Optimized Strict Multiscale Frangi Prefiltering for Segmentation

Towards an automated PCSVN extraction

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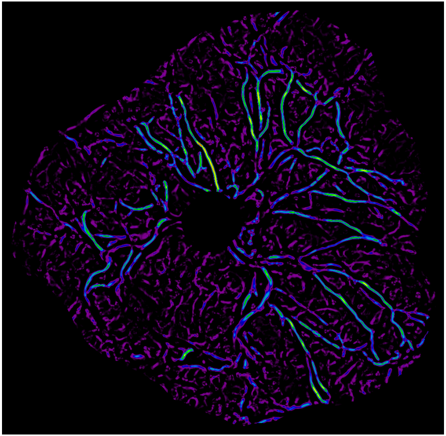
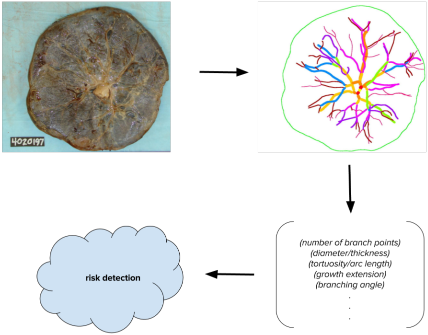
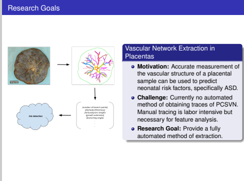


Research Goals



Vascular Network Extraction in Placentas

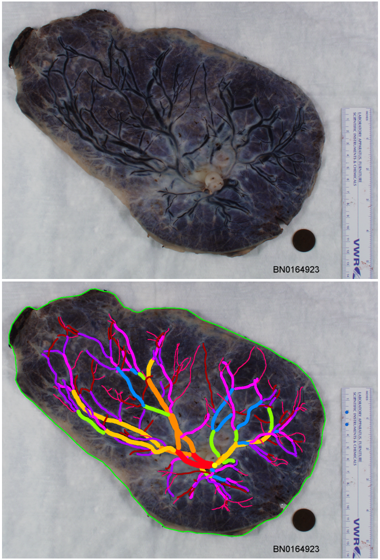
- **Motivation:** Accurate measurement of the vascular structure of a placental sample can be used to predict neonatal risk factors, specifically ASD.
- **Challenge:** Currently no automated method of obtaining traces of PCSVN. Manual tracing is labor intensive but necessary for feature analysis.
- **Research Goal:** Provide a fully automated method of extraction.



1. In the figure, a manual trace of the placental chorionic vascular surface network (PCSVN) is performed. This trace is measured in multiple ways. Those measurements are turned into a feature vector, which can be used to predict a risk. Refer to Boruta paper.
2. Manual tracing requires like 5 hours or something and requires training. There is some guesswork that’s done in it too and some limitations in the ground truth itself (will cover later)

Our image domain

- The PCSVN is a connected network of veins and arteries on the surface of the placenta
- Ground truth for 201 samples from private NCS dataset
- Placentas have been formalin-fixed, so arteries are more prominent.
- Placental images are comparatively noisy



The Image Processing Problem

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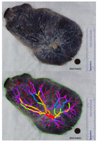
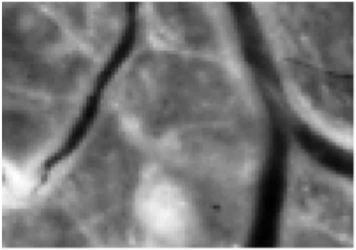
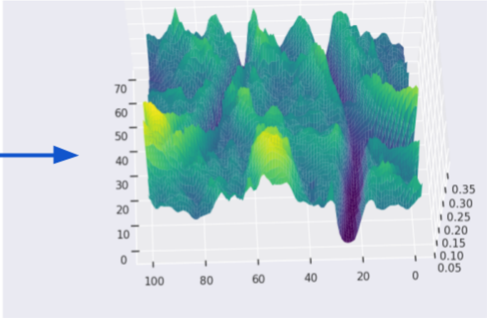


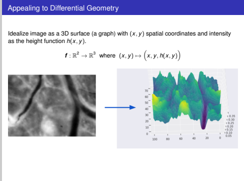
1. The surface of the placenta has a lot of changes in color/topology apart from the PCSVN so a lot of techniques that work elsewhere for vascular segmentation seem to fail here. Thus segmentation is more complicated that than say, an eyeball MRI (like original Frangi paper)
2. Mention colors are simply vessel widths (3 to 19 odds) are part of the tracing protocol
3. NCS is national children's study
4. formalin is basically formaldehyde and water, basically dried out. other samples i had were barium injected so the vessels were kind of whiter. other samples are kind of red against red. etc. etc.
5. Pictures taken from top down, some glare, some inconsistencies.

Idealize image as a 3D surface (a graph) with (x, y) spatial coordinates and intensity as the height function $h(x, y)$.

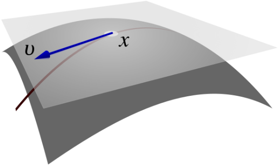
$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^3 \text{ where } (x, y) \mapsto \big(x, y, h(x, y)\big)$$







1. Point of this slide is show that finding curvilinear surfaces is reasonable
2. This point is a lot clearer when you show multiple vessel widths, this is just a little inset
3. Also way clearer later when you show the surface after Gaussian blur, not sure if I should put that here and “lie”/complicate things early on or not.
4. f is the graph, h is the height function
5. Technically we just map from an open set but whatever



Meusnier's Theorem

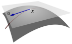
If you look at a point on the surface and fix a tangent vector, then all surface curves through that point with that velocity will have the same curvature there. So the curvature is intrinsic to the surface. Call it **normal curvature.**

Definition

Extremal values of normal curvature are called **principal curvatures** of the surface at that point. The extremizing tangent vectors are **principal directions.**

Theorem of Olinde Rodrigues

These principal curvatures/directions are the eigenvalues/eigenvectors of a particular map called the **Weingarten map**.



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1. "Let's just pretend we're dealing with this as a continuous surface for now"
2. Weingarten map also called shape operator. Also can just define the second fundamental form and use that matrix (for our purposes)
3. Note that all this is true for *any* kind of surface, but we really just care about graphs.
4. If you want to get into notation, you can do so as far as explicitly showing what the Weingarten map is (requires Gauss map). You probably can avoid showing any setup of Meusnier– defining curves and so on.

