

date: 2025-02-20

TFE4188 - Lecture 6

Oversampling and Sigma-Delta ADCs

Goal for today

Understand **why** there are different ADCs

Introduction to **oversampling** and **delta-sigma** modulators

A few **examples**

1999, R. Walden: Analog-to-digital converter survey and analysis

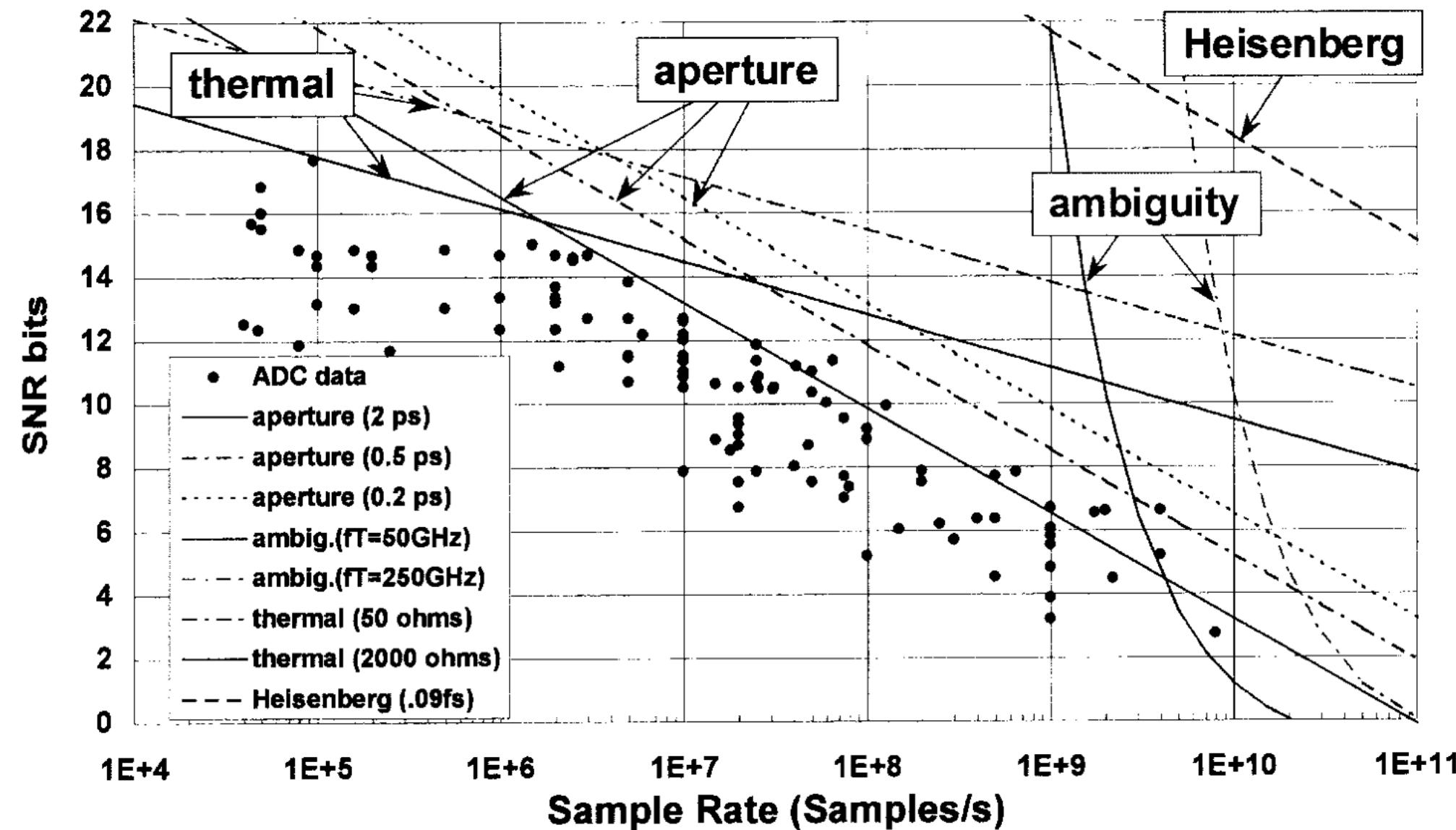
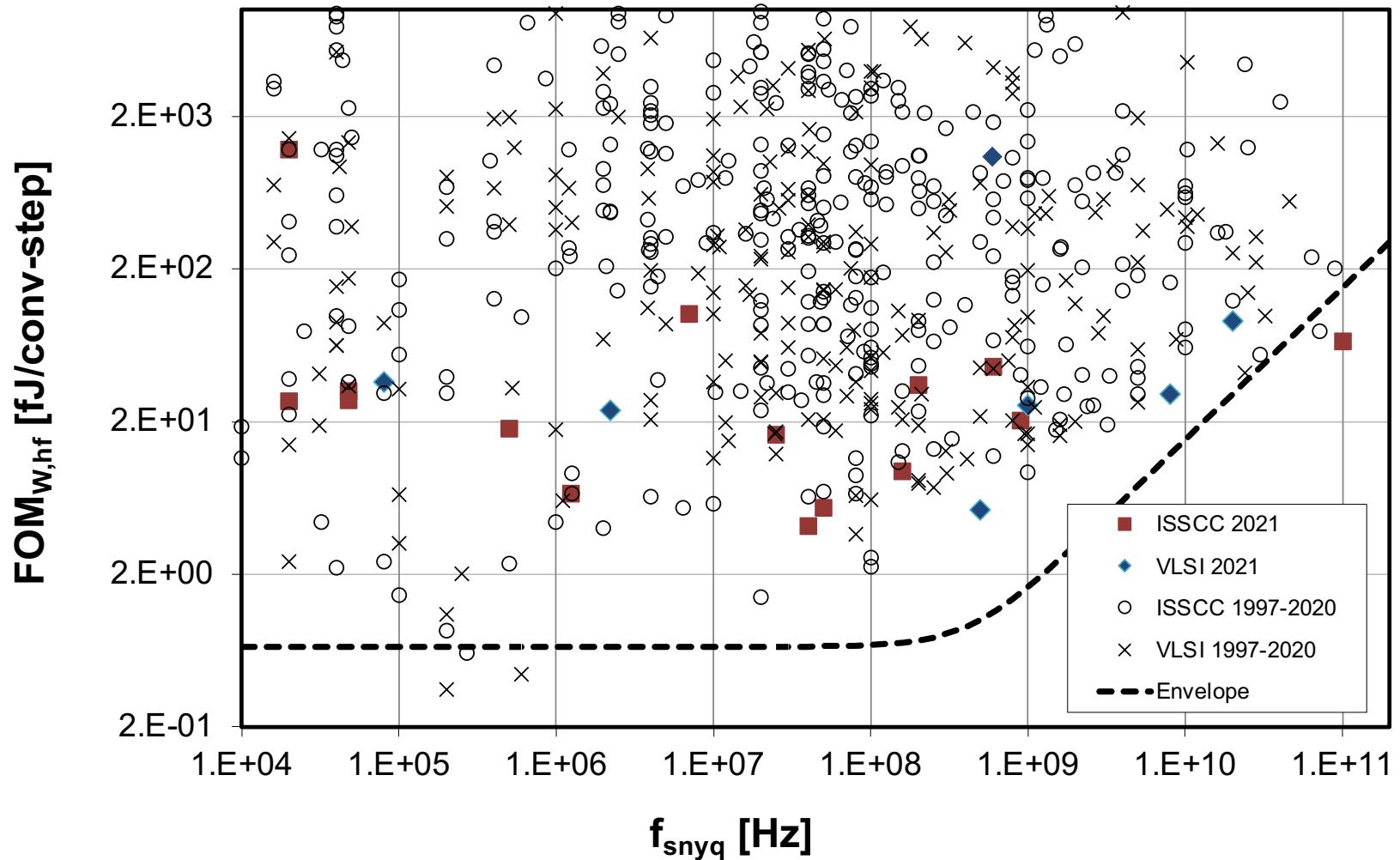


Fig. 7. Signal-to-noise ratio according to $\text{SNR_bits} = (\text{SNR(dB)} - 1.76)/6.02$. Three sets of curves show performance limiters due to thermal noise, aperture uncertainty, and comparator ambiguity. The Heisenberg limit is also displayed.

B. Murmann, ADC Performance Survey 1997-2023

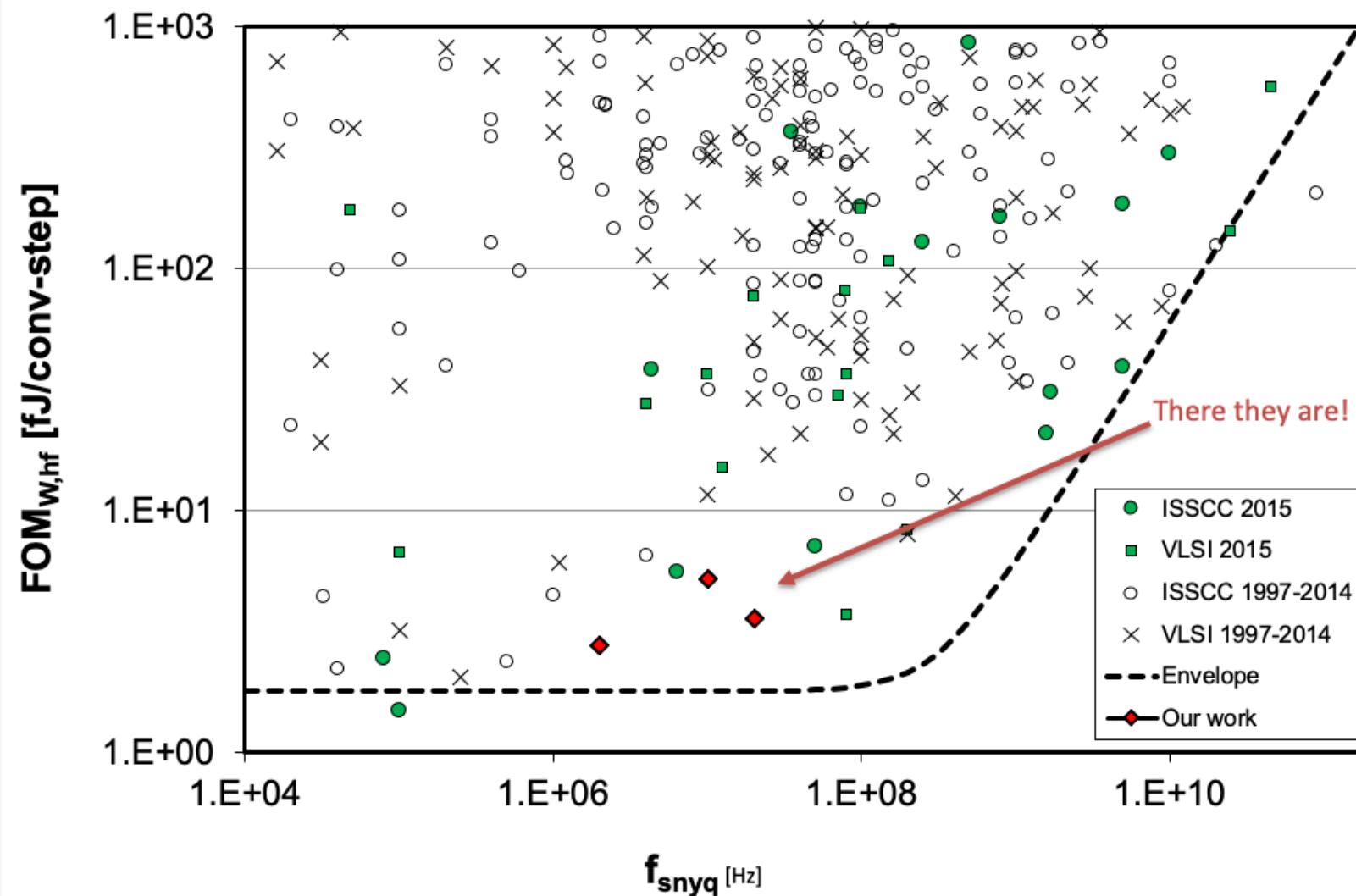


$$FOM_W = \frac{P}{2^B f_s}$$

Below 10 fJ/conv.step is good.

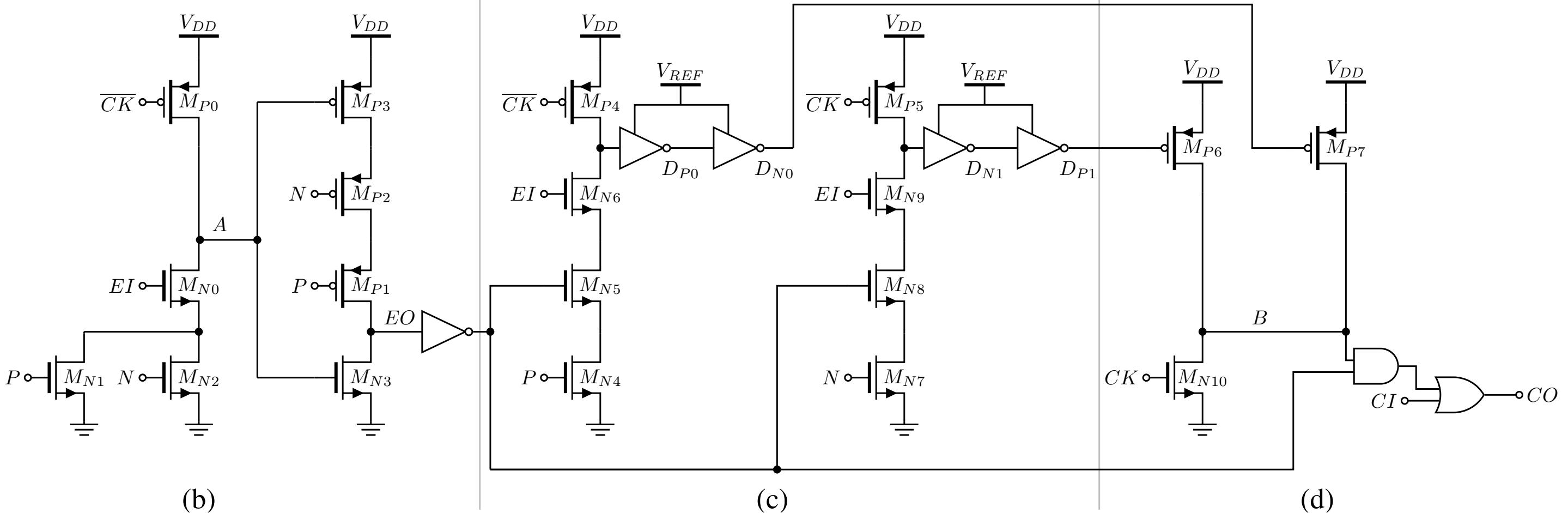
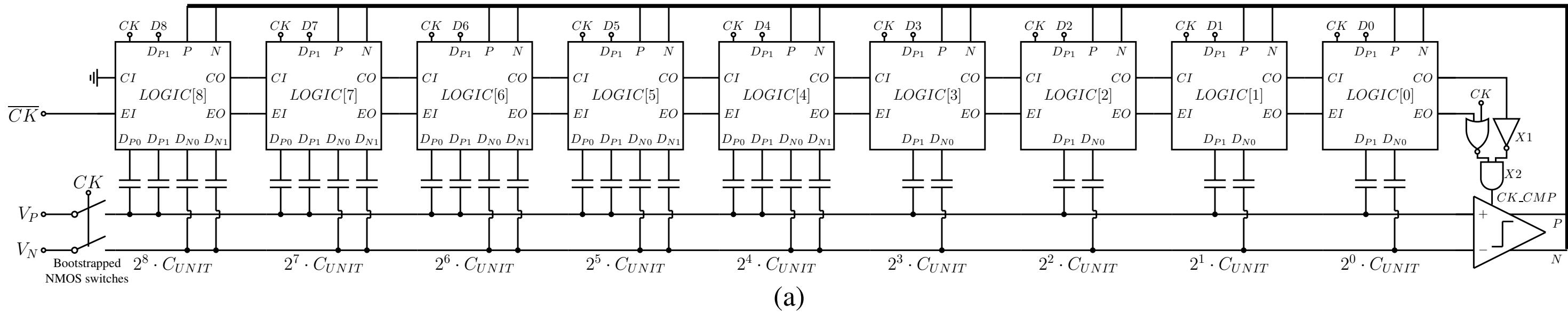
Below 1 fJ/conv.step is extreme.

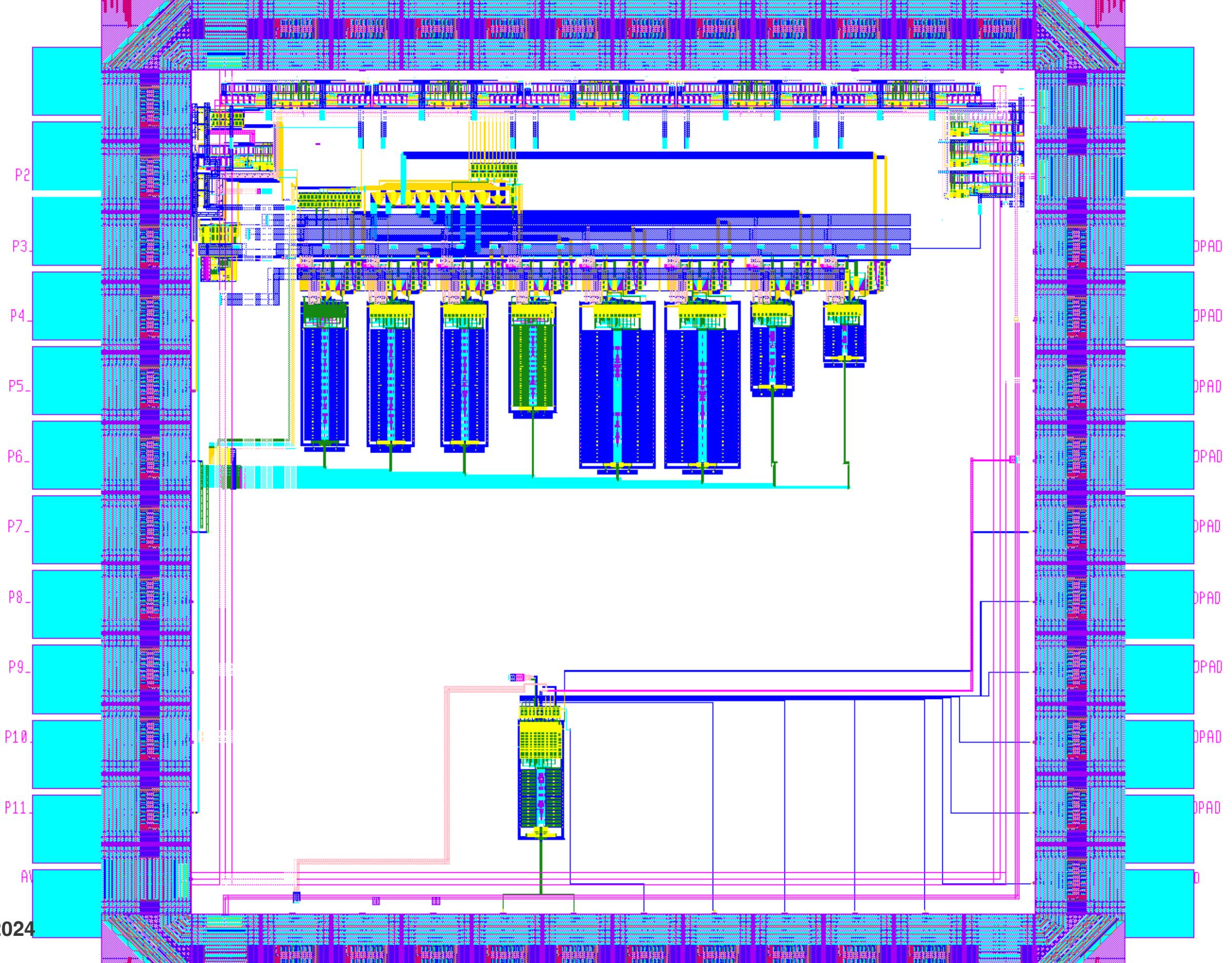
People from NTNU have made some of
the worlds best ADCs

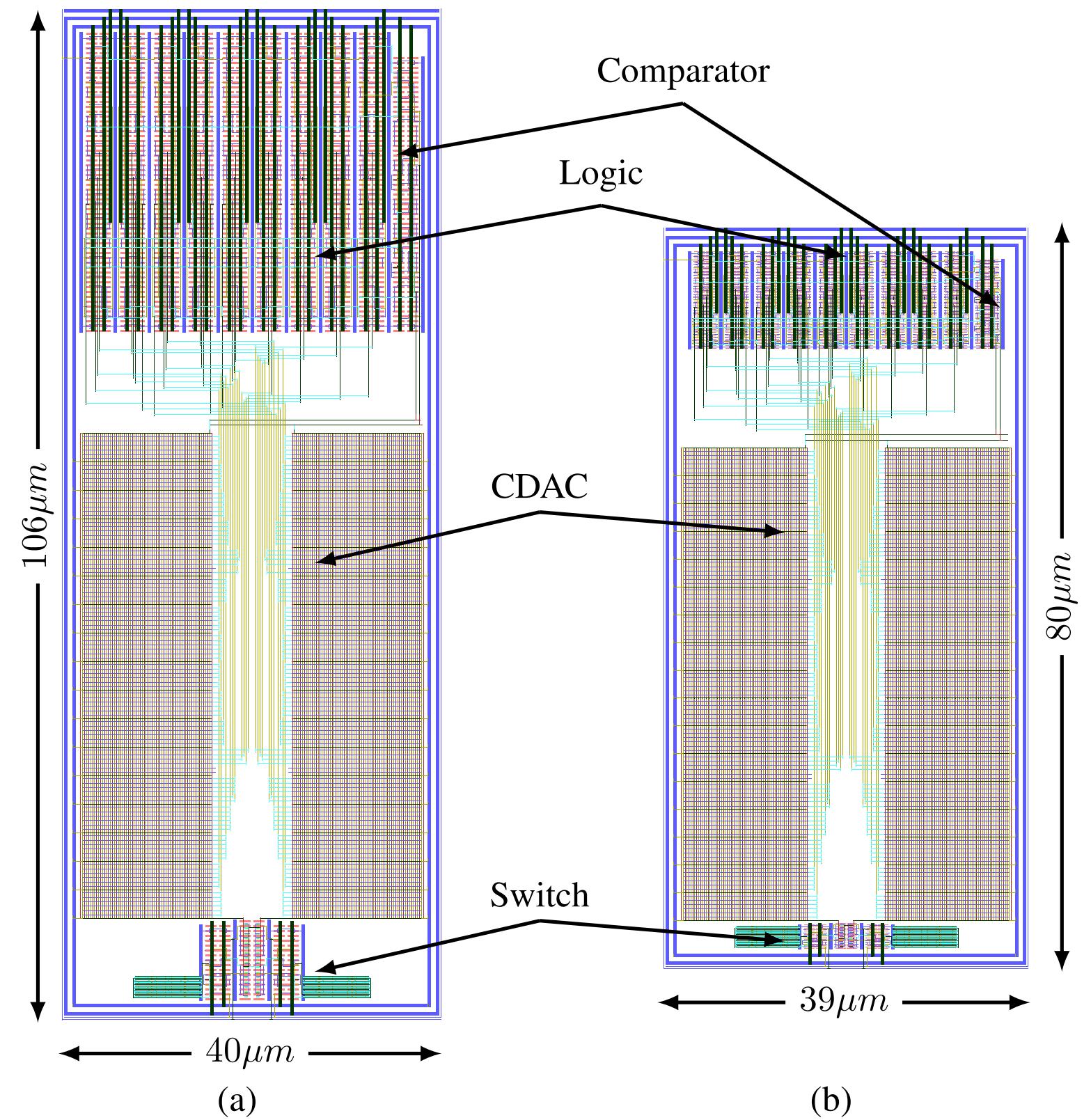


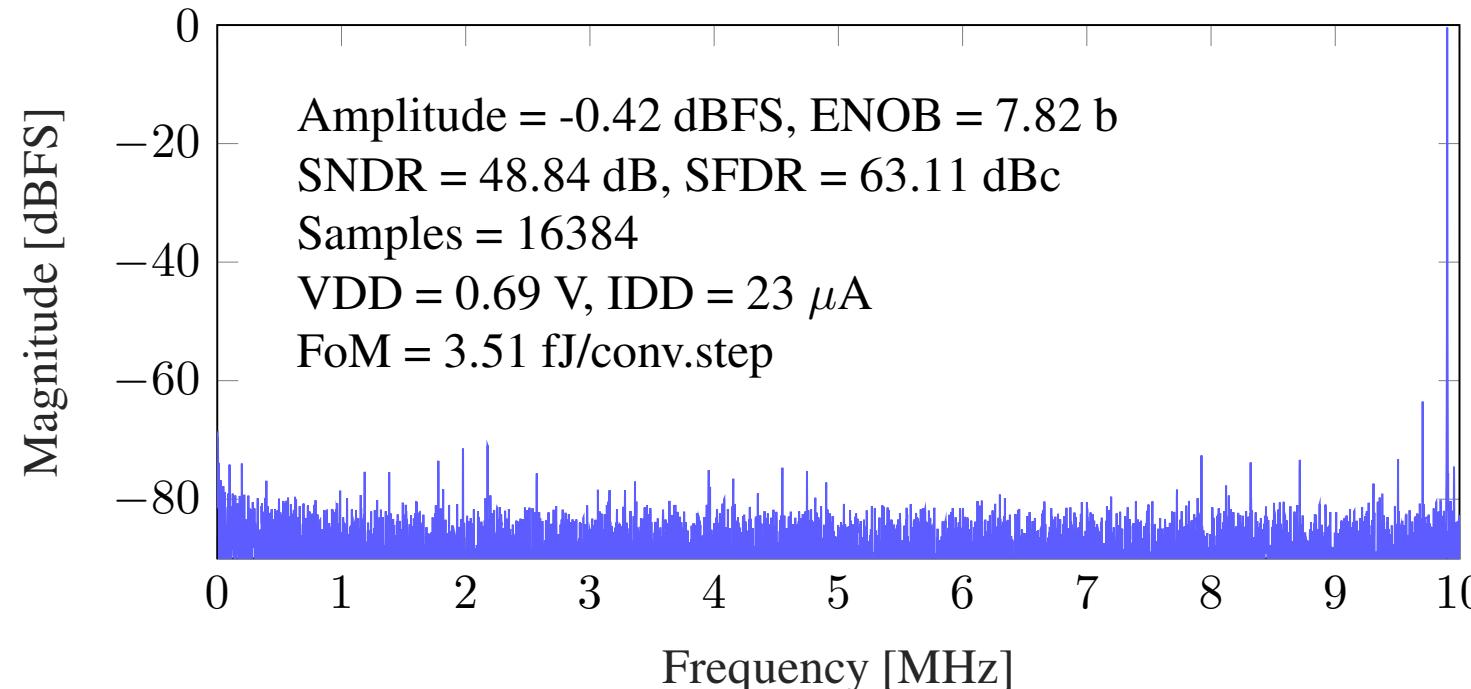
- [1] A Compiled 9-bit 20-MS/s 3.5-fJ/
conv.step SAR ADC in 28-nm FDSOI for
Bluetooth Low Energy Receivers
- [2] A 68 dB SNDR Compiled Noise-
Shaping SAR ADC With On-Chip CDAC
Calibration

