

# Optimized Strict Multiscale Frangi Prefiltering for Segmentation

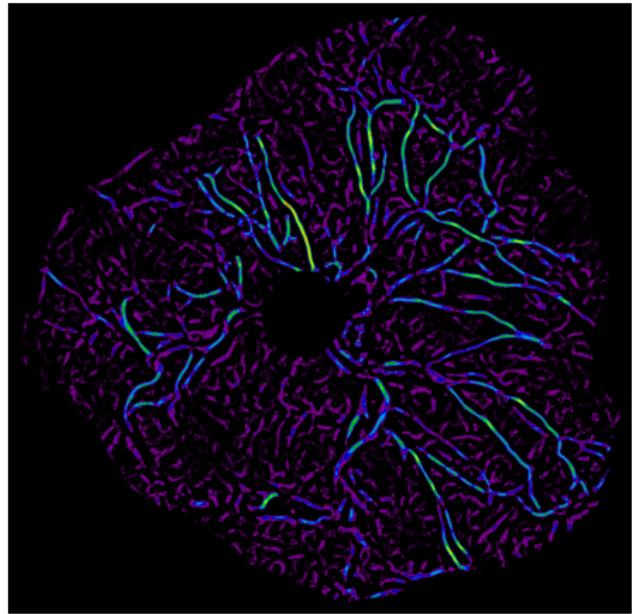
## Towards an automated PCSVN extraction

Luke Wukmer

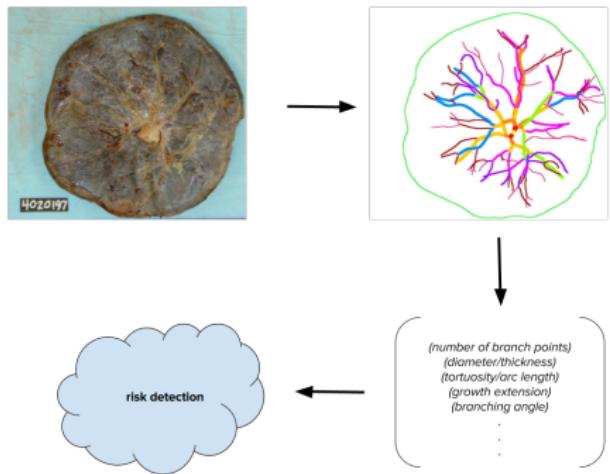
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# Research Goals



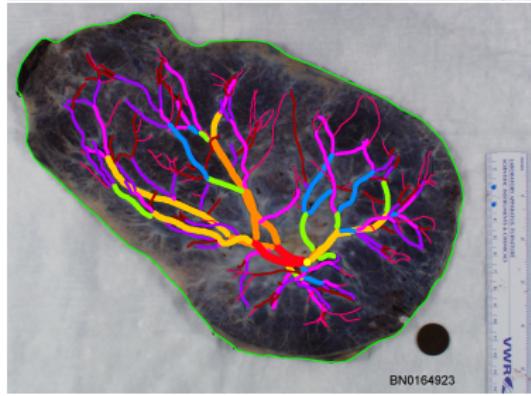
## Vascular Network Extraction in Placentas

- **Motivation:** Accurate measurement of the vascular structure of a placental sample can be used to predict neonatal risk factors, specifically ASD.
- **Challenge:** Currently no automated method of obtaining traces of PCSVN. Manual tracing is labor intensive but necessary for feature analysis.
- **Research Goal:** Provide a fully automated method of extraction.

# The Image Processing Problem

## Our image domain

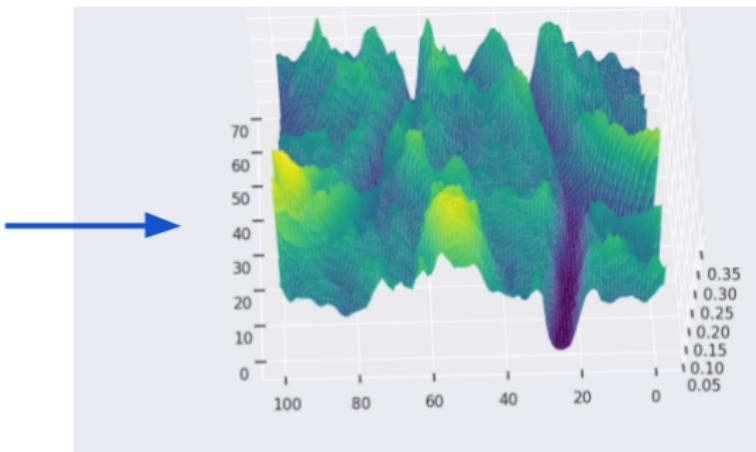
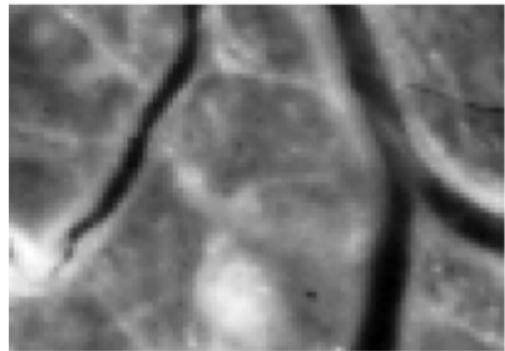
- The PCSVN is a connected network of veins and arteries on the surface of the placenta
- Ground truth for 201 samples from private NCS dataset
- Placentas have been formalin-fixed, so arteries are more prominent.
- Placental images are comparatively noisy

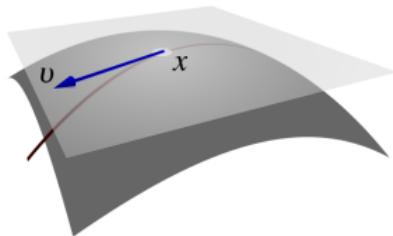


# Appealing to Differential Geometry

Idealize image as a 3D surface (a graph) with  $(x, y)$  spatial coordinates and intensity as the height function  $h(x, y)$ .

$$\mathbf{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ where } (x, y) \mapsto (x, y, h(x, y))$$





## Meusnier's Theorem

If you look at a point on the surface and fix a tangent vector, then all surface curves through that point with that velocity will have the same curvature there. So the curvature is intrinsic to the surface. Call it **normal curvature**.

## Definition

Extremal values of normal curvature are called **principal curvatures** of the surface at that point. The extremizing tangent vectors are **principal directions**.

## Theorem of Olinde Rodrigues

These principal curvatures/directions are the eigenvalues/eigenvectors of a particular map called the **Weingarten map**.

## Weingarten Map for Graphs

Given the graph  $f : U \rightarrow \mathbb{R}^3$  where  $(x, y) \mapsto (x, y, h(x, y))$ , the matrix representation of its Weingarten map is given by

$$\widehat{\mathbf{L}} = \text{Hess}(h)\tilde{\mathbf{G}}$$

where  $\tilde{\mathbf{G}} := \frac{1}{(1+h_x^2+h_y^2)^{3/2}} \begin{bmatrix} 1+h_y^2 & -h_x h_y \\ -h_x h_y & 1+h_x^2 \end{bmatrix}$  and  $\text{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{yx} & h_{yy} \end{bmatrix}$

## Approximating

- In particular, given a point where  $h_x \approx h_y \approx 0$ , we have  $\tilde{\mathbf{G}} \approx \text{Id}$ , and thus  $\widehat{\mathbf{L}} \approx \text{Hess}(h)$ .
- For ease of use, we can simply find eigenvalues of the Hessian instead.
- This gives rise to a class of filters, the so-called Hessian-based filters.

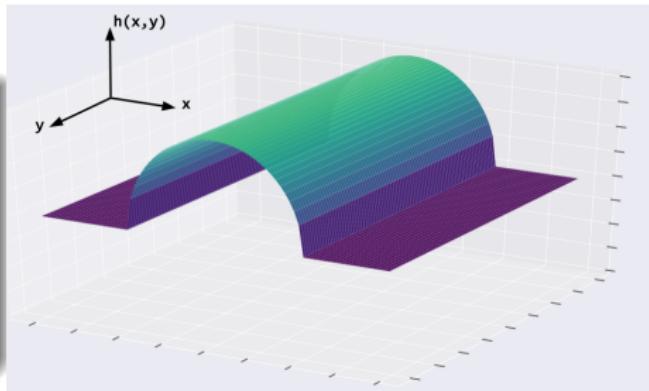
## Example: Finding Weingarten map and Principal Curvatures

Cylindrical Ridge of radius  $r$

Let  $f$  be the graph given by

$f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $f(x, y) = (x, y, h(x, y))$ ,

with  $h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$



$$h_x = \frac{-x}{\sqrt{r^2 - x^2}} \quad , \quad h_{xx} = \frac{-r^2}{(r^2 - x^2)^{3/2}}$$
$$h_y = 0 \quad , \quad h_{yy} = 0 \quad , \quad h_{xy} = 0$$

