Strict Multiscale Frangi Prefiltering for Segmentation

Towards an automated PCSVN extraction

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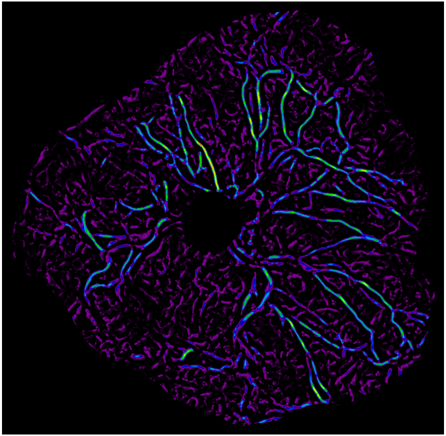
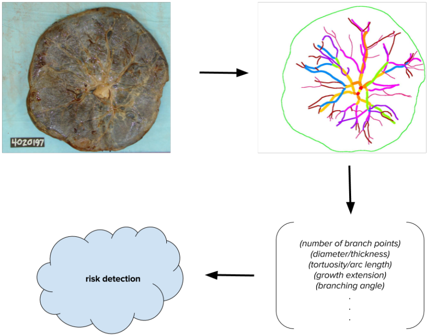
BORATORY APPARATUS, FURNITURE
CIENTIFIC INSTRUMENTS & CHEMICALS



Research Goals



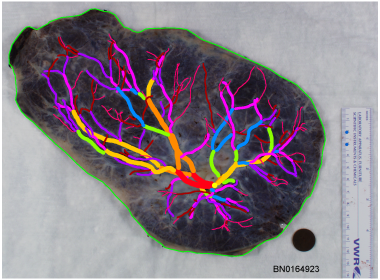
Vascular Network Extraction in Placentas

- **Motivation:** Accurate measurement of the vascular structure of a placental sample can be used to predict neonatal risk factors, specifically ASD.
- **Challenge:** Currently no automated method of obtaining traces of PCSVN. Manual tracing is labor intensive but necessary for feature analysis.
- **Research Goal:** Provide a fully automated method of extraction.

The Image Processing Problem

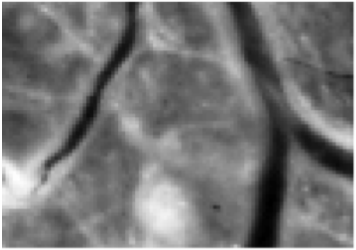
Our image domain

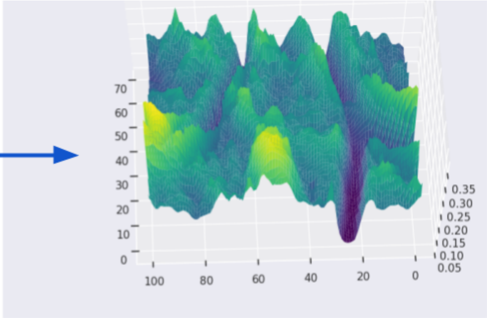
- The PCSVN is a connected network of veins and arteries on the surface of the placenta
- Ground truth for 201 samples from private NCS dataset
- Placentas have been formalin-fixed, so arteries are more prominent.
- Placental images are comparatively noisy

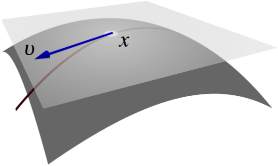


Idealize image as a 3D surface (a graph) with (x, y) spatial coordinates and intensity as the height function $h(x, y)$.

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^3 \text{ where } (x, y) \mapsto \big(x, y, h(x, y)\big)$$







Meusnier's Theorem

If you look at a point on the surface and fix a tangent vector, then all surface curves through that point with that velocity will have the same curvature there. So the curvature is intrinsic to the surface. Call it **normal curvature.**

Definition

Extremal values of normal curvature are called **principal curvatures** of the surface at that point. The extremizing tangent vectors are **principal directions.**

Theorem of Olinde Rodrigues

These principal curvatures/directions are the eigenvalues/eigenvectors of a particular map called the **Weingarten map**.

Relationship Between Hessian and Weingarten Map for Graphs

Weingarten Map for Graphs

Given the graph $f : U \to \mathbb{R}^3$ where $(x, y) \mapsto (x, y, h(x, y))$, the matrix representation of its Weingarten map is given by

$$\widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G}$$

where $\tilde{G} := \frac{1}{(1+h_x^2+h_y^2)^{3/2}} \begin{bmatrix} 1 + h_y^2 & -h_x h_y \\ -h_x h_y & 1 + h_x^2 \end{bmatrix}$ and $\mathrm{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix}$

Approximating

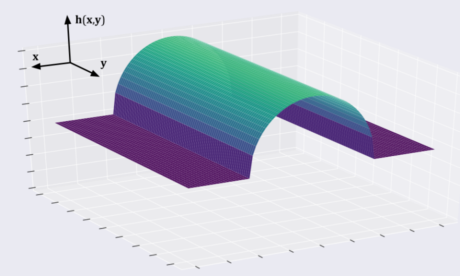
- In particular, given a point where $h_x \approx h_y \approx 0$, we have $\tilde{G} \approx \mathrm{Id}$, and thus $\widehat{\mathrm{L}} \approx \mathrm{Hess}(h)$.
- For ease of use, we can simply find eigenvalues of the Hessian instead.
- This gives rise to a class of filters, the so-called Hessian-based filters.