Relationship Between Hessian and Weingarten Map for Graphs

Weingarten Map for Graphs

Given the graph $f : U \to \mathbb{R}^3$ where $(x, y) \mapsto (x, y, h(x, y))$, the matrix representation of its Weingarten map is given by

$$\widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G}$$

where $\tilde{G} := \frac{1}{(1+h_x^2+h_y^2)^{3/2}} \begin{bmatrix} 1+h_y^2 & -h_x h_y \\ -h_x h_y & 1+h_x^2 \end{bmatrix}$ and $\mathrm{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix}$

Approximating

- In particular, given a point where $h_x \approx h_y \approx 0$, we have $\tilde{G} \approx \mathrm{Id}$, and thus $\widehat{\mathrm{L}} \approx \mathrm{Hess}(h)$.
- For ease of use, we can simply find eigenvalues of the Hessian instead.
- This gives rise to a class of filters, the so-called Hessian-based filters.

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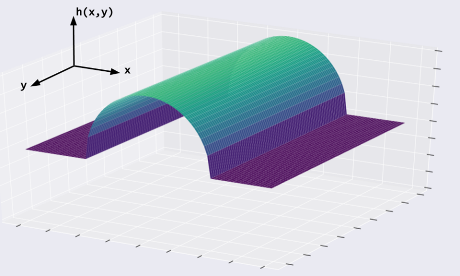
1. Make sure you have the graph definition clearly here. It's at the top but make it more prominent / earlier slidewise
2. Make point about when we're not at a critical point, we don't guarantee any of this (but it seems to work out okay)
3. "Remember this point right here, we will come back to this."

Cylindrical Ridge of radius r

Let f be the graph given by

$$f : \mathbb{R}^2 \to \mathbb{R}^3 \text{ by } f(x, y) = (x, y, h(x, y)),$$

$$\text{with } h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$$



$$h_x = \frac{-x}{\sqrt{r^2 - x^2}} \quad , \quad h_{xx} = \frac{-r^2}{(r^2 - x^2)^{3/2}}$$

$$h_y = 0 \quad , \quad h_{yy} = 0 \quad , \quad h_{xy} = 0$$

Example: Finding Weingarten map and Principal Curvatures

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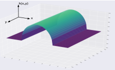

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1. Show the example here. Your example calculates from a different definition, like with Gauss map etc. so maybe rework or decide what you want here.
2. “Calculate boring partial derivatives”
3. Note: the formula for weinmat is on the next slide

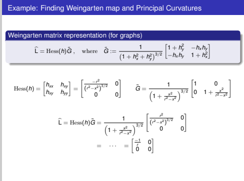
Example: Finding Weingarten map and Principal Curvatures

Weingarten matrix representation (for graphs)

$$\widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G}\,, \quad \text{where} \quad \tilde{G} := \frac{1}{\left(1+h_x^2+h_y^2\right)^{3/2}} \begin{bmatrix} 1+h_y^2 & -h_x h_y \\ -h_x h_y & 1+h_x^2 \end{bmatrix}$$

$$\mathrm{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-r^2}{\left(r^2-x^2\right)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \qquad \tilde{G} = \frac{1}{\left(1+\frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} 1 & 0 \\ 0 & 1+\frac{x^2}{r^2-x^2} \end{bmatrix}$$

$$\begin{aligned} \widehat{\mathrm{L}} = \mathrm{Hess}(h)\tilde{G} &= \frac{1}{\left(1+\frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} \frac{r^2}{\left(r^2-x^2\right)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \\ &= \quad \cdots \quad = \begin{bmatrix} \frac{-1}{r} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$



Here's the steps without dots just in case you need to add it back in:

$$\widehat{\mathsf{L}} = \mathrm{Hess}(h)\tilde{G} = \frac{1}{\left(1+\frac{x^2}{r^2-x^2}\right)^{3/2}}\begin{bmatrix}\frac{r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0\end{bmatrix}$$

$$= \frac{1}{\left(\frac{r^2}{r^2-x^2}\right)^{3/2}}\begin{bmatrix}\frac{r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0\end{bmatrix} = \frac{\left(r^2-x^2\right)^{3/2}}{r^3}\begin{bmatrix}\frac{-r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0\end{bmatrix} = \begin{bmatrix}\frac{-1}{r} & 0 \\ 0 & 0\end{bmatrix}$$

1. This is just some matrix multiplication, exponent rules, and finding common denominators. This would be a good challenge problem for Math 112A minus the matrix.
2. Mostly, just point out it's $\begin{bmatrix}a & 0 \\ 0 & 0\end{bmatrix}$ times $\begin{bmatrix}b & 0 \\ 0 & c\end{bmatrix}$

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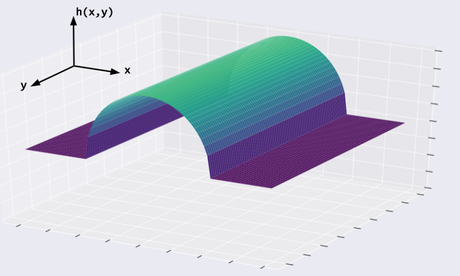
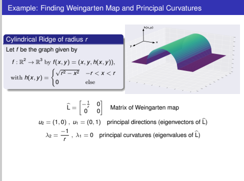


$$\widehat{\mathrm{L}} = \begin{bmatrix} -\frac{1}{r} & 0 \\ 0 & 0 \end{bmatrix}$$ Matrix of Weingarten map

$u_2 = (1, 0)$, $u_1 = (0, 1)$ principal directions (eigenvectors of $\widehat{\mathrm{L}}$)

$\lambda_2 = \dfrac{-1}{r}$, $\lambda_1 = 0$ principal curvatures (eigenvalues of $\widehat{\mathrm{L}}$)



1. **META**: need to show what hessian eigs/eigvecs are and emphasize that at the ridge point it's identical
2. **META**: also can explain (here or later) if there's a stronger signal at the ridge point or if it's literally anywhere on the ridge
3. This is what to stick in somewhere:

$$\mathrm{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-r^2}{(r^2 - x^2)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \tag{1}$$

Note that eigenvalues are still 0 and the top left element (now a function of x), eigenvectors are (can be chosen to be) the same throughout, and if $x = 0$ (at the crest) then $\widehat{\mathrm{L}} = \mathrm{Hess}$ exactly, as expected.

