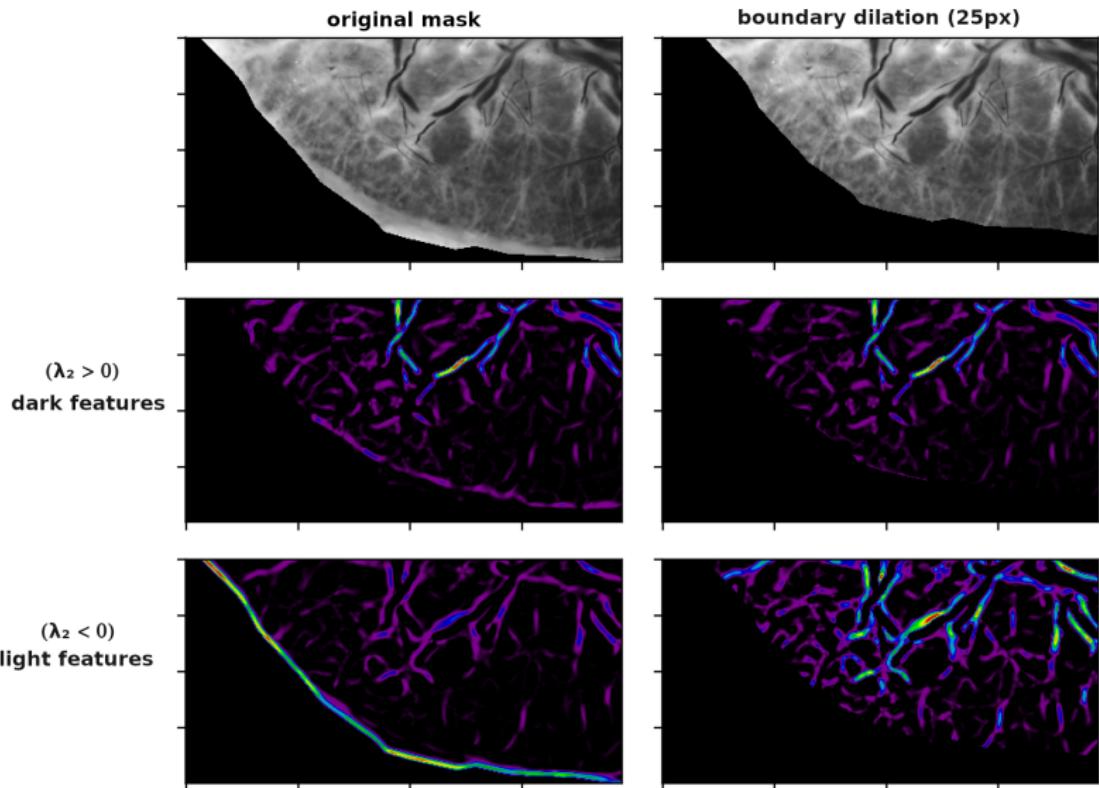


no ucip mask



ucip mask (50px radius)

Preprocessing: Dealing with Boundaries



Cumulative Vesselness Ratio (CVR)

\mathcal{V}_{\max} (standard)

$\beta = 0.50, \gamma = 0.50$

CVR: 0.480

\mathcal{V}_{\max} (loose)

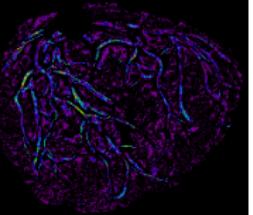
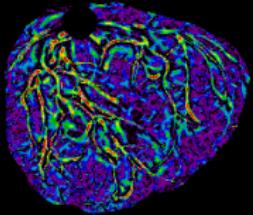
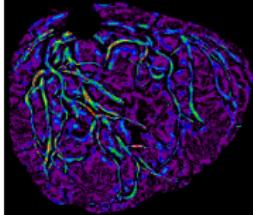
$\beta = 1.00, \gamma = 0.30$

CVR: 0.395

\mathcal{V}_{\max} (strict)

$\beta = 0.10, \gamma = 1.00$

CVR: 0.615



\mathcal{V}_{\max} (Anisotropy Factor)

$\beta = 0.50, \gamma = 0.00$

CVR: 0.196

\mathcal{V}_{\max} (semiloose-beta)

