ABSTRACT

A Beefy Frangi Filter for Noisy Vascular Segmentation and Network Connection in PCSVN

By

Lucas Wukmer

May 2018

Recent statistical analysis of placental features has suggested the usefulness of studying key features of the placental chorionic surface vascular network (PCSVN) as a measure of overall neonatal health. A recent study has suggested that reliable reporting of these features may be useful in identifying risks of certain neurodevelopmental disorders at birth. The necessary features can be extracted from an accurate tracing of the surface vascular network, but such tracings must still be done manually, with significant user intervention. Automating this procedure would not only allow more data acquisition to study the potential effects of placental health on later conditions, but may ideally serve as a real-time diagnostic for neonatal risk factors as well.

Much work has been to develop reliable vascular extraction methods for well-known image domains (such as retinal MRA images) using Hessian-based filters, namely the (multiscale) Frangi filter. It is desirable to extend these arguments to placental images, but this approach is greatly hindered by the inherent irregularity of the placental surface as a whole, which introduces significant noise into the image domain. A recent attempt was made to apply an additional local curvilinear filter to the Frangi result in an effort to remove some noise from the final extraction.

Here we propose an alternate extraction method. First, we use arguments from Frangis original paper to provide a proper selection of parameters for our particular image domain. Using the same arguments from differential geometry that gave rise to the Frangi filter, we calculate the leading principal direction (eigenvector of the Hessian) to indicate the directionality of curvilinear

features at a particular scale. We are then able to apply an appropriately-oriented morphological filter to our Frangi targets at select scales to remove noise. This approach differs significantly from previous efforts in that morphological filtering will take place at each scale space, rather than being performed one time following multiscale synthesis. Noise removal performed in this way is expected to aide in coherent interpretation of targets that should appear in a connected network.

Finally, we discuss an important advancement in implementation–scale space conversion for differentiation (i.e. gaussian blur) via Fast Fourier Transform (FFT) rather than a more traditional convolution with a gaussian kernel, which offers a significant speedup. This thesis will also contain a general, in depth summary of both multiscale Hessian filters and scale-space theory.

We demonstrate the effectiveness of our improved vascular extraction technique on several of the following image domains: a private database of barium-injected samples provided by University of Rochester, uninjected/raw placental samples from Placental Analytics LLC, a collection of simulated images, the DRIVE and STARE databases of retinal MRAs, and a new collection of computer-generated images with significant curvilinear content.

Time permitting, this research will be extended to include a method of network connection, so that a logically connected vascular network is realized (i.e. network completion).

A Beefy Frangi Filter for Noisy Vascular Segmentation and Network Connection in PCSVN

A THESIS

Presented to the Department of Department of Mathematics and Statistics

California State University, Long Beach

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Applied Mathematics

Committee Members:

Jen-Mei Chang, Ph.D. (Chair)
James von Brecht, Ph.D.
William Ziemer, Ph.D.

College Designee:

Tangan Gao, Ph.D.

By Lucas Wukmer

B.S., 2013, University of California, Los Angeles

May 2018

WE, THE UNDERSIGNED MEMBERS OF THE COMMITTEE,

HAVE APPROVED THIS THESIS

A Beefy Frangi Filter for Noisy Vascular Segmentation and Network Connection in PCSVN

By

Lucas Wukmer

COMMITTEE MEMBERS

Jen-Mei Chang, Ph.D. (Chair)	Mathematics and Statistics
James von Brecht, Ph.D.	Mathematics and Statistics
William Ziemer, Ph.D.	Mathematics and Statistics

ACCEPTED AND APPROVED ON BEHALF OF THE UNIVERSITY

Tangan Gao, Ph.D.
Department Chair, Mathematics and Statistics

California State University, Long Beach

May 2018

ACKNOWLEDGEMENTS

Acknowledgments go here.

TABLE OF CONTENTS

APPENDIX Page

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The Applied Problem

Reference Nen's paper and latest autism risk paper.

Context in Image Processing

- brief background of math image processing methods
- what's been tried in this applied problem
  - nen [1]
  - catalina's paper
  - kara's paper
  - other domains

Research Goals

Segue from previous paragraph, talk about strengths and weaknesses of other methods and what this research aims to accomplish. Include 'research questions' that could allow a reader to answer the question "will this research work for my problem?". "Elevator pitch" maybe goes here.