What makes a state-of-the-art ADC

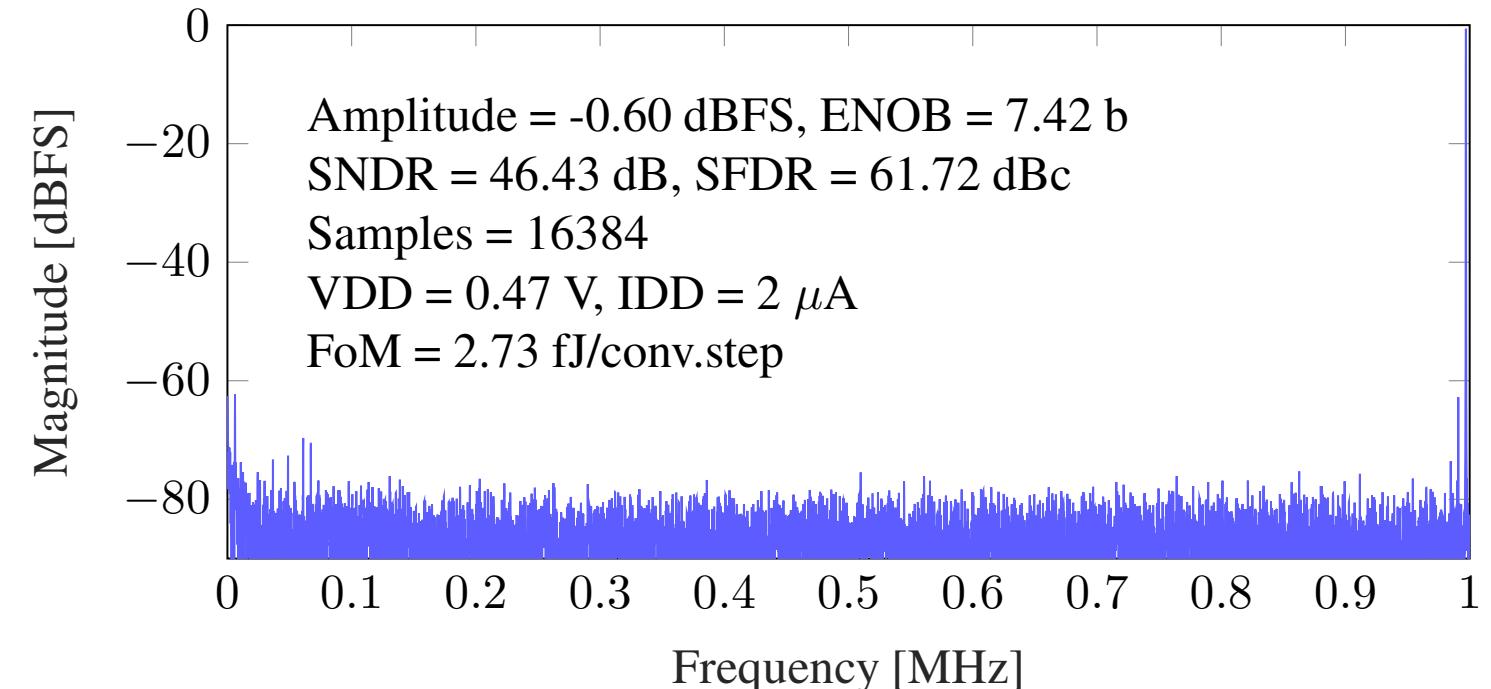




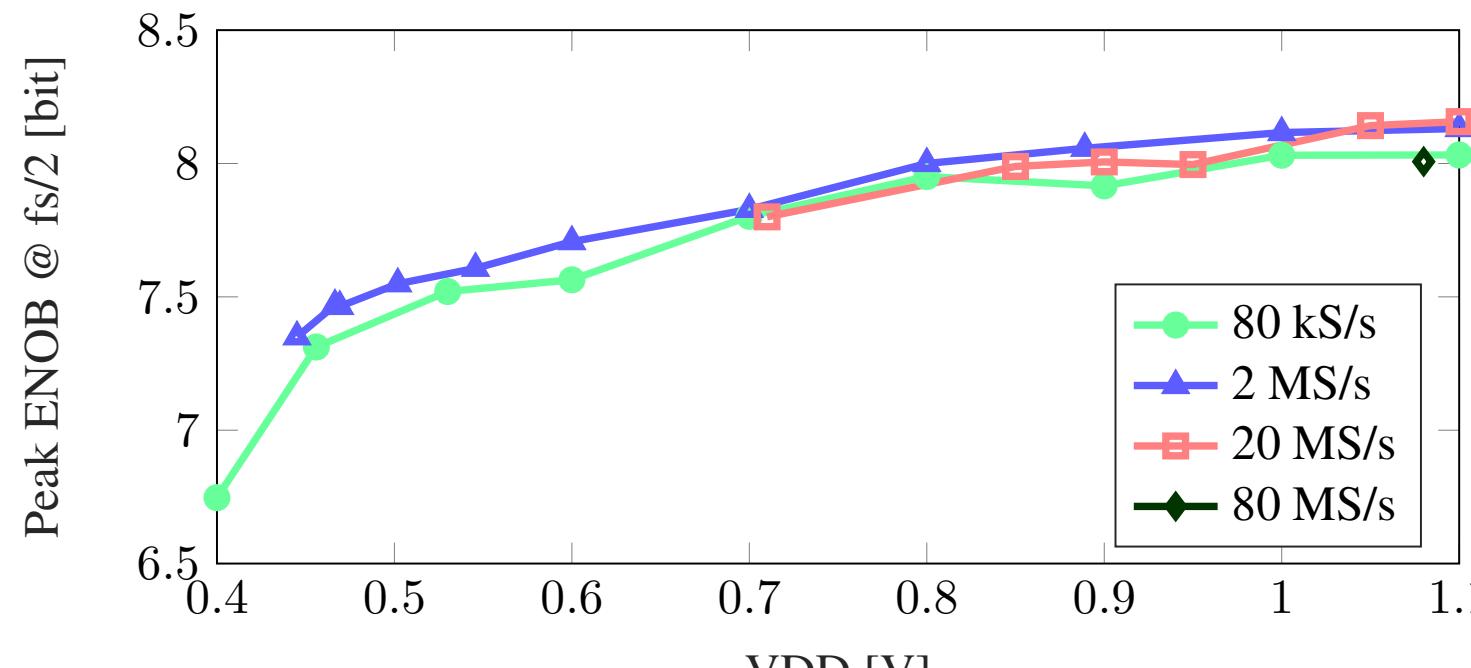




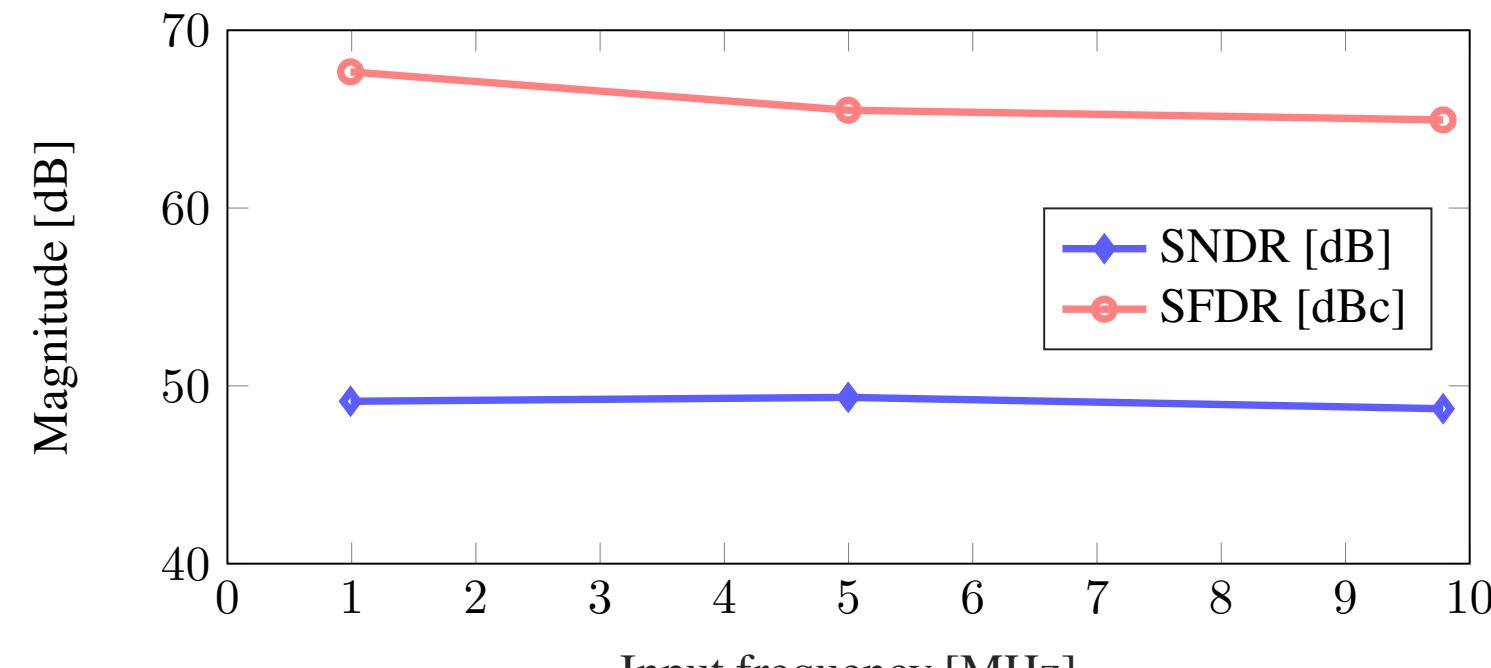
(a)



(b)

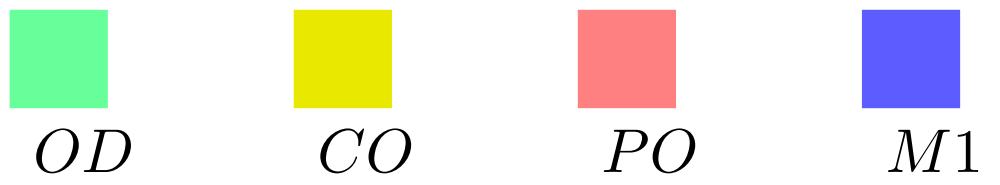
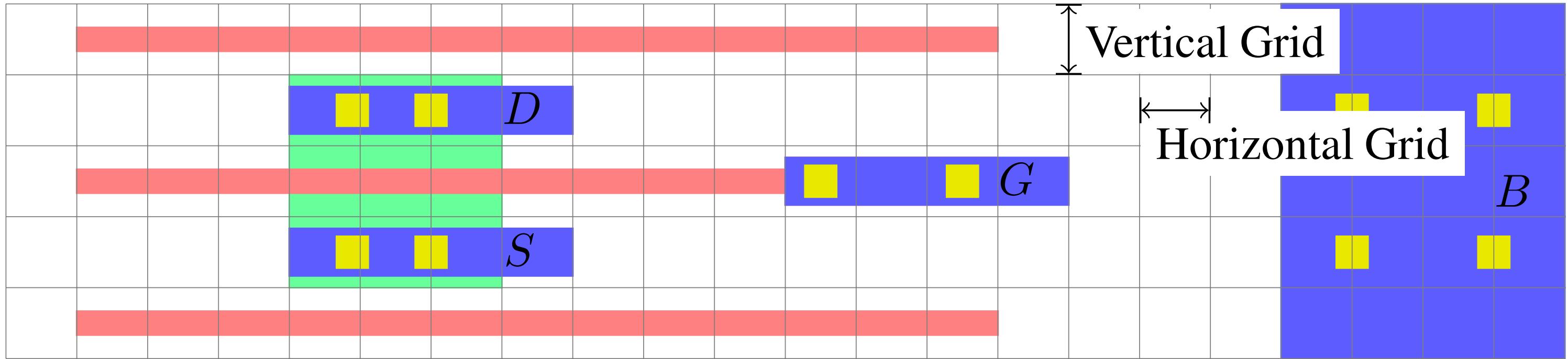


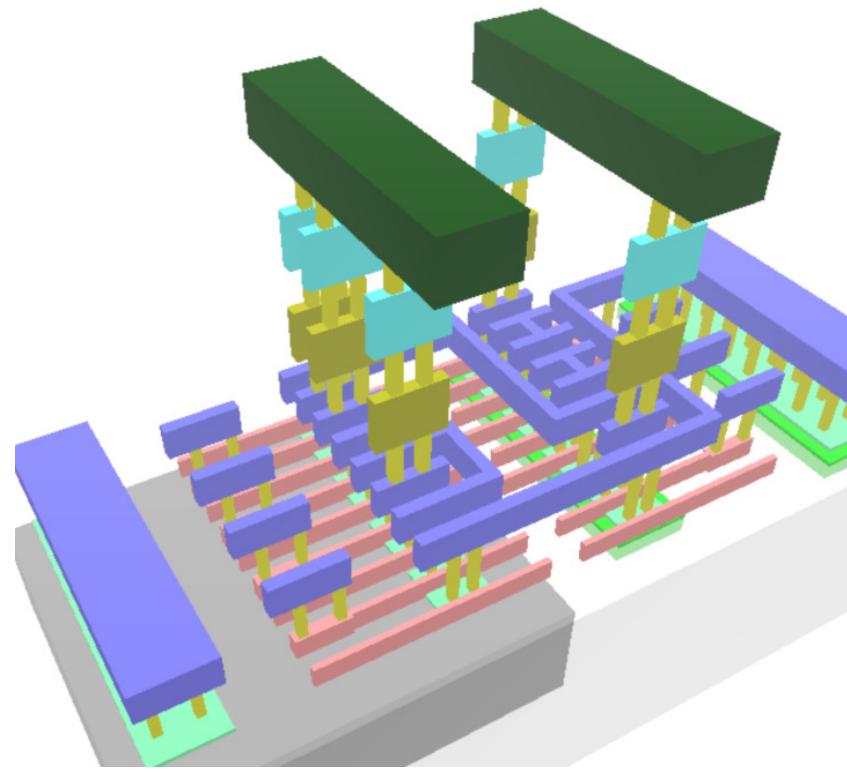
(c)



(d)

	Weaver [5]	Harpe [9]	Patil [10]	Liu [11]	This work	
Technology (nm)	90	90	28 FDSOI	28	28	FDSOI
Fsample (MS/s)	21	2	No sampling	100	2	20
Core area (mm ²)	0.18	0.047	0.0032	0.0047	0.00312	
SNDR (dB)	34.61	57.79	40	64.43	46.43	48.84
SFDR (dBc)	40.81	72.33	30	75.42	61.72	63.11
ENOB (bits)	5.45	6.7 - 9.4	6.35	10.41	7.42	7.82
Supply (V)	0.7	0.7	0.65	0.9	0.47	0.69
Pwr (μ W)	1110	1.64 -3.56	24	350	0.94	15.87
Compiled	Yes	No	No	No	Yes	
FoM (fJ/c.step)	838	2.8 - 6.6	3.7	2.6	2.7	3.5





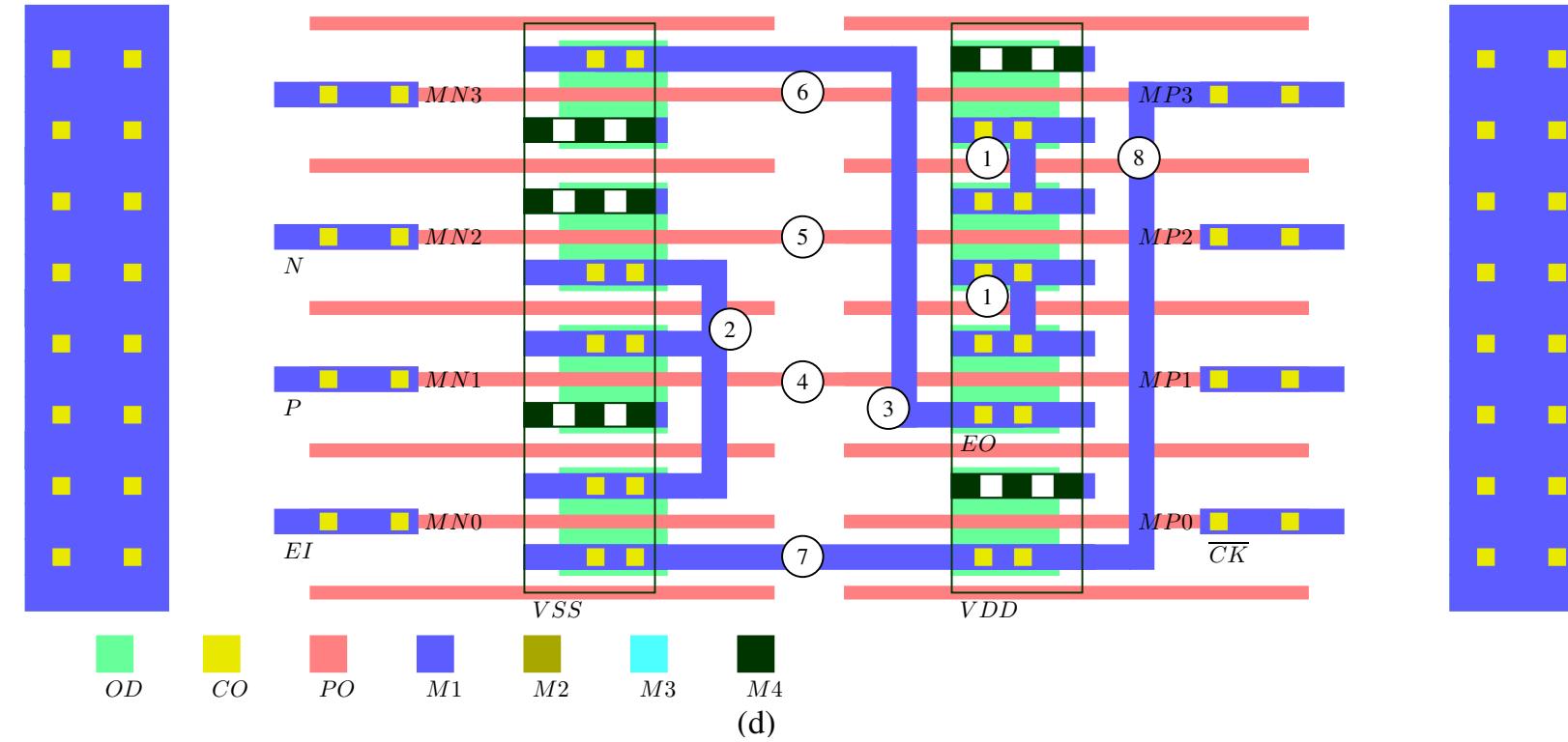
(a)

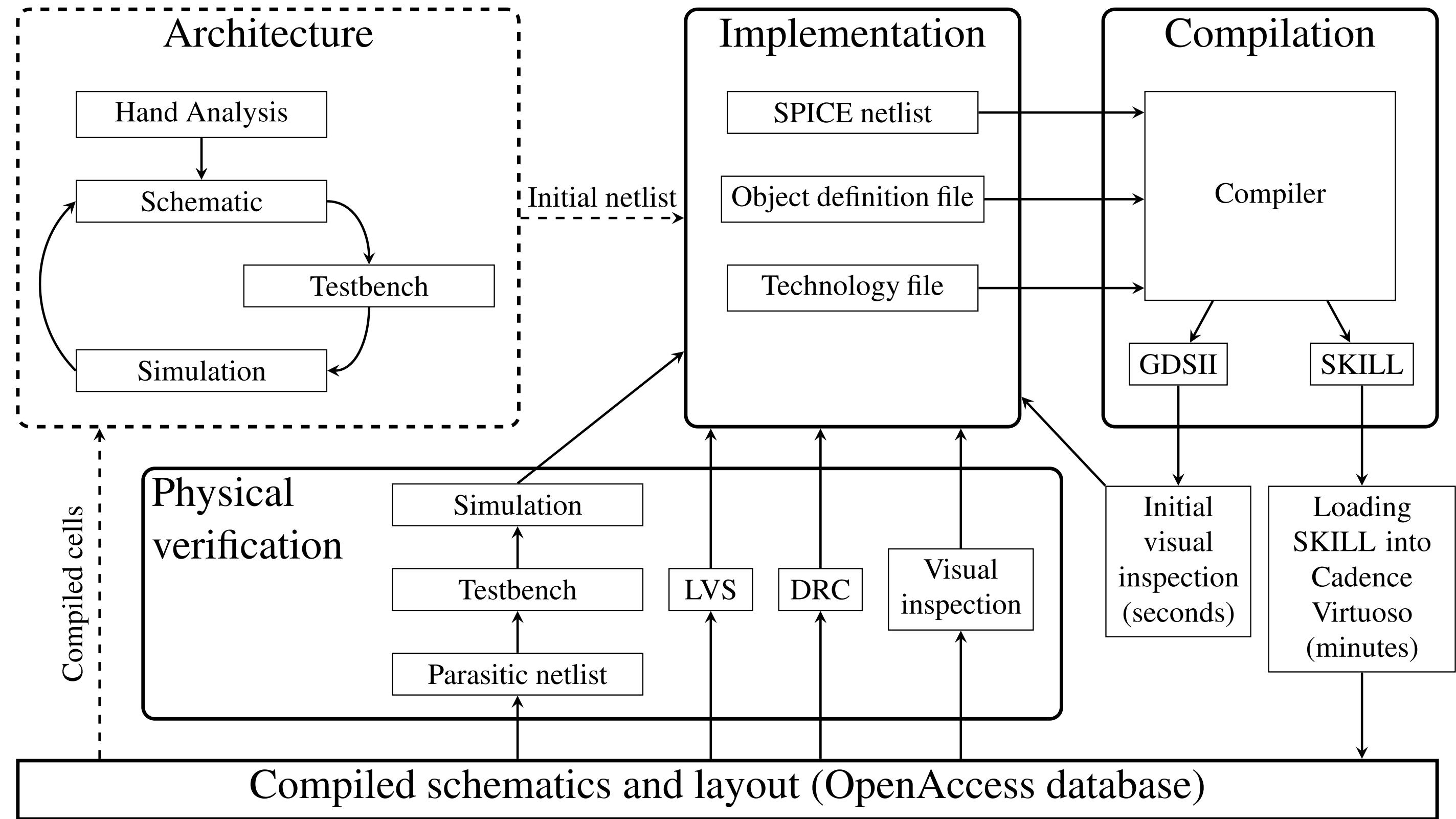
```
.SUBCKT SAREMX1_CV P N EI EO CK_N AVDD AVSS
  MN0 N3 EI A AVSS NCHDL
  MN1 N3 P AVSS AVSS NCHDL
  MN2 AVSS N N3 AVSS NCHDL
  MN3 EO A AVSS AVSS NCHDL
  MP0 AVDD CK_N A AVSS PCHDL
  MP1 N2 P EO AVSS PCHDL
  MP2 N1 N N2 AVSS PCHDL
  MP3 AVDD A N1 AVSS PCHDL
.ENDS
```