# Example: Finding Weingarten map and Principal Curvatures

## Weingarten matrix representation (for graphs)

$$\widehat{\mathbf{L}} = \text{Hess}(h)\tilde{\mathbf{G}}, \quad \text{where} \quad \tilde{\mathbf{G}} := \frac{1}{(1 + h_x^2 + h_y^2)^{3/2}} \begin{bmatrix} 1 + h_y^2 & -h_x h_y \\ -h_x h_y & 1 + h_x^2 \end{bmatrix}$$

$$\text{Hess}(h) = \begin{bmatrix} h_{xx} & h_{xy} \\ h_{xy} & h_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{\mathbf{G}} = \frac{1}{\left(1 + \frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 + \frac{x^2}{r^2-x^2} \end{bmatrix}$$

$$\begin{aligned} \widehat{\mathbf{L}} &= \text{Hess}(h)\tilde{\mathbf{G}} = \frac{1}{\left(1 + \frac{x^2}{r^2-x^2}\right)^{3/2}} \begin{bmatrix} \frac{r^2}{(r^2-x^2)^{3/2}} & 0 \\ 0 & 0 \end{bmatrix} \\ &= \dots = \begin{bmatrix} \frac{-1}{r} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

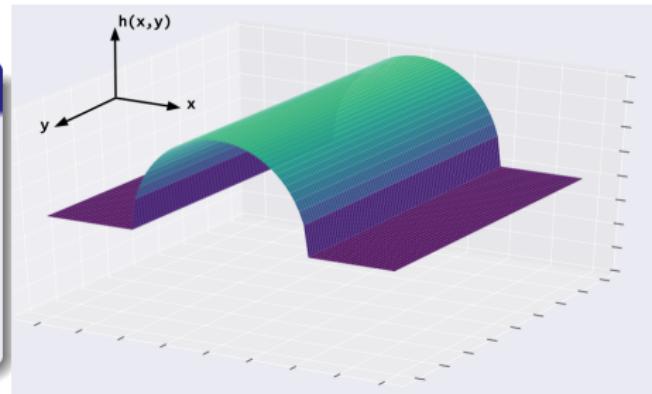
## Example: Finding Weingarten Map and Principal Curvatures

### Cylindrical Ridge of radius $r$

Let  $f$  be the graph given by

$f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $f(x, y) = (x, y, h(x, y))$ ,

with  $h(x, y) = \begin{cases} \sqrt{r^2 - x^2} & -r < x < r \\ 0 & \text{else} \end{cases}$



$$\hat{L} = \begin{bmatrix} -\frac{1}{r} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{Matrix of Weingarten map}$$

$u_2 = (1, 0)$ ,  $u_1 = (0, 1)$  principal directions (eigenvectors of  $\hat{L}$ )

$\lambda_2 = \frac{-1}{r}$ ,  $\lambda_1 = 0$  principal curvatures (eigenvalues of  $\hat{L}$ )

## The (Uniscale) Frangi Filter

$$\mathcal{V}(x, y) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) & \text{otherwise} \end{cases}$$

where

$$A := |\lambda_1 / \lambda_2| \quad (\text{Anisotropy Measure})$$

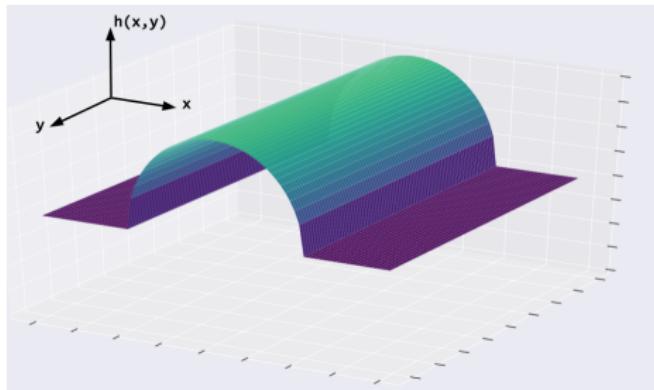
$$\text{and } S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad (\text{Structureness Measure})$$

for eigenvalues  $\lambda_1, \lambda_2$  of the Hessian (at point  $(x,y)$ ),  $|\lambda_1| \leq |\lambda_2|$   
and  $\beta$  and  $c$  are parameters

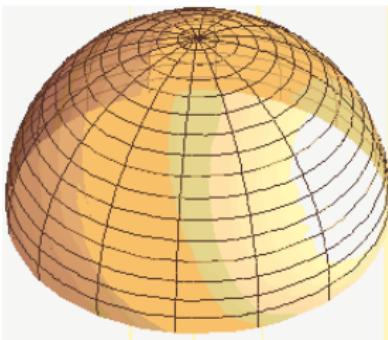
## Anisotropy Factor

$$\exp\left(-\frac{A^2}{2\beta^2}\right) \quad , \quad A := \left| \frac{\lambda_1}{\lambda_2} \right| \quad (1)$$

- For selecting anisotropic content (lines not blobs)
- When  $A$  is very close to 0,  $|\lambda_2| \gg |\lambda_1|$ , and the factor is  $\approx 1$ .
- Choosing parameter: Frangi suggested  $\beta = \frac{1}{2}$  as a reasonable default.



(a) Anisotropic



(b) Isotropic

## Structureness Factor

$$\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \quad , \quad S := \sqrt{\lambda_1^2 + \lambda_2^2} \quad (2)$$

- Purpose: Filter out numerically insignificant values.
- Important to pick a reasonable value for the image at hand / image domain.
- Frangi suggested “half the maximum value of the Hessian norm”. We will define the parameter  $\gamma$  and define  $c(\gamma) = \gamma S_{\max}$ , since  $S_{\max}$  is the Frobenius norm of the Hessian.

## Frangi filter

$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{s^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases} \quad (3)$$

$$\text{where } A := |\lambda_1/\lambda_2| \quad \text{and} \quad S := \sqrt{\lambda_1^2 + \lambda_2^2}, \quad |\lambda_2| \geq |\lambda_1| \quad (4)$$

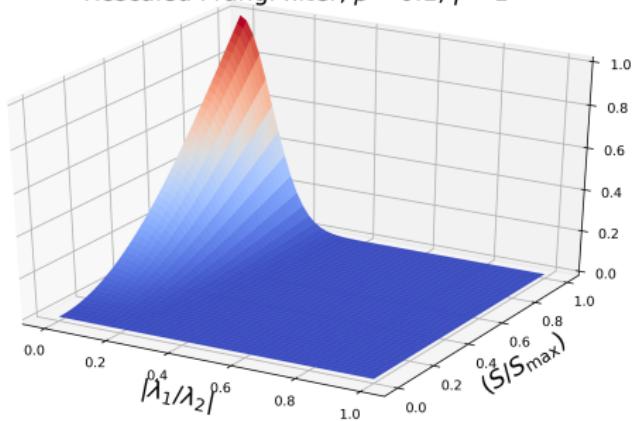
- $\lambda_2 > 0$  means the point would be a local minimum (at a critical point). We are looking for a local maximum by looking for bright curvilinear objects. You can switch this ( $\lambda_2 < 0$  as the case) if you are looking for dark curvilinear.
- Point of  $\exp(\dots)$  and  $1 - \exp(\dots)$  structure is that the filter decays rapidly as anisotropy or structureness decrease.

# Frangi filter anatomy: Choosing Parameters

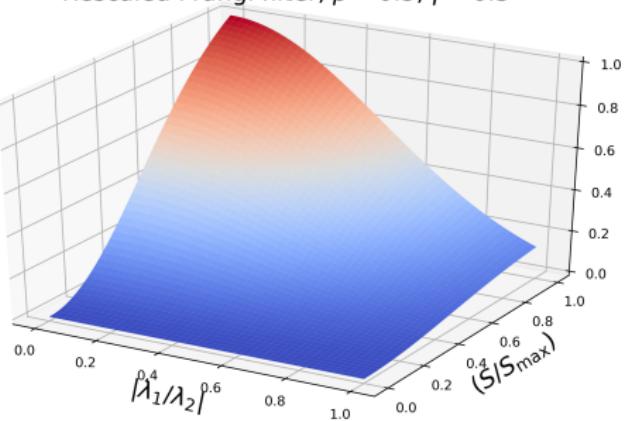
$$\mathcal{V}_\sigma(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{s^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

where  $A := |\lambda_1/\lambda_2|$  and  $S := \sqrt{\lambda_1^2 + \lambda_2^2}$ ,  $|\lambda_2| \geq |\lambda_1|$

Rescaled Frangi filter,  $\beta = 0.1, \gamma = 1$



Rescaled Frangi filter,  $\beta = 0.5, \gamma = 0.5$



# Linear Scale Space Theory for Kids

- Obviously the image is not actually a continuous surface. It is a particular sampling  $I$  of the surface.
- We want to create a “family of derived images” with a “resolution” parameter  $\sigma \geq 0$ , ideally from some operator  $T_\sigma$  acting on the image  $I$ .

$$K(x, y; \sigma) = T_\sigma$$

## Some Axioms

- Linear shift and rotational invariance
- Semigroup property  $T_{\sigma_1 + \sigma_2} I = T_{\sigma_1} I + T_{\sigma_2} I$
- Continuity of scale parameter  $\sigma$
- Causality condition

## Moral

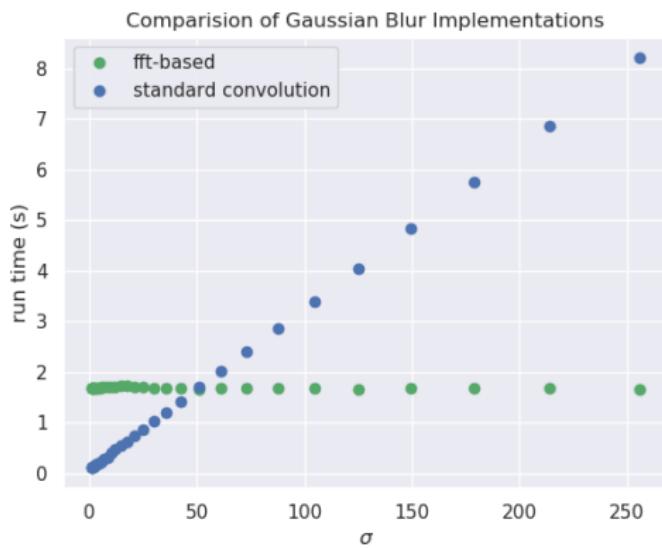
Convolution by Gaussian solves these problems.