The (Uniscale) Frangi Filter

$$\mathcal{V}(x, y) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) & \text{otherwise} \end{cases}$$

where

$$A := |\lambda_1 / \lambda_2| \quad \text{(Anisotropy Measure)}$$

$$\text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad \text{(Structureness Measure)}$$

for eigenvalues λ_1, λ_2 of the Hessian (at point (x,y)), $|\lambda_1| \leq |\lambda_2|$
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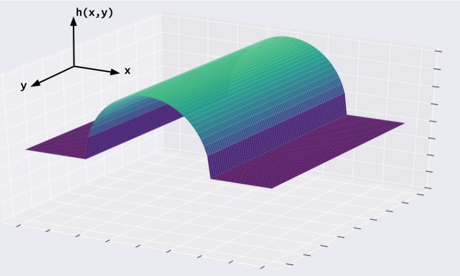
1. Mostly point out how it all fits together. Two simple functions of the principal curvatures, put into exponential forms, with a parameter to control each.
2. “We’ll go through this step by step and look at each part more closely”
3. Mention that if you wanted to use the “real” principal curvatures here you’re more than welcome to
4. Don’t need to spend too much time on this slide, we’ll put it back again

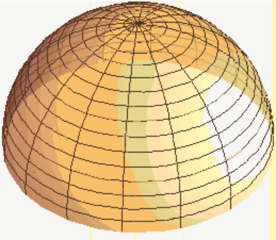
Anisotropy Factor

$$\exp\left(-\frac{A^2}{2\beta^2}\right) \quad , \quad A := \left|\frac{\lambda_1}{\lambda_2}\right| \tag{2}$$

- For selecting anisotropic content (lines not blobs)
- When A is very close to 0, $|\lambda_2| \gg |\lambda_1|$, and the factor is ≈ 1.
- Choosing parameter: Frangi suggested $\beta = \frac{1}{2}$ as a reasonable default.



(a) Anisotropic


(b) Isotropic

Frangi filter anatomy

Structureness Factor

$$\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \quad , \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \tag{3}$$

- Purpose: Filter out numerically insignificant values.
- Important to pick a reasonable value for the image at hand / image domain.
- Frangi suggested “half the maximum value of the Hessian norm”. We will define the parameter γ and define $c(\gamma) = \gamma S_{\text{max}}$, since S_{max} is the Frobenius norm of the Hessian.

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1. For example, we don't want our filter to report a point as curvilinear structure if λ_1/λ_2 is large, but we can't differentiate between $\lambda_2 = 100, \lambda_1 = .01$ and $\lambda_2 = .001, \lambda_1 = .000001$
2. This is actually critically important to define, entire filter will be extremely noisy otherwise. Less important in MRI images, but more important for our context where there is significant noise. Talk about the standard implementation where c=15 is the default for seemingly no reason.
3. I'm not sure when/how to introduce γ instead but it should be prominent enough that I don't have to worry about using it exclusively in later slides

Frangi filter anatomy: Putting it together

Frangi filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \tag{4}$$

$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1| \tag{5}$$

- $\lambda_2 > 0$ means the point would be a local minimum (at a critical point). We are looking for a local maximum by looking for bright curvilinear objects. You can switch this ($\lambda_2 < 0$ as the case) if you are looking for dark curvilinear.
- Point of $\exp(\ldots)$ and $1 - \exp(\ldots)$ structure is that the filter decays rapidly as anisotropy or structureness decrease.

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1. **META**: Really look into the bright/dark curvilinear subject and try to explain better.
2. **META**: Significance of sign of principal curvature (real one from Weingarten map)?
3. **META**: Get rid of the sigma until after scale space theory?
4. If it’s too long to get to this point, maybe start showing samples already

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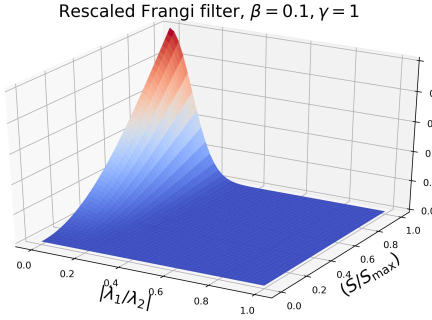

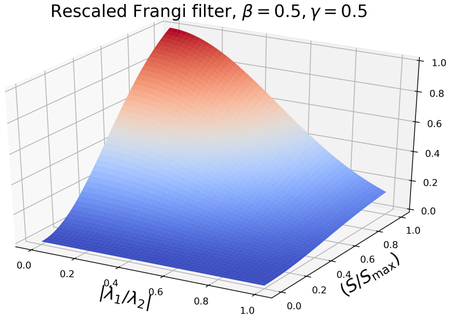

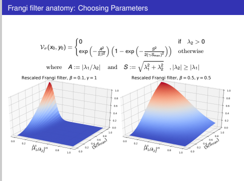

1. We will soon show how choices of parameters manifest in our actual research problem here, but we have one more thing to take care of first

Linear Scale Space Theory for Kids

- Obviously the image is not actually a continuous surface. It is a particular sampling I of the surface.
- We want to create a “family of derived images” with a “resolution” parameter $\sigma \geq 0$, ideally from some operator T_σ acting on the image I.

$$K(x, y; \sigma) = T_\sigma$$

Some Axioms

- Linear shift and rotational invariance
- Semigroup property $T_{\sigma_1+\sigma_2}\mathrm{I} = T_{\sigma_1}\mathrm{I} + T_{\sigma_2}\mathrm{I}$
- Continuity of scale parameter σ
- Causality condition

Moral

Convolution by Gaussian solves these problems.

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- Causality condition

Moral

Convolution by Gaussian solves these problems.

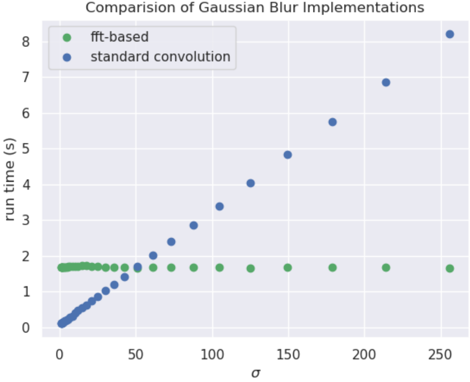
1. Maybe spend an extra slide on causality condition. Basically, as resolution decreases (scale increases) then local minima do not decrease, and local maxima do not increase.
2. Prepare a couple more details to get into here.

Implementation Detail: Calculating Discrete Hessian

$$K(x, y; \sigma) = T_\sigma u_0 = G_\sigma \star u_0$$

$$\text{where} \quad G_\sigma(x, y) := \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Convolution by gaussian matrix is slow
- Calculate in frequency space as a multiplication according to the convolution theorem.
- While you're at it, use a FFT.

