

date: 2025-02-13

TFE4188 - Lecture 5

Switched-Capacitor Circuits

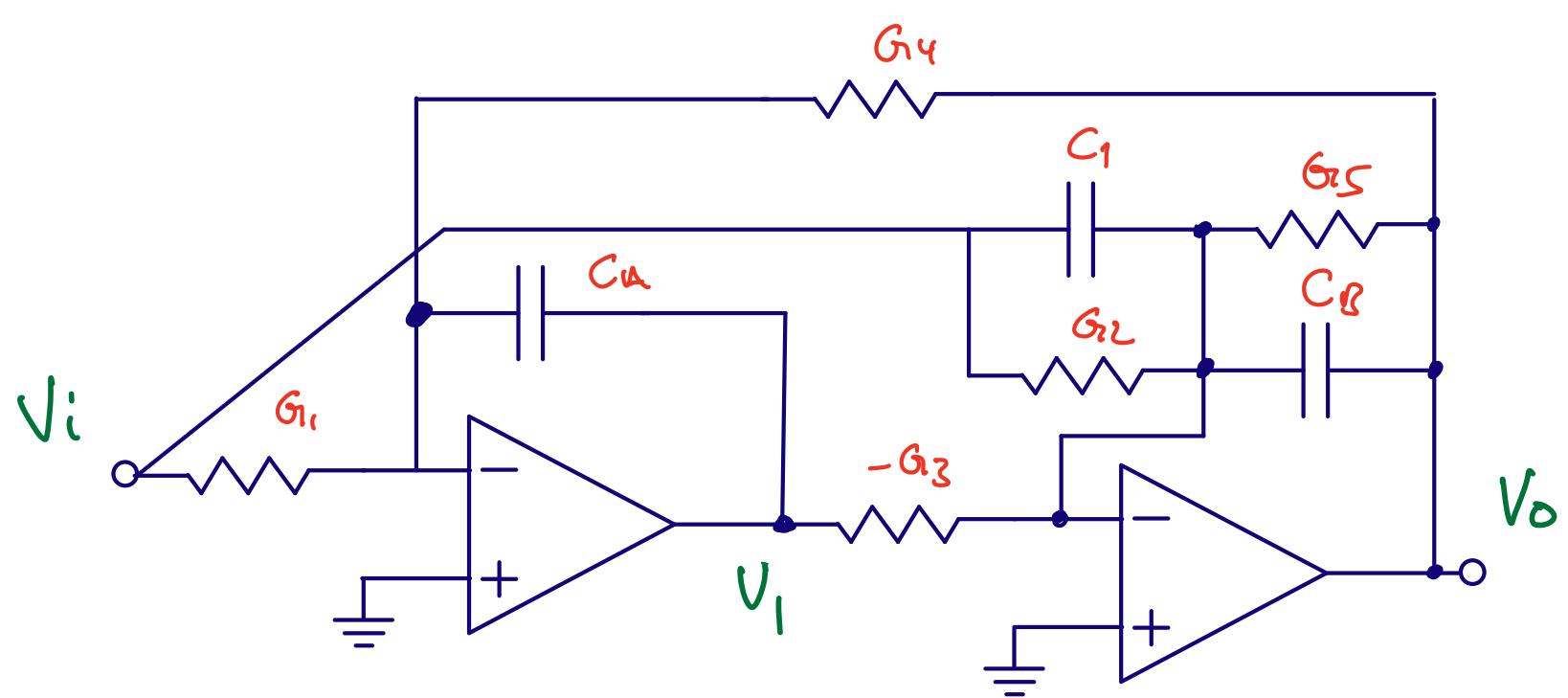
Goal

Understand **why** we would use switched capacitor circuits

Introduction to **discrete-time**, and **switched capacitor principles** and the **circuits** we need