# Implementation Detail: Calculating Discrete Hessian

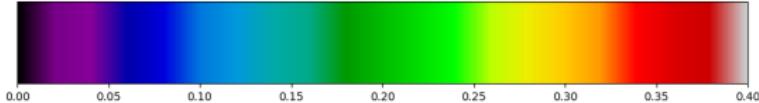
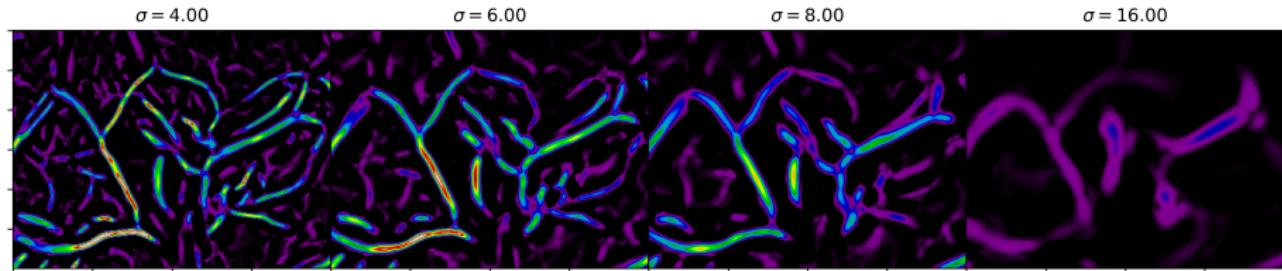
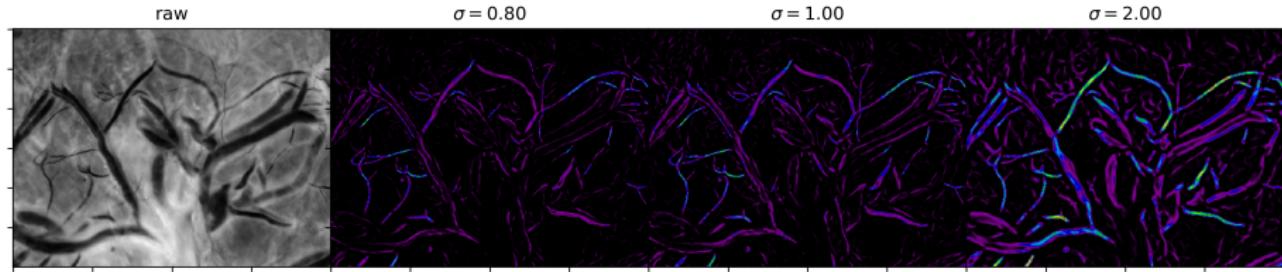
$$K(x, y; \sigma) = T_\sigma u_0 = G_\sigma \star u_0$$

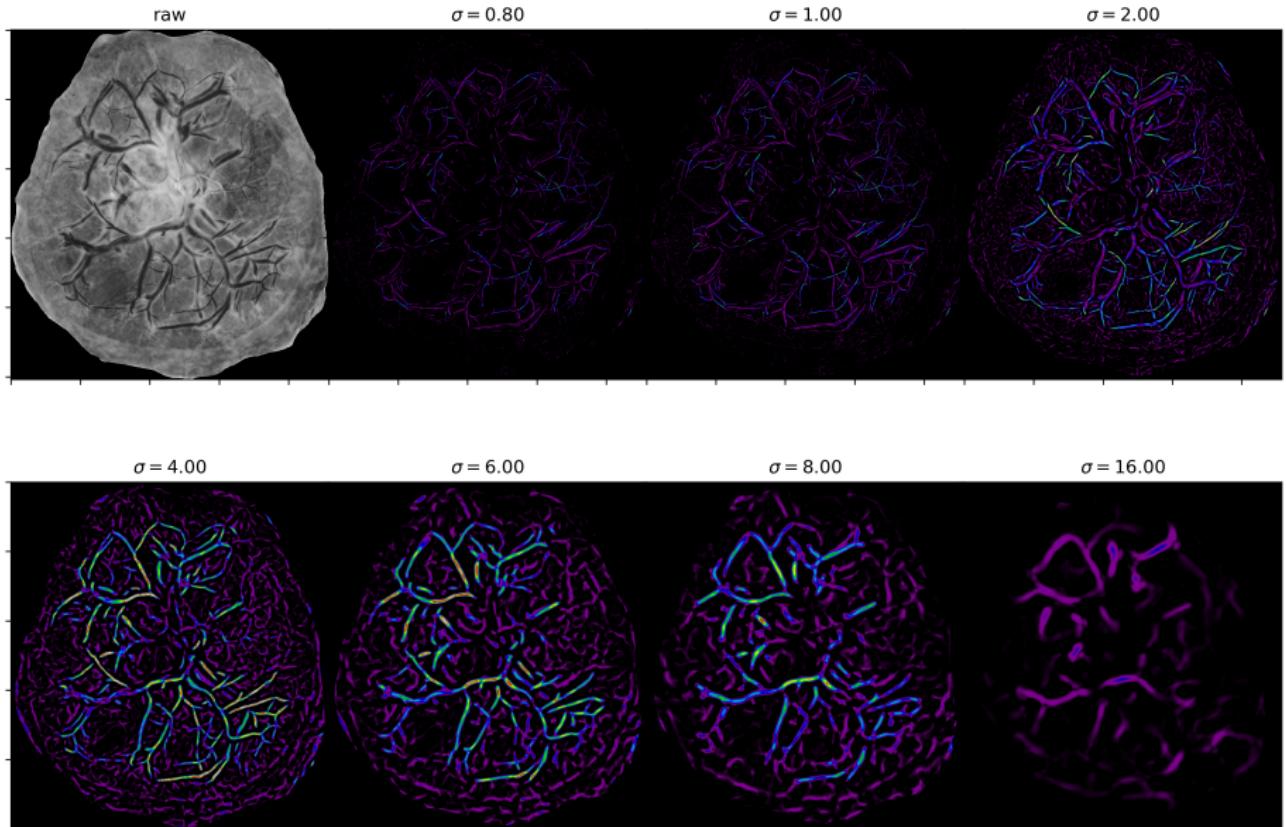
where  $G_\sigma(x, y) := \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$

- Convolution by gaussian matrix is slow
- Calculate in frequency space as a multiplication according to the convolution theorem.
- While you're at it, use a FFT.



# Frangi Filter Anatomy: Scale parameter





## Multiscale Frangi filter

For a set of scales to probe,  $\Sigma := \{\sigma_0, \sigma_1, \dots, \sigma_n\}$ , the set of  $n$  scales at which to probe,

$$\mathcal{V}_{\max}(x_0, y_0) := \max_{\sigma \in \Sigma} \{\mathcal{V}_\sigma(x_0, y_0)\} \quad (5)$$

where  $\mathcal{V}_\sigma$  is the Frangi vesselness measure at scale  $\sigma$  for the pixel  $(x_0, y_0)$

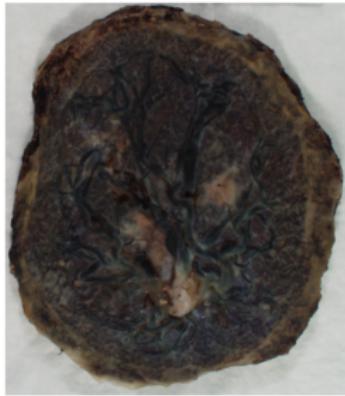
### Notes

- The default “merging” strategy suggested by Frangi.
- Alternatively, we can process each scale by itself too if we want to. (Keep track of  $\mathcal{V}_{\arg\max}$ ,  $\mathcal{V}_\Sigma$ , etc.)