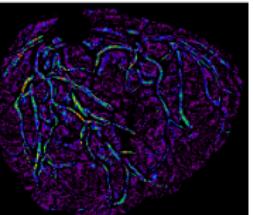
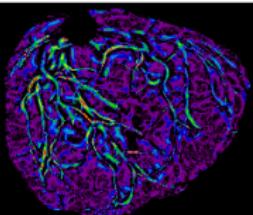
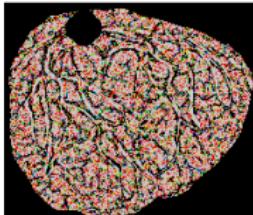
$\beta = 1.00, \gamma = 0.50$

CVR: 0.451

\mathcal{V}_{\max} (semistrict-beta)

$\beta = 0.10, \gamma = 0.50$

CVR: 0.580



\mathcal{V}_{\max} (Structureness Factor)

$\beta = \text{inf}, \gamma = 0.50$

CVR: 0.437

\mathcal{V}_{\max} (semiloose-gamma)

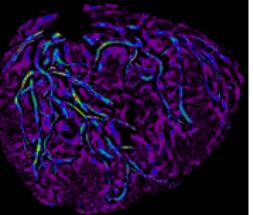
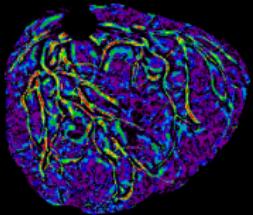
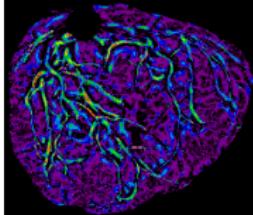
$\beta = 0.50, \gamma = 0.30$

CVR: 0.421

\mathcal{V}_{\max} (semistrict-gamma)