Cylindrical Ridge of radius r

Let f be the graph given by

$$f : \mathbb{R}^2 \to \mathbb{R}^3 \text{ by } f(x, y) = (x, y, h(x, y)),$$

$$\text{with } h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$$



$$h_x = \frac{-x}{\sqrt{r^2 - x^2}} \quad , \quad h_{xx} = \frac{-r^2}{(r^2 - x^2)^{3/2}}$$

$$h_y = 0 \quad , \quad h_{yy} = 0 \quad , \quad h_{xy} = 0$$

Weingarten matrix representation (for graphs)

$$\widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G}\,, \quad \text{where} \quad \tilde{G} := \frac{1}{\left(1+h_x^2+h_y^2\right)^{3/2}}\begin{bmatrix} 1+h_y^2 & -h_x h_y \\ -h_x h_y & 1+h_x^2 \end{bmatrix}$$

$$\mathrm{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-r^2}{\left(r^2-x^2\right)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \qquad \tilde{G} = \frac{1}{\left(1+\frac{x^2}{r^2-x^2}\right)^{3/2}}\begin{bmatrix} 1 & 0 \\ 0 & 1+\frac{x^2}{r^2-x^2} \end{bmatrix}$$

$$\begin{aligned} \widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G} &= \frac{1}{\left(1+\frac{x^2}{r^2-x^2}\right)^{3/2}}\begin{bmatrix} \frac{r^2}{\left(r^2-x^2\right)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \\ &= \quad \cdots \quad = \begin{bmatrix} \frac{-1}{r} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

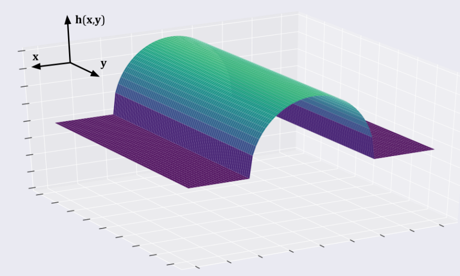
Example: Finding Weingarten Map and Principal Curvatures

Cylindrical Ridge of radius r

Let f be the graph given by

$$f : \mathbb{R}^2 \to \mathbb{R}^3 \text{ by } f(x, y) = (x, y, h(x, y)),$$

$$\text{with } h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$$



$$\widehat{\mathrm{L}} = \begin{bmatrix} -\frac{1}{r} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{Matrix of Weingarten map}$$

$$u_2 = (1, 0)\ ,\ u_1 = (0, 1) \quad \text{principal directions (eigenvectors of } \widehat{\mathrm{L}})$$

$$\lambda_2 = \frac{-1}{r}\ ,\ \lambda_1 = 0 \quad \text{principal curvatures (eigenvalues of } \widehat{\mathrm{L}})$$

The (Uniscale) Frangi Filter

$$\mathcal{V}(x,y) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) & \text{otherwise} \end{cases}$$

where

$$A := |\lambda_1/\lambda_2| \quad \text{(Anisotropy Measure)}$$

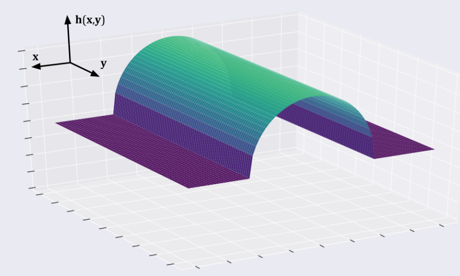
$$\text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad \text{(Structureness Measure)}$$

for eigenvalues λ_1, λ_2 of the Hessian (at point (x,y)), $|\lambda_1| \leq |\lambda_2|$
and β and c are parameters

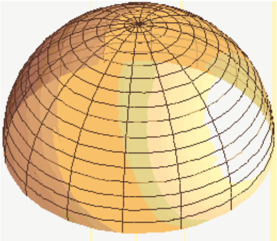
Anisotropy Factor

$$\exp\left(-\frac{A^2}{2\beta^2}\right) \quad , \quad A := \left|\frac{\lambda_1}{\lambda_2}\right| \tag{1}$$

- For selecting anisotropic content (lines not blobs)
- When A is very close to 0, $|\lambda_2| \gg |\lambda_1|$, and the factor is ≈ 1.
- Choosing parameter: Frangi suggested $\beta = \frac{1}{2}$ as a reasonable default.



(a) Anisotropic



(b) Isotropic

Frangi filter anatomy

Structureness Factor

$$\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \quad , \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \tag{2}$$

- Purpose: Filter out numerically insignificant values.
- Important to pick a reasonable value for the image at hand / image domain.
- Frangi suggested “half the maximum value of the Hessian norm”. We will define the parameter γ and define $c(\gamma) = \gamma S_{\text{max}}$, since S_{max} is the Frobenius norm of the Hessian.

Frangi filter anatomy: Putting it together

Frangi filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \tag{3}$$

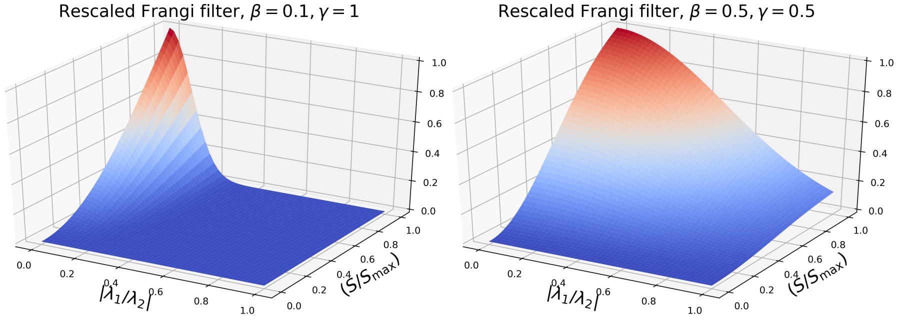
$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1| \tag{4}$$

- $\lambda_2 > 0$ means the point would be a local minimum (at a critical point). We are looking for a local maximum by looking for bright curvilinear objects. You can switch this ($\lambda_2 < 0$ as the case) if you are looking for dark curvilinear.
- Point of $\exp(...)$ and $1 - \exp(...)$ structure is that the filter decays rapidly as anisotropy or structureness decrease.

