Optimized Strict Multiscale Frangi Prefiltering for Segmentation

Towards an automated PCSVN extraction

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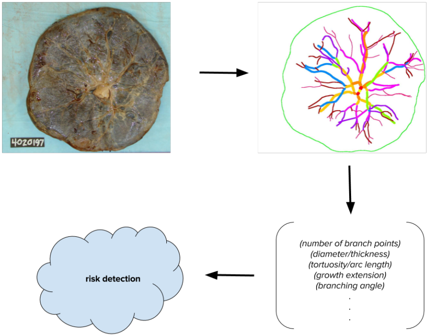
Research Goals



Vascular Network Extraction in Placentas

- **Motivation:** Accurate measurement of the vascular structure of a placental sample can be used to predict neonatal risk factors, specifically ASD.
- **Challenge:** Currently no automated method of obtaining traces of PCSVN. Manual tracing is labor intensive but necessary for feature analysis.
- **Research Goal:** Provide a fully automated method of extraction.

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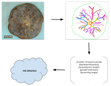

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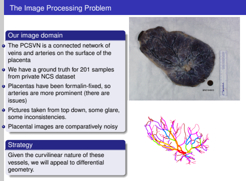
1. In the figure, a manual trace of the placental chorionic vascular surface network (PCSVN) is performed. This trace is measured in multiple ways. Those measurements are turned into a feature vector, which can be used to predict a risk. Refer to Boruta paper.
2. Manual tracing requires like 5 hours or something and requires training. There is some guesswork that's done in it too and some limitations in the ground truth itself (will cover later)

Our image domain

- The PCSVN is a connected network of veins and arteries on the surface of the placenta
- We have a ground truth for 201 samples from private NCS dataset
- Placentas have been formalin-fixed, so arteries are more prominent (there are issues)
- Pictures taken from top down, some glare, some inconsistencies.
- Placental images are comparatively noisy

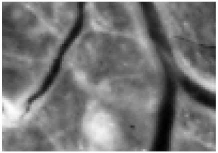
Strategy

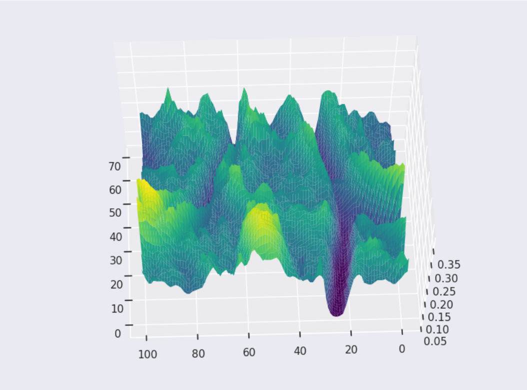
Given the curvilinear nature of these vessels, we will appeal to differential geometry.

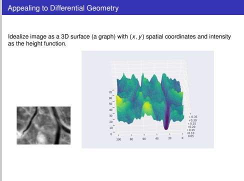


1. The surface of the placenta has a lot of changes in color/topology apart from the PCSVN so a lot of techniques that work elsewhere for vascular segmentation seem to fail here. Thus segmentation is more complicated that than say, an eyeball MRI (like original Frangi paper)
2. Mention colors are simply vessel widths (3 to 19 odds) are part of the tracing protocol. that's really outside of the scope of this thesis, but kept anytime we show a ground truth because they're pretty
3. redo this page with a placenta from NCS, not EARLI

Idealize image as a 3D surface (a graph) with (x, y) spatial coordinates and intensity as the height function.







1. Point of this slide is show that finding curvilinear surfaces is reasonable
2. This point is a lot clearer when you show multiple vessel widths.
3. Also way clearer later when you show the surface after Gaussian blur, not sure if I should put that here and “lie”/complicate things early on or not.
4. Crop graph, center, etc

- (Thm of Meusnier) If you look at a point on the surface and fix a tangent vector, then all surface curves through that point with that velocity will have the same curvature there. So the curvature is intrinsic to the surface, call it normal curvature.
- Varying the tangent vector, we call extremal values of normal curvature the **principal curvatures**. The associated tangent vectors are **principal directions.**
- (Thm. of Olinde Rodrigues) These principal curvatures/directions are the eigenvalues/eigenvectors of a particular map called the **Weingarten map**.