$\beta = 0.50, \gamma = 1.00$

CVR: 0.515

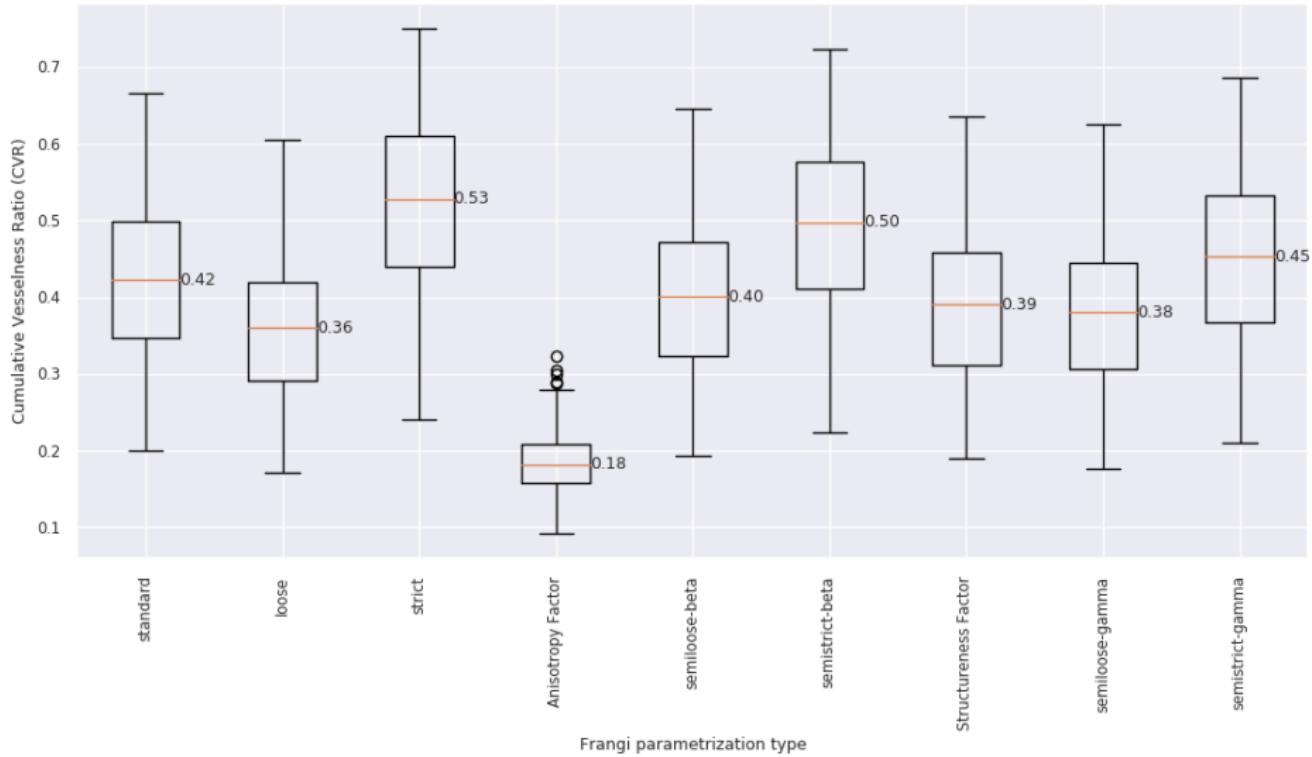


$$\text{CVR}(\mathcal{V}_{\max}) := \frac{\sum_{G \subset I} \mathcal{V}_{\max}(x_0, y_0)}{\sum_I \mathcal{V}_{\max}(x_0, y_0)}$$

where $G \subset I$ is the ground truth

label	β	γ
standard	0.5	0.5
loose	1.0	0.3
strict	0.10	1.0
semiloose- β	1.0	0.5
semistrict- β	0.1	0.5
semiloose- γ	0.5	0.3
semistrict- γ	0.5	1.0
anisotropy only	0.5	0
structureness only	∞	0.5

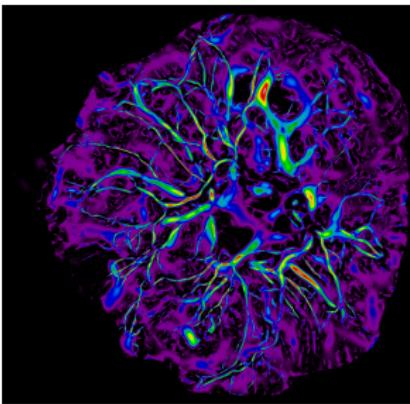
Incidence of Vesselness Score along Traced Vessels (all samples)



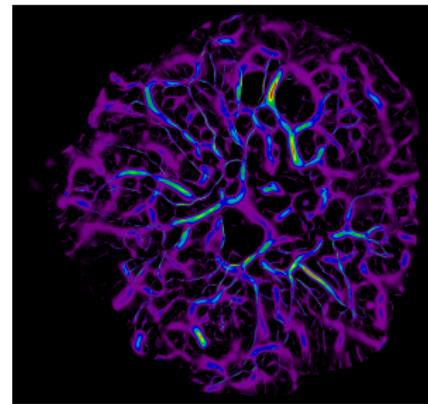
Comparing Frangi Hessian to Frangi with Weingarten Map



(g) Raw Sample



(h) \mathcal{V}_{\max} (Hessian eigvals)



(i) \mathcal{V}_{\max} (Weingarten eigvals)

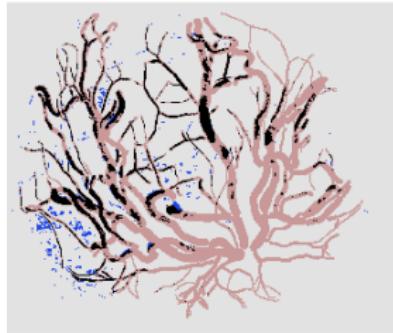
- Finding Hessian eigenvalues is about 16 times faster (25 scales 24 seconds vs. 7 minutes. mean of 25 samples)
- Scale dependency not the same
- Frangi parameters seem less important for Weingarten map

A non-Fangi segmentation method: ISODATA (Ridler-Calvert)