W A W h n y

Active-RC



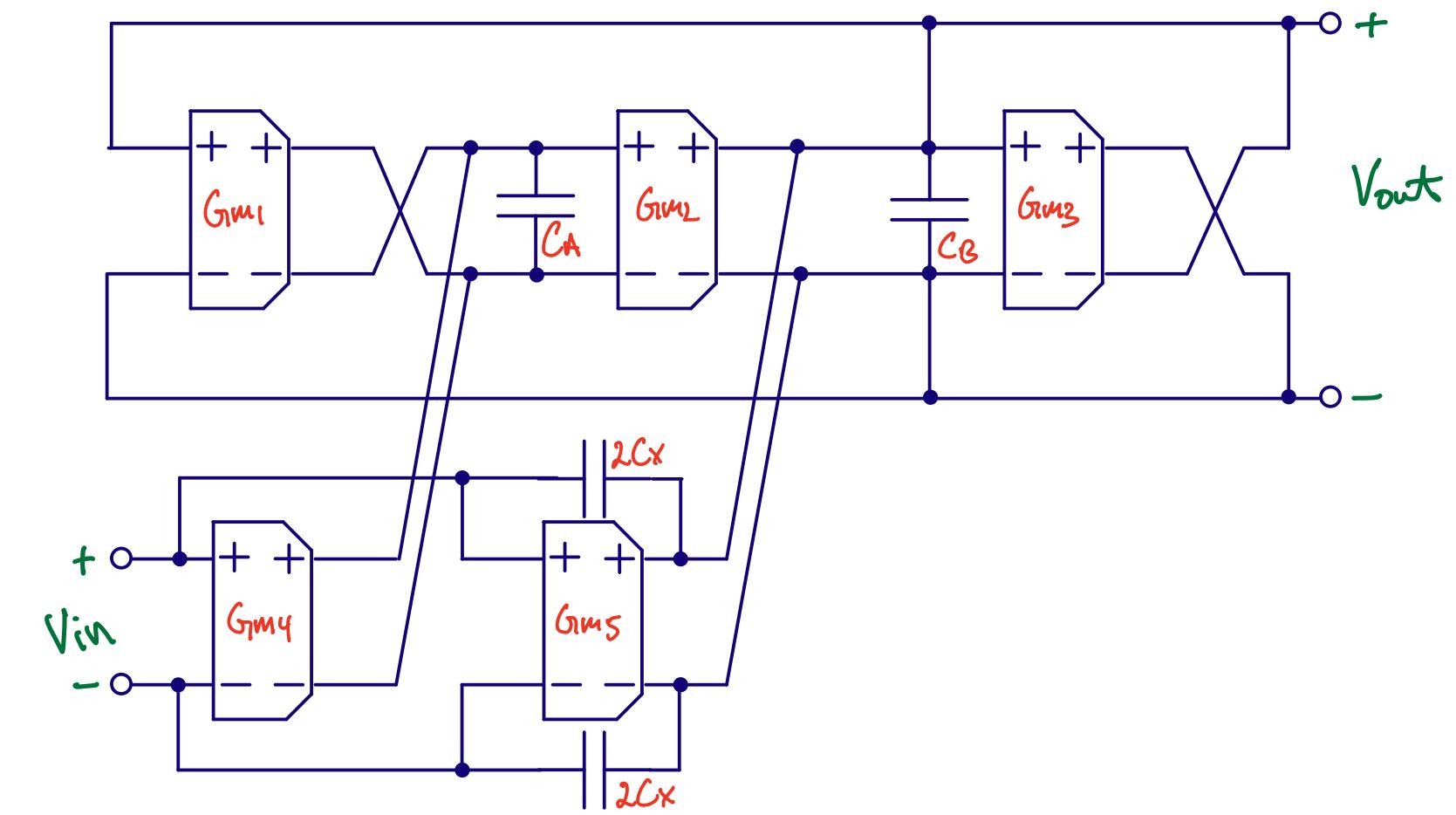
$$H(s) = \frac{\left[\frac{C_1}{C_B} s^2 + \frac{G_2}{C_B} s + \left(\frac{G_1 G_3}{C_A C_B} \right) \right]}{\left[s^2 + \frac{G_5}{C_B} s + \frac{G_3 G_4}{C_A C_B} \right]}$$