Frangi filter anatomy: Choosing Parameters

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1|$$



Linear Scale Space Theory for Kids

- Obviously the image is not actually a continuous surface. It is a particular sampling I of the surface.
- We want to create a “family of derived images” with a “resolution” parameter $\sigma \geq 0$, ideally from some operator T_σ acting on the image I.

$$K(x, y; \sigma) = T_\sigma$$

Some Axioms

- Linear shift and rotational invariance
- Semigroup property $T_{\sigma_1+\sigma_2}\mathrm{I} = T_{\sigma_2} \circ T_{\sigma_1}\mathrm{I}$
- Continuity of scale parameter σ
- Causality condition

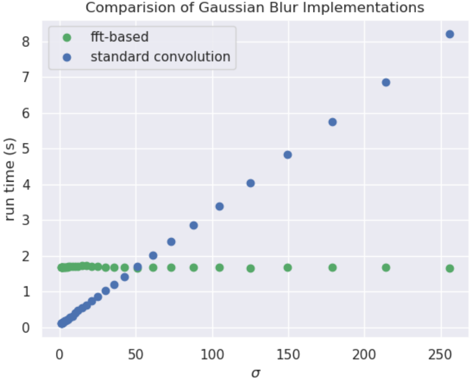
Moral

Convolution by Gaussian solves these problems.

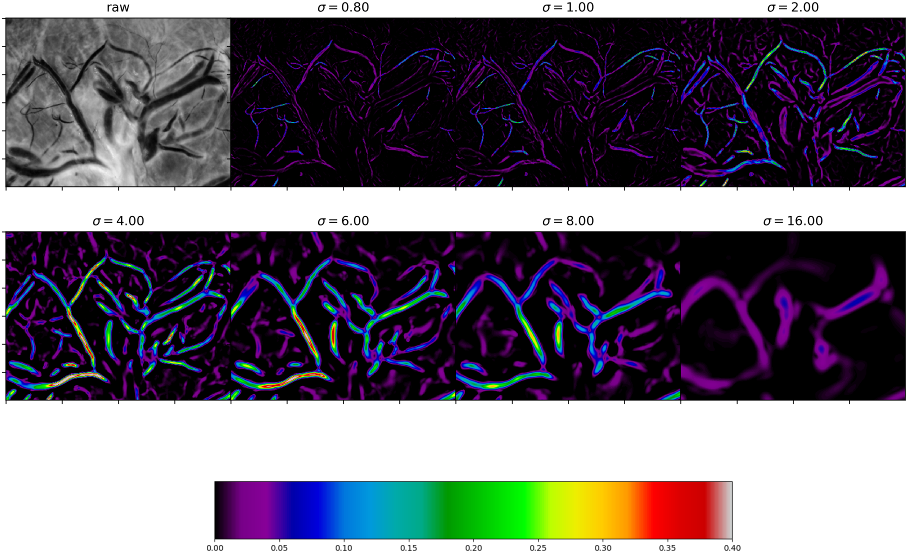
$$K(x, y; \sigma) = T_\sigma u_0 = G_\sigma \star u_0$$

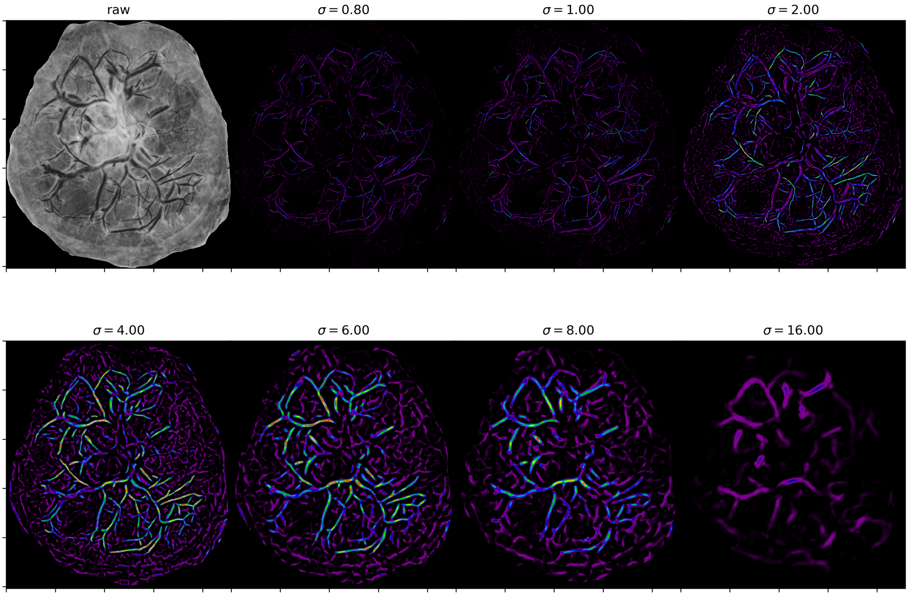
$$\text{where} \quad G_\sigma(x, y) := \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Convolution by gaussian matrix is slow
- Calculate in frequency space as a multiplication according to the convolution theorem.
- While you're at it, use a FFT.