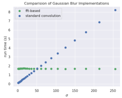


Implementation Detail: Calculating Discrete Hessian

$$K(x,y;\sigma) = T_\sigma u_0 = G_\sigma \star u_0$$

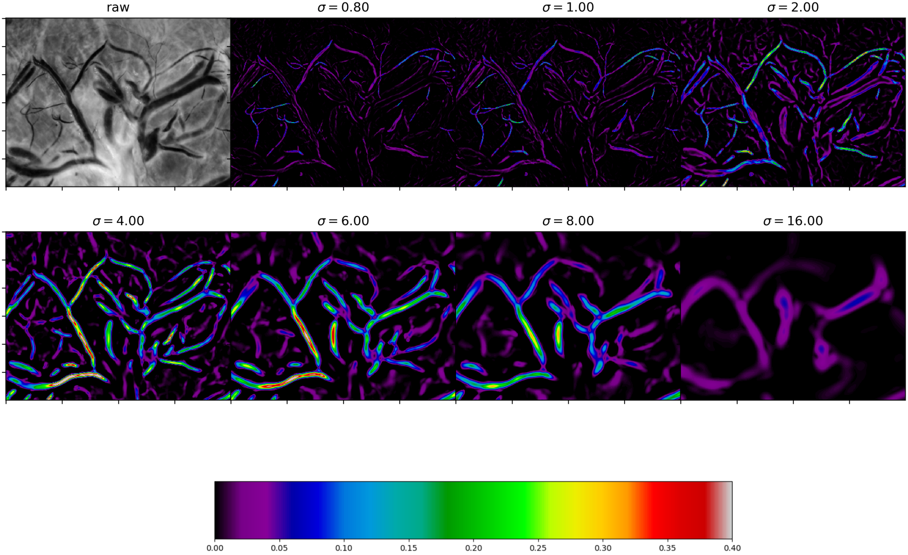
$$\text{where} \quad G_\sigma(x,y) := \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

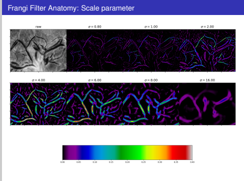
- Convolution by gaussian matrix is slow
- Calculate in frequency space as a multiplication according to the convolution theorem.
- While you're at it, use a FFT.



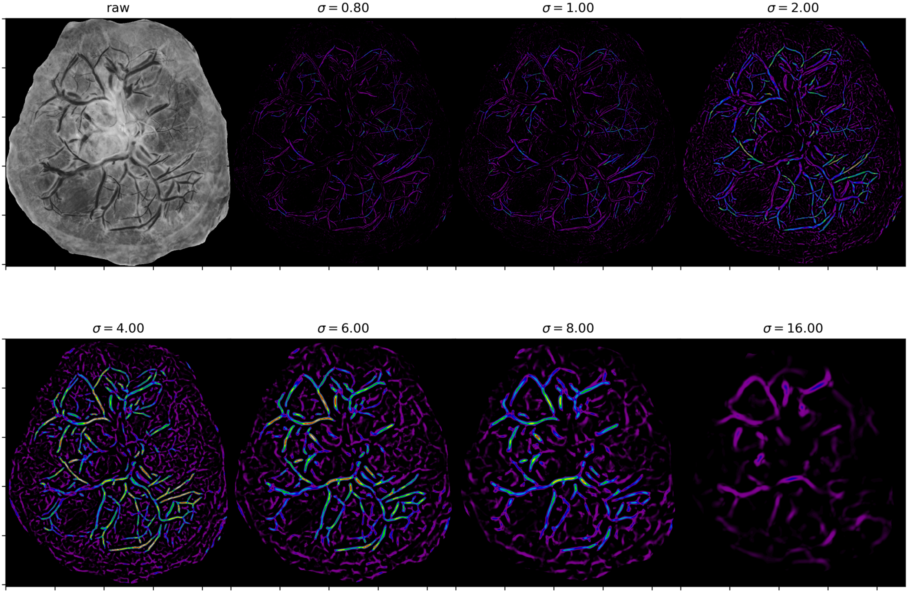
1. Add some of the math but not too much
2. Why is this so much faster now?
3. This is short. Mention MSE findings

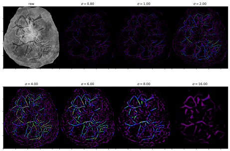
Frangi Filter Anatomy: Scale parameter





1. Mention general relationship between vessel width.
2. Describe relative strength of outputs and what's too large.
3. This is unscaled output


raw
σ = 0.80
σ = 1.00
σ = 2.00
σ = 4.00
σ = 6.00
σ = 8.00
σ = 16.00



1. Same thing but whole plate. Show this for like 5 seconds.
2. “This is basically just to show that these trends work throughout the whole image, etc”

Multiscale Frangi filter

For a set of scales to probe, $\Sigma := \{\sigma_0, \sigma_1, \cdots, \sigma_n\}$, the set of n scales at which to probe,

$$\mathcal{V}_{\max}(x_0, y_0) := \max_{\sigma \in \Sigma} \{\mathcal{V}_\sigma(x_0, y_0)\} \tag{6}$$

where $\mathcal{V}_\sigma$ is the Frangi vesselness measure at scale σ for the pixel (x_0, y_0)

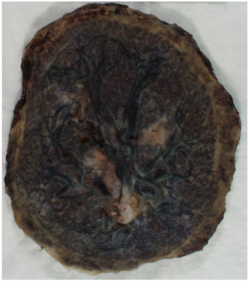
Notes

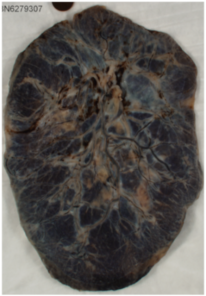
- The default "merging" strategy suggested by Frangi.
- Alternatively, we can process each scale by itself too if we want to. (Keep track of $\mathcal{V}_{\text{argmax}}, \mathcal{V}_\Sigma$, etc.)

What scales to use?

- Logarithmic spacing makes some intuitive sense.
- Experiment to determine what is large / small enough.
- Smaller scales for smaller features, larger scales for larger features.

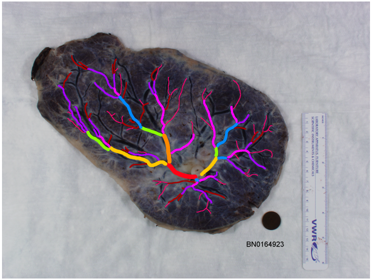
The data set

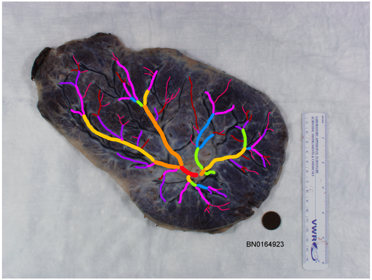


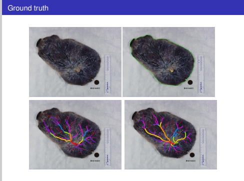


1. Just talk about how some are better than others, some are bad perfusion, some have issues with bordering and UCIP point (default methods didn't work well). A few have very nonstandard vessel widths.
2. We'll look at some of the issues with ground truth in detail in a minute
3. Basically, since you've already explained the data set earlier, this is for explaining the wide variation in sample quality
4. Better to have single samples with Frangi outputs to explain how it's not good