$$\omega_{p|z} \propto \frac{G}{C} = \frac{1}{RC}$$

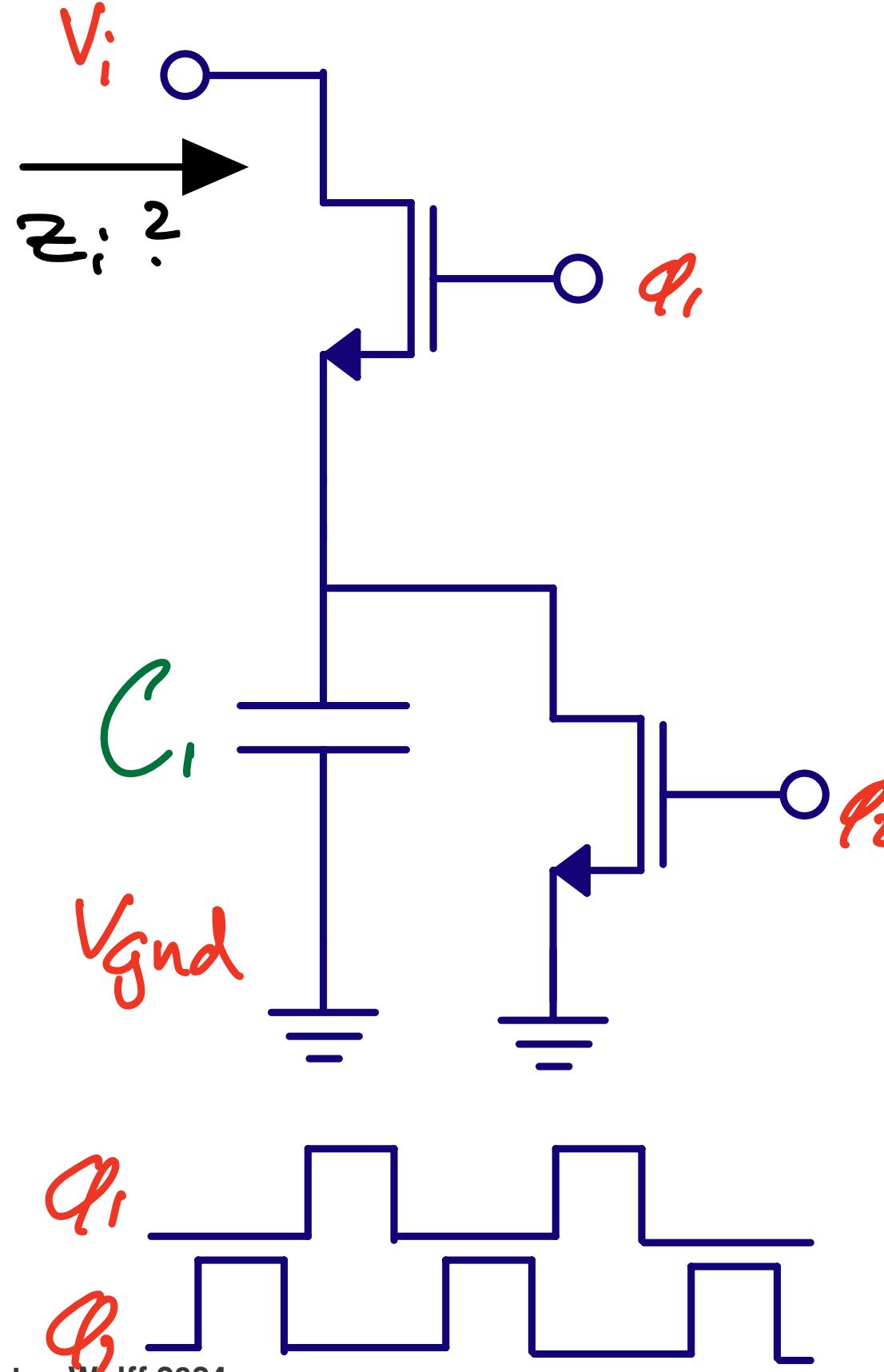
Gm-C

$$H(s) = \frac{\left[s^2 \frac{C_X}{C_X+C_B} + s \frac{G_{m5}}{C_X+C_B} + \frac{G_{m2}G_{m4}}{C_A(C_X+C_B)} \right]}{\left[s^2 + s \frac{G_{m2}}{C_X+C_B} + \frac{G_{m1}G_{m2}}{C_A(C_X+C_B)} \right]}$$