(b)

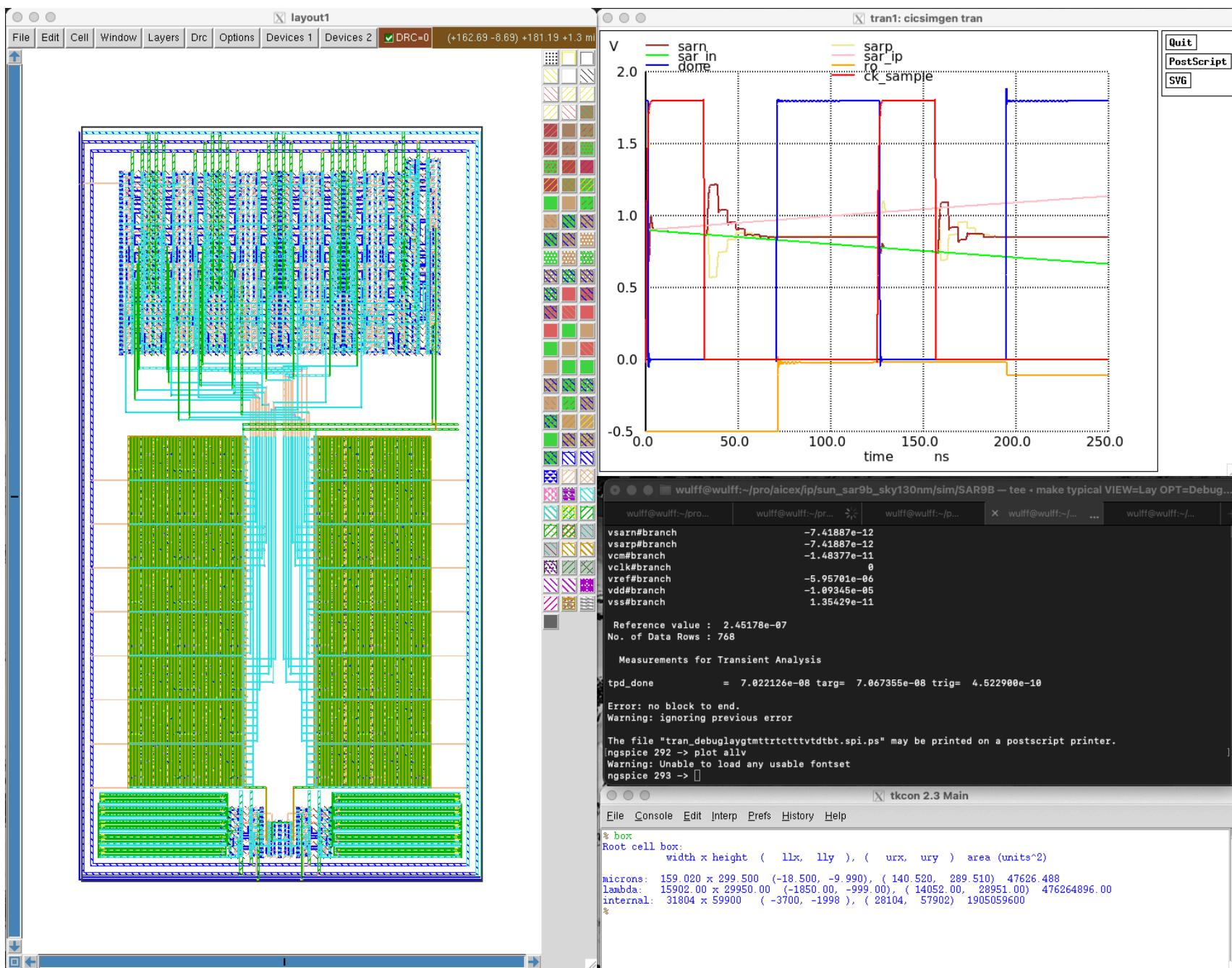
```
{
  "name": "SAREMX1_CV",
  "class": "Layout::LayoutDigitalCell",
  "addConnectivityRoutes": [
    ["M1", "N1|N2", "| |", "", 1],
    ["M1", "N3", "- |", "", 2],
    ["M1", "EO", "-- | -", "onTopR"], 3
  ],
  "addDirectedRoutes": [
    ["PO", "P", "MN1:G-MP1:G"], 4
    ["PO", "N", "MN2:G-MP2:G"], 5
    ["PO", "A", "MN3:G-MP3:G"], 6
    ["M1", "A", "MN0:S-MP0:S"], 7
    ["M1", "A", "MP0:S-| --MP3:G"] 8
  ]
}
```

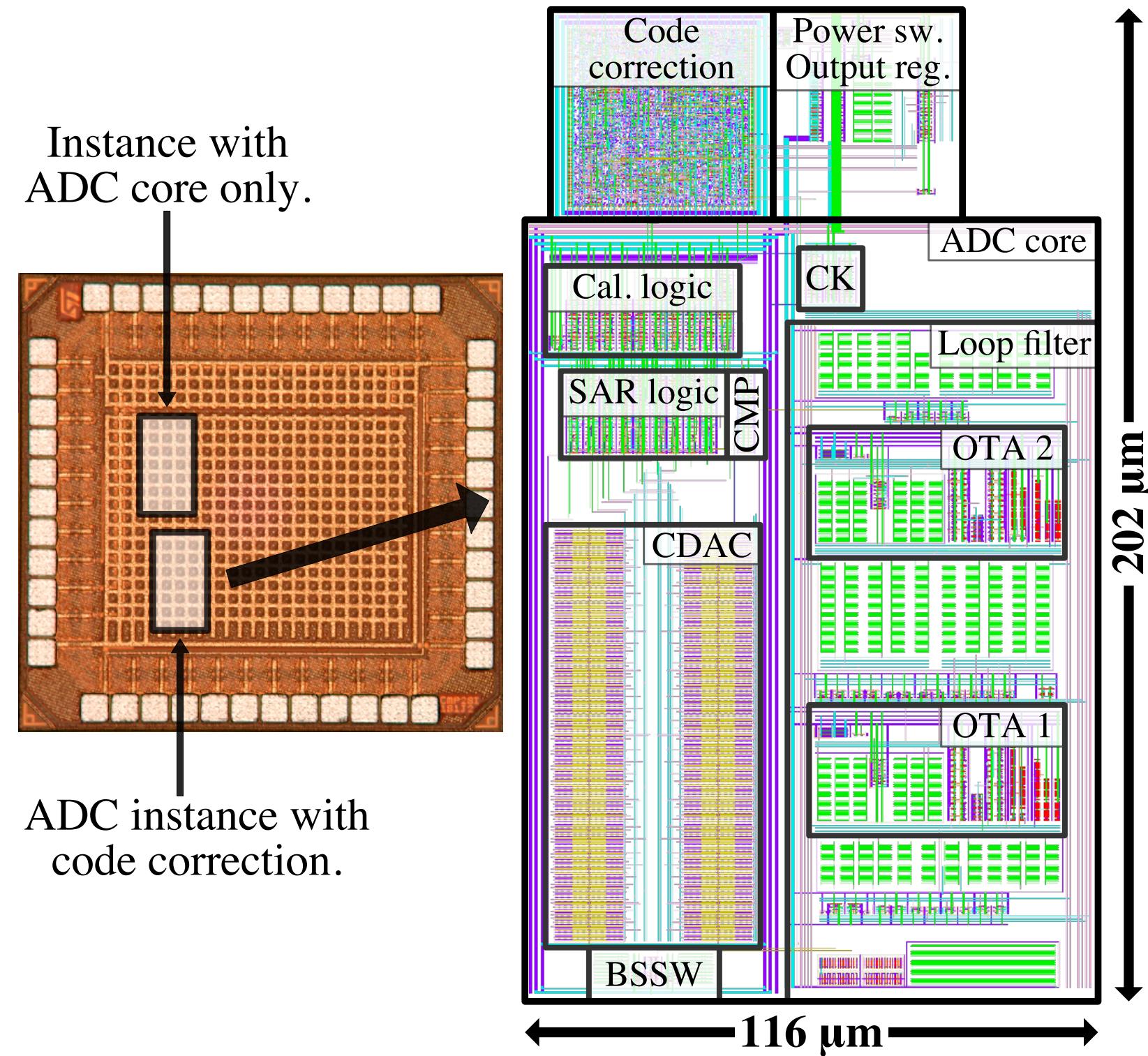
(c)





SUN_SAR9B_SKY130NM





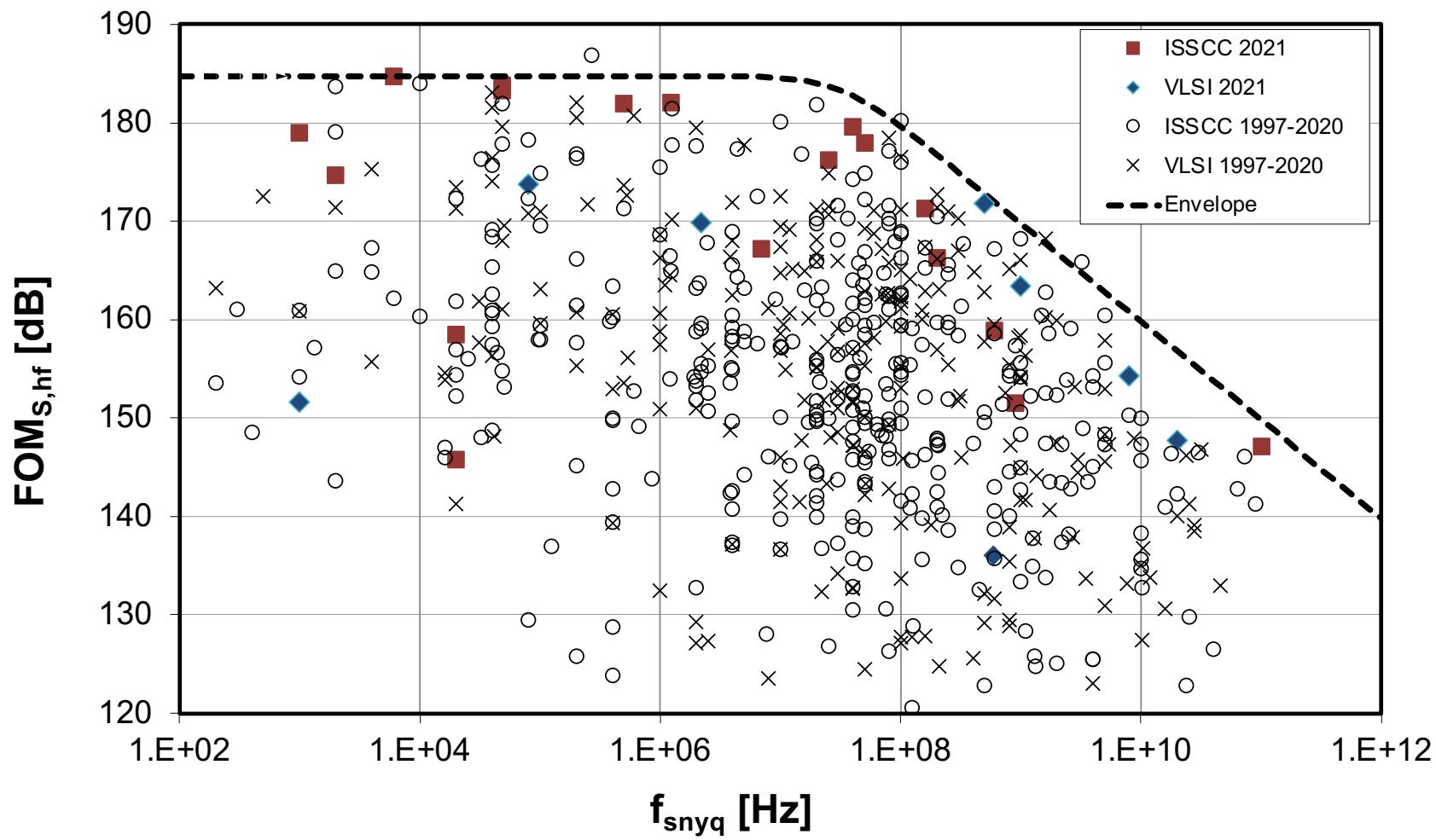
Instance with
ADC core only.

ADC instance with
code correction.

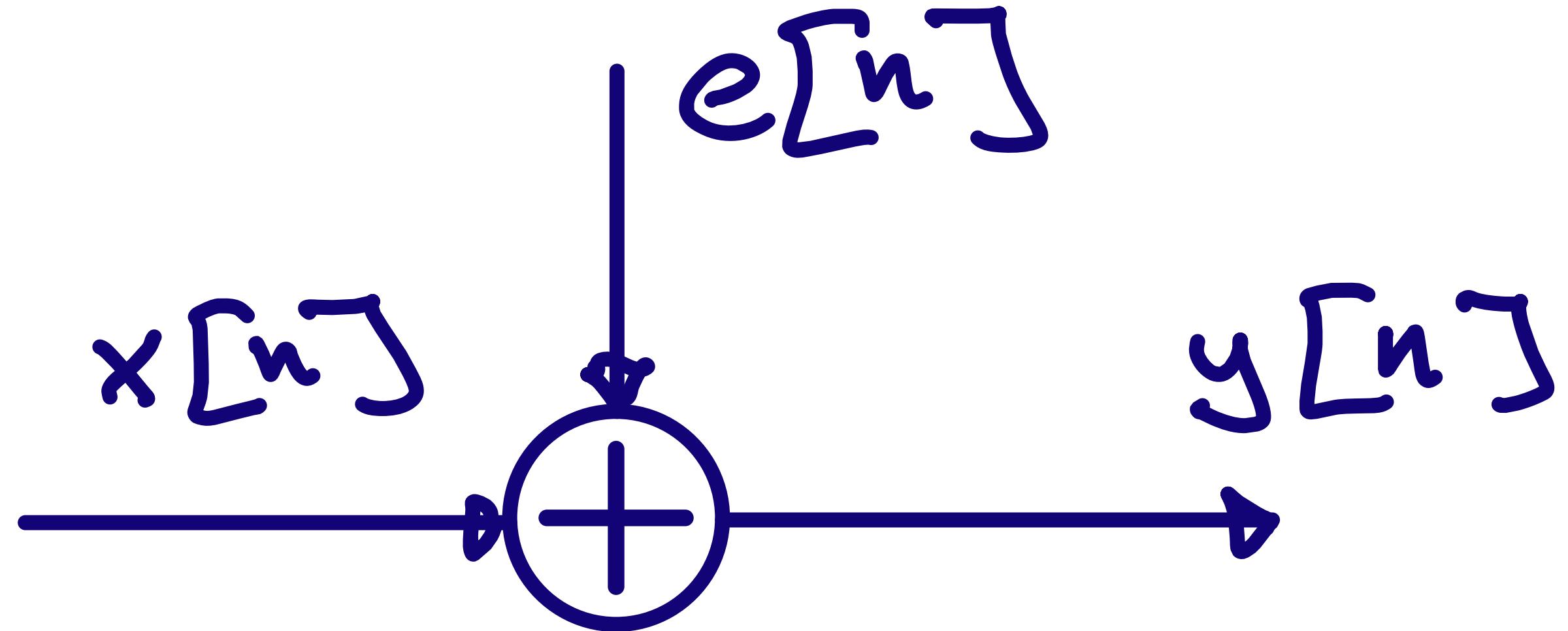
B. Murmann, ADC Performance Survey 1997-2022 (ISSCC & VLSI Symposium)

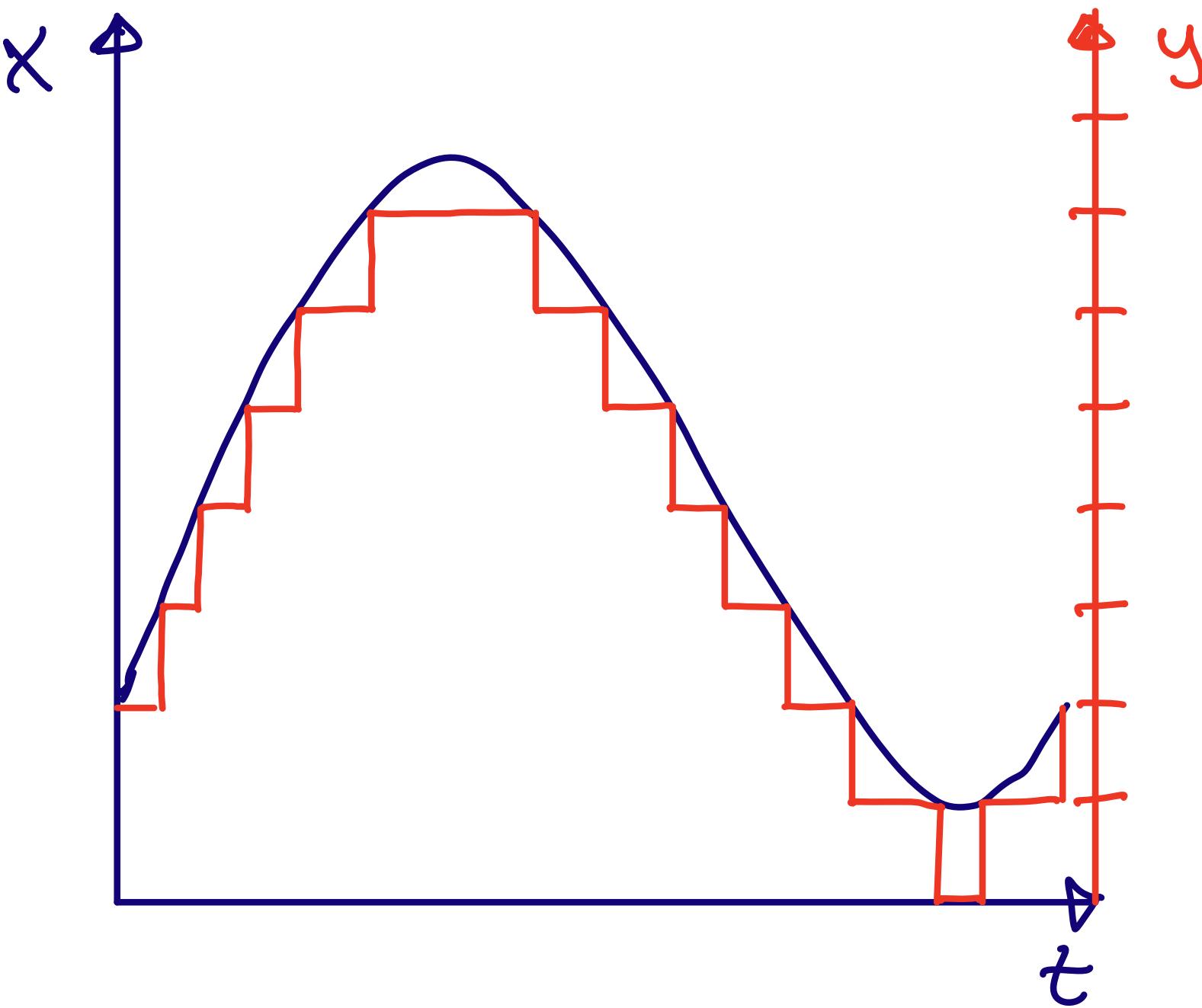
$$FOM_S = SNDR + 10 \log\left(\frac{f_s/2}{P}\right)$$

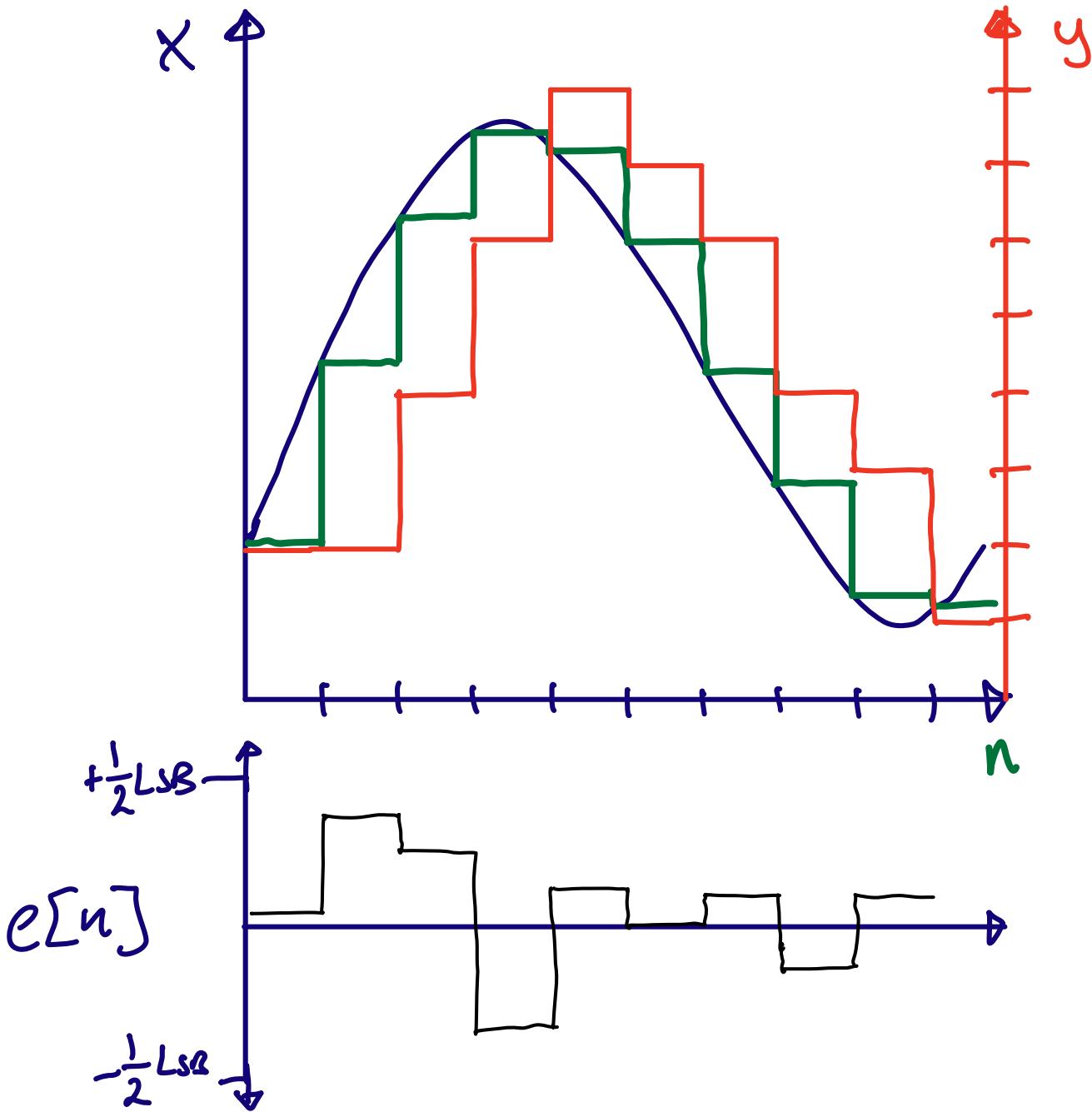
Above 180 dB is extreme



Quantization







See [The intermodulation and distortion due to quantization of sinusoids](#) for details

$$e_n(t) = \sum_{p=1}^{\infty} A_p \sin p\omega t$$

where p is the harmonic index, and

$$A_p = \begin{cases} \delta_{p1} A + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_p(2m\pi A) & , p = \text{odd} \\ 0 & , p = \text{even} \end{cases}$$

$$\delta_{p1} \begin{cases} 1 & , p = 1 \\ 0 & , p \neq 1 \end{cases}$$

and $J_p(x)$ is a Bessel function of the first kind, A is the amplitude of the input signal.