Ground truth

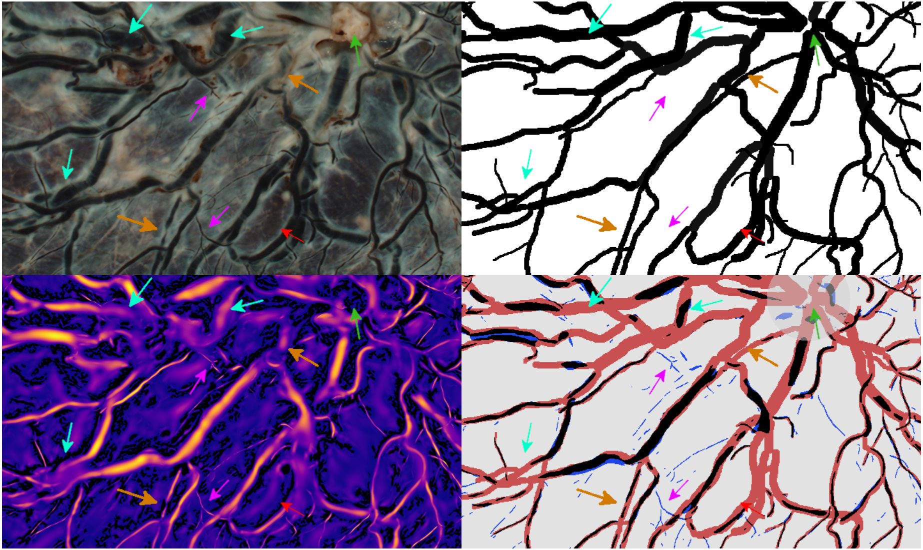


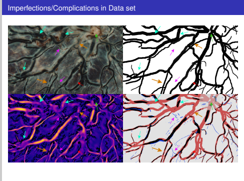




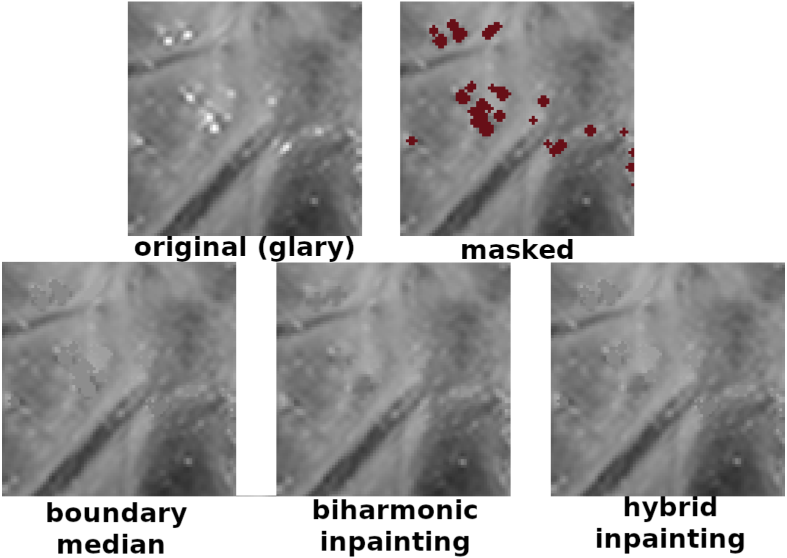
1. Maybe show these cropped, who cares
2. Maybe show enlarged UCIP and border, IDK if these will show up well
3. Maybe show cropped, merged trace by itself on one slide (quickly)

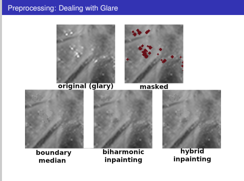
Imperfections/Complications in Data set





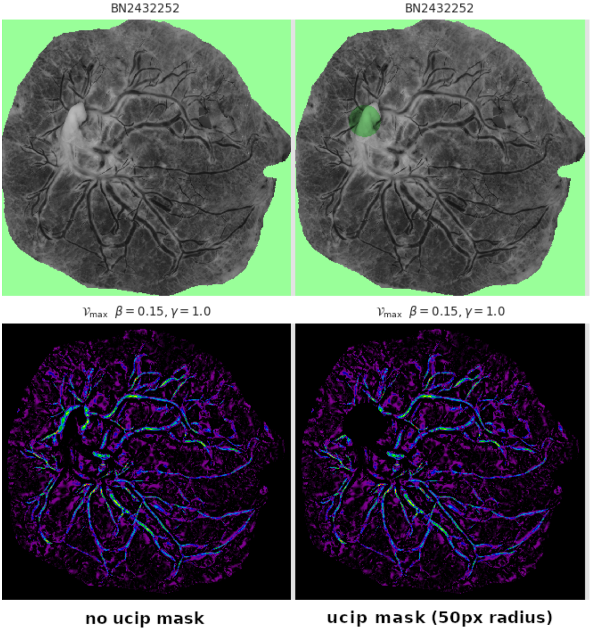
1. The 2×2 annotations might be too complicated, might not show up on the projector. Maybe just switch between two slides? or show different colors of annotations on each slide?
2. Talk about:
   - UCIP noise
   - too small and too big
   - bad perfusion
   - crossings

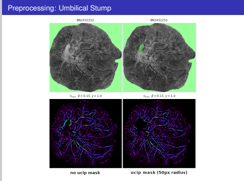

original (glary)
masked
boundary median
biharmonic inpainting
hybrid inpainting



1. Glare isn't catastrophic in most cases, but it can throw off results.
2. We're looking for small abrupt peaks on the surface, want to smooth out.
3. For simplicity, we assume everything higher than $175/255$ is glare
4. MATLAB has a technique called imfill that solves Laplace equation $\nabla f = 0$. We use built-in python that does biharmonic equation $\nabla\nabla f = 0$. It's pretty slow, so we speed it up by using an alternate strategy, where we just take the median of the boundary for smaller masked regions. You can see the lowest portion of the middle mask is fixed in the hybrid method.
5. Times to process a representative sample (whole thing not this inset):
   - biharmonic inpainting: 22 seconds
   - boundary median 4 seconds
   - hybrid method 6 seconds
6. Would be nice to have implementation details on the slide but I'm worried you can't