$$\omega_{p|z} \propto \frac{G_m}{C}$$



Switched capacitor



$$Q_{\phi 2\$} = C_1 V_{GND} = 0$$

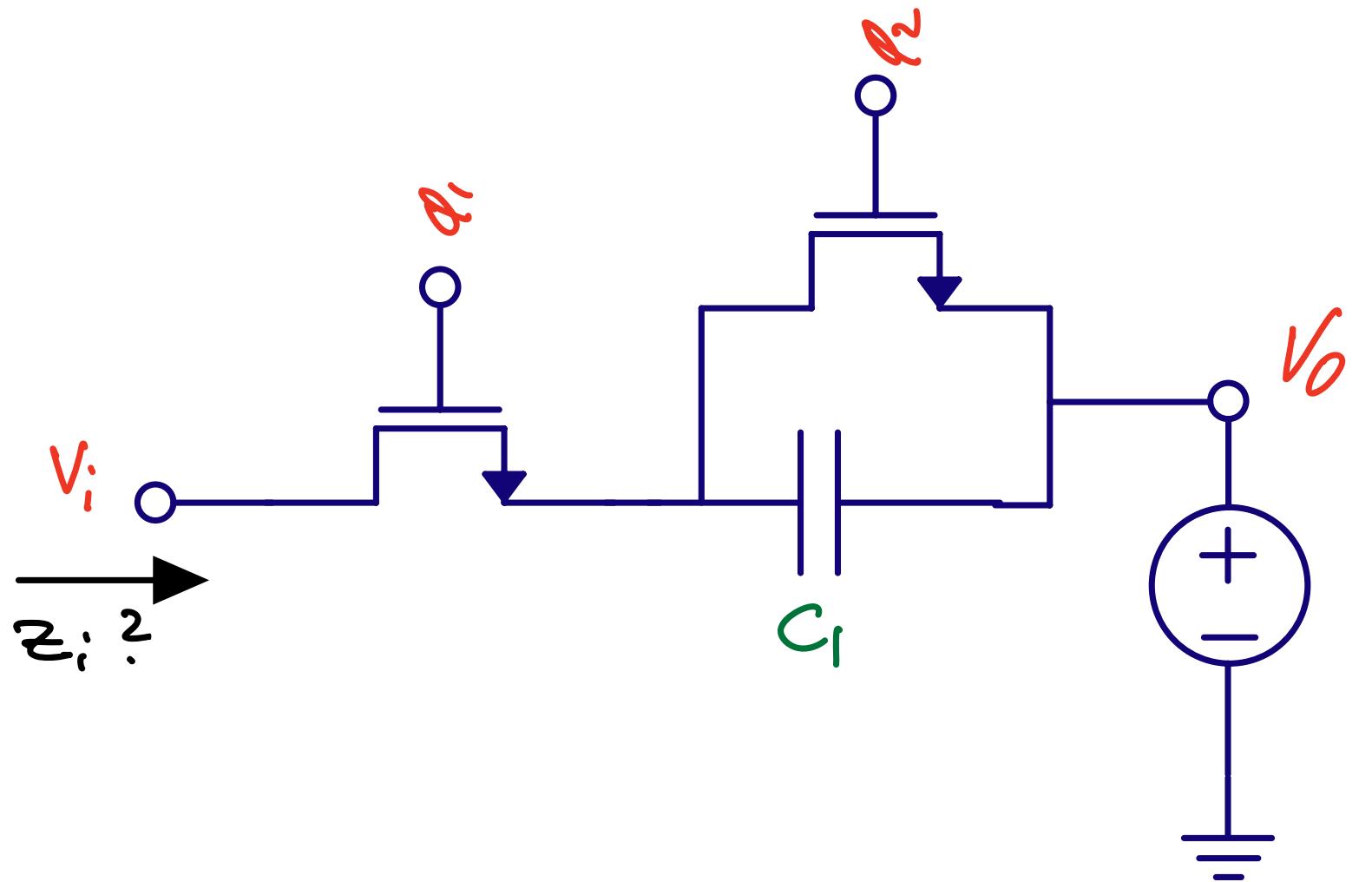
$$Q_{\phi 1\$} = C_1 V_I$$

$$Z_I = (V_I - V_{GND})/I_I$$

$$I_I = \frac{Q}{dt} = Q f_\phi$$

$$Z_I = \frac{V_I - V_{GND}}{(Q_{\phi 1\$} - Q_{\phi 2\$}) f_\phi}$$

$$Z_I = \frac{V_I}{(V_I C - 0) f_\phi} = \frac{1}{C_1 f_\phi}$$

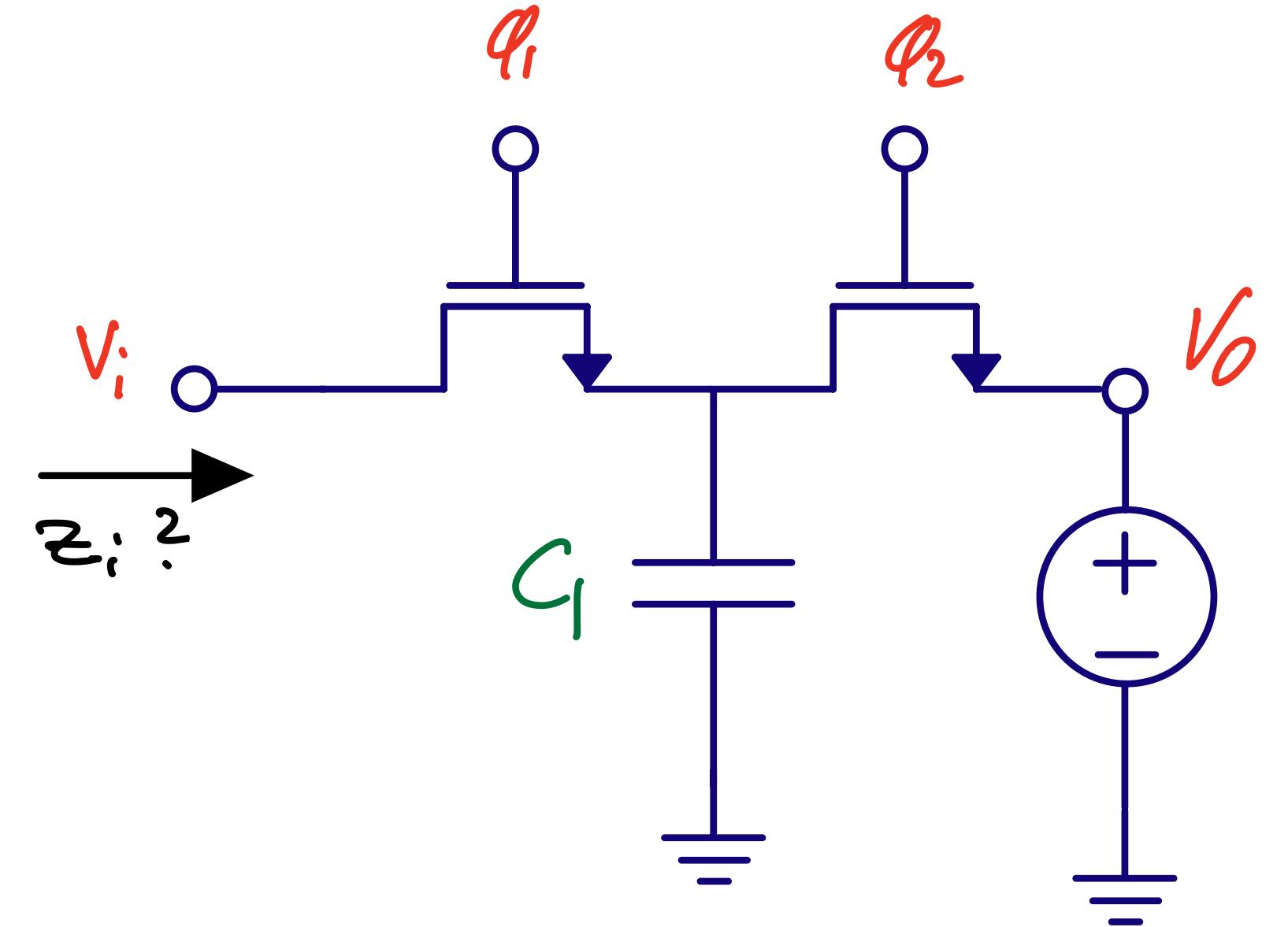


$$Z_I = \frac{V_I - V_O}{(Q_{\phi 1\$} - Q_{\phi 2\$}) f_\phi}$$

$$Q_{\phi 1\$} = C_1 (V_I - V_O)$$

$$Q_{\phi 2\$} = 0$$

$$Z_I = \frac{V_I - V_O}{(C_1 (V_I - V_O)) f_\phi} = \frac{1}{C_1 f_\phi}$$