Roadmap

Outline of the thesis ("firstly" bullshit)

CHAPTER 2

MATHEMATICAL METHODS

(Note to self: this can be 50 pages long. Don't focus on brevity–rather make it basically self-contained, within reason)

Overview of Differential Geometry in Image Processing

Basics, Definitions

For theoretical purposes, we may view any 2D grayscale image as a continuous surface $L : \mathbb{R}^2 \to \mathbb{R}$ with $L \in \mathcal{C}^2\left(\mathbb{R}^2\right)$. In situations where we wish to discuss a discrete image, we may refer to $L_0 \in \mathbb{R}^{m \times n}$. That is, L_0 is a matrix corresponding to the m-by-n digital grayscale image.

Viewing the surface in R^3, we define the Hessian $\mathcal{H}$ of the surface L at a point (x,y) on the surface as the matrix of its second partial derivatives:

$$\mathcal{H}(x,y) = \begin{bmatrix} L_{xx}(x,y) & L_{xy}(x,y) \\ L_{yx}(x,y) & L_{yy}(x,y) \end{bmatrix} \tag{2.1}$$

At any point (x,y) we denote the two eigenpairs of $\mathcal{H}(x,y)$ as

$$\mathcal{H}u_i = \kappa_i u_i\ , \quad i = 1,2 \tag{2.2}$$

where κ_i and u_i are known as the *principal curvatures* and *principal directions* of $L(x,y)$, respectively, and we label such that $|\kappa_2| \geq |\kappa_1|$. Notably, $\mathcal{H}(x,y)$ is a real, symmetric matrix (since $L_{xy} = L_{yx}$ and L is a real function)and thus its eigenvalues are real and its eigenvectors are orthonormal to each other, as given by following lemma:

Lemma 2.1.1 (Principal Axis Theorem?)**.** *Let A be a real, symmetric matrix. The eigenvalues of A are real and its eigenvectors are orthonormal to each other.*

Proof. Let $x \neq 0$ so that $Ax = \lambda x$. Then

$$\begin{aligned}
\|Ax\|_2^2 = \langle Ax, Ax \rangle &= (Ax)^* Ax \\
&= x^* A^* Ax = x^* A^T Ax = x * AAx \\
&= x^* A \lambda x = \lambda x^* Ax \\
&= \lambda x^* \lambda x = \lambda^2 x^* x = \lambda^2 \|x\|_2^2
\end{aligned}$$

Upon rearrangement, we have $\lambda^2 = \frac{\|Ax\|_2^2}{\|x\|_2^2} \geq 0 \implies \lambda$ is real.

To prove that a set of orthonormalizable eigenvectors exists, let A be real, symmetric as above and consider the eigenpairs $Av_1 = \lambda_1 v_1, Av_2 = \lambda_2 v_2$ with $v_1, v_2 \neq 0$. [1]

In the case that $\lambda_1 \neq \lambda_2$, we have

$$\begin{aligned}
(\lambda_1 - \lambda_2) v_1^T v_2 &= \lambda_1 v_1^T v_2 - \lambda_2 v_1^T v_2 \\
&= (\lambda_1 v_1)^T v_2 - v_1^T (\lambda_2 v_2) \\
&= (Av_1)^T v_2 - v_1^T (Av_2) \\
&= v_1^T A^T v_2 - v_1^T Av_2 \\
&= v_1^T Av_2 - v_1^T Av_2 = 0
\end{aligned}$$

Since $\lambda_1 \neq \lambda_2$, we conclude that $v_1^T v_2 = 0$.

In the case that $\lambda_1 = \lambda_2 =: \lambda$, we can define (as in Gram-Schmidt orthogonalization) $u = v_2 - \frac{v_1^T v_2}{v_1^T v_1} v_1$. This is an eigenvector for $\lambda = \lambda_2$, as

$$\begin{aligned}
Au &= A \left(v_2 - \frac{v_1^T v_2}{v_1^T v_1} v_1 \right) \\
&= Av_2 - \frac{v_1^T v_2}{v_1^T v_1} Av_1 \\
&= \lambda v_2 - \frac{v_1^T v_2}{v_1^T v_1} \lambda v_1 \\
&= \lambda \left(v_2 - \frac{v_1^T v_2}{v_1^T v_1} v_1 \right) = \lambda u
\end{aligned}$$