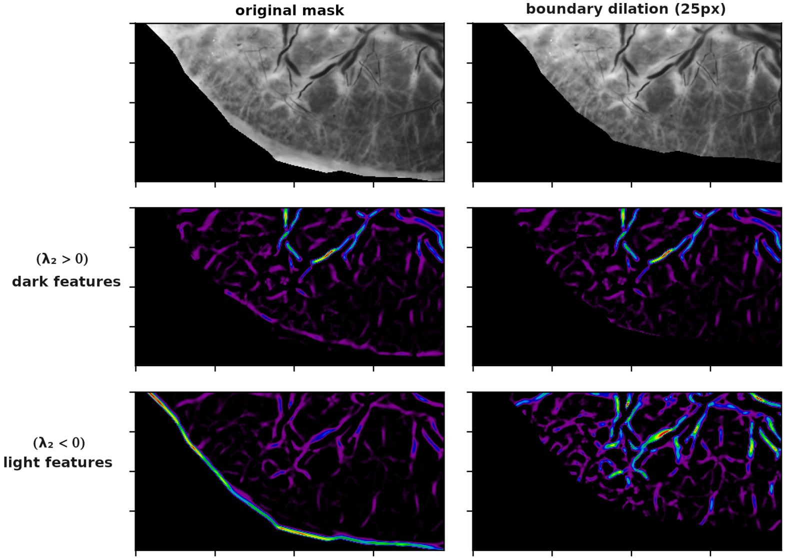
Preprocessing: Umbilical Stump



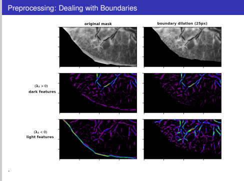


1. It was hard to remove these in an automated way that worked for all samples. You could do a white top hat and have it work a good amount of the time, but I ran out of time to do it effectively. There isn't any good data here due to diffgeo anyway as far as I can tell. Tracing protocol seems to estimate a lot.
2. This was with a radius of 50 pixels
3. This prevents area around the stump from being misconstrued as part of vascular network.
4. Basically picked this particular one because you can see that the radius we picked doesn't exactly fix the issue, but it does make it better. Hard to tell that the signal is *too* much better, but we see (also from the lack of reporting these as false positives) a minor improvement in precision and MCC scores throughout.
5. Zoom in would be nicer if you have time

Preprocessing: Dealing with Boundaries

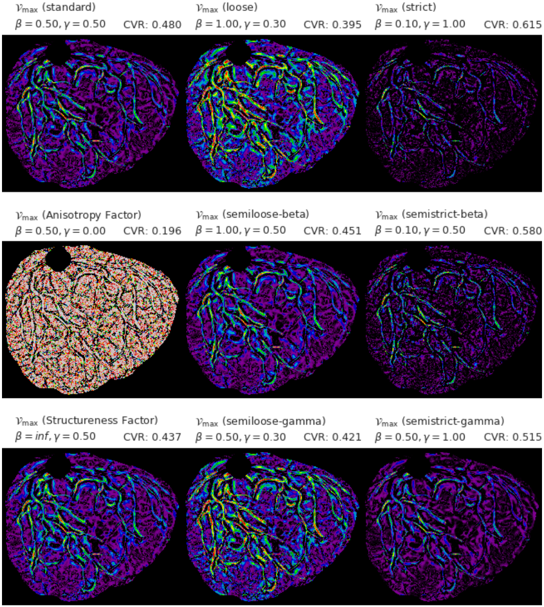


.



1. Prevent curvilinear content around the edges from being interpreted as curivilinear (especially with bounding structureness).
2. This can be especially significant with certain choices of dark/light BG and at small or large scales. Doesn't matter sooooo much, it's more of a quality of life thing.
3. Mostly because we want the background to be ignored, not treated as minimum (or maximum) intensity.
4. Since we already know where the mask is, we can just dilate it by a fixed amount (this is by 25 pixel radius)
5. Urgent: You need to describe watershedding procedure
6. Note the different response for light features when the border is removed

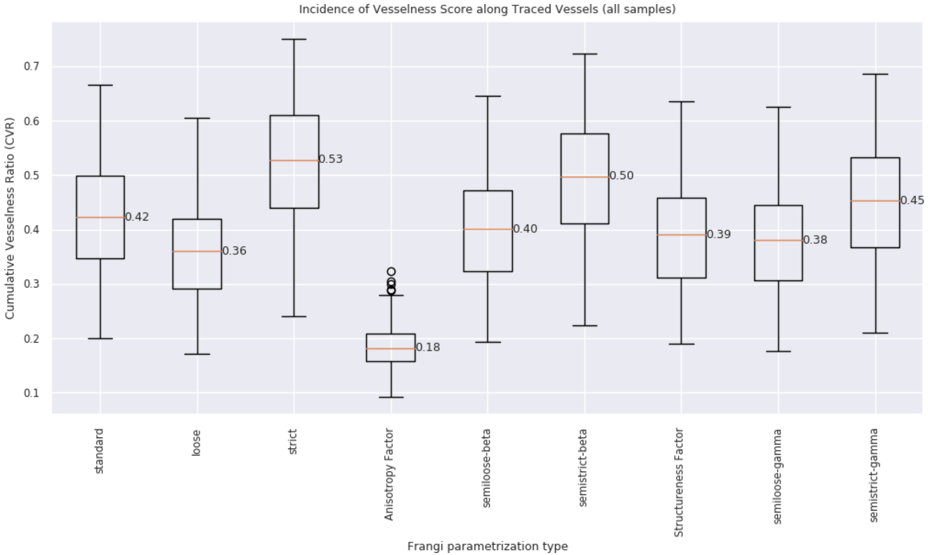
Cumulative Vesselness Ratio (CVR)



$$\mathrm{CVR}\,(\mathcal{V}_{\max}) := \frac{\sum_{G \subset \mathrm{I}} \mathcal{V}_{\max}(x_0, y_0)}{\sum_{\mathrm{I}} \mathcal{V}_{\max}(x_0, y_0)}$$

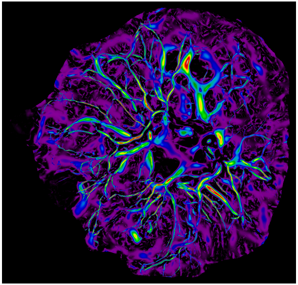
where $G \subset \mathrm{I}$ is the ground truth

label	β	γ
standard	0.5	0.5
loose	1.0	0.3
strict	0.10	1.0
semiloose-β	1.0	0.5
semistrict-β	0.1	0.5
semiloose-γ	0.5	0.3
semistrict-γ	0.5	1.0
anisotropy only	0.5	0
structureness only	∞	0.5

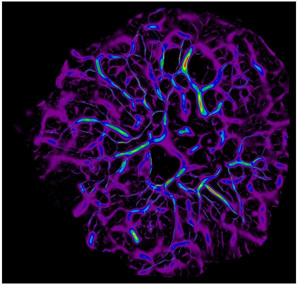

Incidence of Vesselness Score along Traced Vessels (all samples)
Cumulative Vesselness Ratio (CVR)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.42
0.36
0.53
0.18
0.40
0.50
0.39
0.38
0.45
standard
loose
strict
Anisotropy Factor
semiloose-beta
semistrict-beta
Structureness Factor
semiloose-gamma
semistrict-gamma
Frangi parametrization type