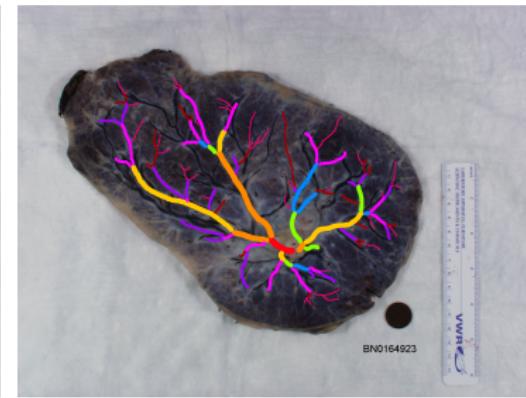
### What scales to use?

- Logarithmic spacing makes some intuitive sense.
- Experiment to determine what is large / small enough.
- Smaller scales for smaller features, larger scales for larger features.

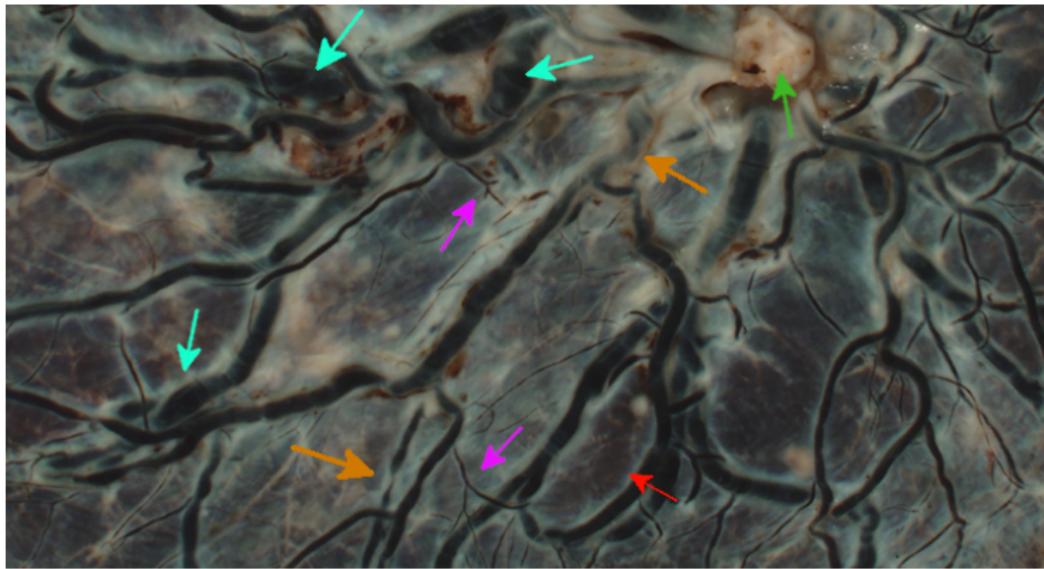
# The data set



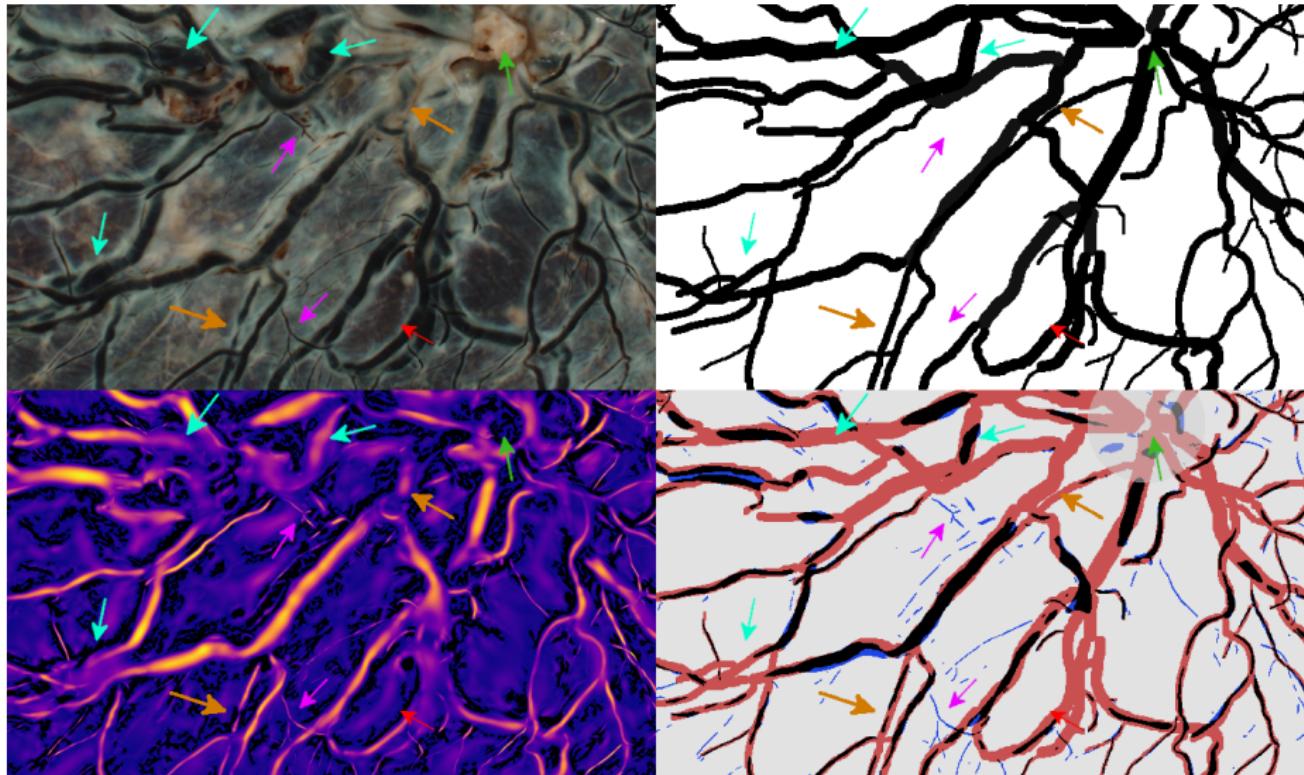
# Ground truth



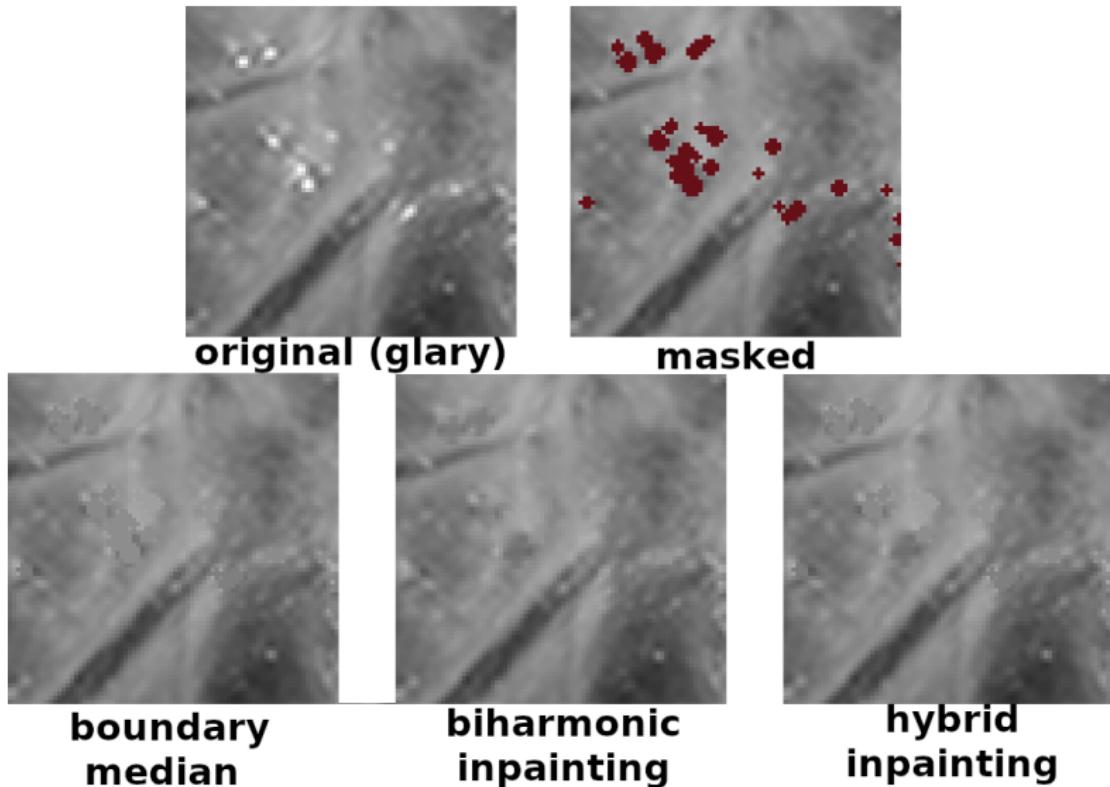
## Imperfections in Data set (1/2)