[1] To simplify notation, we simplify our argument to consider two explicit eigenvectors only, since we're only concerned with the 2×2 matrix $\mathcal{H}$ anyway.

and is perpendicular to v_1, since

$$\begin{aligned} v_1^T u &= v_1^T \left(v_2 - \frac{v_1^T v_2}{v_1^T v_1} v_1 \right) \\ &= v_1^T v_2 - \left(\frac{v_1^T v_2}{v_1^T v_1} \right) v_1^T v_1 \\ &= v_1^T v_2 - v_1^T v_2 (1) = 0. \end{aligned}$$

□

Thus we see that the two principal directions exist and are orthogonal at each point (x,y) within the continuous image $L(x,y)$.

Note: you could also prove (which might be useful for later) that these vary continuously along paths in $\mathbb{R}^2$ *except at points where* $\mathcal{H}(x,y)$ *is singular.*

Calculating Derivatives of Discrete Images

proof that derivatives should be taken on gaussian blur.

Frangi Filter

Intro to Hessian-based filters

Hessian-based filters are a family of curvilinear filters that employ the Hessian and its eigenspaces to determine regions of significant curvature within an image. Several such filters exist –see Sato [13] and Lorenz[10] for more info within Frangi's paper

Overview of Frangi vesselness measure

The Frangi filter is a widely used [citation needed] Hessian-based filter that relies on the principal curvatures–that is, the eigenvalues of $\mathcal{H}_\sigma(x,y)$ at some particular scale σ at each point (x,y) in the image.

Implementation Details: Convolution Speedup via FFT

As described above, the actual computation of derivatives is achieved via convolution with a gaussian. In practice, this is very slow for large scales. In **TODO** .

- find image processing papers that find hessian from FFT / who uses this?

- with above: downsides?
- SIDE BY SIDE comparison?

2D Discrete Fourier Transform Convolution Theorem . [2]

Theorem 2.3.1 (2D DFT Convolution Theorem)**.** *Given two discrete functions are sequences with the same length*[3]*, that is: $f(x,y)$ and $h(x,y)$ for integers $0 < x < M$ and $0 < y < N$, we can take the discrete fourier transform (DFT) of each:*

$$F(u,v) := \mathcal{D}\{f(x,y)\} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)} \tag{2.3}$$

$$H(u,v) := \mathcal{D}\{h(x,y)\} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} h(x,y)e^{-2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)} \tag{2.4}$$

and given the convolution of the two functions

$$(f \star h)(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m,y-n) \tag{2.5}$$

then $(f \star h)(x,y)$ and $MN \cdot F(u,v)H(u,v)$ are transform pairs, i.e.

$$(f \star h)(x,y) = \mathcal{D}^{-1}\{MN \cdot F(u,v)H(u,v)\} \tag{2.6}$$

The proof follows from the definition of convolution, substituting in the inverse-DFT of f and h, and then rearrangement of finite sums.

[2] the following was adapted in a large part from DFT: an owner's manual. cite?

[3] If they're not actually the same length, DIP-GW suggests to make the final length at least $P = A + C - 1$ and $Q = B + D - 1$ in the case that the sizes are $A \times B$ and $C \times D$ for $f(x,y)$ and $h(x,y)$ respectively. Not sure if that matters.

Proof.

$$(f \star h)(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m,y-n) \tag{2.7}$$

$$= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \left(\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} F(p,q)e^{2\pi i\left(\frac{mp}{M}+\frac{nq}{N}\right)}\right)\left(\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} H(u,v)e^{2\pi i\left(\frac{u(x-m)}{M}+\frac{v(y-n)}{N}\right)}\right) \tag{2.8}$$

$$= \left(\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} H(u,v)e^{2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)\left(\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} F(p,q)\left(\sum_{m=0}^{M-1} e^{2\pi i\left(\frac{m(p-u)}{M}\right)}\right)\left(\sum_{n=0}^{N-1} e^{2\pi i\left(\frac{n(q-v)}{N}\right)}\right)\right) \tag{2.9}$$

$$= \left(\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} H(u,v)e^{2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)\left(\sum_{p=0}^{M-1}\sum_{q=0}^{N-1} F(p,q)\left(M\cdot\hat{\delta}_M(p-u)\right)\left(N\cdot\hat{\delta}_M(q-v)\right)\right) \tag{2.10}$$

$$= \left(\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} H(u,v)e^{2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)\cdot MNF(u,v) \tag{2.11}$$

$$= MN\cdot\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)H(u,v)e^{2\pi i\left(\frac{ux}{M}+\frac{vy}{N}\right)} \tag{2.12}$$

$$= MN\cdot\mathcal{D}^{-1}\{FH\} \tag{2.13}$$

where

$$\hat{\delta}_N(k) = \begin{cases} 1 & \text{when } k=0 \mod N \\ 0 & \text{else} \end{cases} \tag{2.14}$$