Frangi Filter Anatomy: Scale parameter




raw
$\sigma = 0.80$
$\sigma = 1.00$
$\sigma = 2.00$
$\sigma = 4.00$
$\sigma = 6.00$
$\sigma = 8.00$
$\sigma = 16.00$

Multiscale Frangi filter

For a set of scales to probe, $\Sigma := \{\sigma_0, \sigma_1, \cdots, \sigma_n\}$, the set of n scales at which to probe,

$$\mathcal{V}_{\max}(x_0, y_0) := \max_{\sigma \in \Sigma} \{\mathcal{V}_\sigma(x_0, y_0)\} \tag{5}$$

where $\mathcal{V}_\sigma$ is the Frangi vesselness measure at scale σ for the pixel (x_0, y_0)

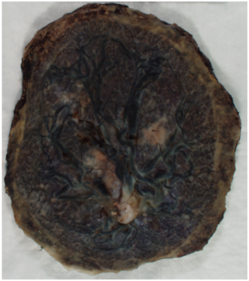
Notes

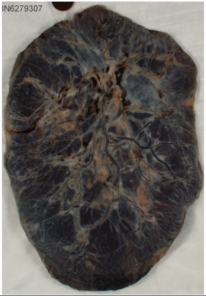
- The default "merging" strategy suggested by Frangi.
- Alternatively, we can process each scale by itself too if we want to. (Keep track of $\mathcal{V}_{\mathrm{argmax}}$, $\mathcal{V}_\Sigma$, etc.)

What scales to use?

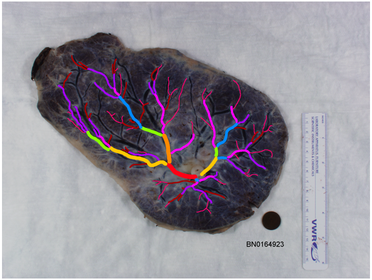
- Logarithmic spacing makes some intuitive sense.
- Experiment to determine what is large / small enough.
- Smaller scales for smaller features, larger scales for larger features.

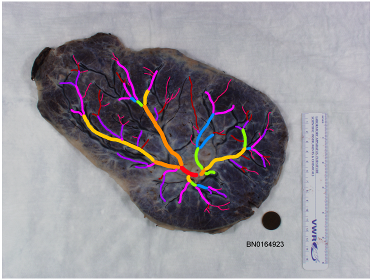
The data set



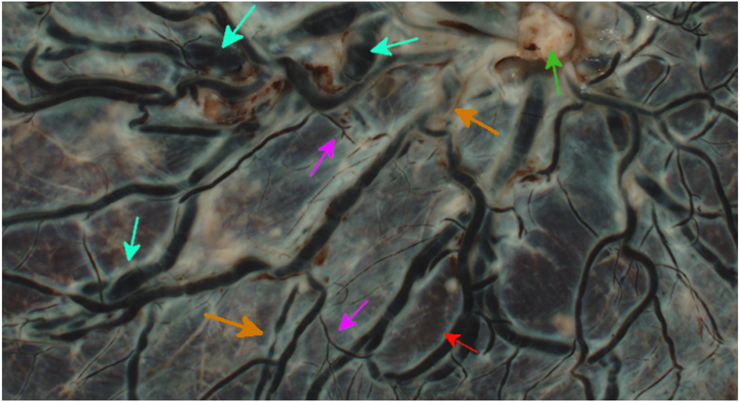


Ground truth

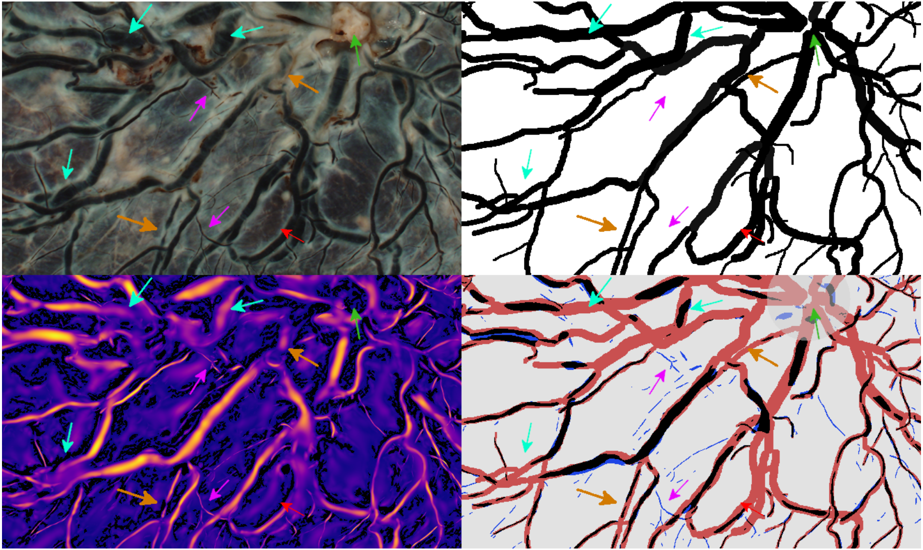




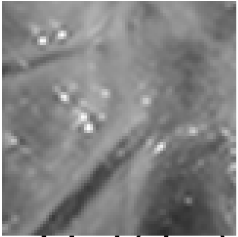
Imperfections in Data set (1/2)