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1. "Let's just pretend we're dealing with this as a continuous surface for now"
2. Weingarten map also called shape operator. Also can just define the second fundamental form and use that matrix (for our purposes)
3. Note that all this is true for *any* kind of surface, but we really just care about graphs.
4. If you want to get into notation, you can do so as far as explicitly showing what the Weingarten map is (requires Gauss map). You probably can avoid showing any setup of Meusnier– defining curves and so on.

Relationship Between Hessian and Weingarten Map for Graphs

Weingarten Map for Graphs

Given the graph $f : U \to \mathbb{R}^3$ where $(x, y) \mapsto (x, y, h(x, y))$, the matrix representation of its Weingarten map is given by

$$\widehat{\mathrm{L}} = \mathrm{Hess}(h(x, y))\tilde{G}\,, \quad \text{where} \quad \tilde{G} := \frac{1}{\left(1 + h_x^2 + h_y^2\right)^{3/2}} \begin{bmatrix} 1 + h_y^2 & -h_x h_y \\ -h_x h_y & 1 + h_x^2 \end{bmatrix} \tag{1}$$

In particular, given a point $u = (x, y) \in U \subset \mathbb{R}^2$ where $h_x \approx h_y \approx 0$, we have $\tilde{G} \approx \mathrm{Id}$, and thus $\widehat{\mathrm{L}} \approx \mathrm{Hess}(h)$.

Approximating

- For ease of use, we can simply find eigenvalues of the Hessian instead.
- Talk about that more.
- This gives rise to a class of filters, the so-called Hessian-based filters.

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1. Define hessian as the second derivative matrix
2. Make sure you have the graph definition clearly here. It’s at the top but make it more prominent / earlier slidewise
3. Make point about when we’re not at a critcial point, we don’t guarantee any of this but it seems to work out okay
4. Fix notation in general

Show the example here. Your example calculates from a different definition, like with Gauss map etc. so maybe rework or decide what you want here.

The (Uniscale) Frangi Filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) & \text{otherwise} \end{cases} \tag{2}$$

where

$$A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \tag{3}$$

for eigenvalues λ_1, λ_2 of the Hessian (defined at each point)

$$\mathrm{H} = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix} \quad \text{such that } \mathrm{H}u_i = \lambda_i u_i \ , \ |\lambda_1| \leq |\lambda_2| \tag{4}$$

- We'll go through this step by step.

Frangi filter anatomy: Anisotropy Factor

$$\exp\left(-\frac{A^2}{2\beta^2}\right) \quad , \quad A := |\lambda_1/\lambda_2| \tag{5}$$

- This is the main thing that selects curvilinear points.
- When A is very close to 0, $\lambda_2 \gg \lambda_1$, and the surface is very anisotropic.
- Frangi suggested $\beta = \frac{1}{2}$ as a reasonable default.
- Show pictures of a curvilinear structure here, or a hangar.

Frangi filter anatomy: Structureness Factor

$$\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \quad , \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \tag{6}$$

- Purpose is to filter out numerically small values, we only want significant curvilinear content (S large).
- Important to pick a reasonable value for the image at hand / image domain.
- Frangi suggested "half the maximum value of the Hessian norm". We will define the parameter γ and define $c(\gamma) = \gamma S_{\text{max}}$, since S_{max} is the Frobenius norm of the Hessian.