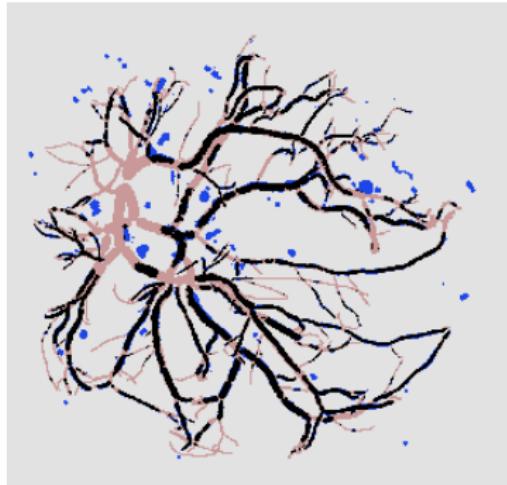
$$\alpha_{\text{ISO}} = \arg \min_{\alpha} \left(\frac{1}{2} \left[\text{mean} \{ I(x, y) \mid I(x, y) \leq \alpha \} + \text{mean} \{ I(x, y) \mid I(x, y) > \alpha \} \right] \right)$$

ISODATA
(Frangi-less)

MCC: 0.39
precision: 85.44%



Since the vascular structure in our image domain is darker than the background, we select pixels where $I(x, y) < \alpha_{\text{ISO}}$.



- TP
- FP
- FN
- TN

Scoring Methods

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

f

$$\text{MCC} = \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$

Fixed Threshold (thresh-low and thresh-high)

Pick some $0 < \alpha < 1$, then extract $\mathcal{V}_{\max} > \alpha$

- α must be manually selected
- We picked $\alpha = 0.3$ and $\alpha = 0.2$

scalewise (nonzero) percentile filtering (snz-p)

For each scale, filter under ρ th percentile: $\mathcal{V}_\sigma > \alpha_\rho$.

- Use *nonzero* percentile for each scale
- Smaller scales get in easier
- Good for unscaled Frangi, unknown filter response
- Very noisy if Σ chosen poorly

Don't throw anything away!

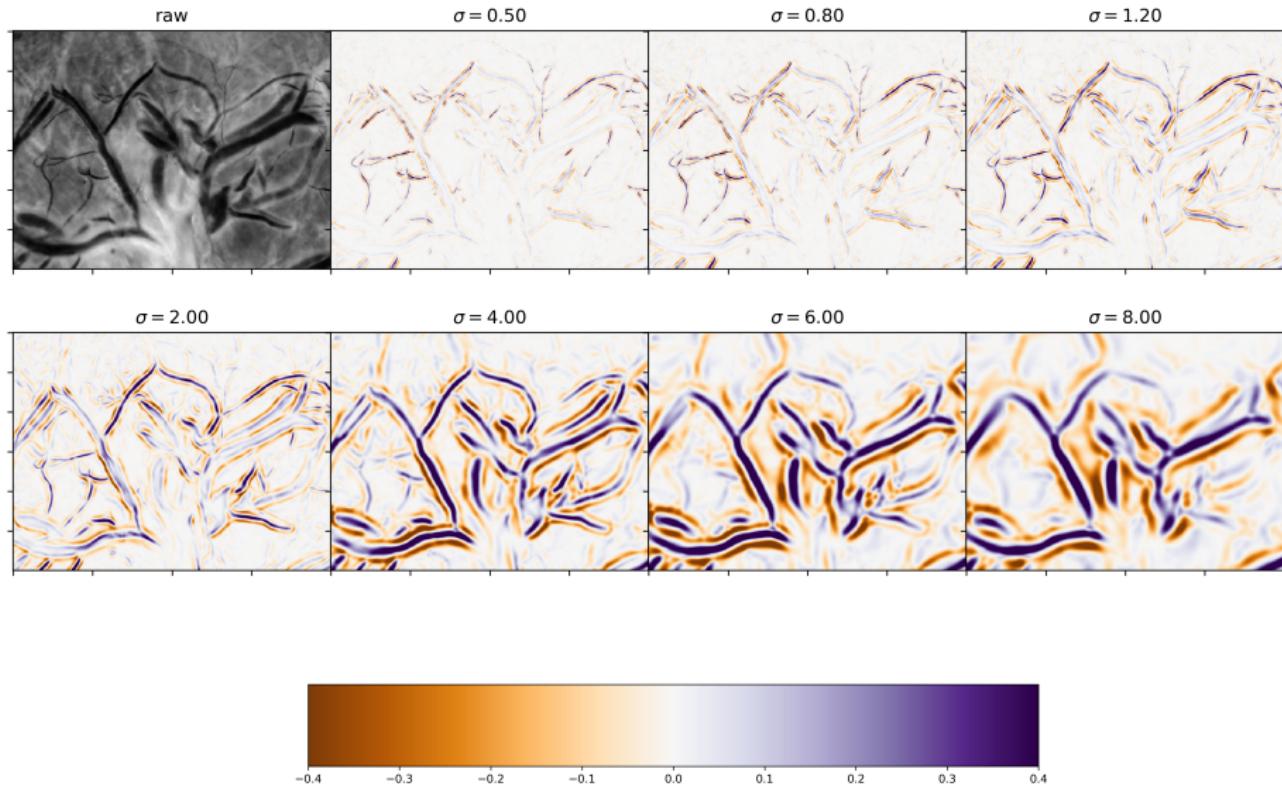
$$\mathcal{V}_\sigma^{(+)}(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_\sigma^{(-)}(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 < 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