## Imperfections in Data set (2/2)

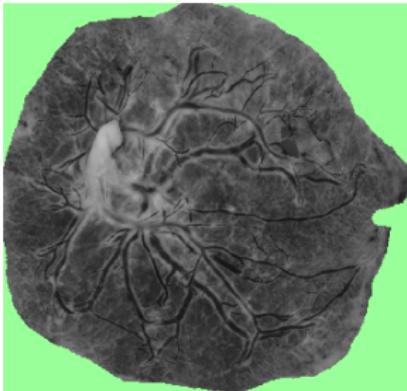


## Preprocessing: Dealing with Glare



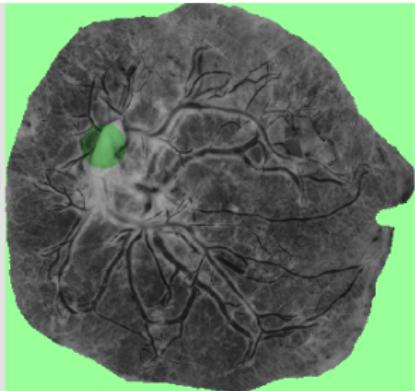
# Preprocessing: Umbilical Stump

BN2432252

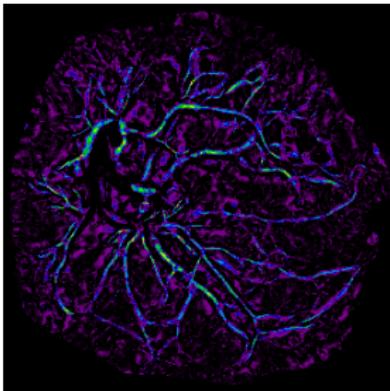


$\mathcal{V}_{\max}$   $\beta = 0.15, \gamma = 1.0$

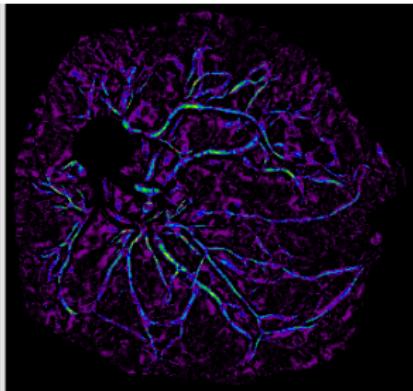
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$\mathcal{V}_{\max}$   $\beta = 0.15, \gamma = 1.0$

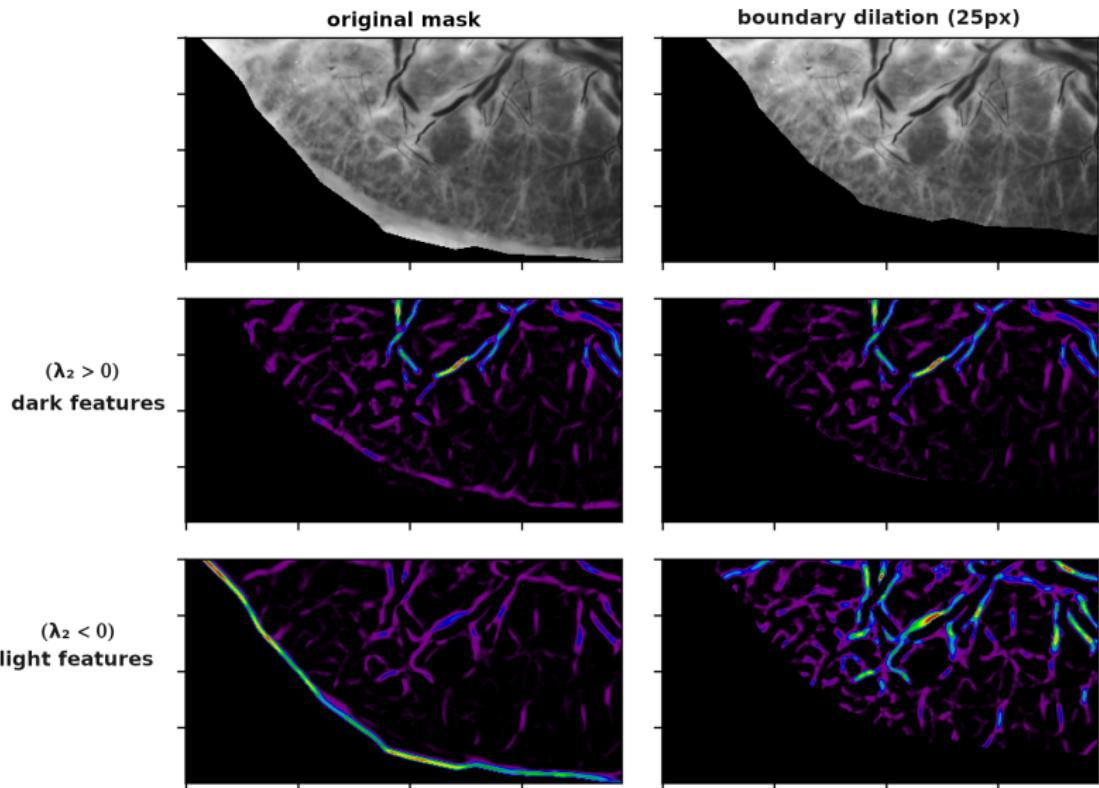


no ucip mask



ucip mask (50px radius)

# Preprocessing: Dealing with Boundaries



# Cumulative Vesselness Ratio (CVR)

$\mathcal{V}_{\max}$  (standard)

$\beta = 0.50, \gamma = 0.50$

CVR: 0.480

$\mathcal{V}_{\max}$  (loose)

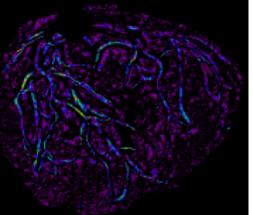
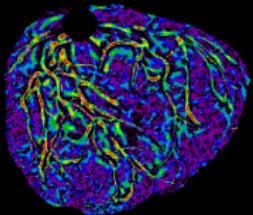
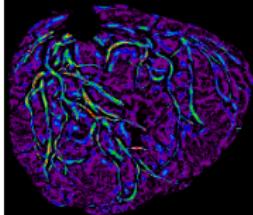
$\beta = 1.00, \gamma = 0.30$

CVR: 0.395

$\mathcal{V}_{\max}$  (strict)

$\beta = 0.10, \gamma = 1.00$

CVR: 0.615



$\mathcal{V}_{\max}$  (Anisotropy Factor)

$\beta = 0.50, \gamma = 0.00$

CVR: 0.196

$\mathcal{V}_{\max}$  (semiloose-beta)

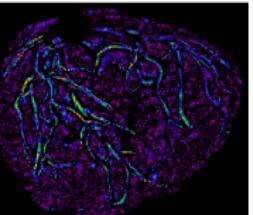
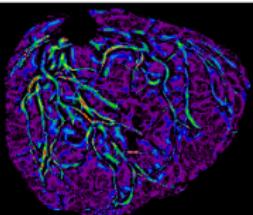
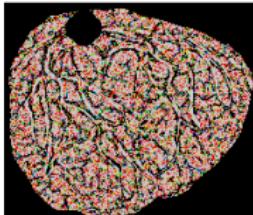
$\beta = 1.00, \gamma = 0.50$

CVR: 0.451

$\mathcal{V}_{\max}$  (semistrict-beta)

$\beta = 0.10, \gamma = 0.50$

CVR: 0.580



$\mathcal{V}_{\max}$  (Structureness Factor)

$\beta = \text{inf}, \gamma = 0.50$

CVR: 0.437

$\mathcal{V}_{\max}$  (semiloose-gamma)

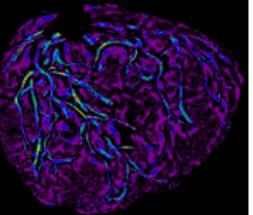
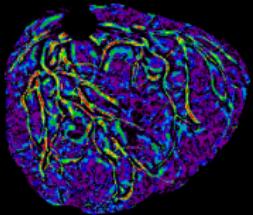
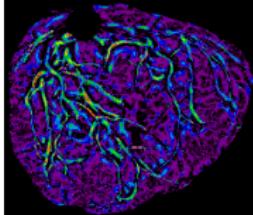
$\beta = 0.50, \gamma = 0.30$

CVR: 0.421

$\mathcal{V}_{\max}$  (semistrict-gamma)