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1. For example, we don't want our filter to report a point as curvilinear structure if λ_1/λ_2 is large, but we can't differentiate between $\lambda_2 = 100, \lambda_1 = .01$ and $\lambda_2 = .001, \lambda_1 = .000001$
2. This is actually critically important to define, entire filter will be extremely noisy otherwise. Less important in MRI images, but more important for our context where there is significant noise. Talk about the standard implementation where c=15 is the default for seemingly no reason.
3. I'm not sure when/how to introduce γ instead but it should be prominent enough that I don't have to worry about using it exclusively in later slides

Frangi filter anatomy: Putting it together

Frangi filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \tag{7}$$

$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1| \tag{8}$$

- $\lambda_2 > 0$ means the point would be a local minimum (at a critical point). We are looking for a local maximum by looking for bright curvilinear objects. You can switch this ($\lambda_2 < 0$ as the case) if you are looking for dark curvilinear.
- Point of $\exp(\ldots)$ and $1 - \exp(\ldots)$ structure is that the filter decays rapidly as anisotropy or structureness decrease.

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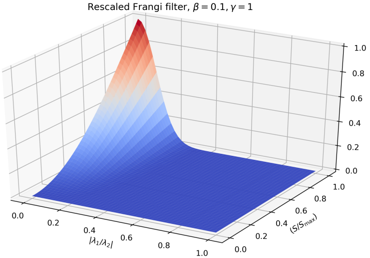
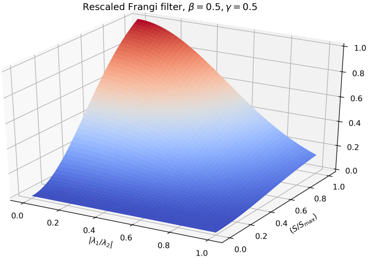
1. Really look into the bright/dark curvilinear subject and try to explain better.
2. Get rid of the sigma until after scale space theory?
3. If it’s too long to get to this point, maybe start showing samples already

Frangi filter anatomy: Choosing Parameters

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \tag{9}$$

$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1| \tag{10}$$





- We will show a lot of data with different choices of parameters later.

- Obviously the image is not actually a continuous surface. It is a particular sampling of a surface.
- We want to embed the image this image with a "resolution" parameter.
- Motivate and say some basic axioms.
- Convolution by Gaussian solves these problems, discrete derivatives are fine too

Axioms

- Linear shift and rotational invariance
- Semigroup property
- Continuity of scale parameter
- Causality condition

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Axioms

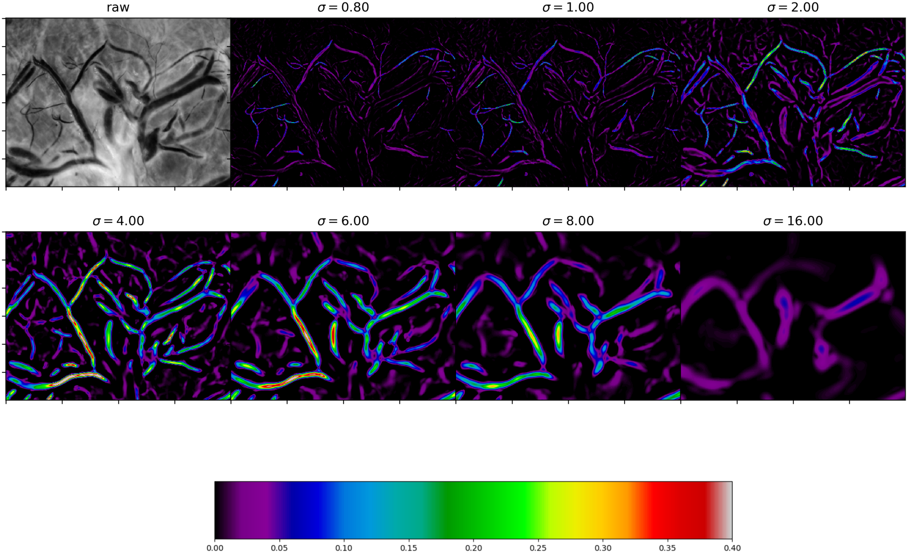
- Linear shift and rotational invariance
- Semigroup property
- Continuity of scale parameter
- Causality condition

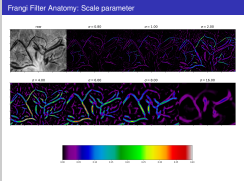
1. Maybe spend an extra slide on causality condition. Basically, as resolution decreases (scale increases) then local minima do not decrease, and local maxima do not increase.
2. Prepare a couple more details to get into here.