□

Above, we make use of the following lemma:

Lemma 2.3.2. *Let j and k be integers and let N be a positive integer. Then*

$$\sum_{n=0}^{N-1} e^{2\pi i\left(\frac{n(j-k)}{N}\right)} = N\cdot\hat{\delta}_N(j-k) \tag{2.15}$$

Proof. For any particular $n \in 0..N-1$,consider the complex number $e^{2\pi in(j-k)/N}$. Note first, that this is an N-th root of unity, since

$$\left(e^{2\pi in(j-k)/N}\right)^N = e^{2\pi in(j-k)} = \left(e^{2\pi i}\right)^{n(j-k)} = 1^{n(j-k)} = 1$$

Thus, we understand that the complex number $e^{2\pi in(j-k)/N}$ is a root of $z^N - 1 = 0$, which we rewrite as

$$z^N - 1 \; = \; (z-1)(z^{n-1} + \cdots + z + 1) \; = \; (z-1)\sum_{n=0}^{N-1} z^n.$$

We consider two cases: in the case that $j-k$ is a multiple of N, we of course have $e^{2\pi in(j-k)/N} = 1$ for any n, and thus

$$\sum_{n=0}^{N-1} e^{2\pi i\left(\frac{n(j-k)}{N}\right)} = \sum_{n=0}^{N-1} (1) = N$$

.

In the case that $j-k$ is *not* a multiple of N, then clearly $\left(e^{2\pi in(j-k)/N}\right) - 1 \neq 0$, and thus it must be that

$$\left(e^{2\pi in(j-k)/N}\right)^N - 1 = 0 \implies \sum_{n=0}^{N-1} \left(e^{2\pi in(j-k)/N}\right)^n = 0$$

Combining these two cases gives the result of the lemma. □

FFT .

As noted, the above result applies to the Discrete Fourier Transform. As noted, we actually achieve a convolution speedup using a Fast Fourier Transform (FFT) instead.

Should I put theory in on this? Yuck.

Comparision–Notes on Speedup .

Here you actually provide some results that this method is faster. Times, compatibililty of results, etc.

Sampling Theorem . I actually don't know. WRAPAROUND ERROR (like from p250) i don't get how that works

Linear Scale Space Theory

Koenderink showed that "any image can be embedded in a one-parameter family of derived images (with resolution as the parameter) in essentially only one unique way" given a few of the so-called *scale space axioms*. They showed in particular that any such family must satisfy the heat equation

$$\Delta K(x,y,\sigma) = K_\sigma(x,y,\sigma) \text{ for } \sigma \geq 0 \text{ such that } K(x,y,0) = u_0(x,y). \tag{2.16}$$

where $K : \mathbb{R}^3 \to \mathbb{R}$ and $u_0 : \mathbb{R}^2 \to \mathbb{R}$: is the original image (viewed as a continuous surface) and σ is a resolution parameter.

Overview of Theory

methods of ”merging”

Other Odds and Ends

Finding the placental plate? Morphology stuff like skeletonization.

CHAPTER 3

RESEARCH PROTOCOL

List. All. Decisions. You Make. Be very explicit.

Pseudocode?

CHAPTER 4

RESULTS AND ANALYSIS

Data Set

Specifics of your data set. Barium placentas, other datasets.

NOTE: FIND YOUR TRACING PROTOCOL.

NOTE: Be specific about each so it's easy to gauge for others whether or not these methods would apply to their own research

Preprocessing

Results

Show stuff who the fuck knows

Answer Research Questions

CHAPTER 5

CONCLUSION

What it did well. What it didn't. People who want to expand–where should they start?

APPENDICES

APPENDIX A

APPENDIX TITLE

Put code here (and on github)

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Nen Huynh. *A filter bank approach to automate vessel extraction with applications*. PhD thesis, California State University, Long Beach, 2013.