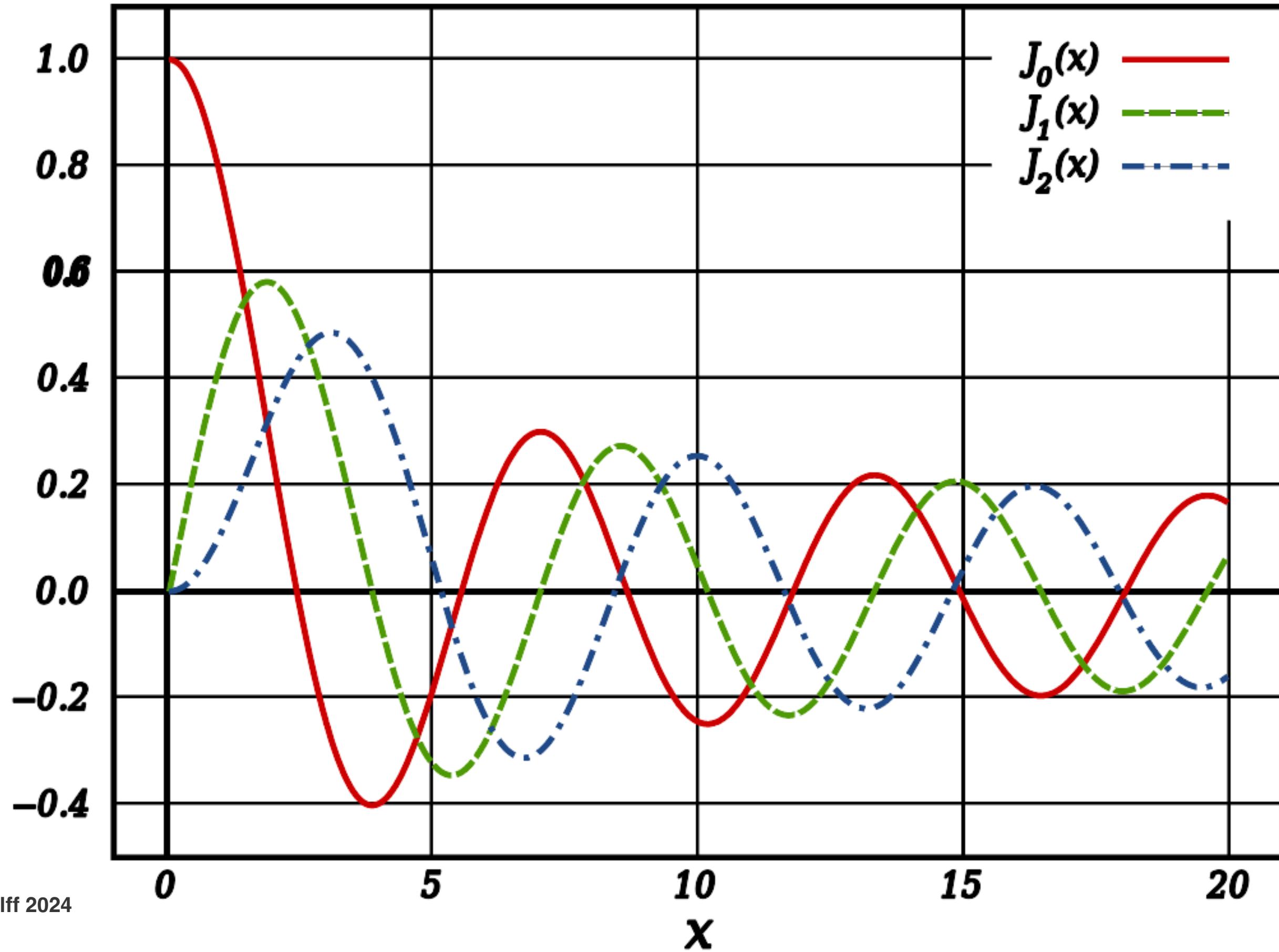
If we approximate the amplitude of the input signal as

$$A = \frac{2^n - 1}{2} \approx 2^{n-1}$$

where n is the number of bits, we can rewrite as

$$e_n(t) = \sum_{p=1}^{\infty} A_p \sin p\omega t$$

$$A_p = \delta_{p1} 2^{n-1} + \sum_{m=1}^{\infty} \frac{2}{m\pi} J_p(2m\pi 2^{n-1}), p = odd$$



$$\overline{e_n(t)} = 0$$

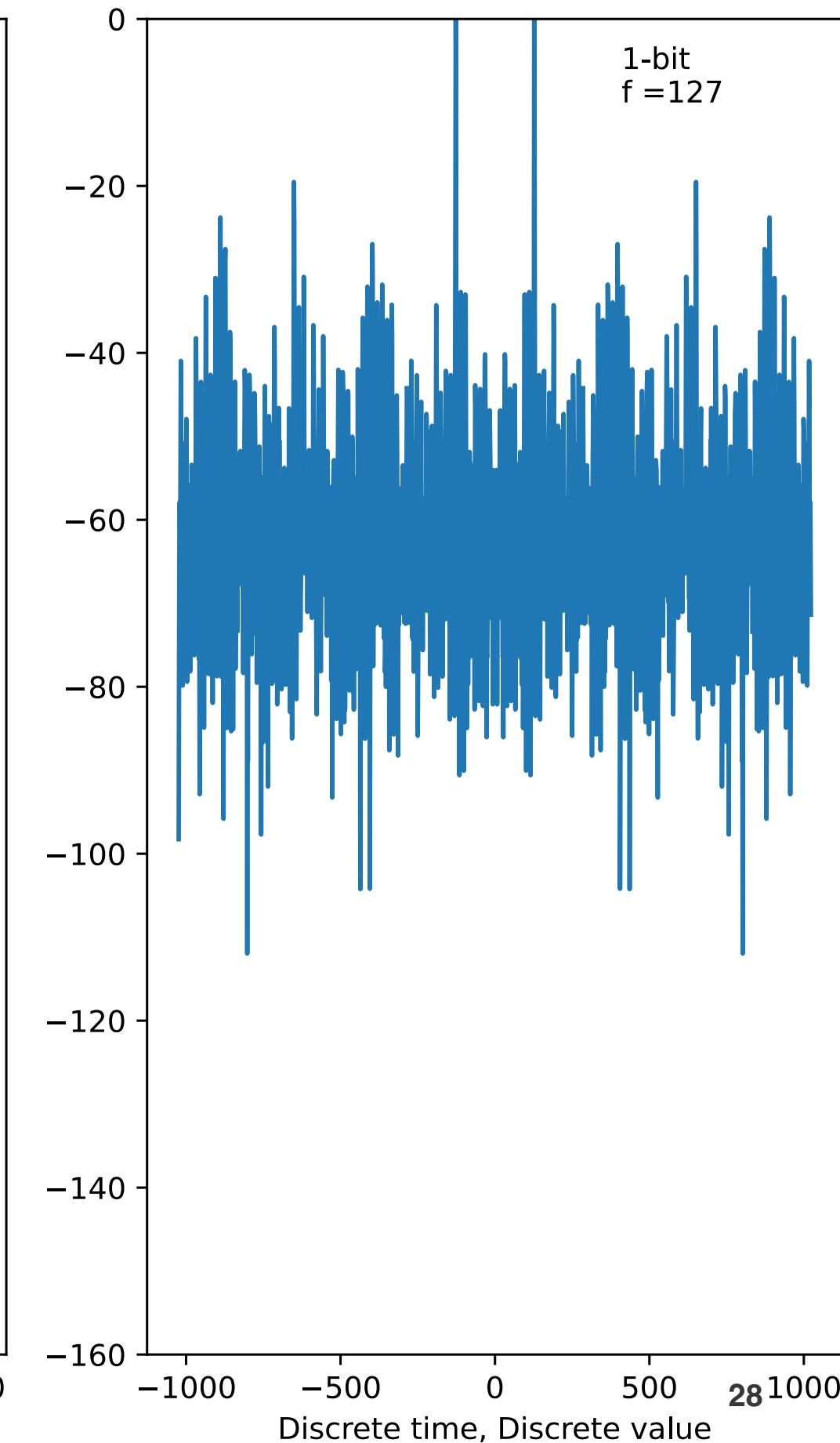
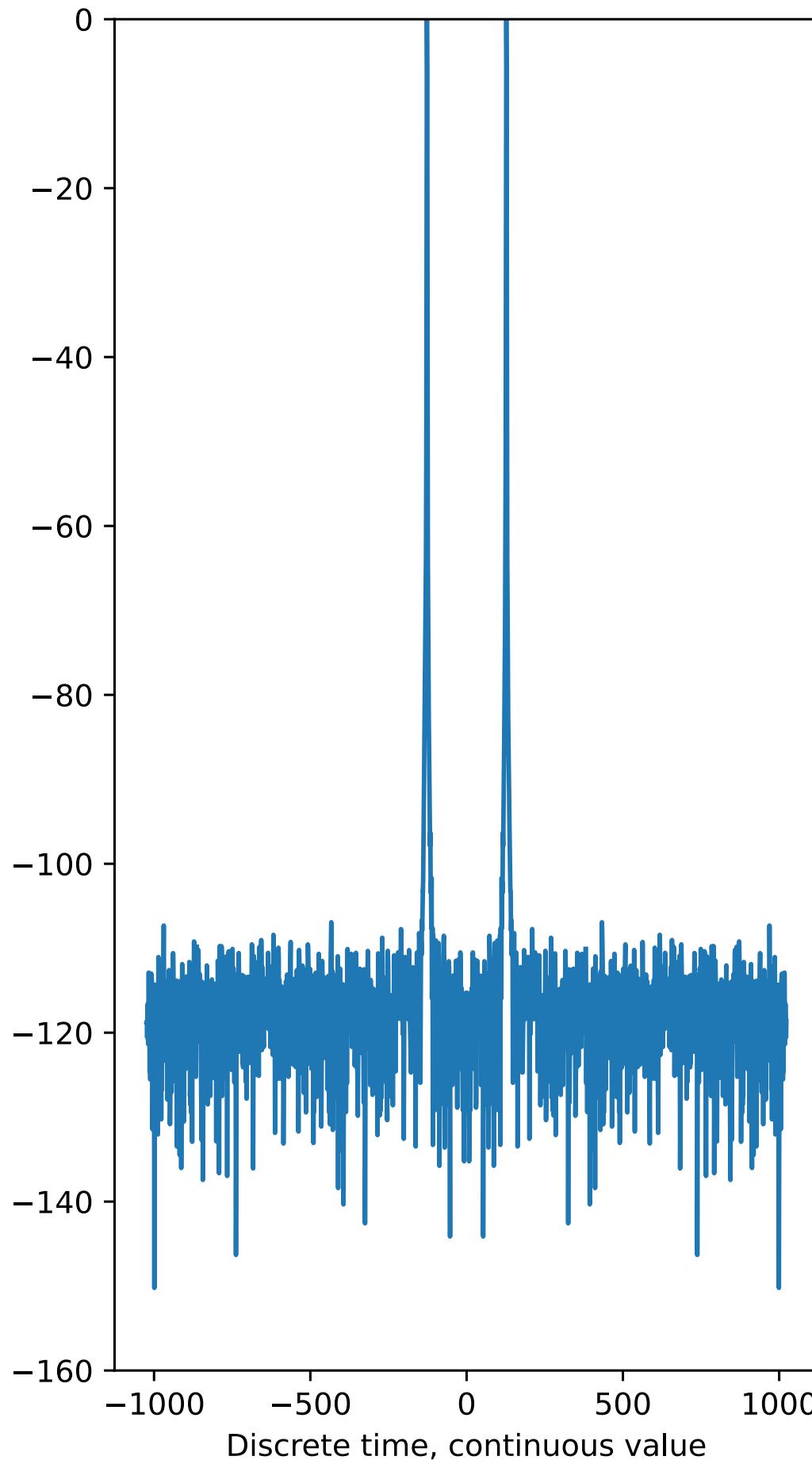
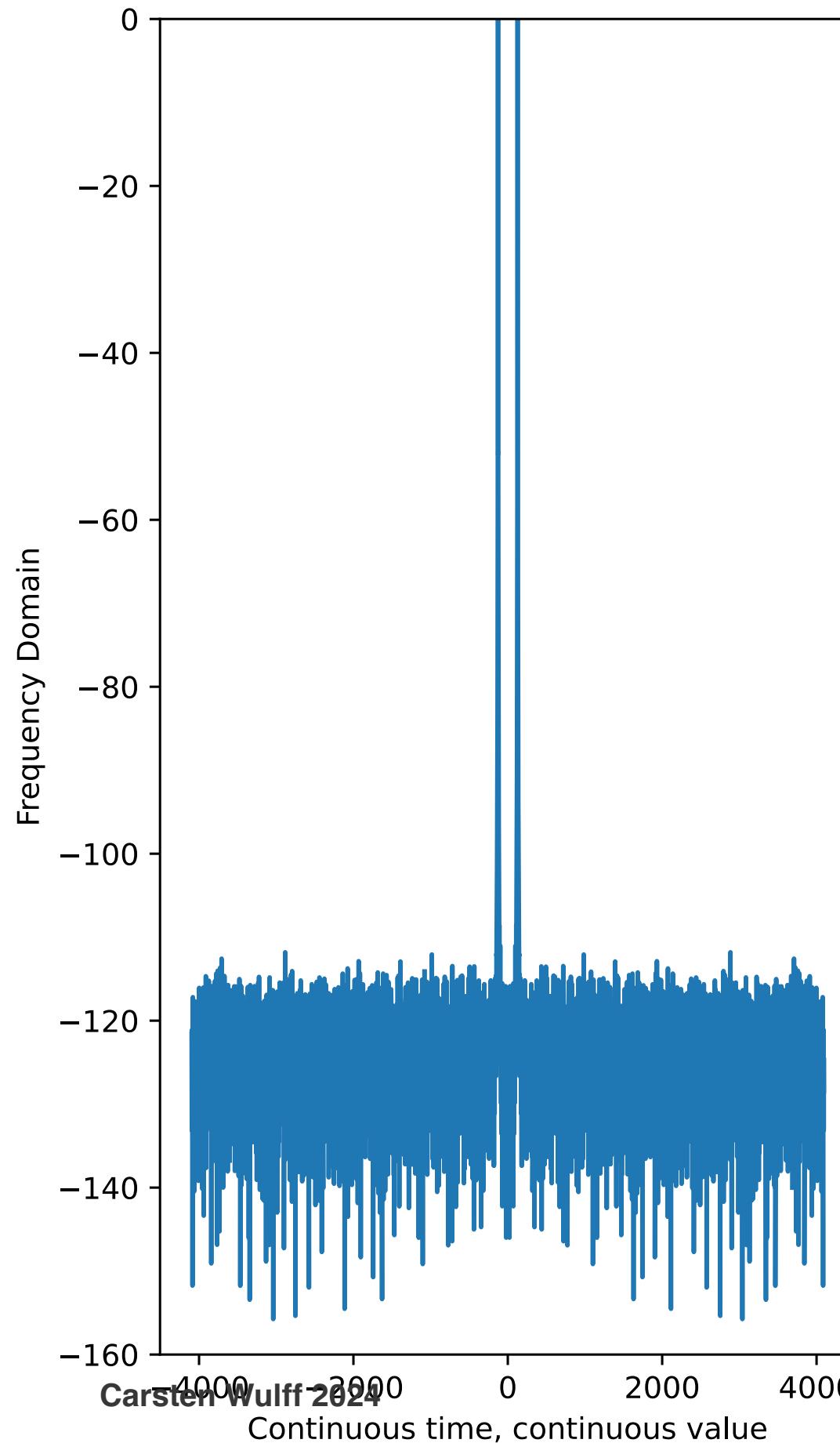
$$\overline{e_n(t)^2} = \frac{\Delta^2}{12}$$

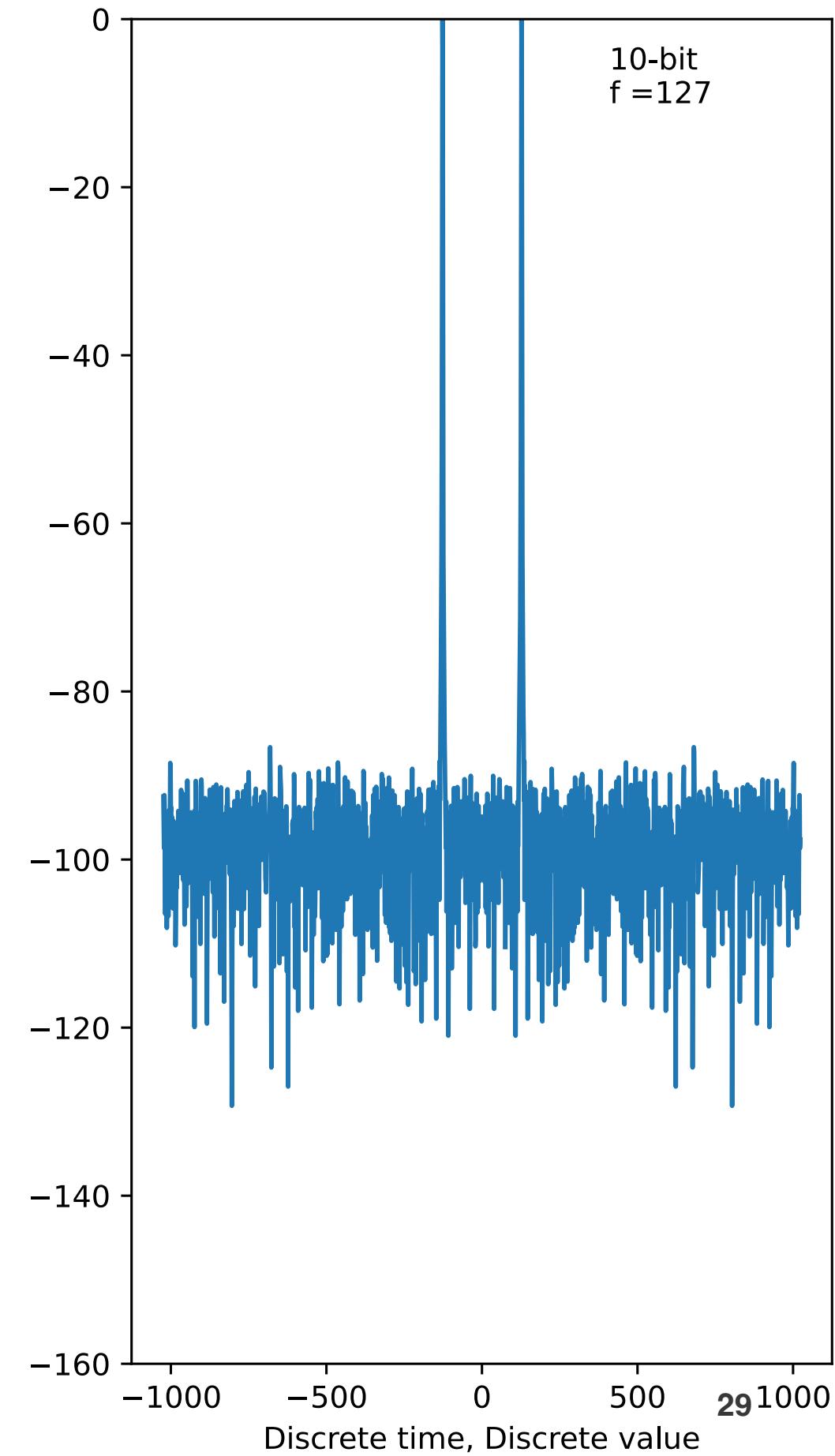
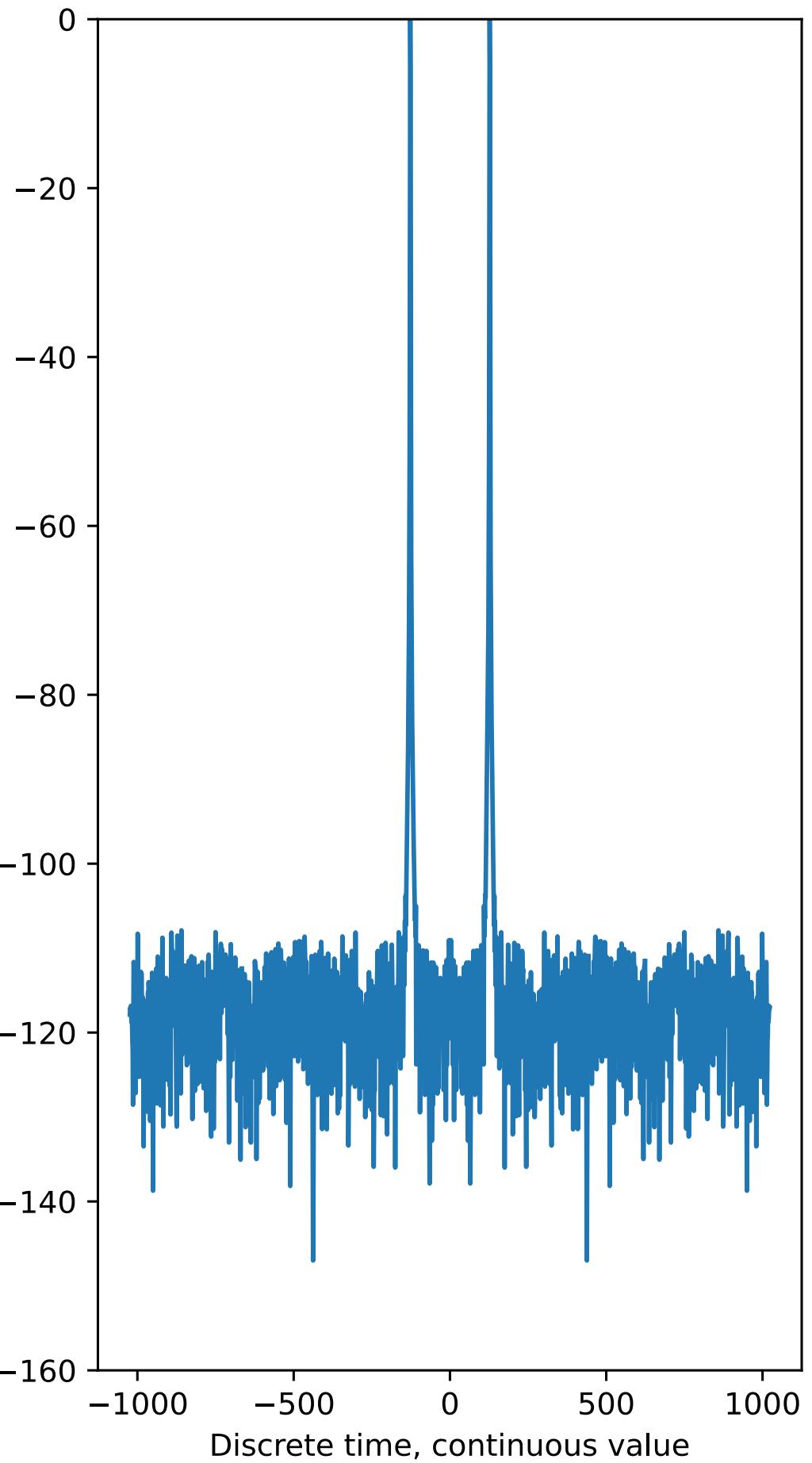
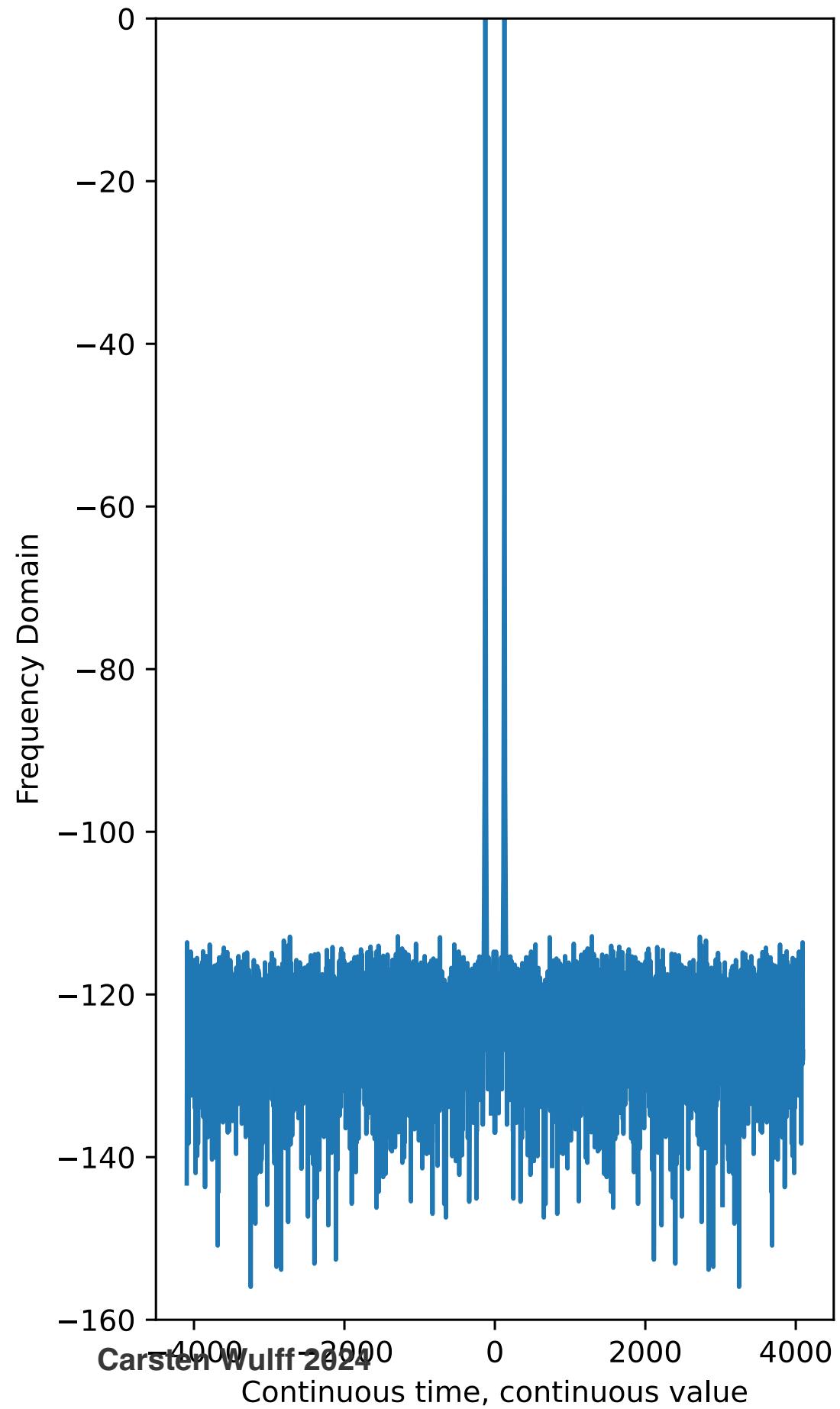
$$SQNR = 10 \log \left(\frac{A^2/2}{\Delta^2/12} \right) = 10 \log \left(\frac{6A^2}{\Delta^2} \right)$$