Comparing Frangi Hessian to Frangi with Weingarten Map

(g) Raw Sample

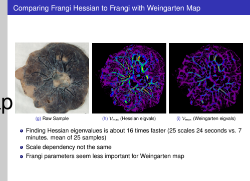


(h) $\mathcal{V}_{\text{max}}$ (Hessian eigvals)



(i) $\mathcal{V}_{\text{max}}$ (Weingarten eigvals)

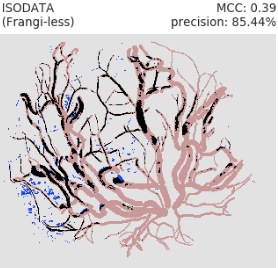
- Finding Hessian eigenvalues is about 16 times faster (25 scales 24 seconds vs. 7 minutes. mean of 25 samples)
- Scale dependency not the same
- Frangi parameters seem less important for Weingarten map



1. Hessian matrix is much easier to find eigenvalues of
2. Admit that this came pretty late into my research adventure so I need to look at this more. Weingarten map gives valid response at higher scales than the Hessian. Better at outlining just the ridge. Would like to compare strict Frangi to just Weingarten map
3. Note, noise is noise; still present.
4. You could potentially do a hybrid, i.e. recalculate significant Frangi responses like this. Also there could be a speedup by not doing *every* non-masked point, i.e. make sure that hessian magnitude is large enough first. that would be fast.
5. Move to late in talk, otherwise rest of this shit kinda feels weak.
6. Also, need to look at `skimage.feature.hessian_matrix_eigvals` to figure out other reasons for speedup/ask von Brecht
7. Scales are kinda wrong for optimal Hessian-Frangi, hence more noise there

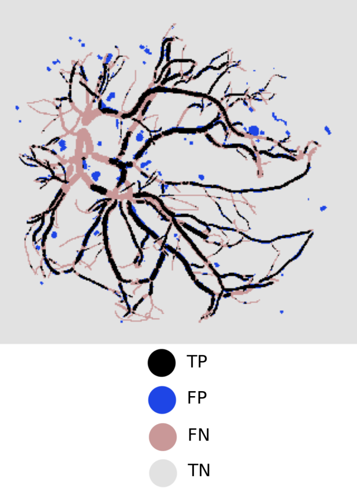
A non-Frangi segmentation method: ISODATA (Ridler-Calvert)

$$\alpha_{\mathrm{ISO}} = \arg\min_{\alpha} \left(\frac{1}{2} \left[\mathrm{mean}\left\{ \mathsf{I}(x,y) \mid \mathsf{I}(x,y) \leq \alpha \right\} + \mathrm{mean}\left\{ \mathsf{I}(x,y) \mid \mathsf{I}(x,y) > \alpha \right\} \right] \right)$$



Since the vascular structure in our image domain is darker than the background, we select pixels where $\mathsf{I}(x, y) < \alpha_{\mathsf{ISO}}$.

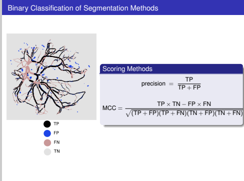
Binary Classification of Segmentation Methods



Scoring Methods

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\text{MCC} = \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$



1. **META**: Use GIMP to merge legend with image and put horizontal under image. maybe add precision and mcc of this particular example
2. There are a lot of different choices in terms of scoring, like accuracy (list more)
3. Rationale behind MCC, the benefit
4. Explain that MCC is good but precision is important too
5. Would be good to have a scoring method based on network, but that's definitely nontrivial to define

Thresholding Methods

Fixed Threshold (`thresh-low` and `thresh-high`)

Pick some $0 < \alpha < 1$, then extract $\mathcal{V}_{\text{max}} > \alpha$

- α must be manually selected
- We picked $\alpha = 0.3$ and $\alpha = 0.2$

scalewise (nonzero) percentile filtering (`snz-p`)

For each scale, filter under *p*th percentile: $\mathcal{V}_\sigma > \alpha_p$.

- Use *nonzero* percentile for each scale
- Smaller scales get in easier
- Good for unscaled Frangi, unknown filter response
- Very noisy if Σ chosen poorly

Don't throw anything away!

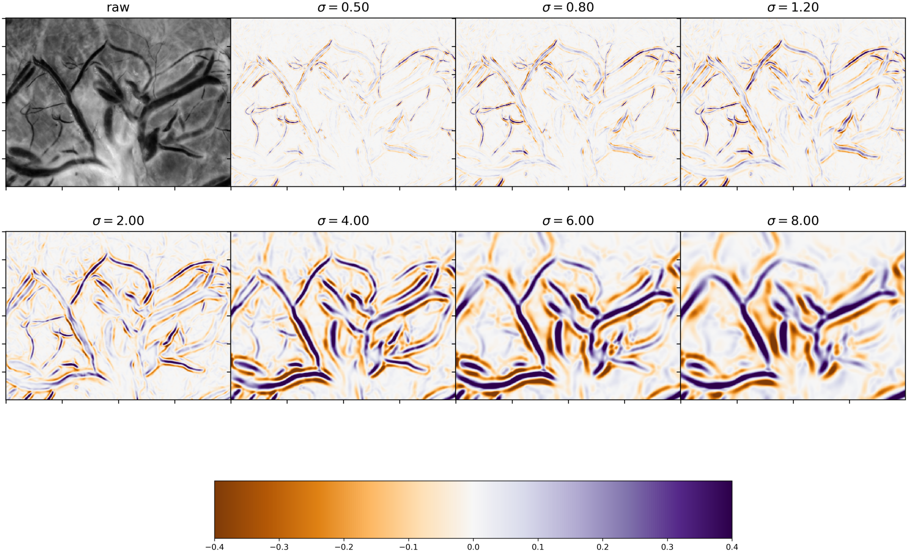
$$\mathcal{V}_\sigma^{(+)}(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_\sigma^{(-)}(x_0, y_0) = \begin{cases} 0 & \text{if} \quad \lambda_2 < 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

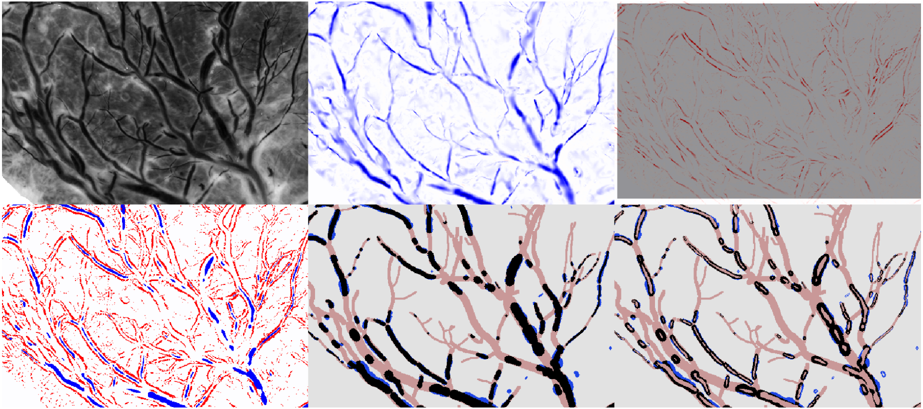
$$\text{where} \quad A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad , |\lambda_2| \geq |\lambda_1|$$