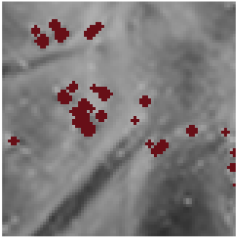
Imperfections in Data set (2/2)



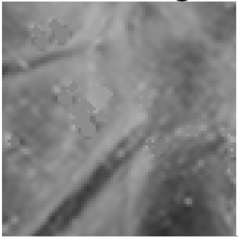
Preprocessing: Dealing with Glare



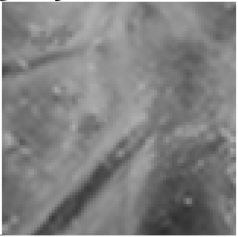
original (glary)



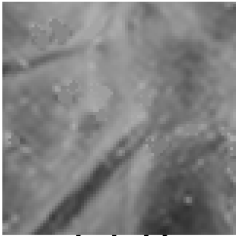
masked



boundary median

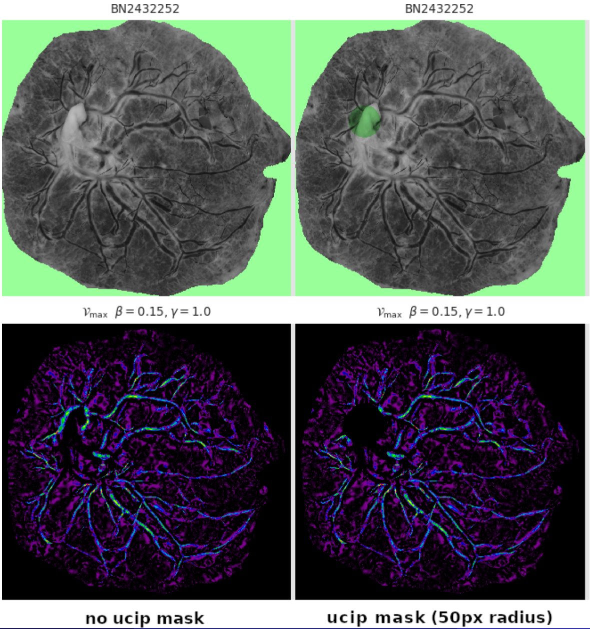


biharmonic inpainting

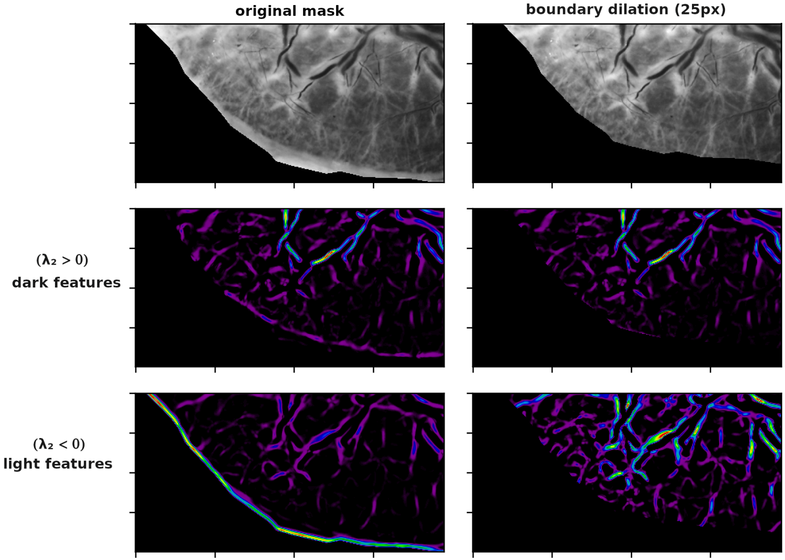


hybrid inpainting

Preprocessing: Umbilical Stump



Preprocessing: Dealing with Boundaries



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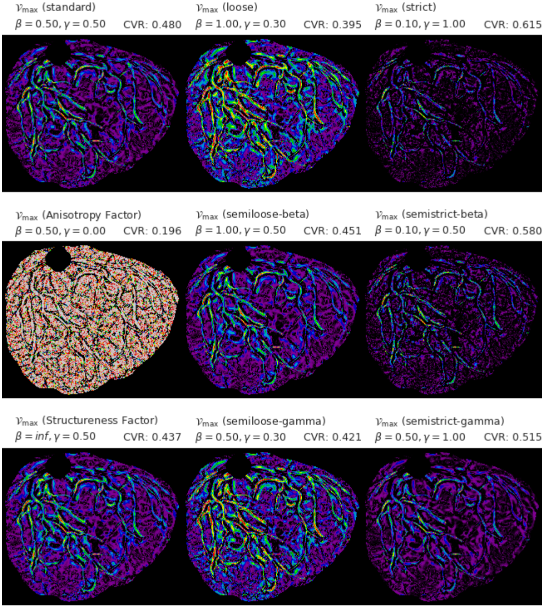
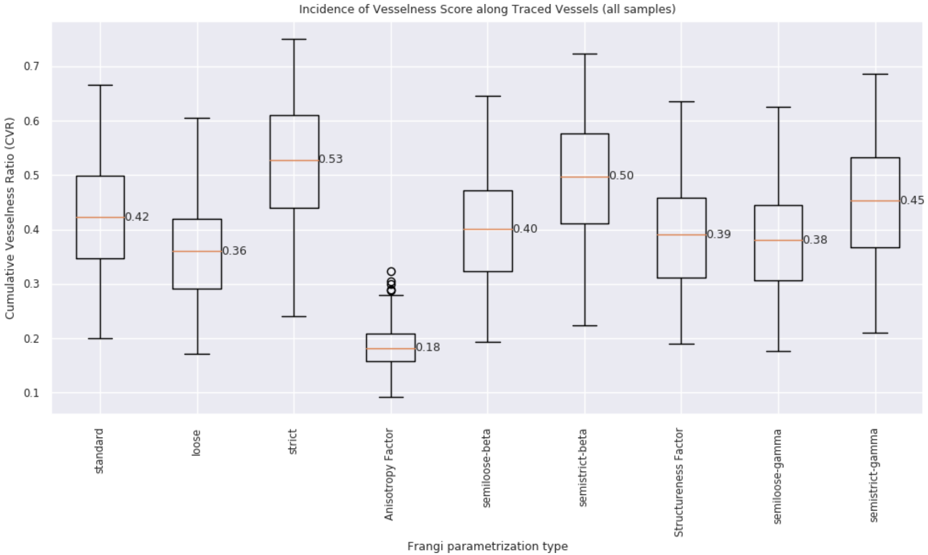
Cumulative Vesselness Ratio (CVR)