$$\Delta = \frac{2A}{2^B}$$

$$SQNR = 10 \log \left(\frac{6A^2}{4A^2/2^B} \right) = 20B \log 2 + 10 \log 6/4$$

$$SQNR \approx 6.02B + 1.76$$





Oversampling

in-band quantization noise for a oversampling ratio (OSR)

$$\overline{e_n(t)^2} = \frac{\Delta^2}{12OSR}$$

$$SQNR = 10 \log\left(\frac{6A^2}{\Delta^2/OSR}\right) = 10 \log\left(\frac{6A^2}{\Delta^2}\right) + 10 \log(OSR)$$

$$SQNR \approx 6.02B + 1.76 + 10 \log(OSR)$$

$$10 \log(2) \approx 3dB$$

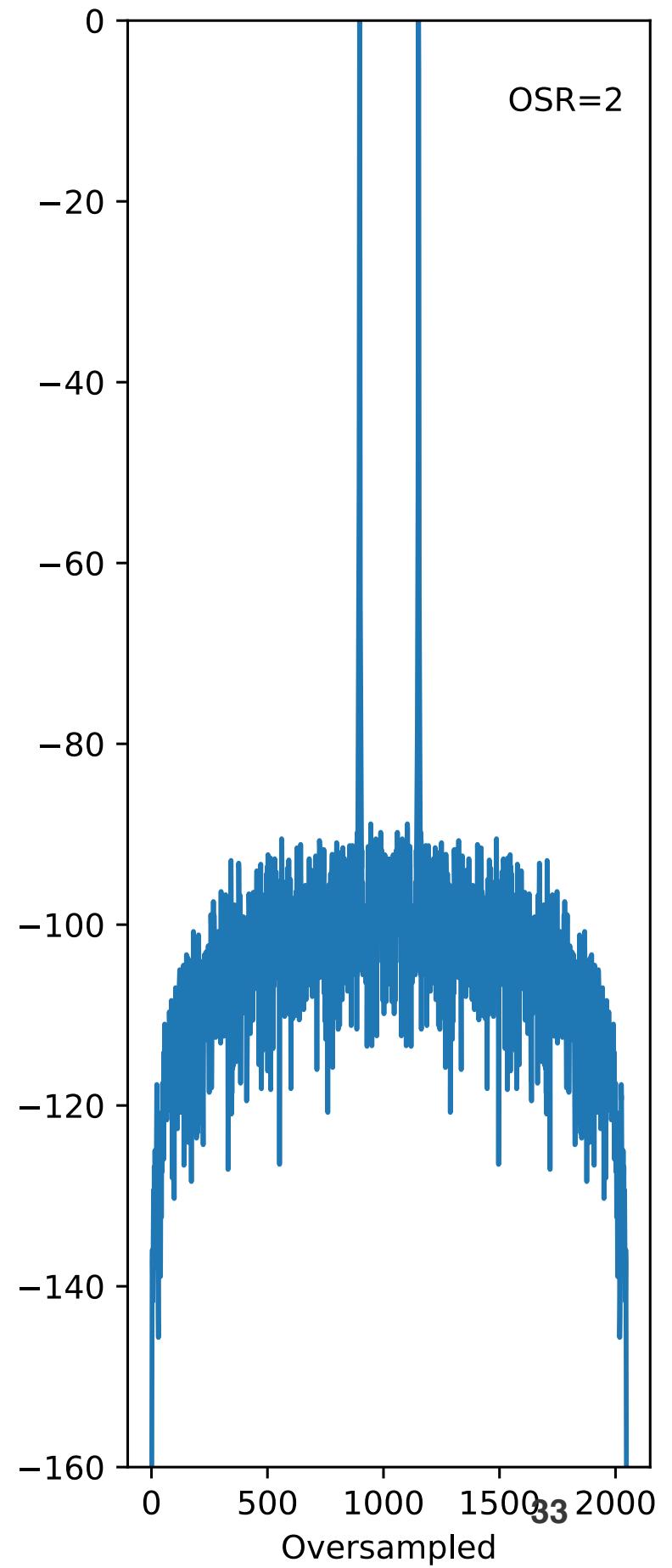
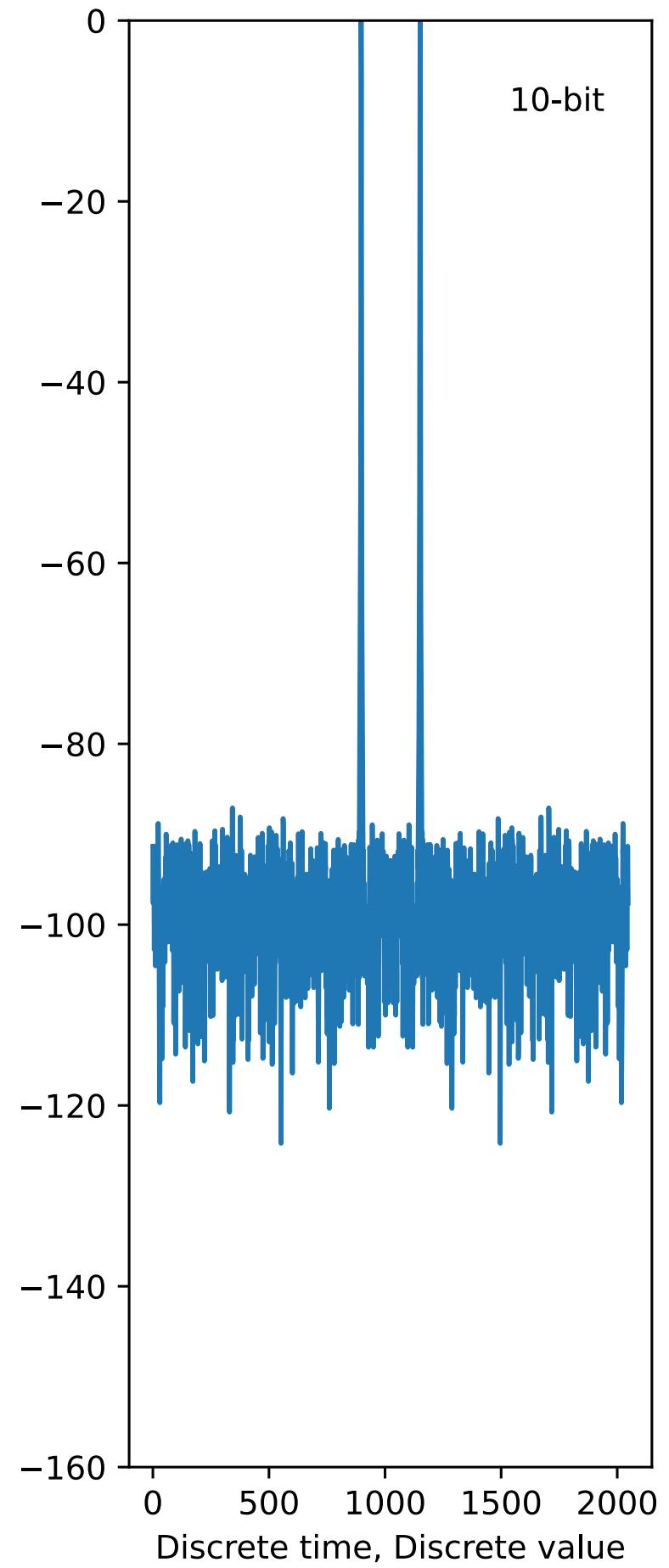
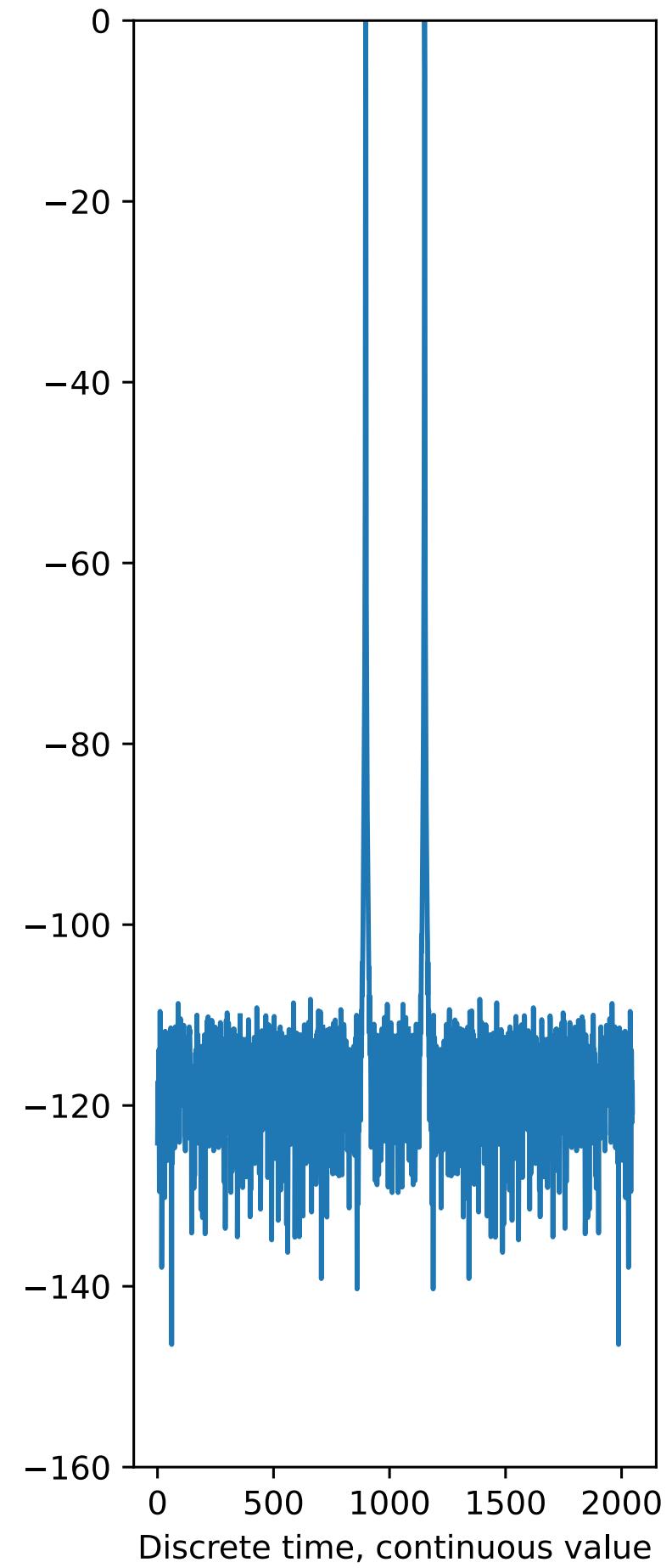
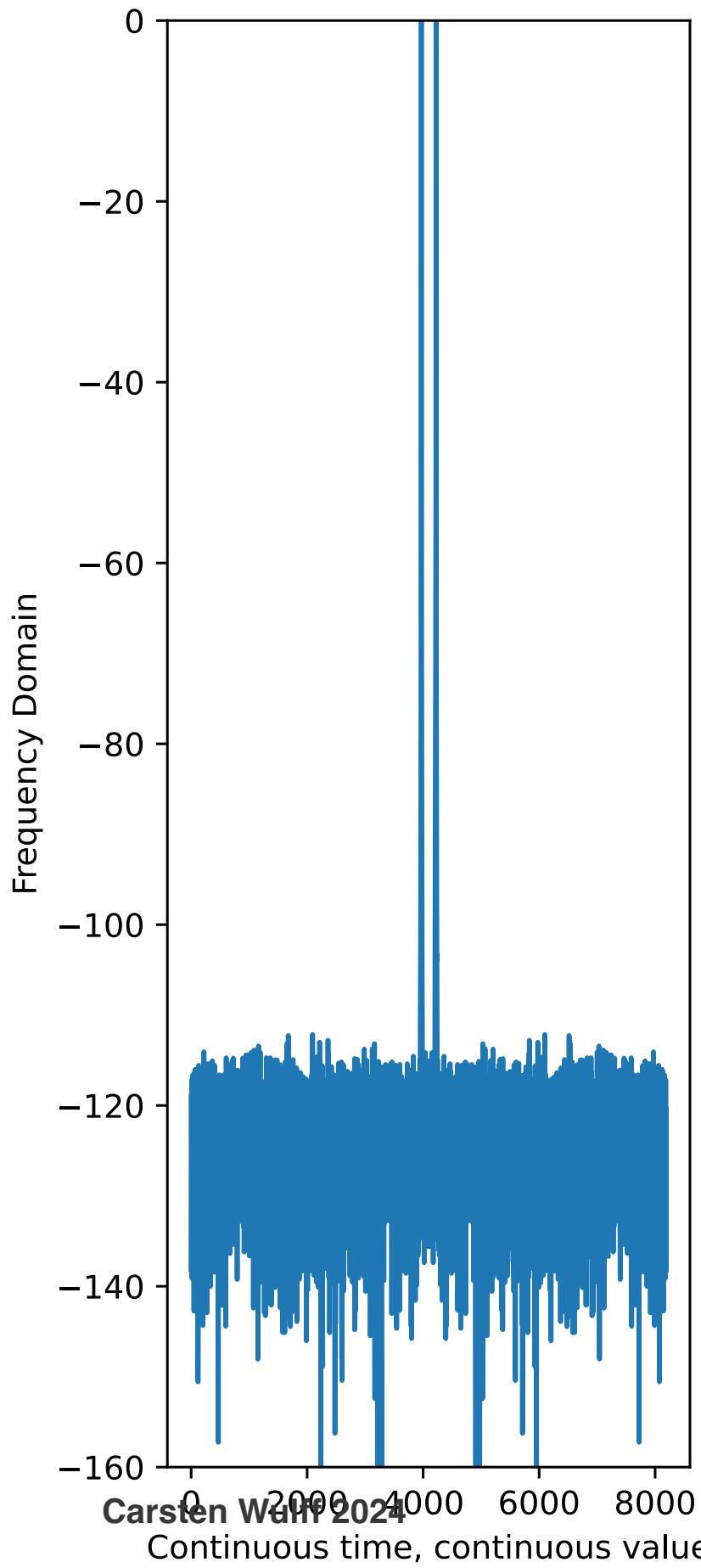
$$10 \log(4) \approx 6dB$$

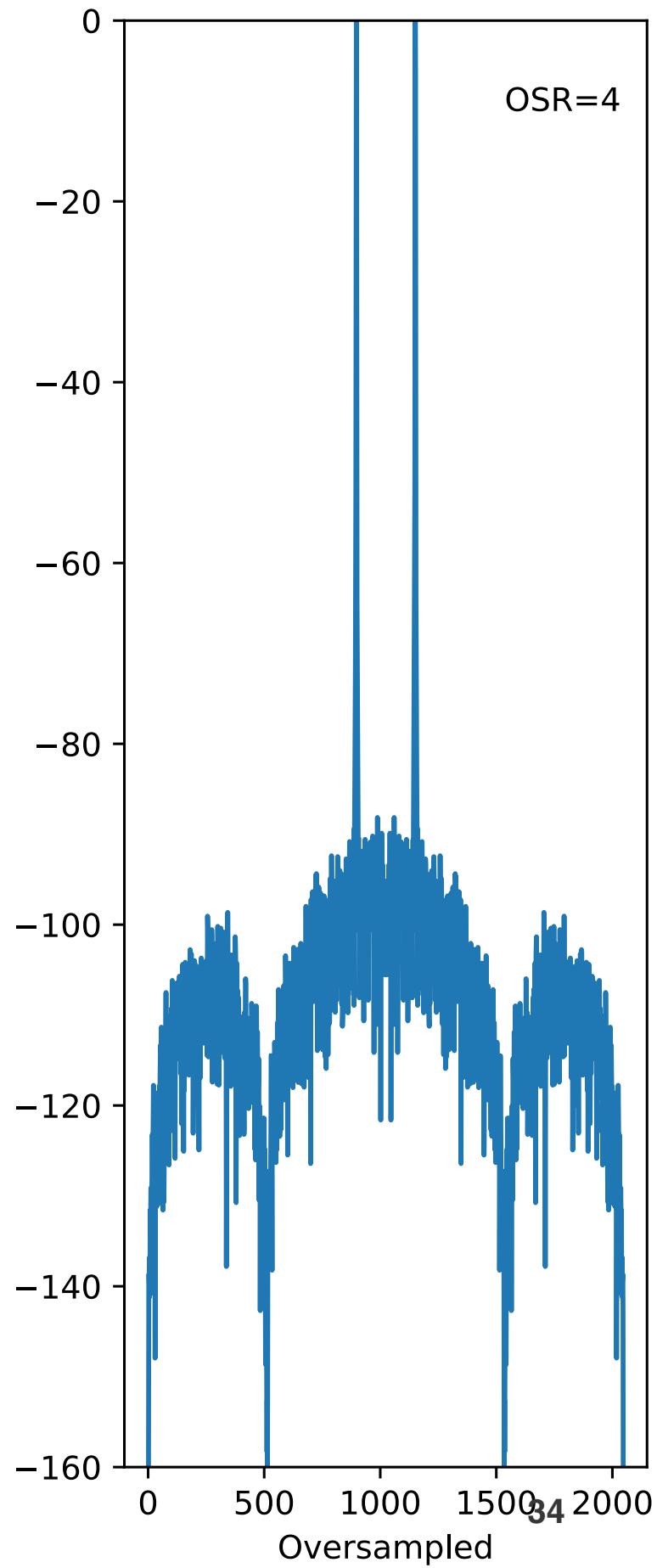
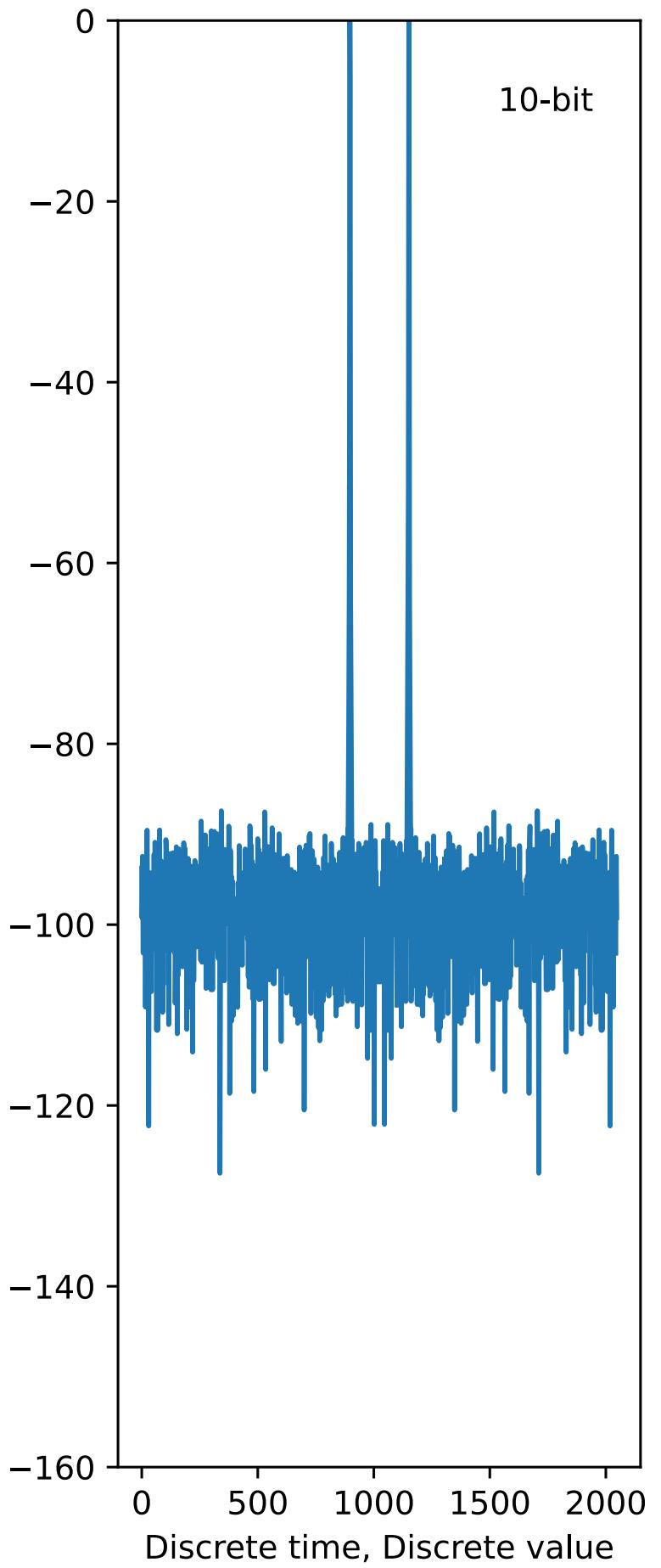
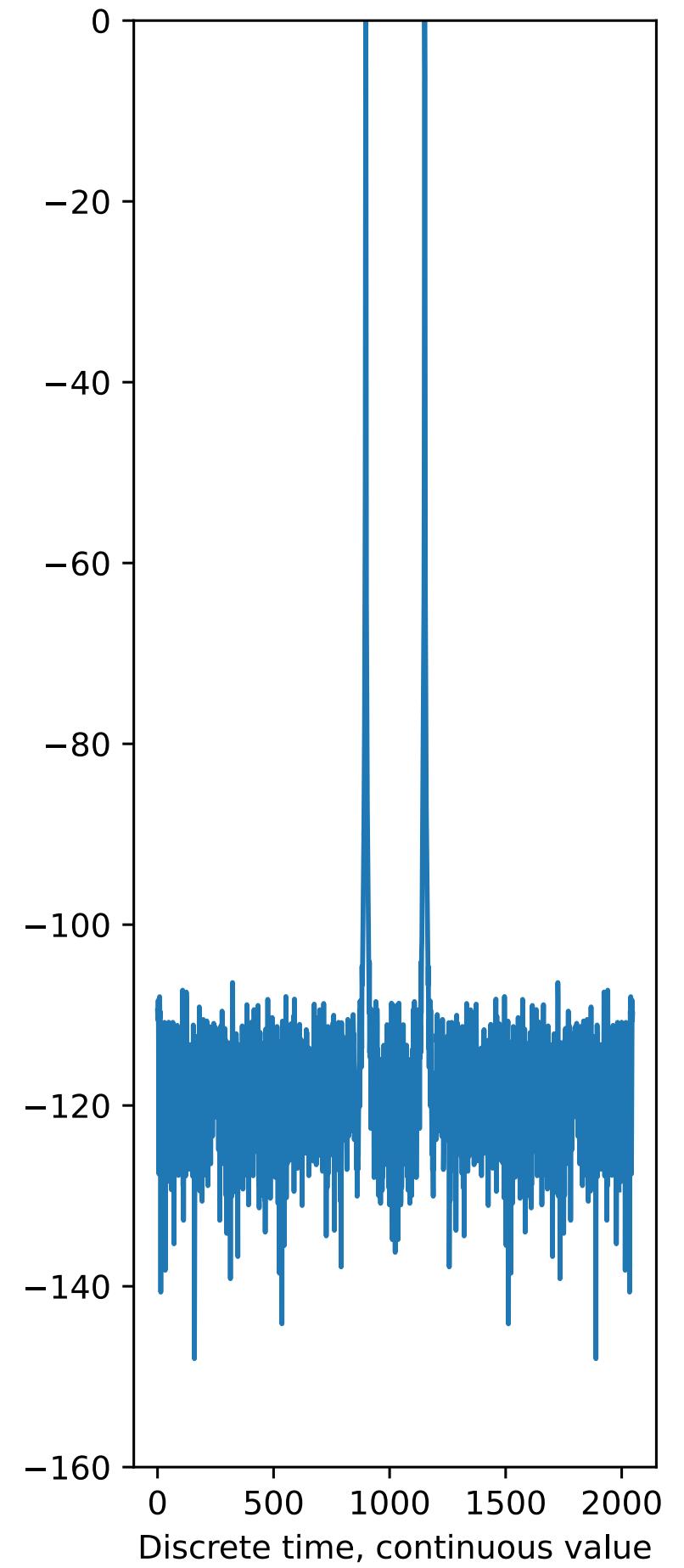
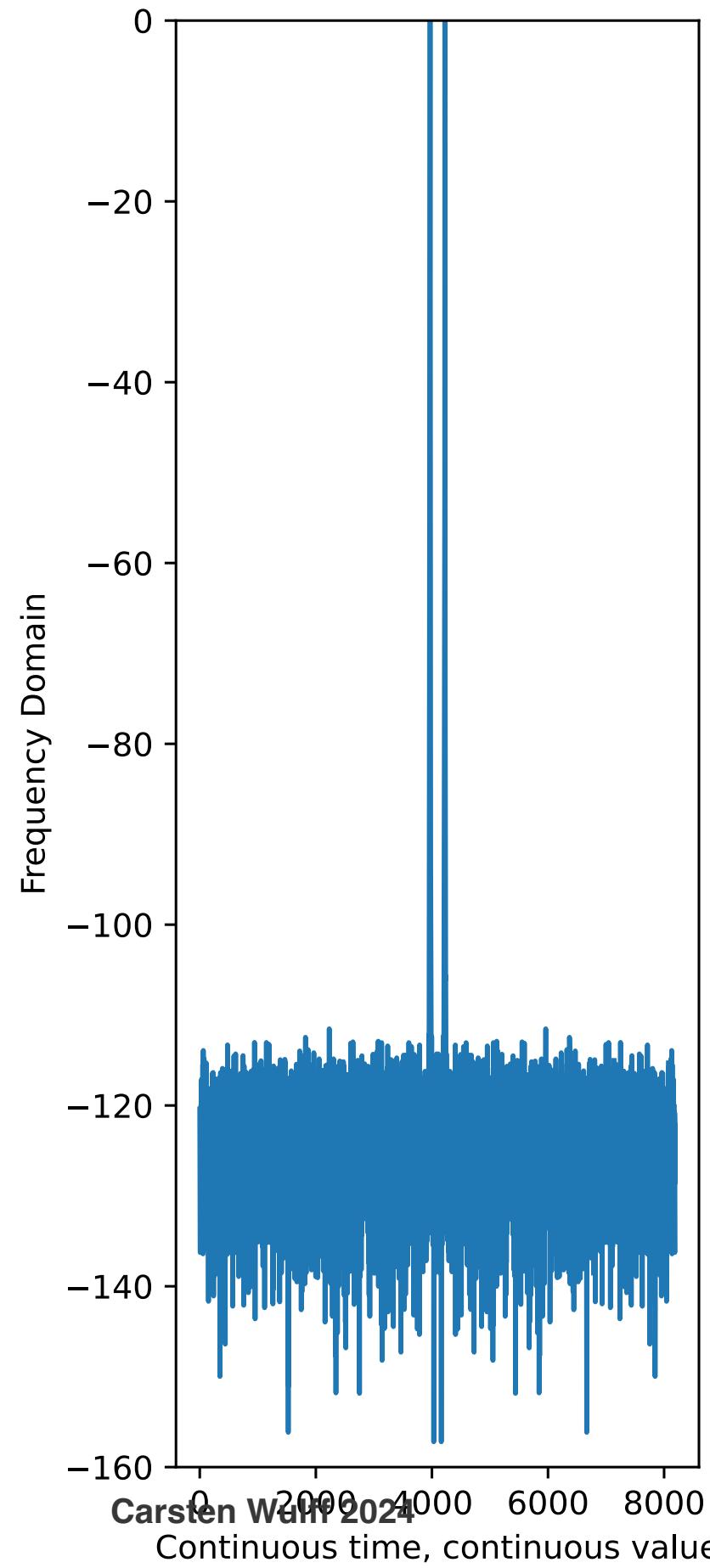
0.5-bit per doubling of OSR

```
def oversample(x,OSR):
    N = len(x)
    y = np.zeros(N)

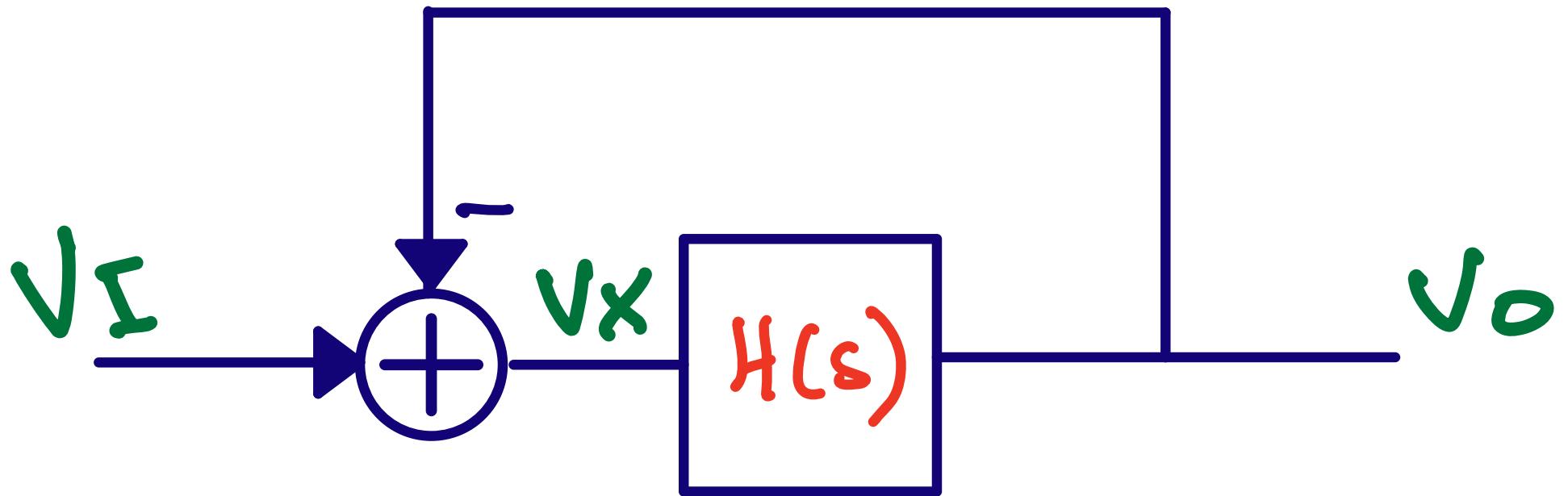
    for n in range(0,N):
        for k in range(0,OSR):
            m = n+k
            if (m < N):
                y[n] += x[m]

    return y
```



