

8 f) with the given $f(x, y)$,

the marginal distribution of x (analogously y) is

$$\begin{aligned} f(x) &= \int_{-\infty}^{\infty} \phi(x) + k|x| \phi(y)|y| \, dy \quad |x| < 1 \\ &= k \phi(x) \quad |x| > 1 \end{aligned}$$

The conditional distribution given $y=0$

is

$$f(x) = k \phi(x) \phi(0)$$

$$\frac{k \phi(x) \phi(0)}{k \phi(0)} = \phi(x) \quad -\infty < x < \infty$$

When $y = \pm 1$,

$$f(x) = \frac{k \phi(x) \phi(\pm 1)}{k \phi(\pm 1)} = \phi(x) \quad -\infty < x < \infty$$

When $y = \pm 0.5$ (analogously -0.5)

$$f(x) = \frac{k(\phi(x) \phi(0.5) + 0.5|x| \phi(x))}{k \phi(0.5) + k 0.5 \int_{-1}^1 \phi(x)|x| \, dx} \quad |x| < 1$$

$$= \frac{k \phi(x) \phi(0.5)}{k \phi(0.5) + 0.5 k \int_{|x| < 1} \phi(x) |x|} \quad |x| > 1$$

where k is s.t

$$\iint_{x, y} \phi(x) \phi(y) \leq 1 + |x| |y| \max\{|x|, |y|\} \\ \int \phi(x) dx = 1$$