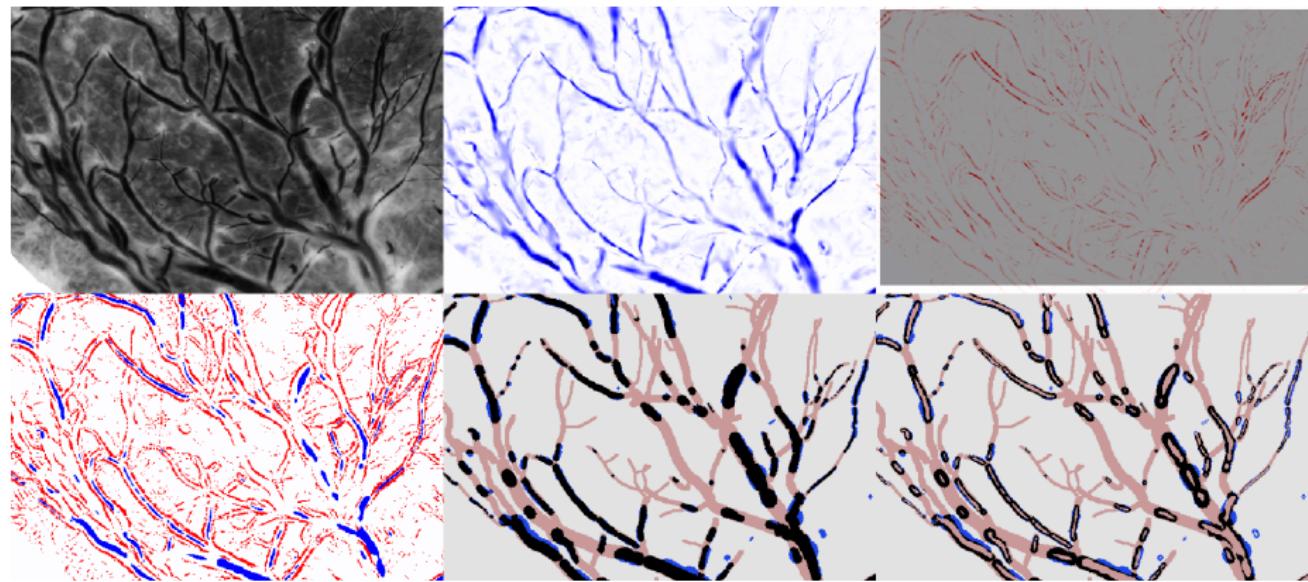
where $A := |\lambda_1/\lambda_2|$ and $S := \sqrt{\lambda_1^2 + \lambda_2^2}$, $|\lambda_2| \geq |\lambda_1|$

- This gives us a $\mathcal{V}_{\max}^{(+)}$ and $\mathcal{V}_{\max}^{(-)}$ with same calculation time.
- We might want to use a subset $\Sigma^{(-)} \subset \Sigma$ (smaller scales only)

Signed Frangi



Trough Filling Method



Example Results (1/4)