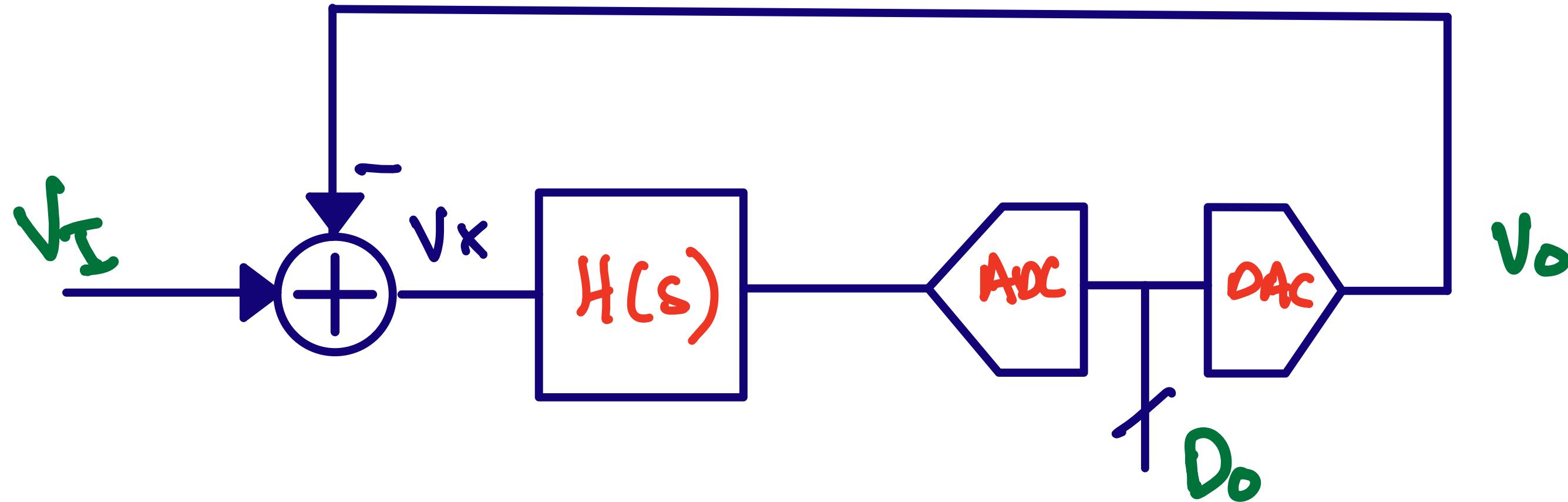


Noise Shaping



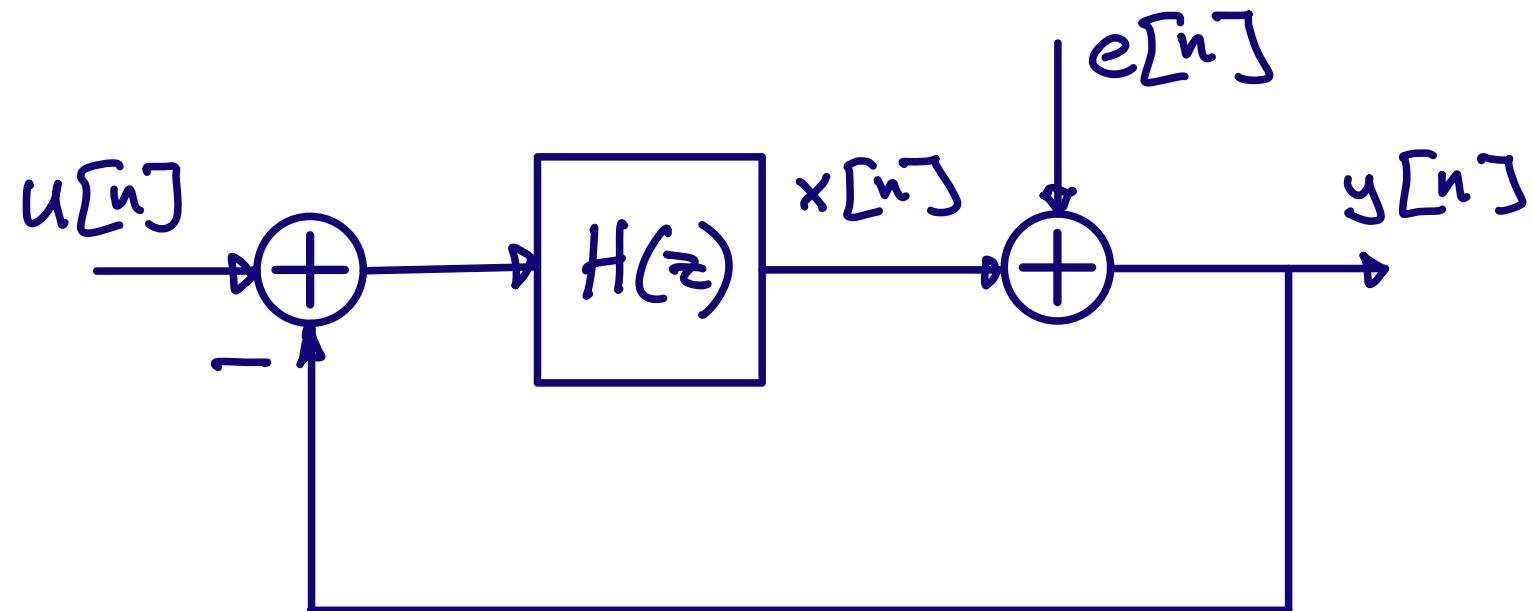
$$V_I - V_O = V_x \quad V_O = V_x H(s)$$

$$V_I = V_O + \frac{V_O}{H(s)} \xrightarrow[H(s)=\infty]{=} V_O = V_I$$



Sample domain

$$y[n] = e[n] + h * (u[n] - y[n])$$



Z-Domain

$$Y(z) = E(z) + H(z) [U(z) - Y(z)]$$

Signal transfer function

Assume U and E are uncorrelated, and E is zero

$$Y = HU - HY$$

$$STF = \frac{Y}{U} = \frac{H}{1+H} = \frac{1}{1+\frac{1}{H}}$$

Noise transfer function

Assume U is zero

$$Y = E + HY \rightarrow NTF = \frac{1}{1+H}$$

Combined transfer function

$$Y(z) = STF(z)U(z) + NTF(z)E(z)$$

First-Order Noise-Shaping

$$H(z) = \frac{1}{z - 1}$$

$$STF = \frac{1/(z - 1)}{1 + 1/(z - 1)} = \frac{1}{z} = z^{-1}$$

$$NFT = \frac{1}{1 + 1/(z - 1)} = \frac{z - 1}{z} = 1 - z^{-1}$$

$$z = e^{sT} \xrightarrow{s=j\omega} e^{j\omega T} = e^{j2\pi f/f_s}$$

$$NTF(f) = 1 - e^{-j2\pi f/f_s}$$

$$= \frac{e^{j\pi f/f_s} - e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s}$$

$$= \sin \frac{\pi f}{f_s} \times 2j \times e^{-j\pi f/f_s}$$

$$|NFT(f)| = \left| 2 \sin \left(\frac{\pi f}{f_s} \right) \right|$$

$$P_s = A^2/2$$

$$P_n = \int_{-f_0}^{f_0} \frac{\Delta^2}{12} \frac{1}{f_s} \left[2 \sin\left(\frac{\pi f}{f_s}\right) \right]^2 dt$$

⋮

$$SQNR = 6.02B + 1.76 - 5.17 + 30 \log(OSR)$$

SQNR and ENOB

$$SQNR_{nyquist} \approx 6.02B + 1.76$$

$$SQNR_{oversample} \approx 6.02B + 1.76 + 10 \log(OSR)$$

$$SQNR_{\Sigma\Delta 1} \approx 6.02B + 1.76 - 5.17 + 30 \log(OSR)$$

$$SQNR_{\Sigma\Delta 2} \approx 6.02B + 1.76 - 12.9 + 50 \log(OSR)$$

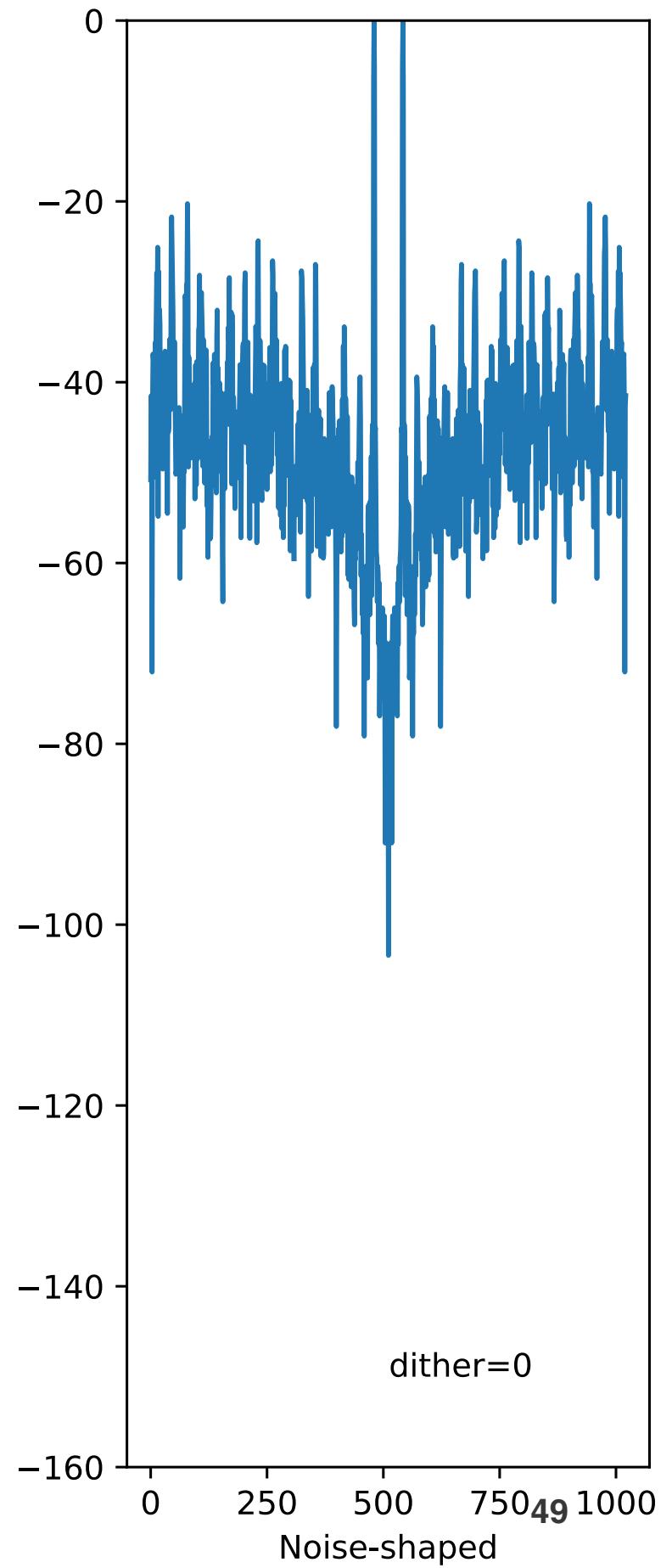
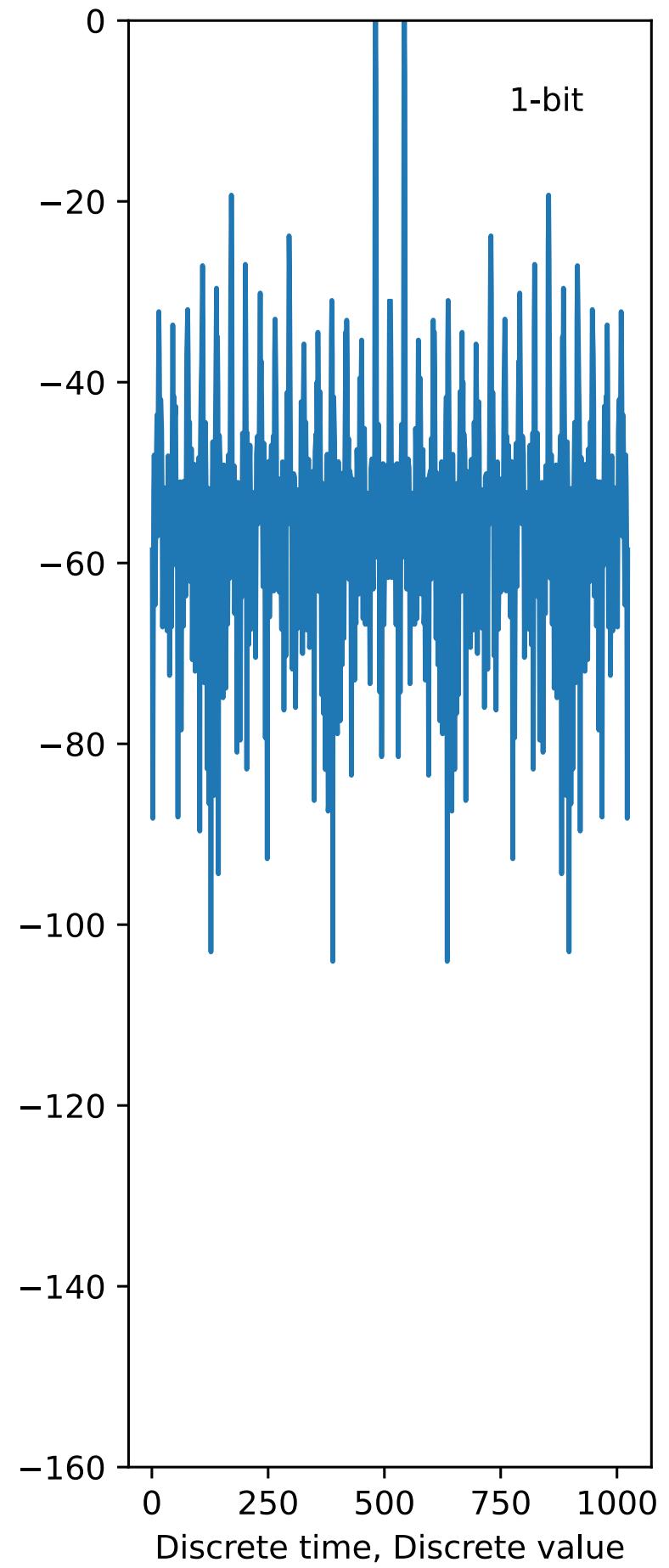
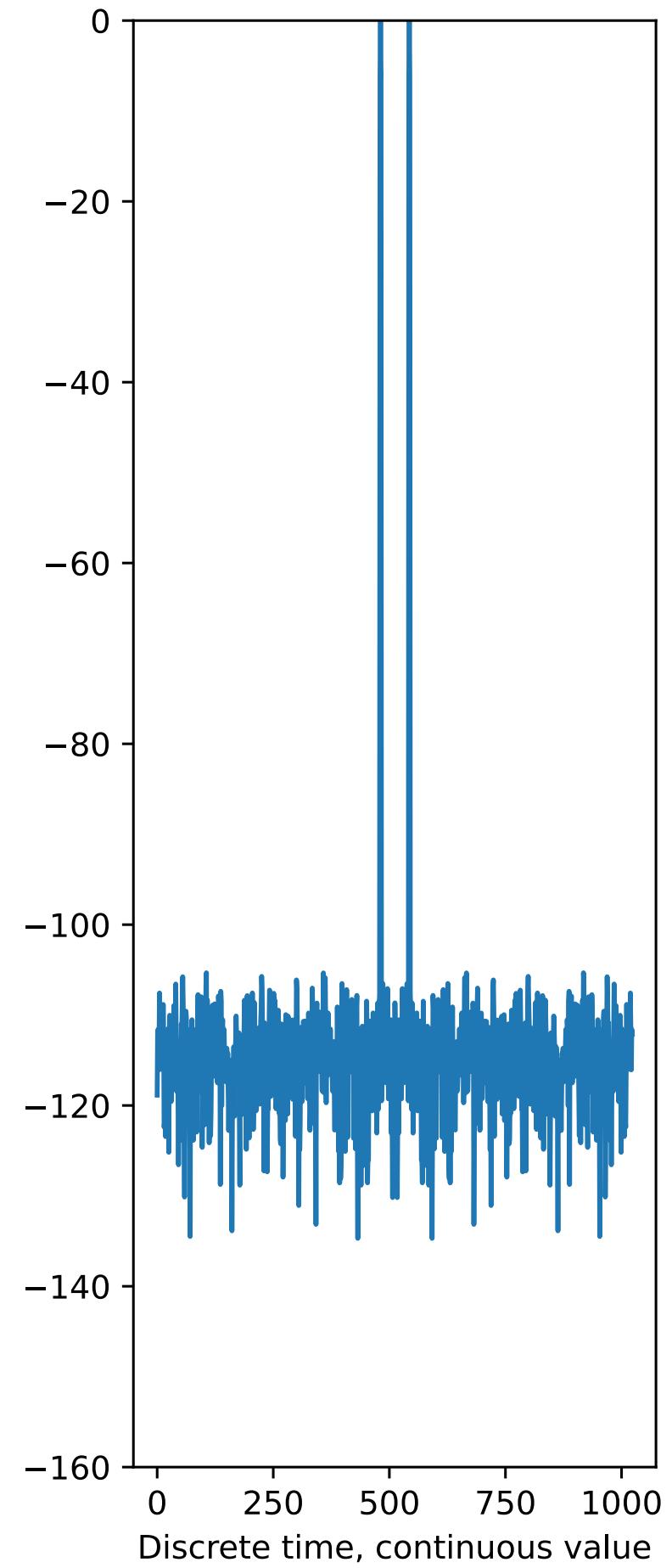
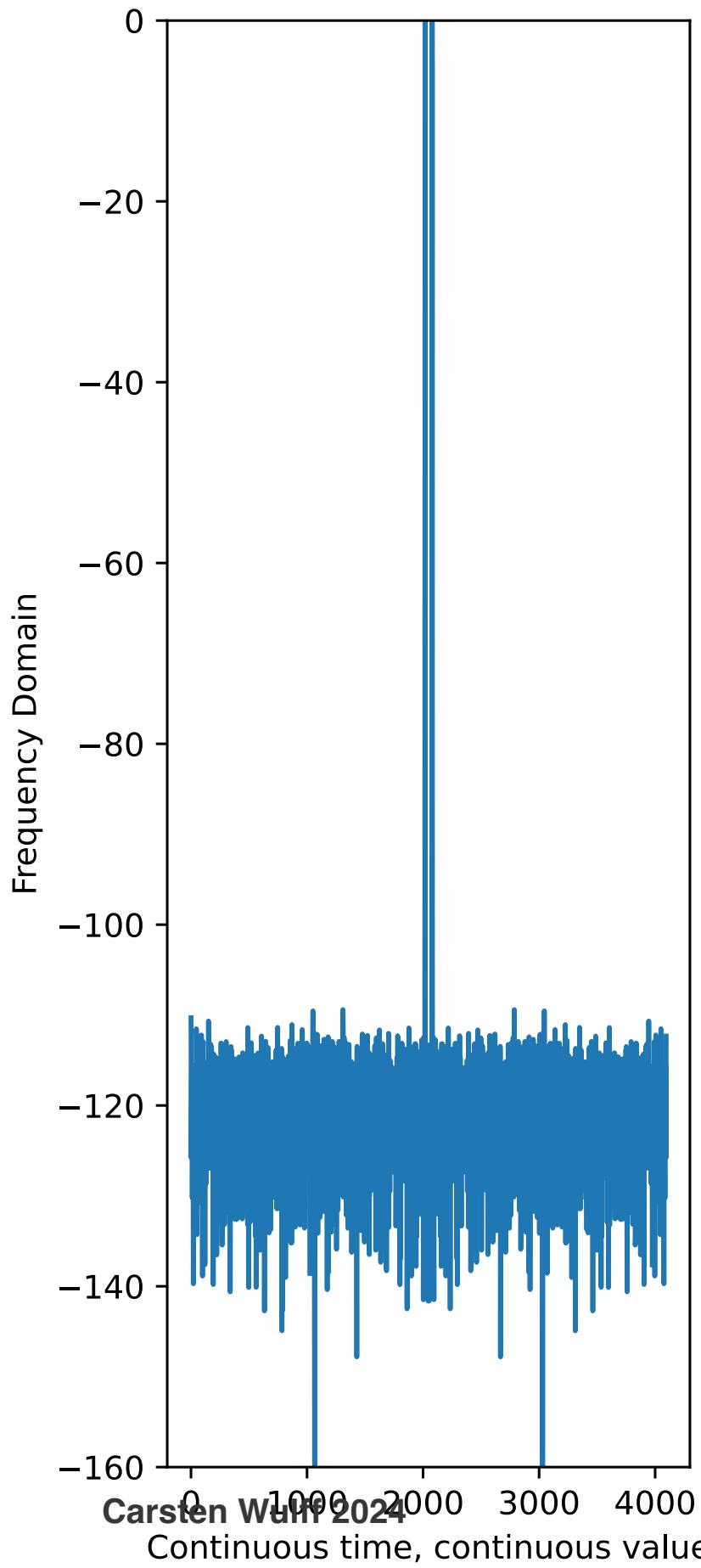
$$ENOB = (SQNR - 1.76)/6.02$$