- This gives us a $\mathcal{V}_{\max}^{(+)}$ and $\mathcal{V}_{\max}^{(-)}$ with same calculation time.
- We might want to use a subset $\Sigma^{(-)} \subset \Sigma$ (smaller scales only)

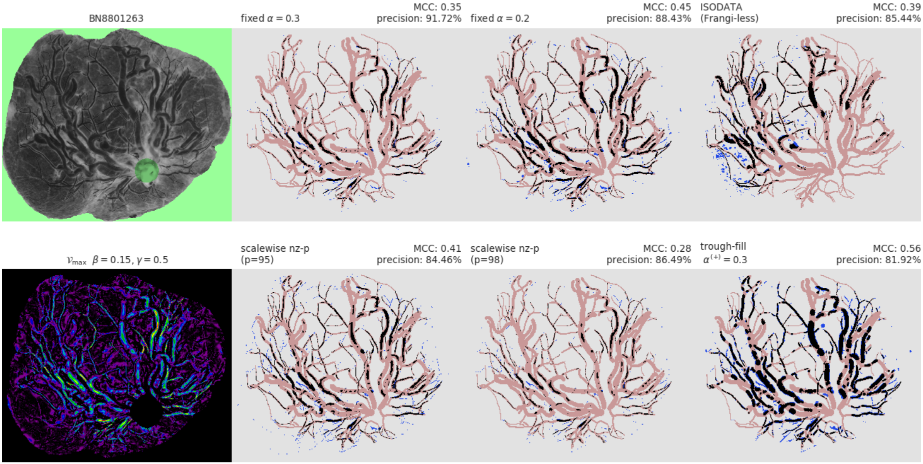
Signed Frangi



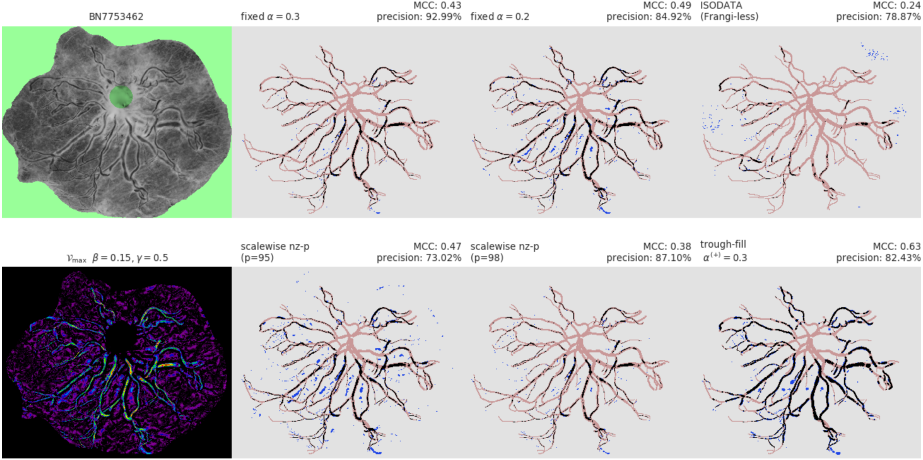
Trough Filling Method



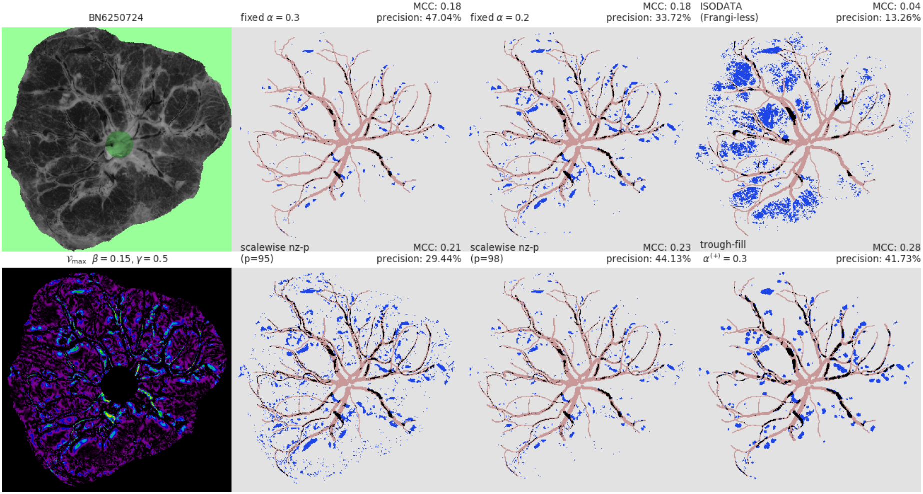
Example Results (1/4)



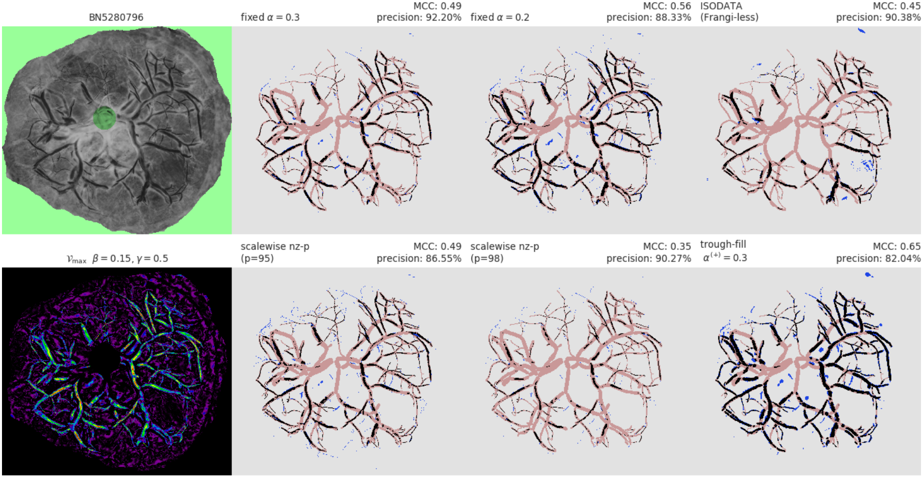
Example Results (2/4)



Example Results (3/4)



Example Results (4/4)



Results on All Samples ($\beta = 0.15, \gamma = 0.5$)