$\beta = 0.50, \gamma = 1.00$

CVR: 0.515

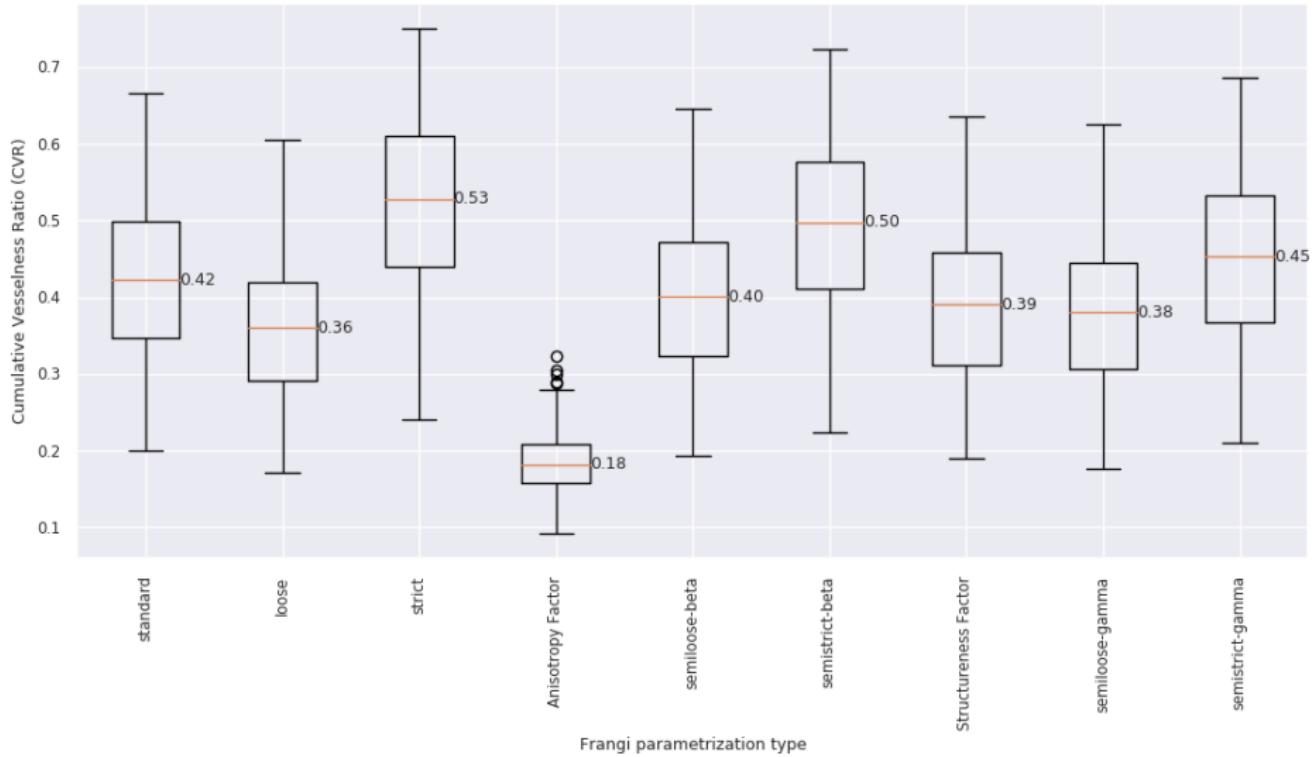


$$\text{CVR}(\mathcal{V}_{\max}) := \frac{\sum_{G \subset I} \mathcal{V}_{\max}(x_0, y_0)}{\sum_I \mathcal{V}_{\max}(x_0, y_0)}$$

where  $G \subset I$  is the ground truth

label	$\beta$	$\gamma$
standard	0.5	0.5
loose	1.0	0.3
strict	0.10	1.0
semiloose- $\beta$	1.0	0.5
semistrict- $\beta$	0.1	0.5
semiloose- $\gamma$	0.5	0.3
semistrict- $\gamma$	0.5	1.0
anisotropy only	0.5	0
structureness only	$\infty$	0.5

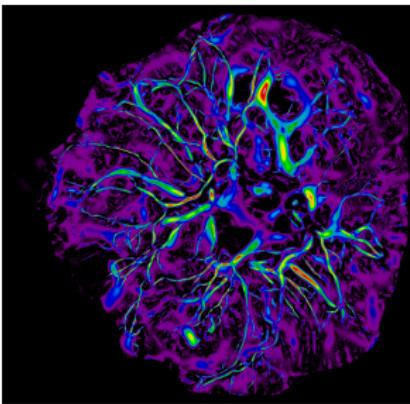
Incidence of Vesselness Score along Traced Vessels (all samples)



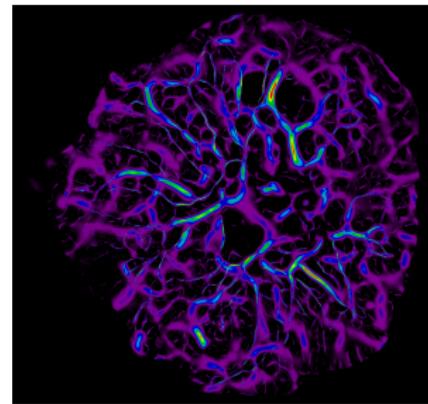
# Comparing Frangi Hessian to Frangi with Weingarten Map



(g) Raw Sample



(h)  $\mathcal{V}_{\max}$  (Hessian eigvals)



(i)  $\mathcal{V}_{\max}$  (Weingarten eigvals)

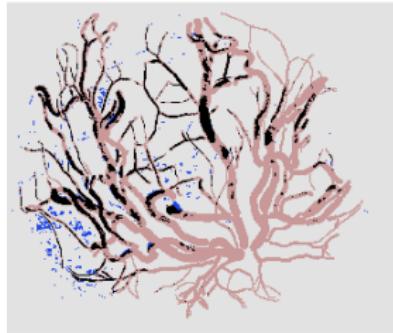
- Finding Hessian eigenvalues is about 16 times faster (25 scales 24 seconds vs. 7 minutes. mean of 25 samples)
- Scale dependency not the same
- Frangi parameters seem less important for Weingarten map

## A non-Fangi segmentation method: ISODATA (Ridler-Calvert)

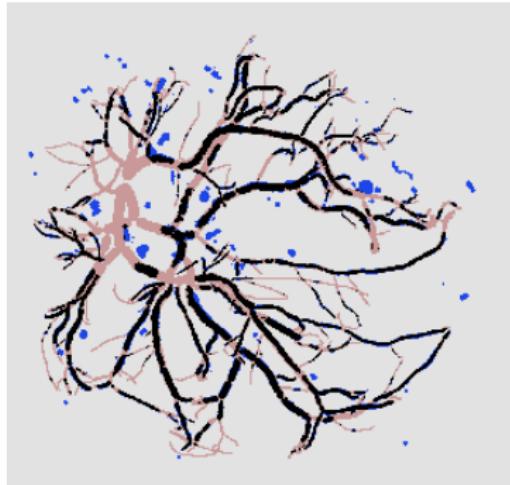
$$\alpha_{\text{ISO}} = \arg \min_{\alpha} \left( \frac{1}{2} \left[ \text{mean} \{ I(x, y) \mid I(x, y) \leq \alpha \} + \text{mean} \{ I(x, y) \mid I(x, y) > \alpha \} \right] \right)$$

ISODATA  
(Frangi-less)

MCC: 0.39  
precision: 85.44%



Since the vascular structure in our image domain is darker than the background, we select pixels where  $I(x, y) < \alpha_{\text{ISO}}$ .



- TP
- FP
- FN
- TN

## Scoring Methods

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\text{MCC} = \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$

## Fixed Threshold (thresh-low and thresh-high)

Pick some  $0 < \alpha < 1$ , then extract  $\mathcal{V}_{\max} > \alpha$

- $\alpha$  must be manually selected
- We picked  $\alpha = 0.3$  and  $\alpha = 0.2$

## scalewise (nonzero) percentile filtering (snz-p)

For each scale, filter under  $\rho$ th percentile:  $\mathcal{V}_\sigma > \alpha_\rho$ .

- Use *nonzero* percentile for each scale
- Smaller scales get in easier
- Good for unscaled Frangi, unknown filter response
- Very noisy if  $\Sigma$  chosen poorly

Don't throw anything away!

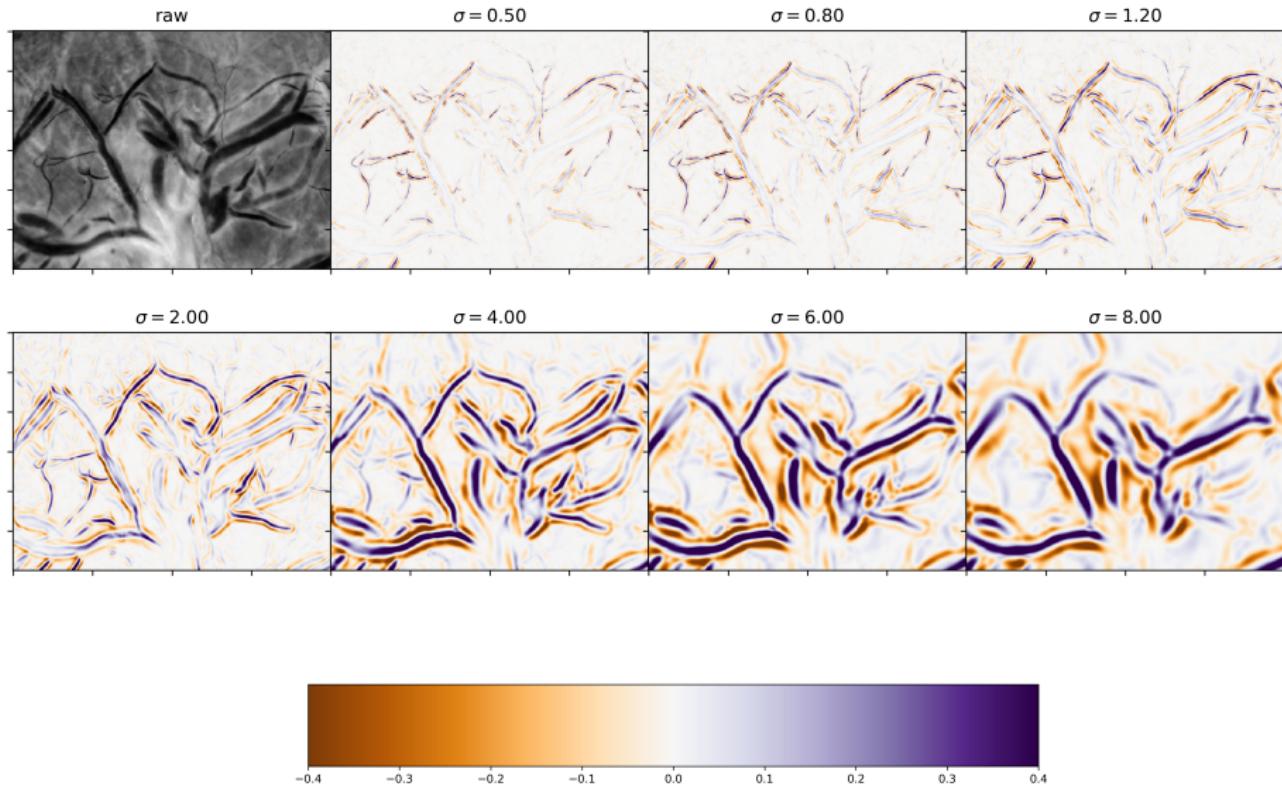
$$\mathcal{V}_\sigma^{(+)}(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