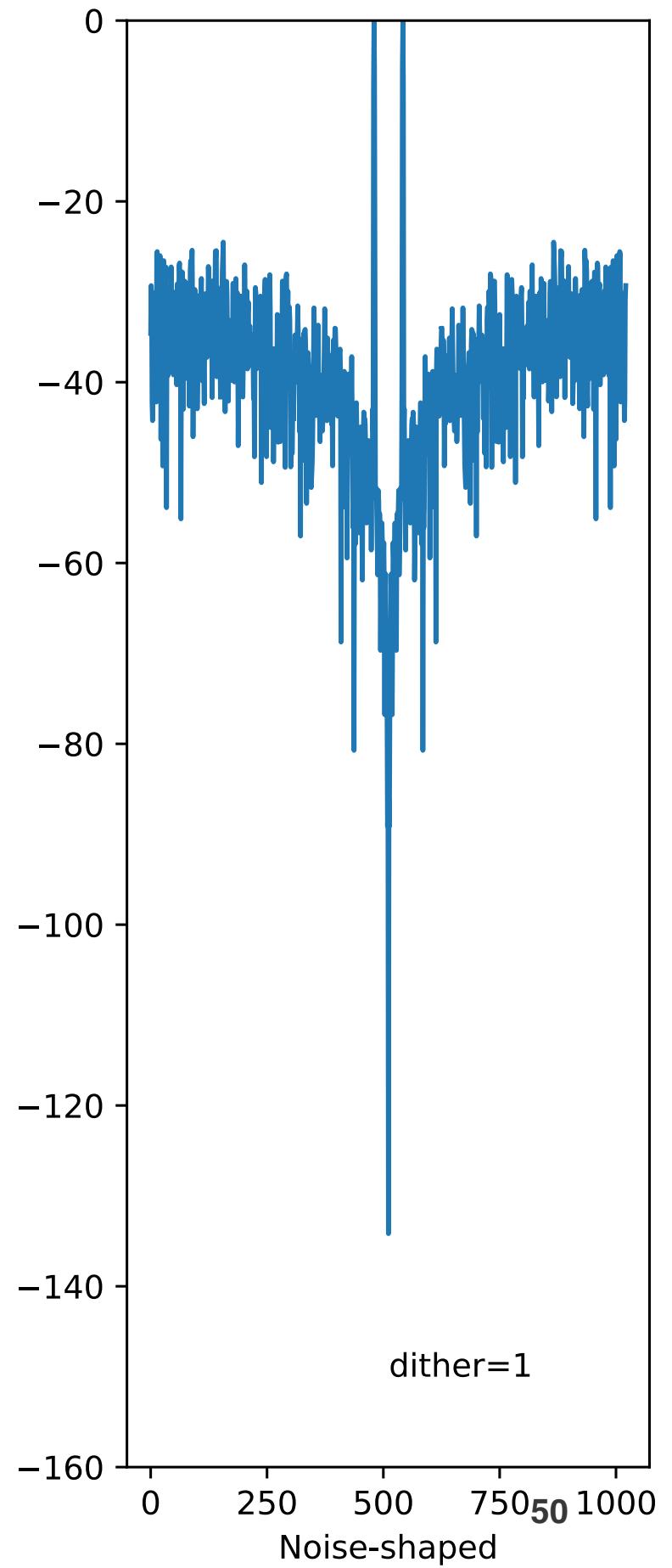
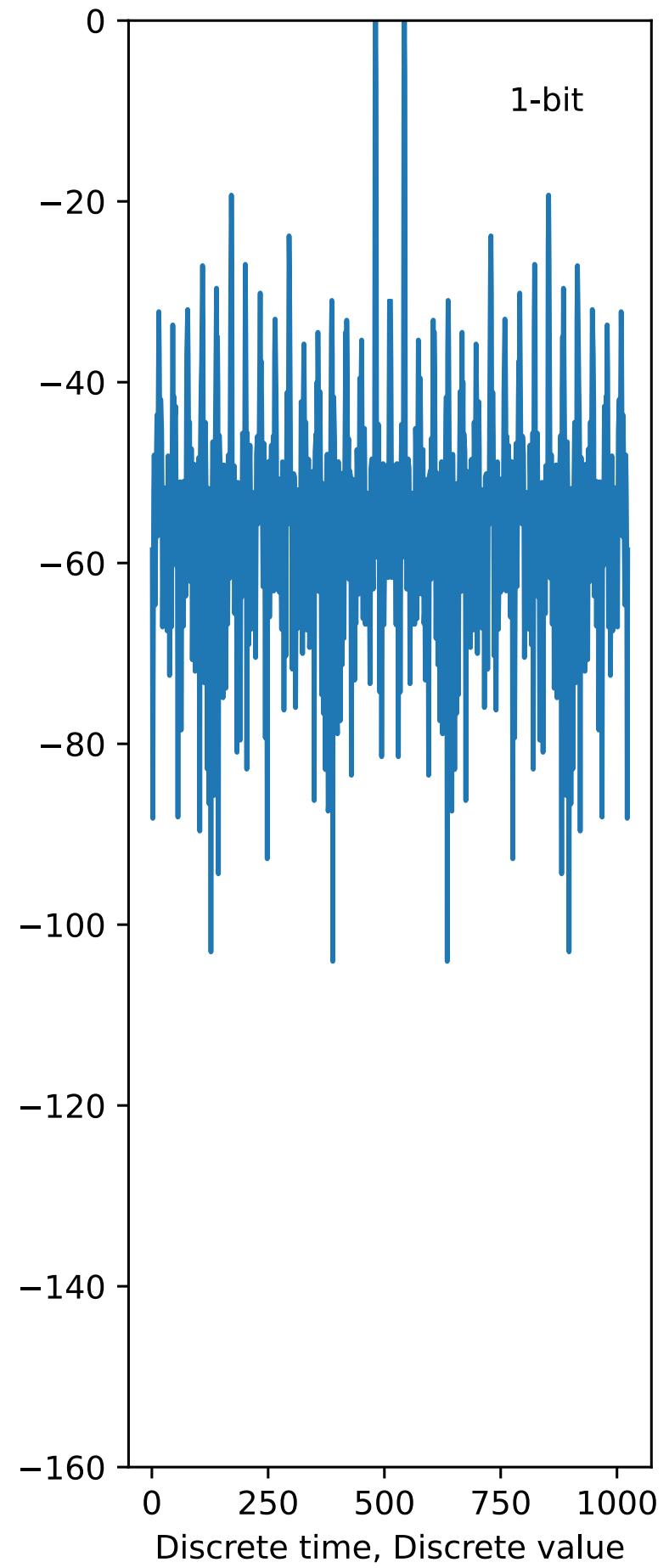
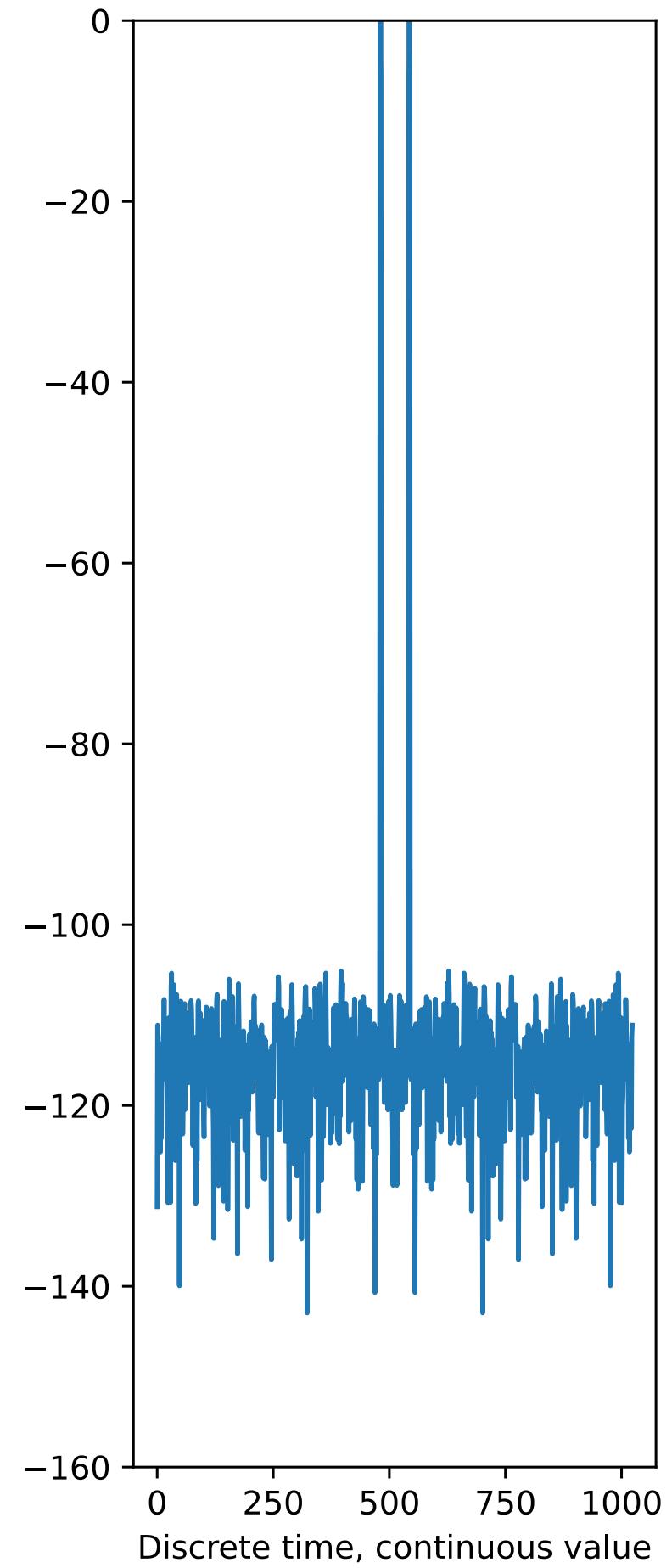
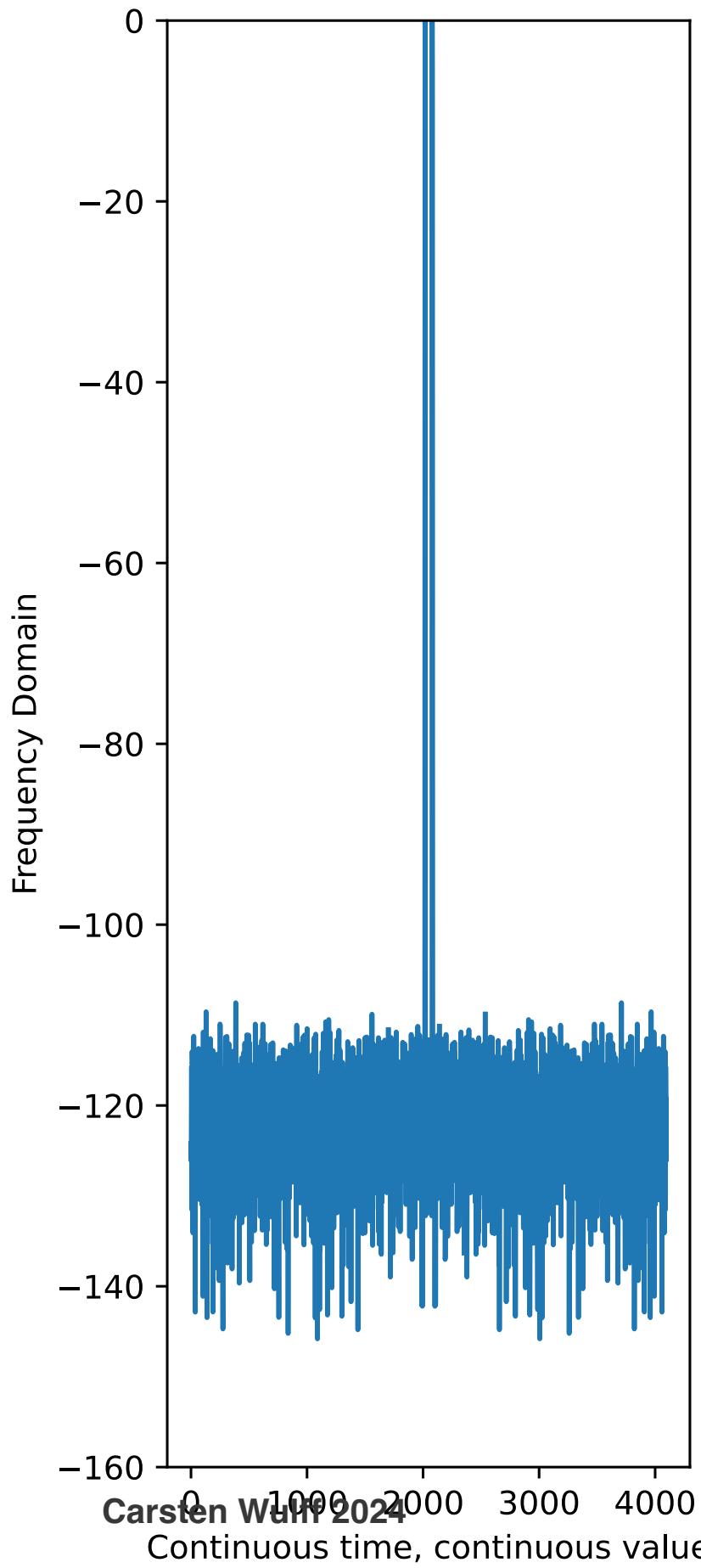
Assume 1-bit quantizer, what would be the maximum ENOB?

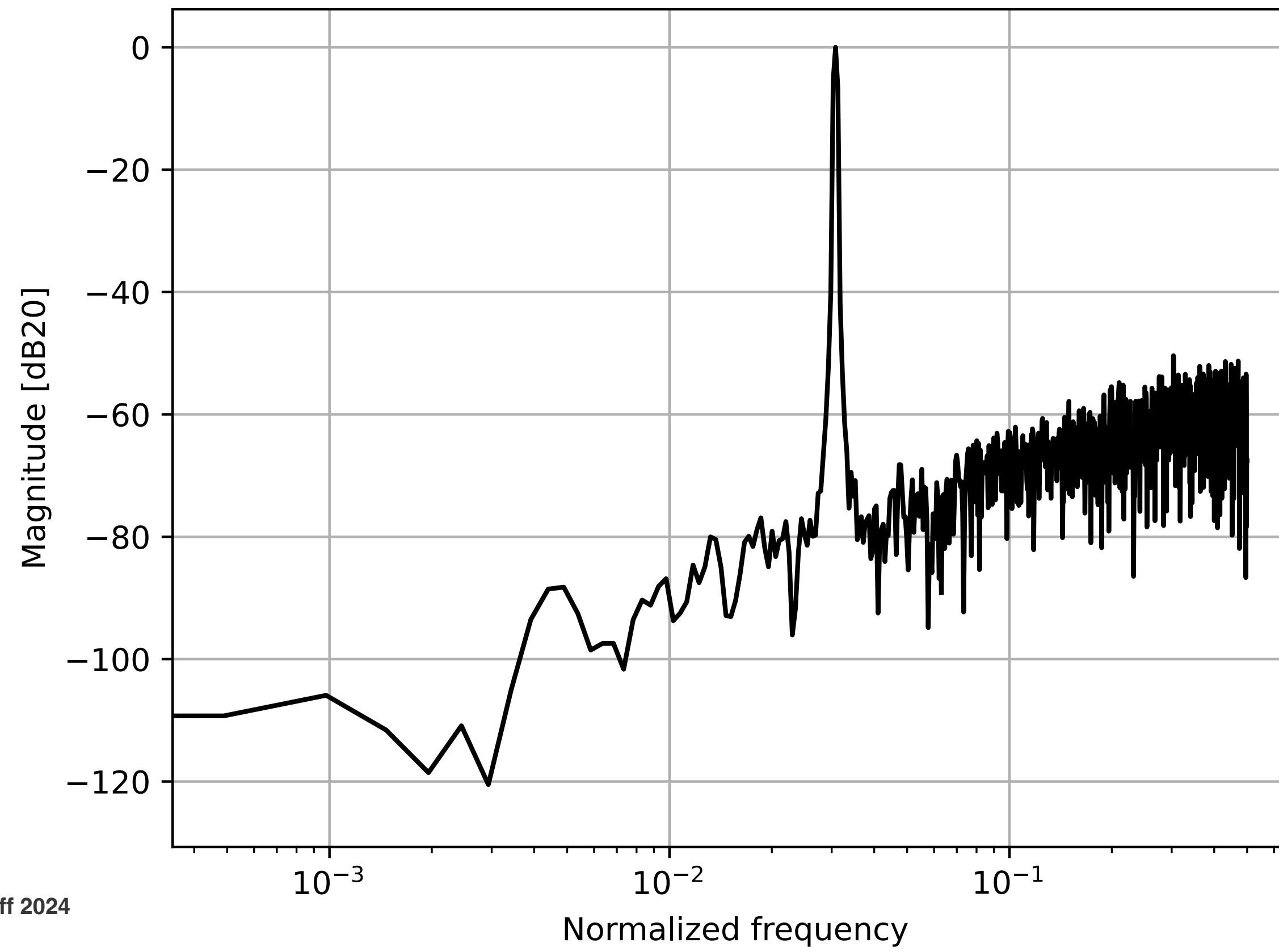
OSR	Oversampling	First-Order	Second Order
4	2	3.1	3.9
64	4	9.1	13.9
1024	6	15.1	23.9

examples

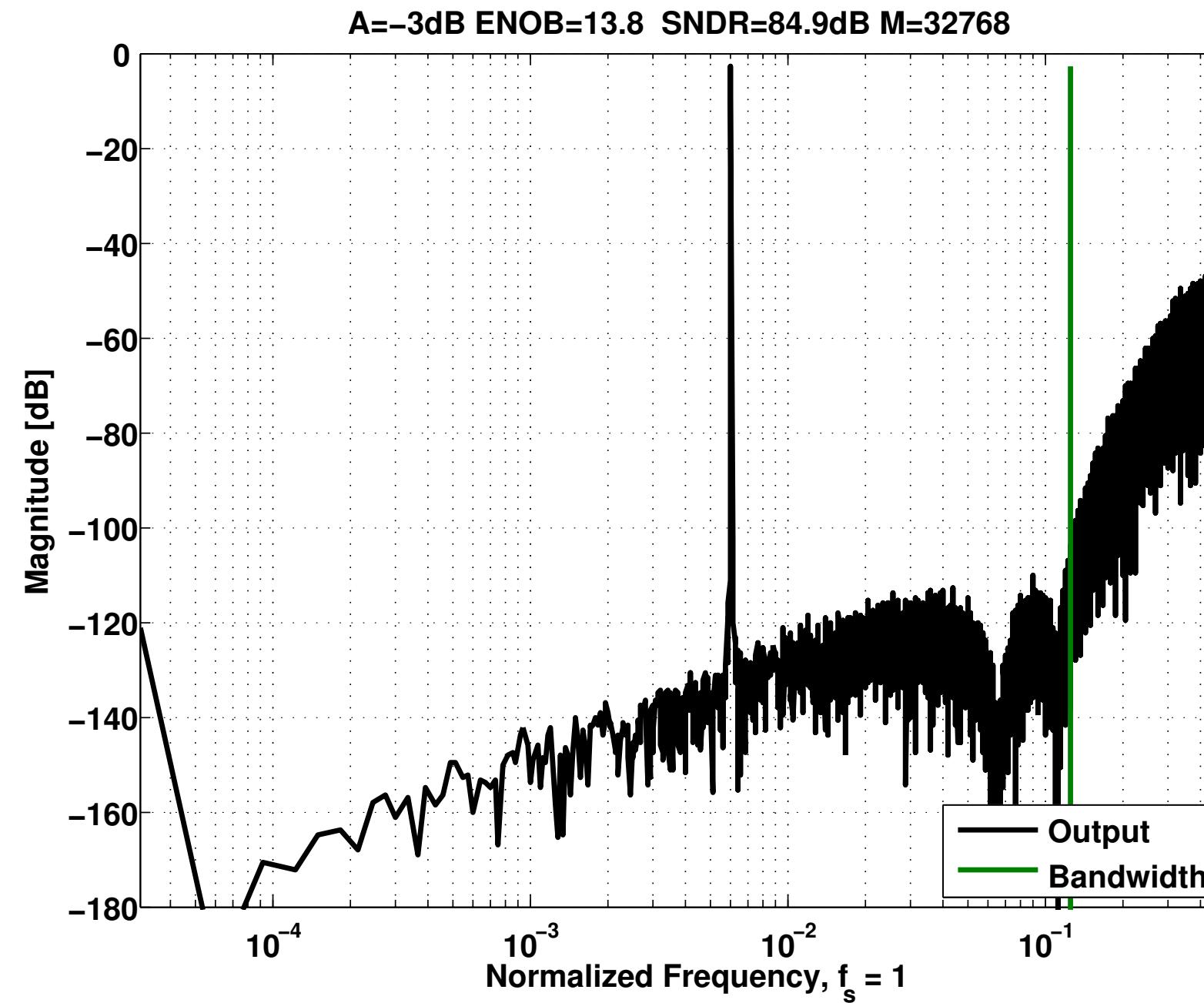
```
# u is discrete time, continuous value input
M = len(u)
y_sd = np.zeros(M)
x = np.zeros(M)
for n in range(1,M):
    x[n] = x[n-1] + (u[n]-y_sd[n-1])
    y_sd[n] = np.round(x[n]*2**bits
+ dither*np.random.randn()/4)/2**bits
```