Implementation Detail: Calculating Discrete Hessian

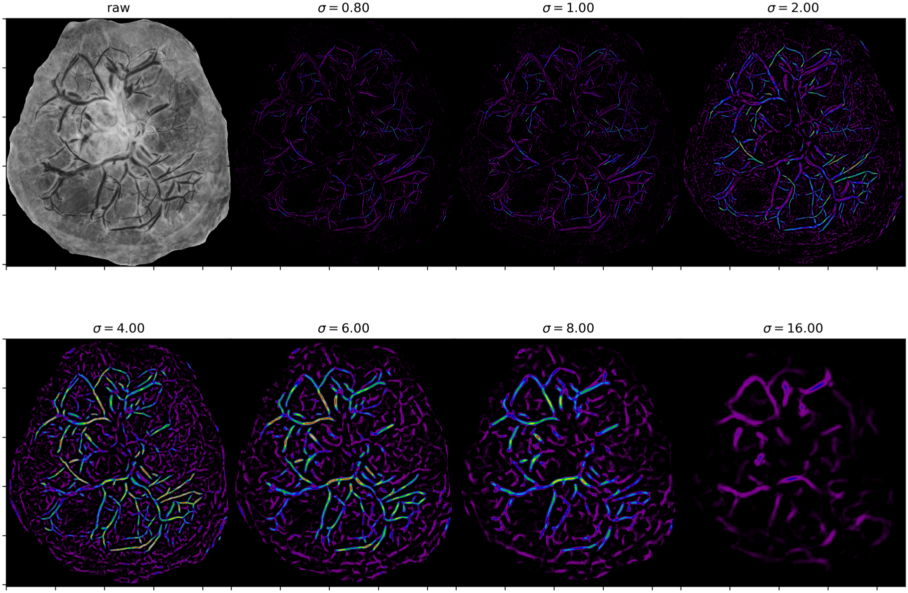
- Convolution by gaussian matrix is slow.
- Calculate in frequency space as a multiplication
- Much much much faster
- Show the speed graph, explain MSE findings

Frangi Filter Anatomy: Scale parameter

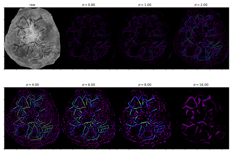




1. Describe relationship between vessel width and (LATER)
2. Describe relative strength of outputs and what's too large
3. Better to come back to these points later after describing research protocol? Or do that earlier?
4. You really should rerun these just to make sure things are the same.



raw
$\sigma = 0.80$
$\sigma = 1.00$
$\sigma = 2.00$
$\sigma = 4.00$
$\sigma = 6.00$
$\sigma = 8.00$
$\sigma = 16.00$



1. Same thing but whole plate. Probably remove this. Otherwise, do some cropping.

Multiscale Frangi filter

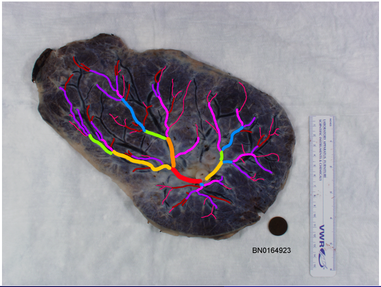
- Definition
- Standard Merging strategy
- It's faster so can pick more
- Logarithmic spacing is kind of sensible
- We can process each scale by itself too