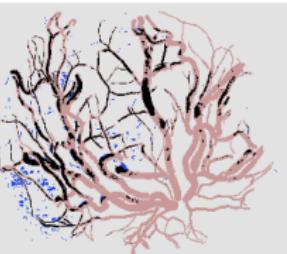
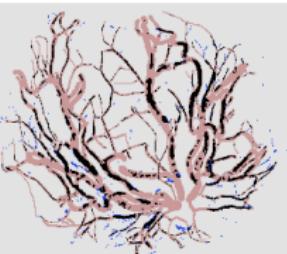
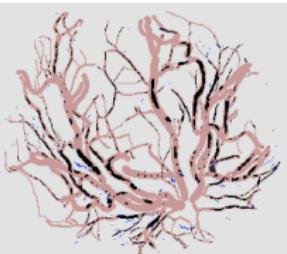
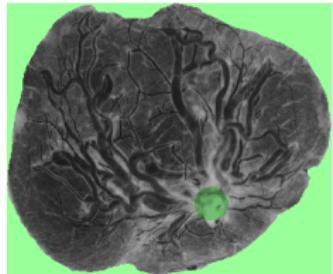
BN8801263

fixed $\alpha = 0.3$

MCC: 0.35
precision: 91.72% fixed $\alpha = 0.2$

MCC: 0.45
precision: 88.43% ISODATA
(Frangi-less)

MCC: 0.39
precision: 85.44%



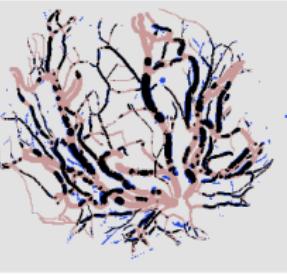
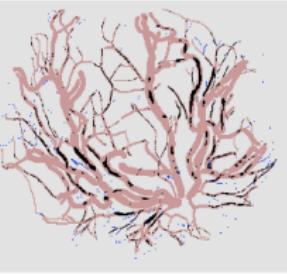
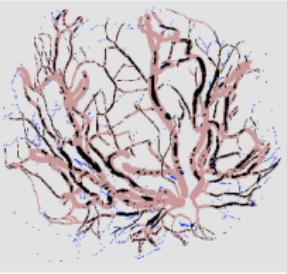
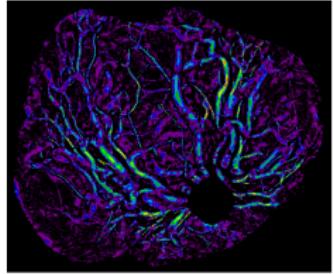
λ_{max} $\beta = 0.15$, $\gamma = 0.5$

scalewise nz-p
(p=95)

MCC: 0.41
precision: 84.46% scalewise nz-p
(p=98)

MCC: 0.28
precision: 86.49% trough-fill
 $\alpha^{(+)}$ = 0.3

MCC: 0.56
precision: 81.92%



Example Results (2/4)

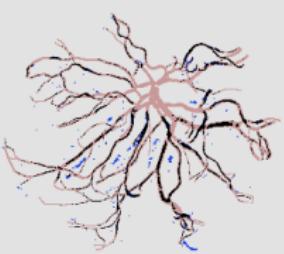
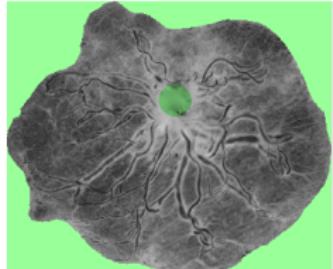
BN7753462

fixed $\alpha = 0.3$

MCC: 0.43
precision: 92.99% fixed $\alpha = 0.2$

MCC: 0.49
precision: 84.92% ISODATA (Frangi-less)

MCC: 0.24
precision: 78.87%



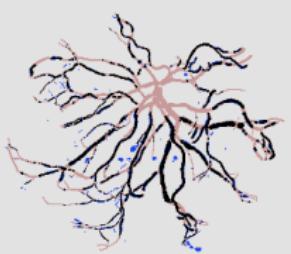
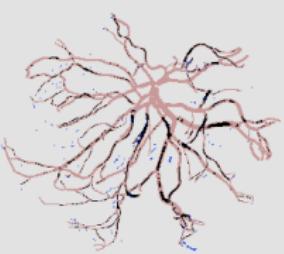
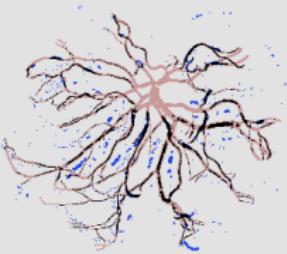
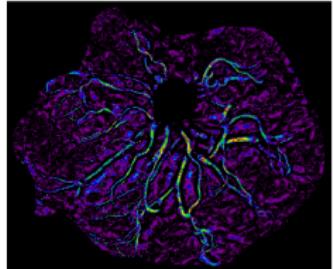
V_{max} $\beta = 0.15$, $\gamma = 0.5$

scalewise nz-p
(p=95)