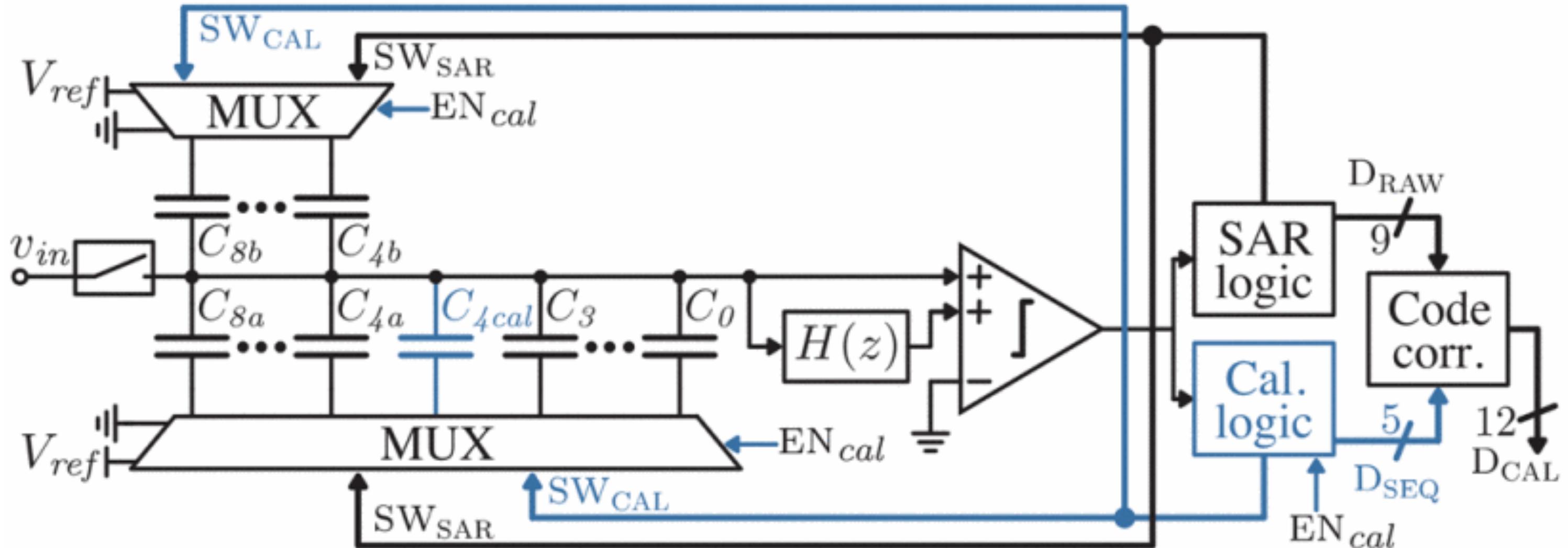


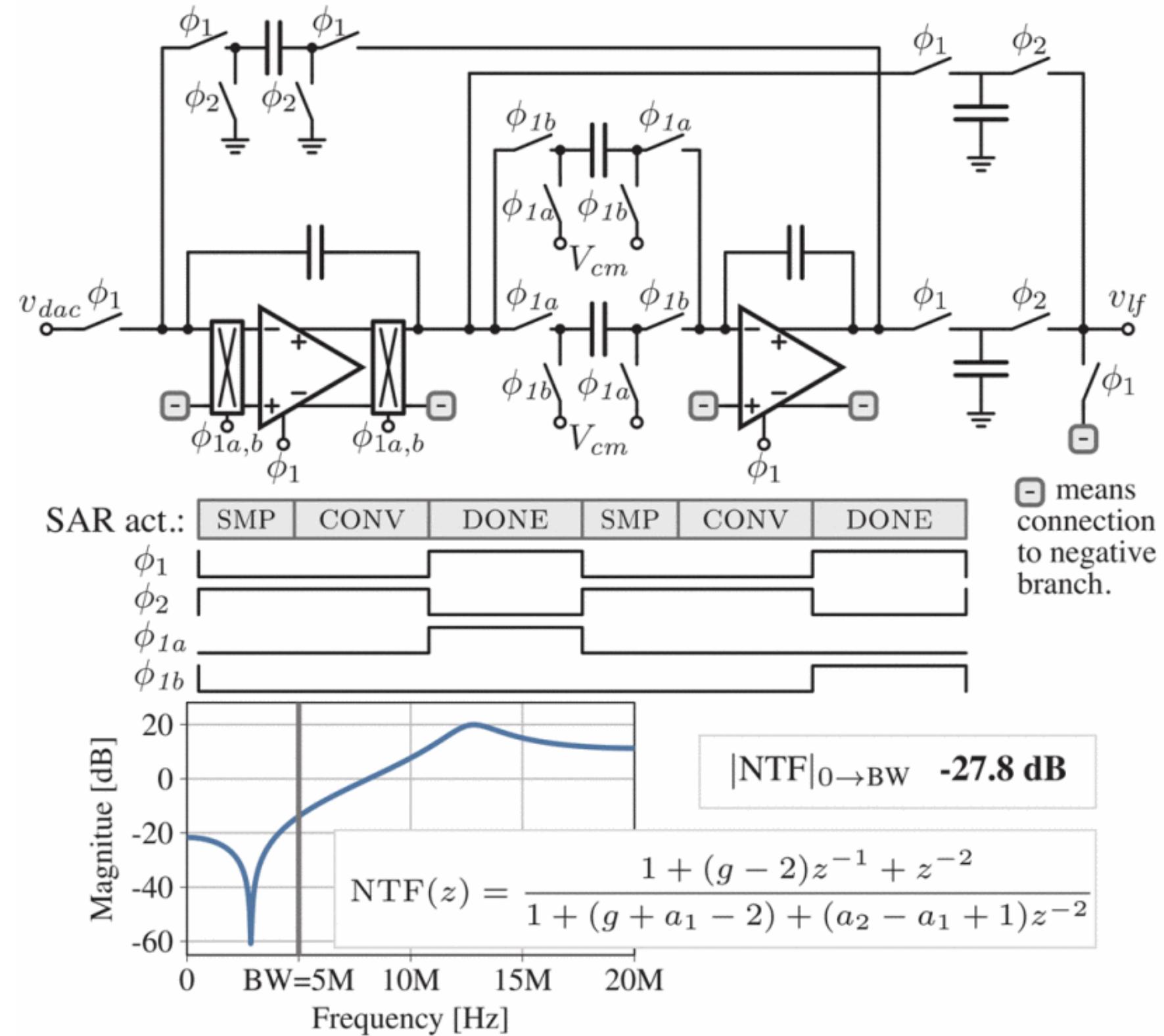


Resonators in Open-Loop Sigma-Delta Modulators



A 68 dB SNDR Compiled Noise-Shaping SAR ADC With On-Chip CDAC Calibration





Design Considerations for a Low-Power Control-Bounded A/D Converter

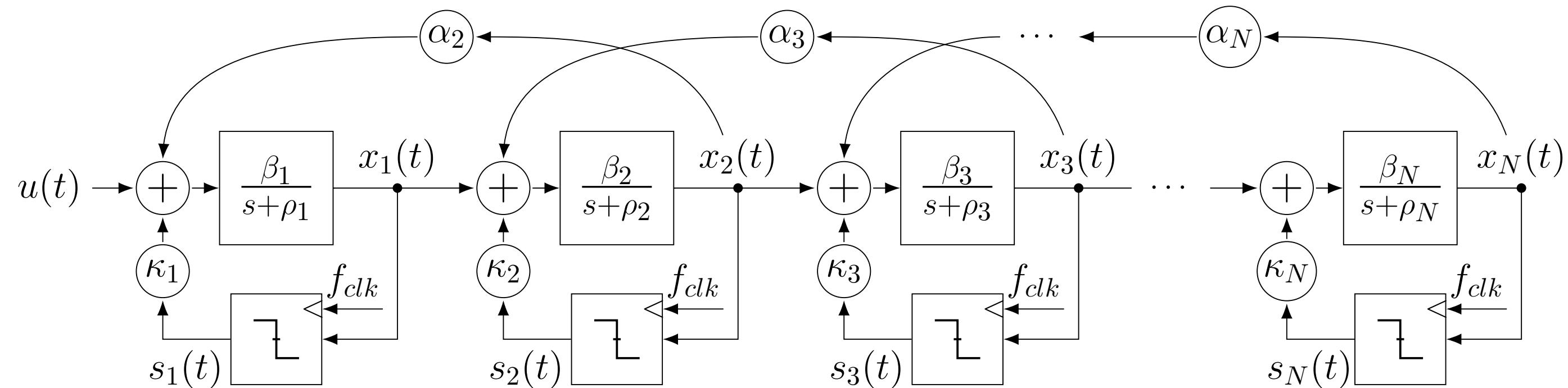
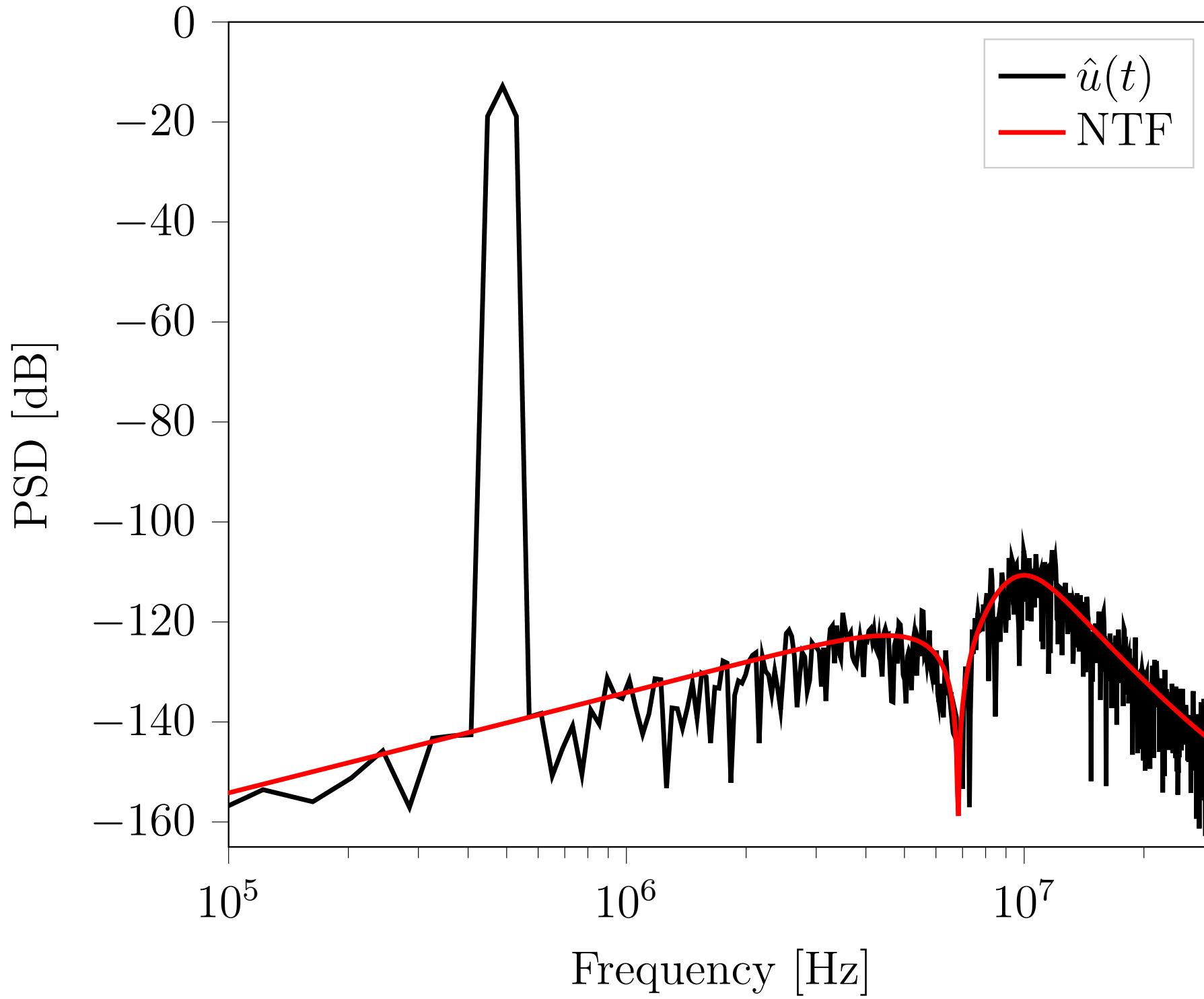
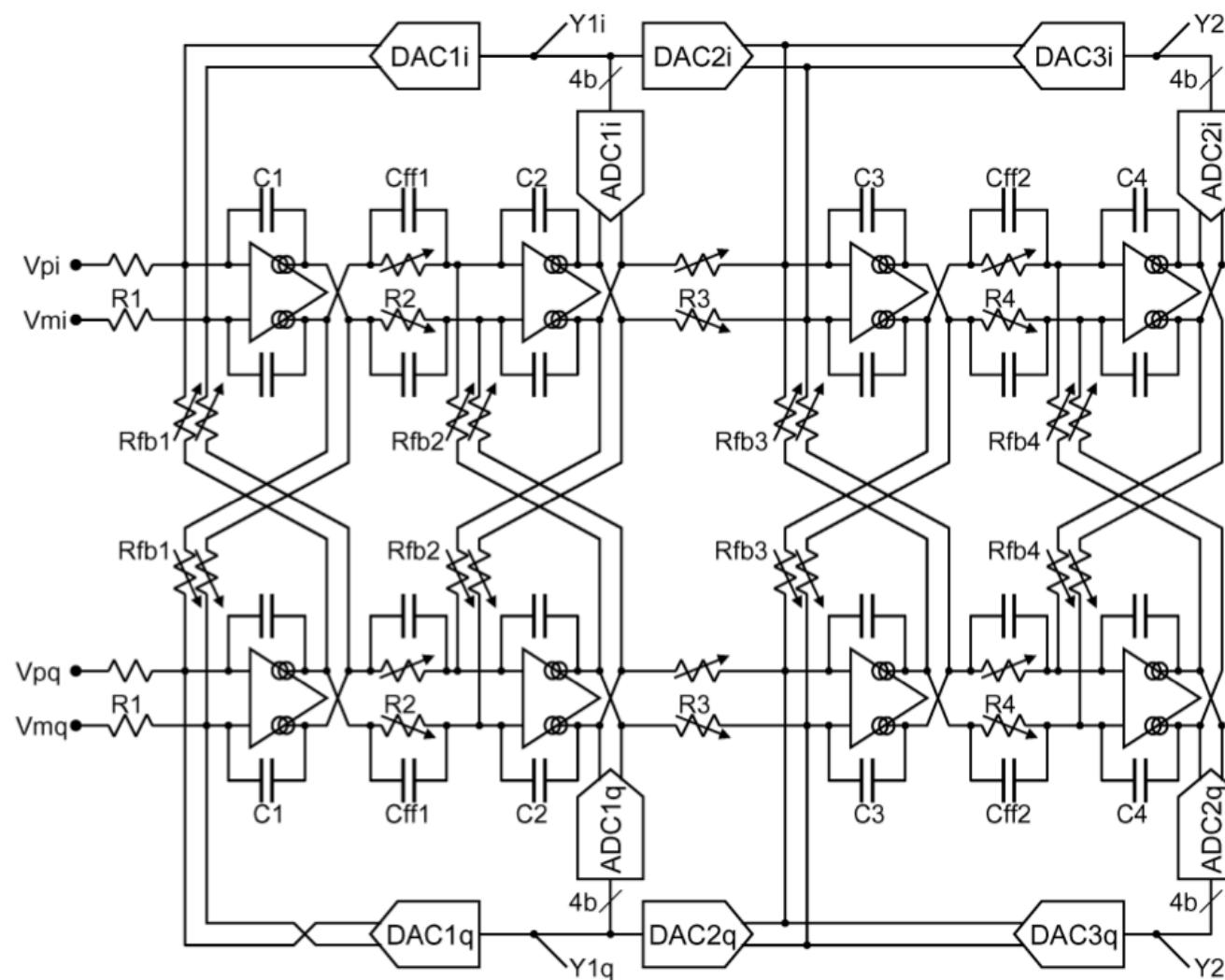


Figure 3.1: The general structure of the Leapfrog ADC



A 56 mW Continuous-Time Quadrature Cascaded Sigma-Delta Modulator With 77 dB DR in a Near Zero-IF 20 MHz Band



sigma-delta modulator design.

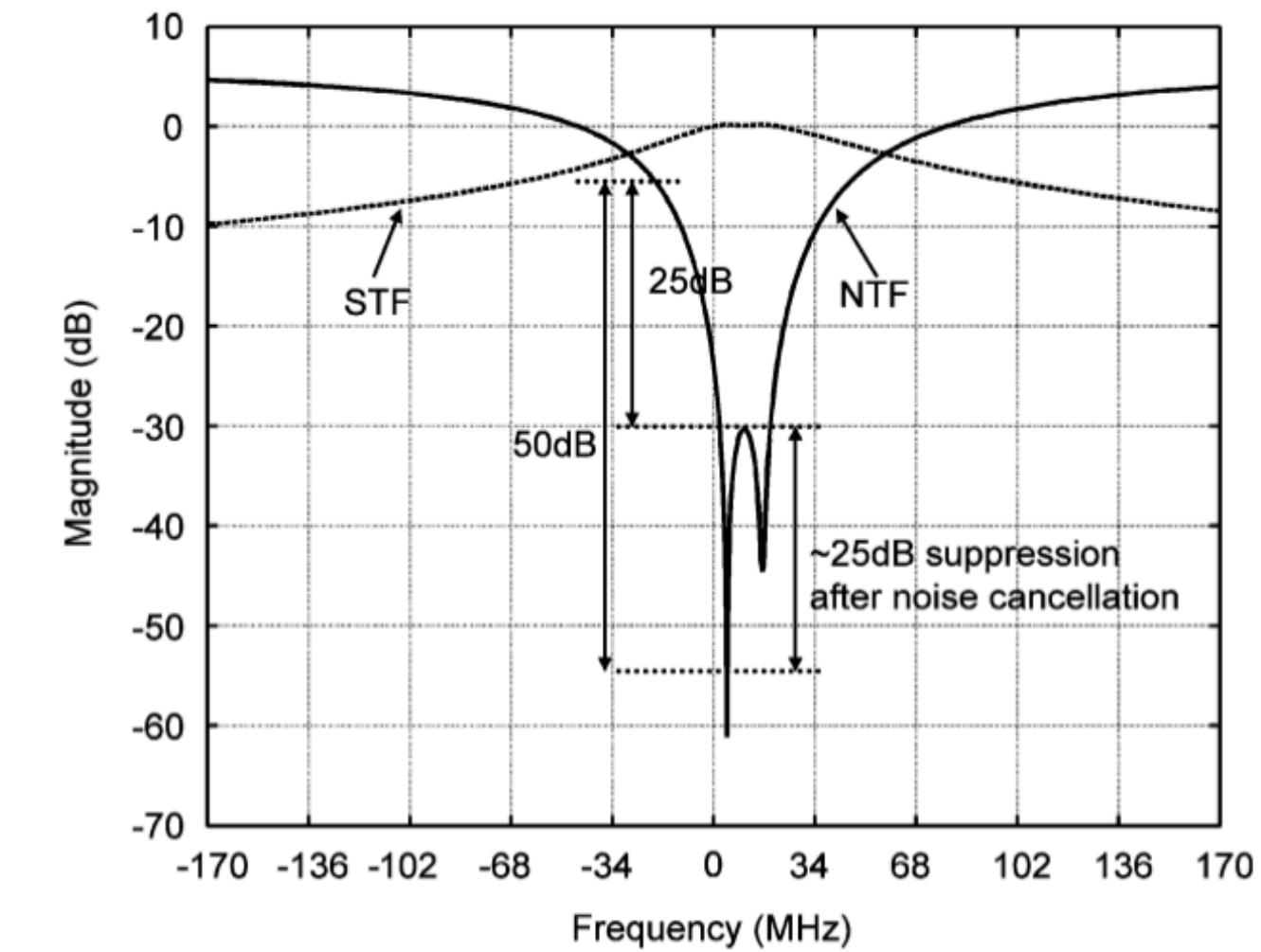
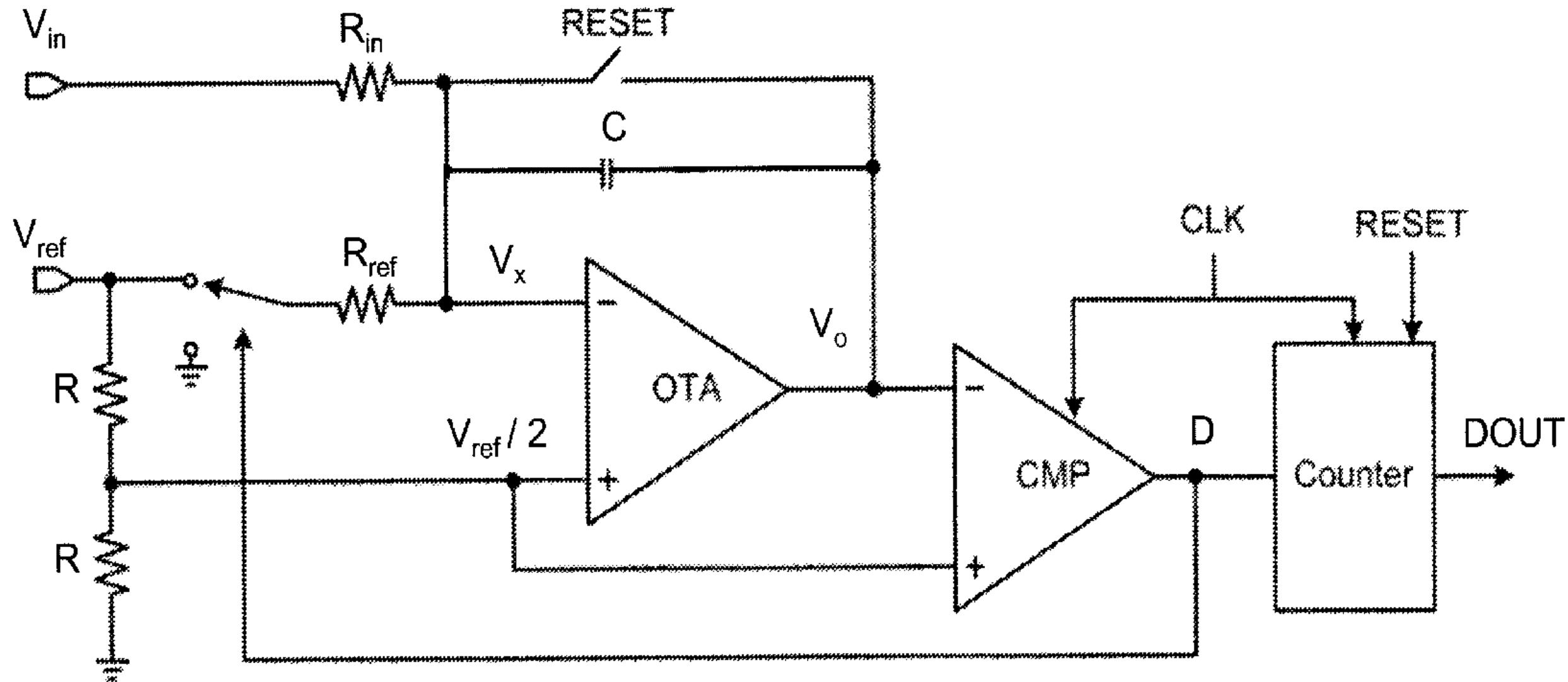


Fig. 9. NTF and STF of first stage.

Analogue-to-digital converter



Thanks!