$$\mathrm{CVR}\left(\mathcal{V}_{\max}\right) := \frac{\sum_{G \subset \mathrm{I}} \mathcal{V}_{\max}(x_0, y_0)}{\sum_{\mathrm{I}} \mathcal{V}_{\max}(x_0, y_0)}$$

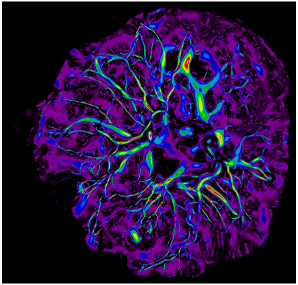
where $G \subset \mathrm{I}$ is the ground truth

label	β	γ
standard	0.5	0.5
loose	1.0	0.3
strict	0.10	1.0
semiloose-β	1.0	0.5
semistrict-β	0.1	0.5
semiloose-γ	0.5	0.3
semistrict-γ	0.5	1.0
anisotropy only	0.5	0
structureness only	∞	0.5

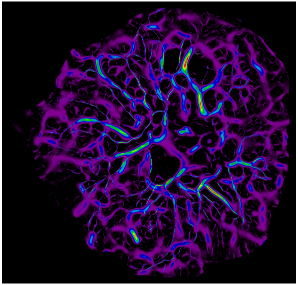

Incidence of Vesselness Score along Traced Vessels (all samples)
Cumulative Vesselness Ratio (CVR)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.42
0.36
0.53
0.18
0.40
0.50
0.39
0.38
0.45
standard
loose
strict
Anisotropy Factor
semiloose-beta
semistrict-beta
Structureness Factor
semiloose-gamma
semistrict-gamma
Frangi parametrization type

Comparing Frangi Hessian to Frangi with Weingarten Map

(g) Raw Sample



(h) $\mathcal{V}_{\max}$ (Hessian eigvals)

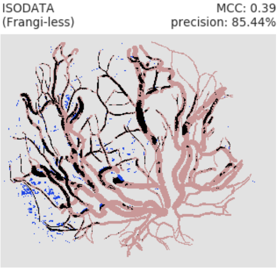


(i) $\mathcal{V}_{\max}$ (Weingarten eigvals)

- Finding Hessian eigenvalues is about 16 times faster (25 scales 24 seconds vs. 7 minutes. mean of 25 samples)
- Scale dependency not the same
- Frangi parameters seem less important for Weingarten map

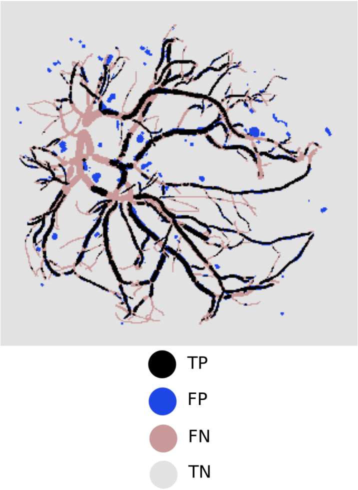
A non-Frangi segmentation method: ISODATA (Ridler-Calvert)

$$\alpha_{\text{ISO}} = \operatorname*{arg\,min}_{\alpha} \left(\frac{1}{2} \left[\text{mean} \left\{ \mathsf{I}(x, y) \mid \mathsf{I}(x, y) \leq \alpha \right\} + \text{mean} \left\{ \mathsf{I}(x, y) \mid \mathsf{I}(x, y) > \alpha \right\} \right] \right)$$



Since the vascular structure in our image domain is darker than the background, we select pixels where $\mathsf{I}(x, y) < \alpha_{\mathsf{ISO}}$.

Binary Classification of Segmentation Methods



Scoring Methods

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\text{MCC} = \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$

Thresholding Methods

Fixed Threshold (`thresh-low` and `thresh-high`)

Pick some $0 < \alpha < 1$, then extract $\mathcal{V}_{\text{max}} > \alpha$

- α must be manually selected
- We picked $\alpha = 0.3$ and $\alpha = 0.2$

scalewise (nonzero) percentile filtering (`snz-p`)

For each scale, filter under *p*th percentile: $\mathcal{V}_\sigma > \alpha_p$.

- Use *nonzero* percentile for each scale
- Smaller scales get in easier
- Good for unscaled Frangi, unknown filter response
- Very noisy if Σ chosen poorly

Signed Frangi

Don't throw anything away!