$$\mathcal{V}_\sigma^{(-)}(x_0, y_0) = \begin{cases} 0 & \text{if } \lambda_2 < 0 \\ \exp\left(-\frac{A^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2(\gamma S_{\max})^2}\right)\right) & \text{otherwise} \end{cases}$$

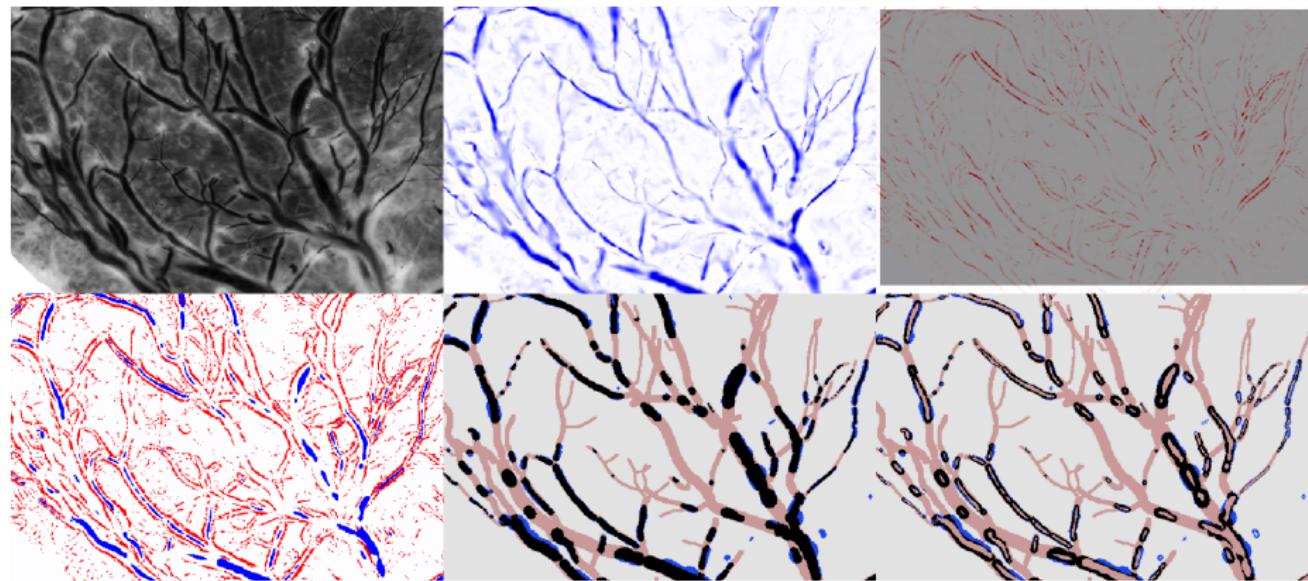
where  $A := |\lambda_1/\lambda_2|$  and  $S := \sqrt{\lambda_1^2 + \lambda_2^2}$ ,  $|\lambda_2| \geq |\lambda_1|$

- This gives us a  $\mathcal{V}_{\max}^{(+)}$  and  $\mathcal{V}_{\max}^{(-)}$  with same calculation time.
- We might want to use a subset  $\Sigma^{(-)} \subset \Sigma$  (smaller scales only)

# Signed Frangi



# Trough Filling Method



# Example Results (1/4)

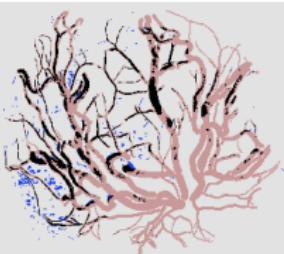
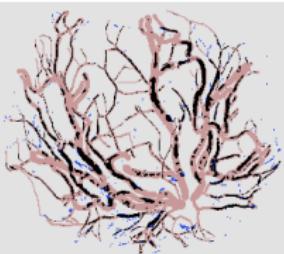
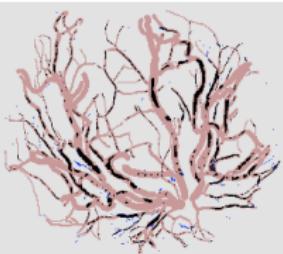
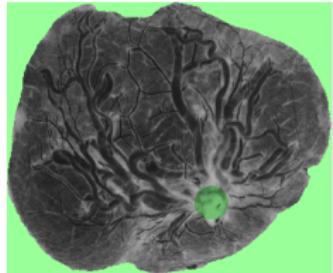
BN8801263

fixed  $\alpha = 0.3$

MCC: 0.35  
precision: 91.72% fixed  $\alpha = 0.2$

MCC: 0.45  
precision: 88.43% ISODATA  
(Frangi-less)

MCC: 0.39  
precision: 85.44%



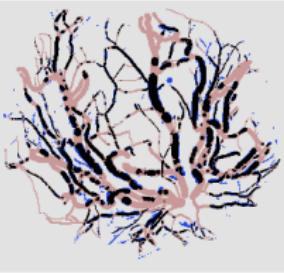
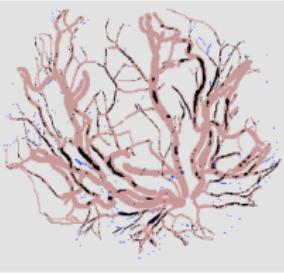
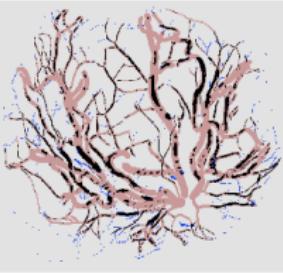
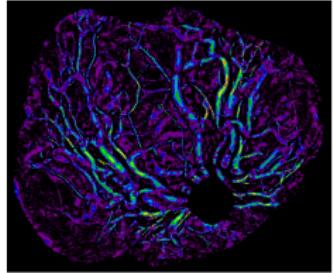
$\mathcal{V}_{\max}$   $\beta = 0.15$ ,  $\gamma = 0.5$

scalewise nz-p  
(p=95)

MCC: 0.41  
precision: 84.46% scalewise nz-p  
(p=98)

MCC: 0.28  
precision: 86.49% trough-fill  
 $\alpha^{(+)} = 0.3$

MCC: 0.56  
precision: 81.92%



## Example Results (2/4)

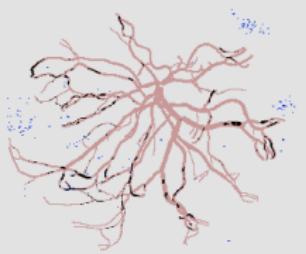
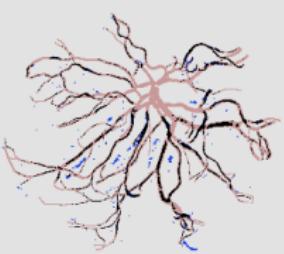
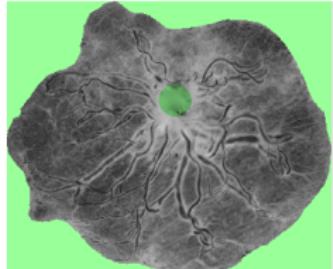
BN7753462

fixed  $\alpha = 0.3$

MCC: 0.43  
precision: 92.99% fixed  $\alpha = 0.2$

MCC: 0.49  
precision: 84.92% ISODATA  
(Frangi-less)