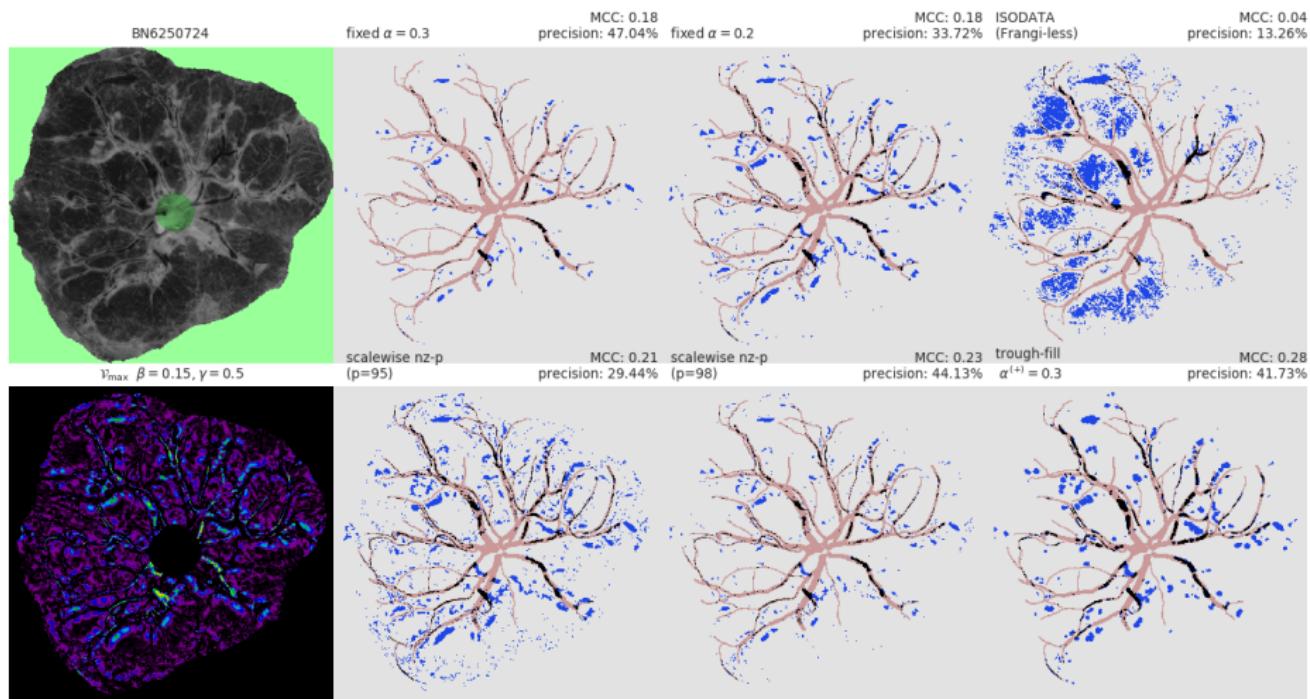
MCC: 0.47
precision: 73.02% scalewise nz-p
(p=98)

MCC: 0.38
precision: 87.10% trough-fill
 $\alpha^{(+)}$ = 0.3

MCC: 0.63
precision: 82.43%

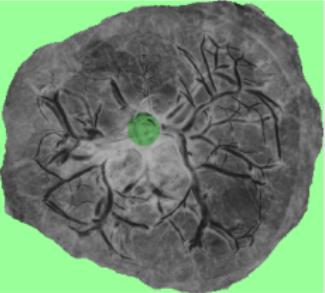


Example Results (3/4)

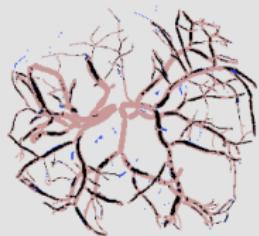


Example Results (4/4)