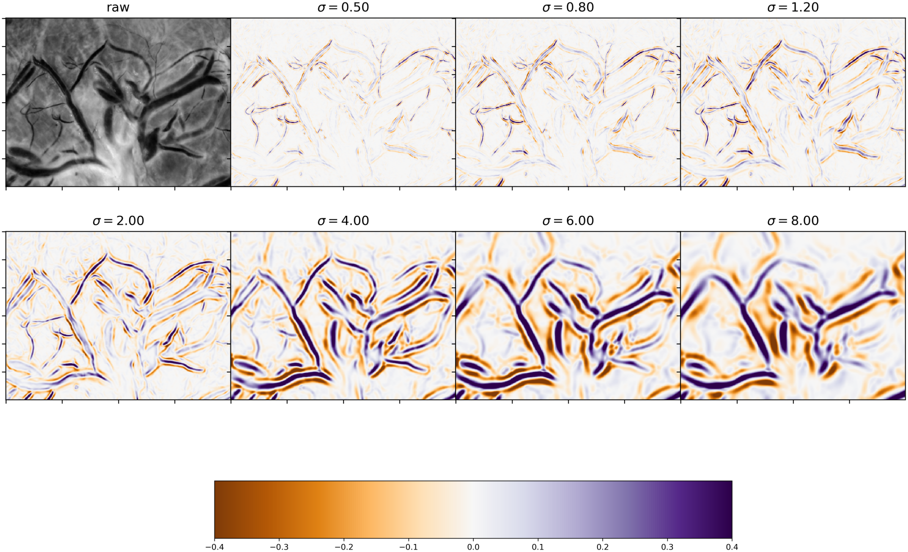
$$\mathcal{V}_\sigma^{(+)}(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_\sigma^{(-)}(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 < 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

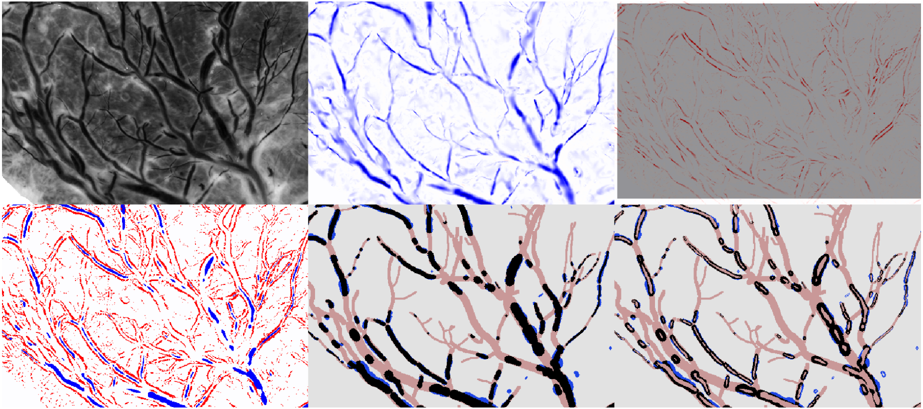
$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1|$$

- This gives us a $\mathcal{V}_{\max}^{(+)}$ and $\mathcal{V}_{\max}^{(-)}$ with same calculation time.
- We might want to use a subset $\Sigma^{(-)} \subset \Sigma$ (smaller scales only)