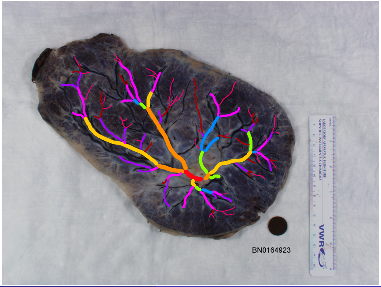
$$\mathcal{V}_{\mathrm{max}}(x_0, y_0) := \max_{\sigma \in \Sigma} \{\mathcal{V}_\sigma(x_0, y_0)\} \tag{11}$$

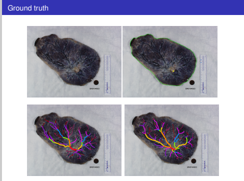
where $\Sigma := \{\sigma_0, \sigma_1, \cdots, \sigma_n\}$ is the set of n scales at which to probe, and $\mathcal{V}_\sigma$ is the Frangi vesselness measure at scale σ for the pixel (x_0, y_0)

The data set

Ground truth

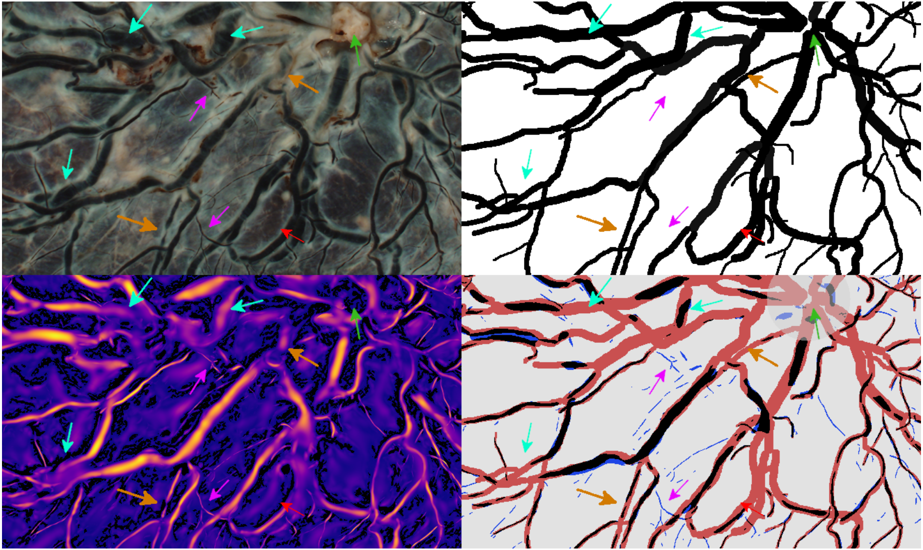






1. Maybe show these cropped, who cares
2. Maybe show enlarged UCIP and border, IDK if these will show up well
3. Maybe show cropped, merged trace by itself on one slide (quickly)

Imperfections/Complications in Data set



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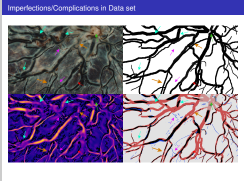

1. The 2×2 annotations might be too complicated, might not show up on the projector. Maybe just switch between two slides? or show different colors of annotations on each slide?
2. Talk about:
   - UCIP noise
   - too small and too big
   - bad perfusion
   - crossings

Preprocessing: Glare

Preprocessing: Umbilical Stump

Cumulative Vesselness Ratio

Just show a few parametrization methods, don't show all

Fixed threshold of Vmax

Scalewise percentile filtering

Trough-filling (1/2)

Show definition of signed Frangi, inset

Trough-filling (2/2)

A non-Frangi segmentation method: ISODATA

This represents a good idea that doesn't rely on diffgeo or local anything

Binary Classification (1/2)

Confusion matrix with TP, FP, TN, FN

MCC and Precision Scores

Example Segmentation Results

Spend a couple slides here talking about

- parametrization choices effect
- threshold choices effect
- talk about “good” false positives and “bad” false positives
- talk about good samples and bad samples

Boxplots

Conclusion

Appendix

Put some proofs / extra things here