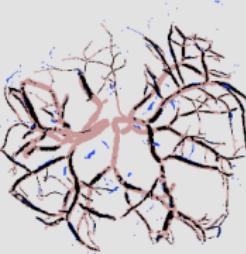
BN5280796



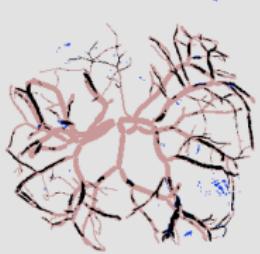
fixed $\alpha = 0.3$



MCC: 0.49
precision: 92.20% fixed $\alpha = 0.2$

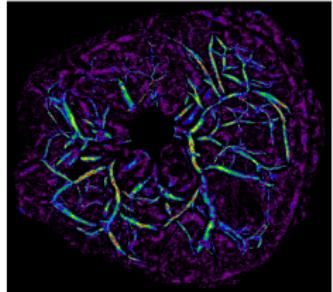


MCC: 0.56
ISODATA
precision: 88.33% (Frangi-less)

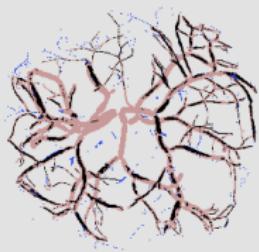


MCC: 0.45
precision: 90.38%

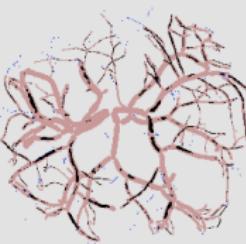
γ_{\max} , $\beta = 0.15$, $\gamma = 0.5$



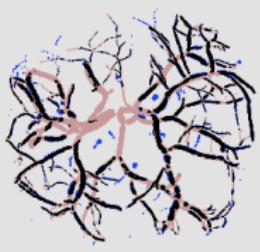
scalewise nz-p
(p=95)



MCC: 0.49
precision: 86.55% (p=98)

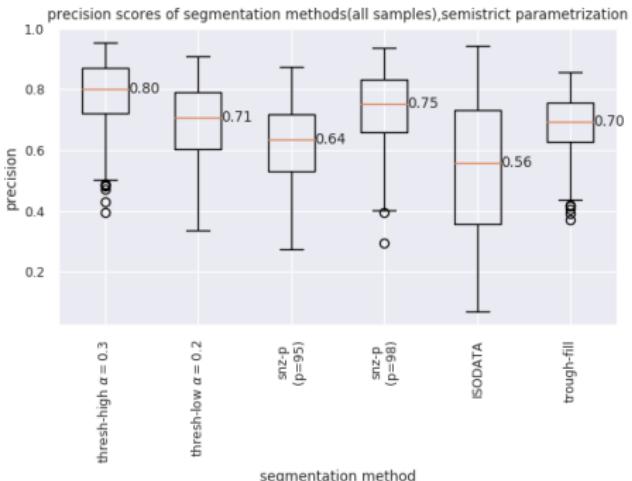
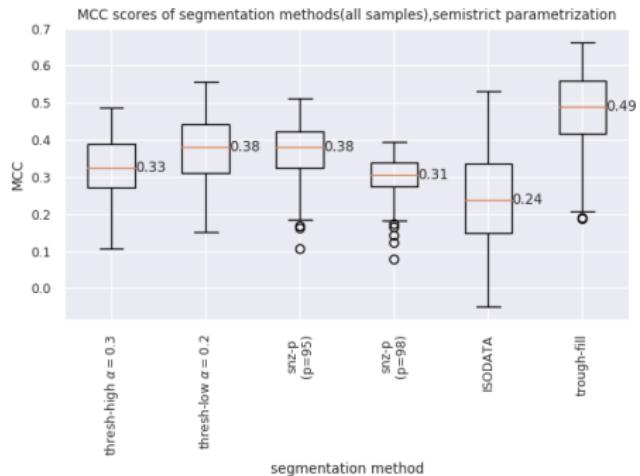


MCC: 0.35
trough-fill
precision: 90.27%
 $\alpha^{(+)}/\alpha^{-} = 0.3$



MCC: 0.65
precision: 82.04%

Results on All Samples ($\beta = 0.15$, $\gamma = 0.5$)



Results on All Samples ($\beta = 0.10, \gamma = 1.0$)