MCC: 0.24  
precision: 78.87%



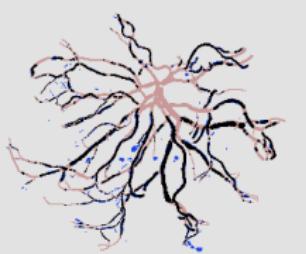
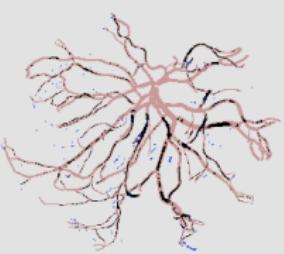
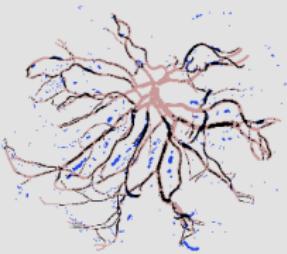
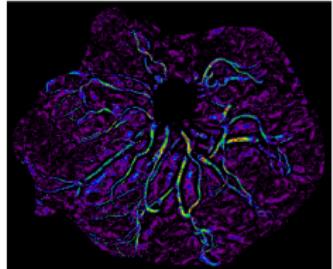
$V_{max}$   $\beta = 0.15$ ,  $\gamma = 0.5$

scalewise nz-p  
(p=95)

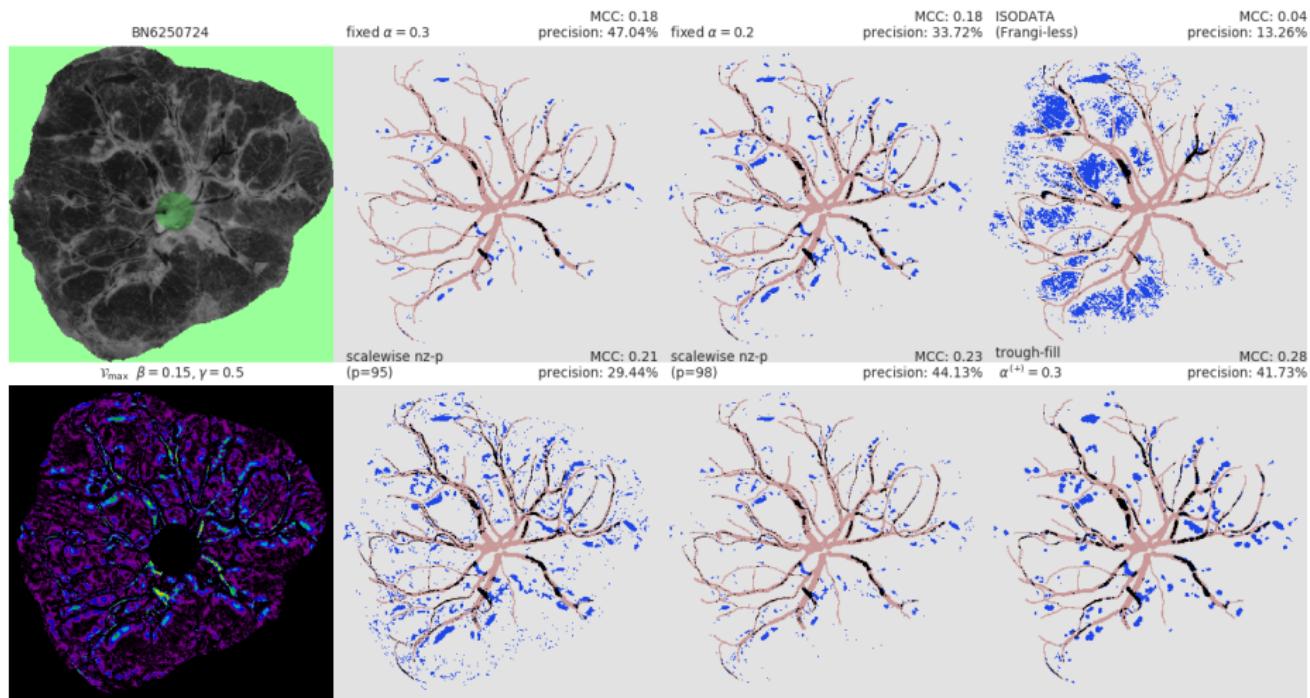
MCC: 0.47  
precision: 73.02% scalewise nz-p  
(p=98)

MCC: 0.38  
precision: 87.10% trough-fill  
 $\alpha^{(+)}$  = 0.3

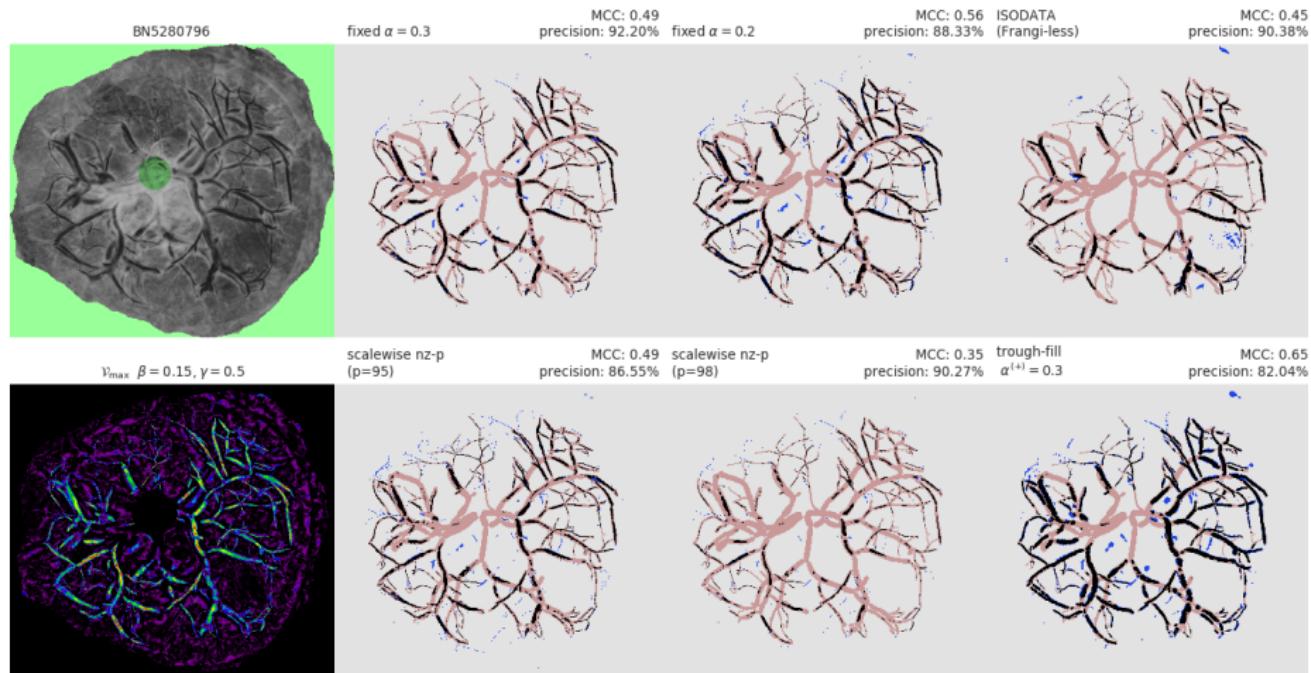
MCC: 0.63  
precision: 82.43%



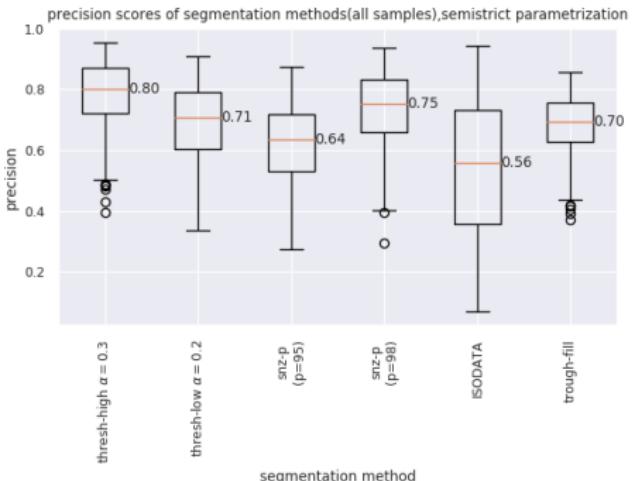
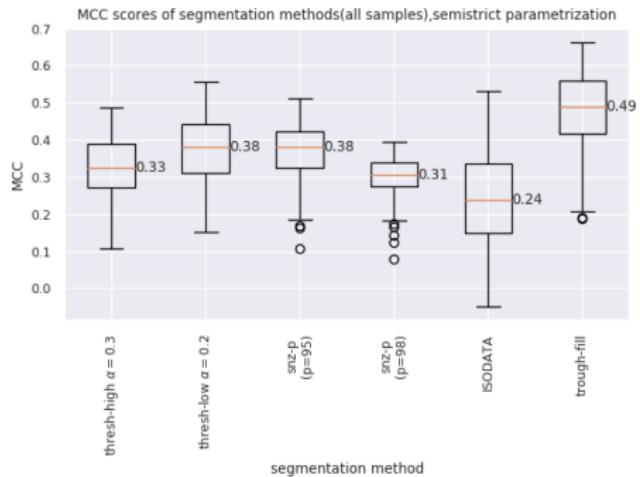
## Example Results (3/4)



## Example Results (4/4)



# Results on All Samples ( $\beta = 0.15$ , $\gamma = 0.5$ )



# Results on All Samples ( $\beta = 0.15, \gamma = 0.5$ )