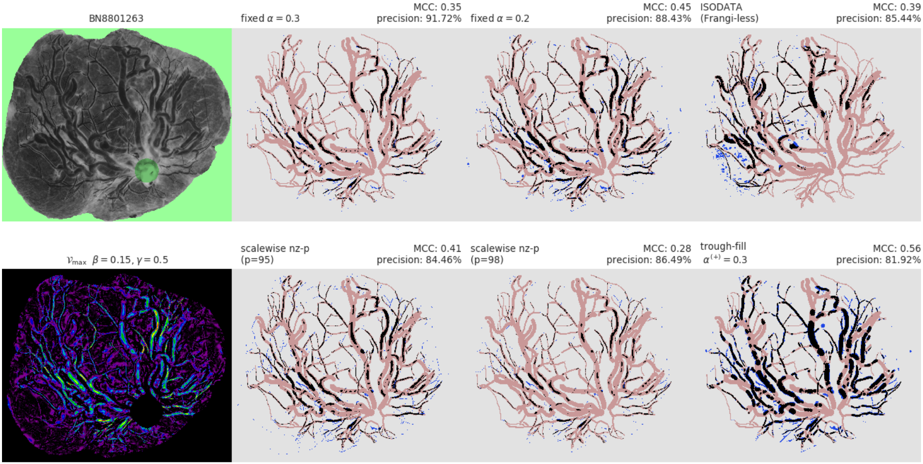
Signed Frangi



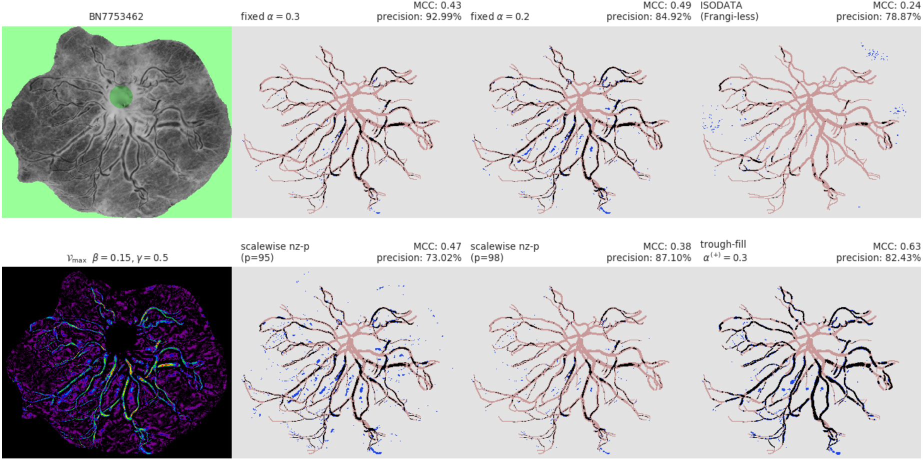
Trough Filling Method

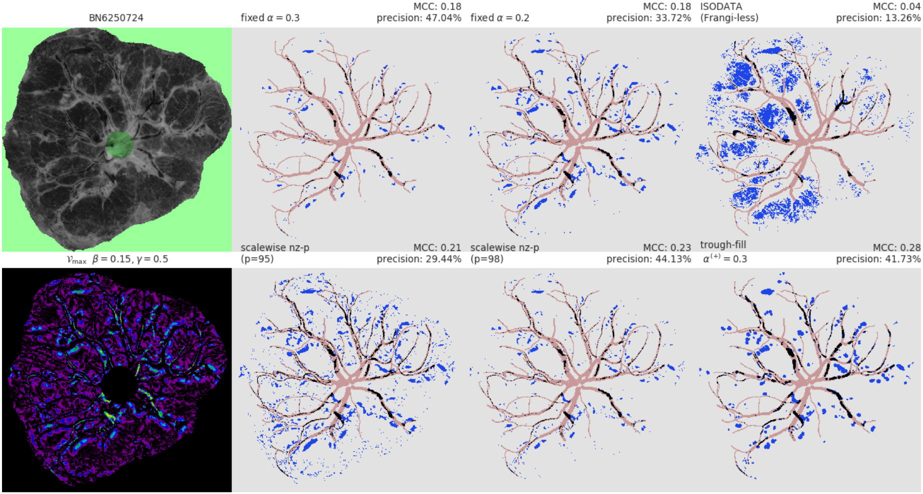


Example Results (1/4)

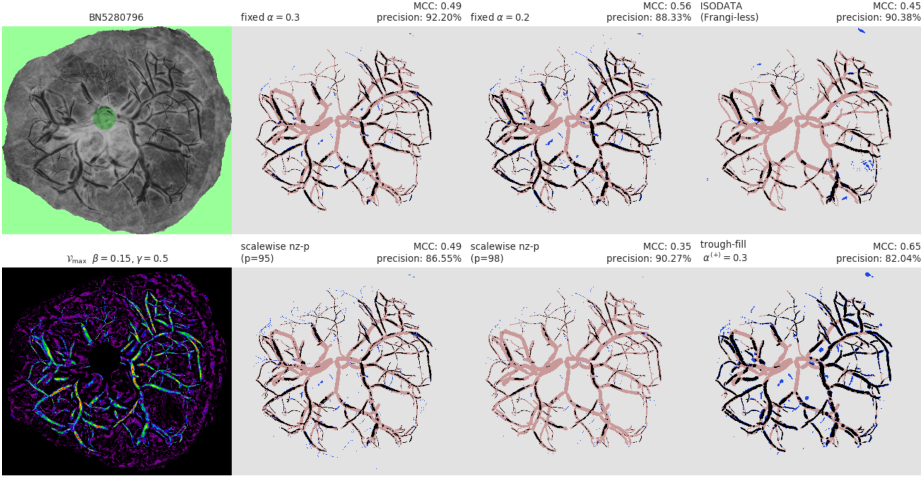


Example Results (2/4)

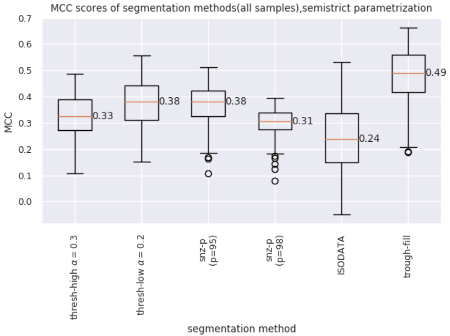


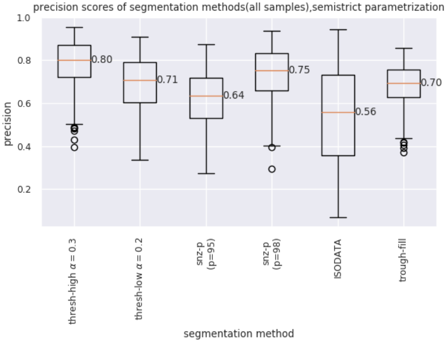

BN6250724
fixed $\alpha = 0.3$
MCC: 0.18
precision: 47.04%
fixed $\alpha = 0.2$
MCC: 0.18
precision: 33.72%
ISODATA
(Frangi-less)
MCC: 0.04
precision: 13.26%
$\mathcal{V}_{\max}$ $\beta = 0.15, \gamma = 0.5$
scalewise nz-p
(p=95)
MCC: 0.21
precision: 29.44%
scalewise nz-p
(p=98)
MCC: 0.23
precision: 44.13%
trough-fill
$\alpha^{(+)} = 0.3$
MCC: 0.28
precision: 41.73%

Example Results (4/4)



Results on All Samples ($\beta = 0.15, \gamma = 0.5$)





Results on All Samples ($\beta = 0.10, \gamma = 1.0$)