$$Z_I = \frac{V_I - V_O}{(Q_{\phi 1\$} - Q_{\phi 2\$}) f_\phi}$$

$$Q_{\phi 1\$} = C_1 V_I$$

$$Q_{\phi 2\$} = C_1 V_O$$

$$Z_I = \frac{V_I - V_O}{(C_1 V_I - C_1 V_O)) f_\phi} = \frac{1}{C_1 f_\phi}$$

A pipelined 5-Msample/s 9-bit analog-to-digital converter

$$\omega_{p|z} \propto \frac{C_1}{C_2}$$

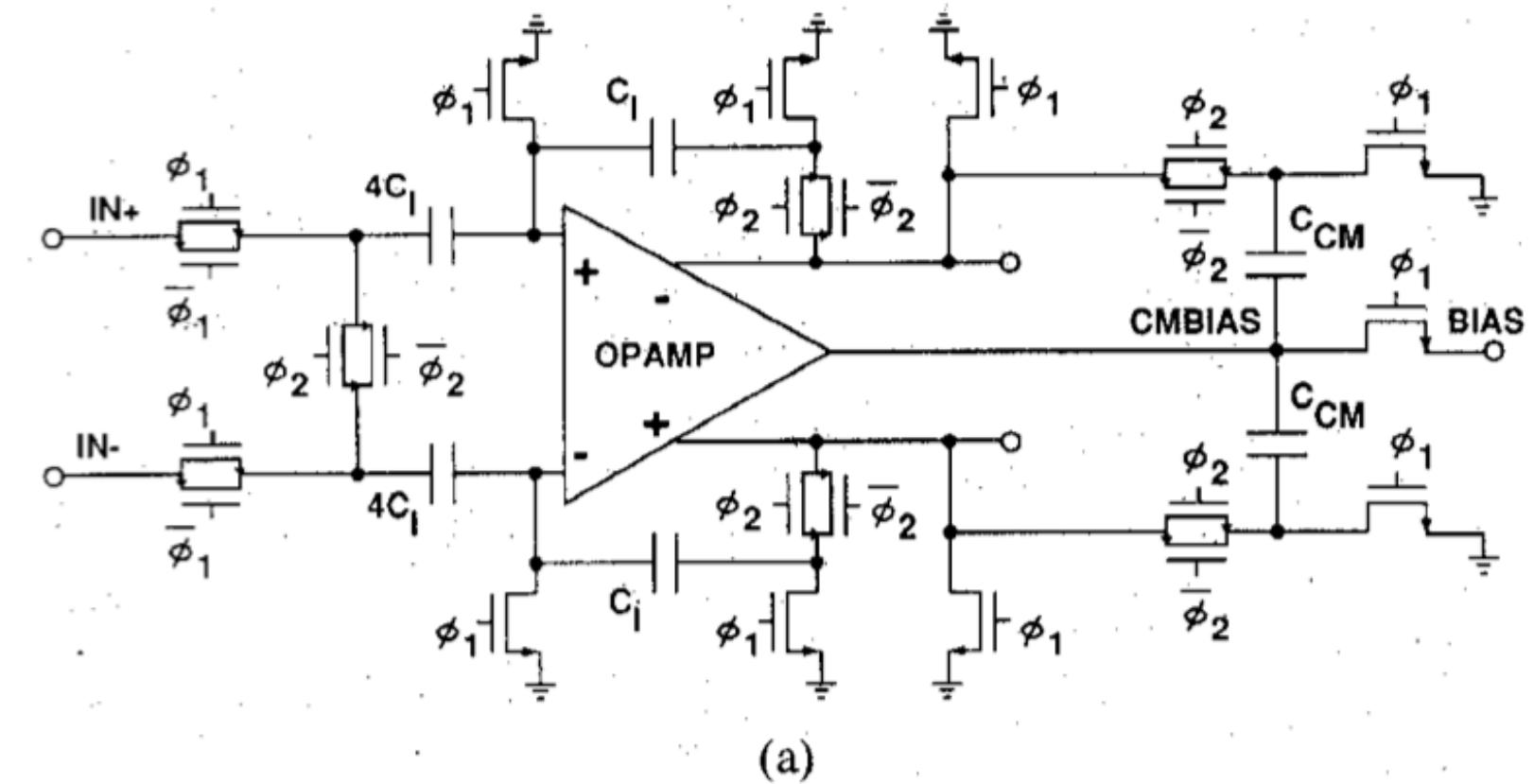


Fig. 6. (a) Schematic of S/H amplifier. (b) Timing diagram of a two-phase nonoverlapping clock.

Discrete-Time Signals

Define x_c as a continuous time, continuous value signal

$$\text{Define } \ell(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

Define

$$x_{sn}(t) = \frac{x_c(nT)}{\tau} [\ell(t - nT) - \ell(t - nT - \tau)]$$

$$\text{Define } x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

Think of a sampled version of an analog signal as an infinite sum of pulse trains where the area under the pulse train is equal to the analog signal.

Why do this?

If $x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$

$$\text{Then } X_{sn}(s) = \frac{1}{\tau} \frac{1 - e^{-s\tau}}{s} x_c(nT) e^{-snT}$$

$$\text{And } X_s(s) = \frac{1}{\tau} \frac{1 - e^{-s\tau}}{s} \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$\text{Thus } \lim_{\tau \rightarrow 0} X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$\text{Or } X_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left(j\omega - \frac{jk2\pi}{T} \right)$$

The spectrum of a sampled signal is an infinite sum of frequency shifted spectra

or equivalently

When you sample a signal, then there will be copies of the input spectrum at every $n f_s$

However, if you do an FFT of a sampled signal, then all those infinite spectra will fold down between $0 \rightarrow f_{s1}/2$ or $-f_{s1}/2 \rightarrow f_{s1}/2$ for a complex FFT

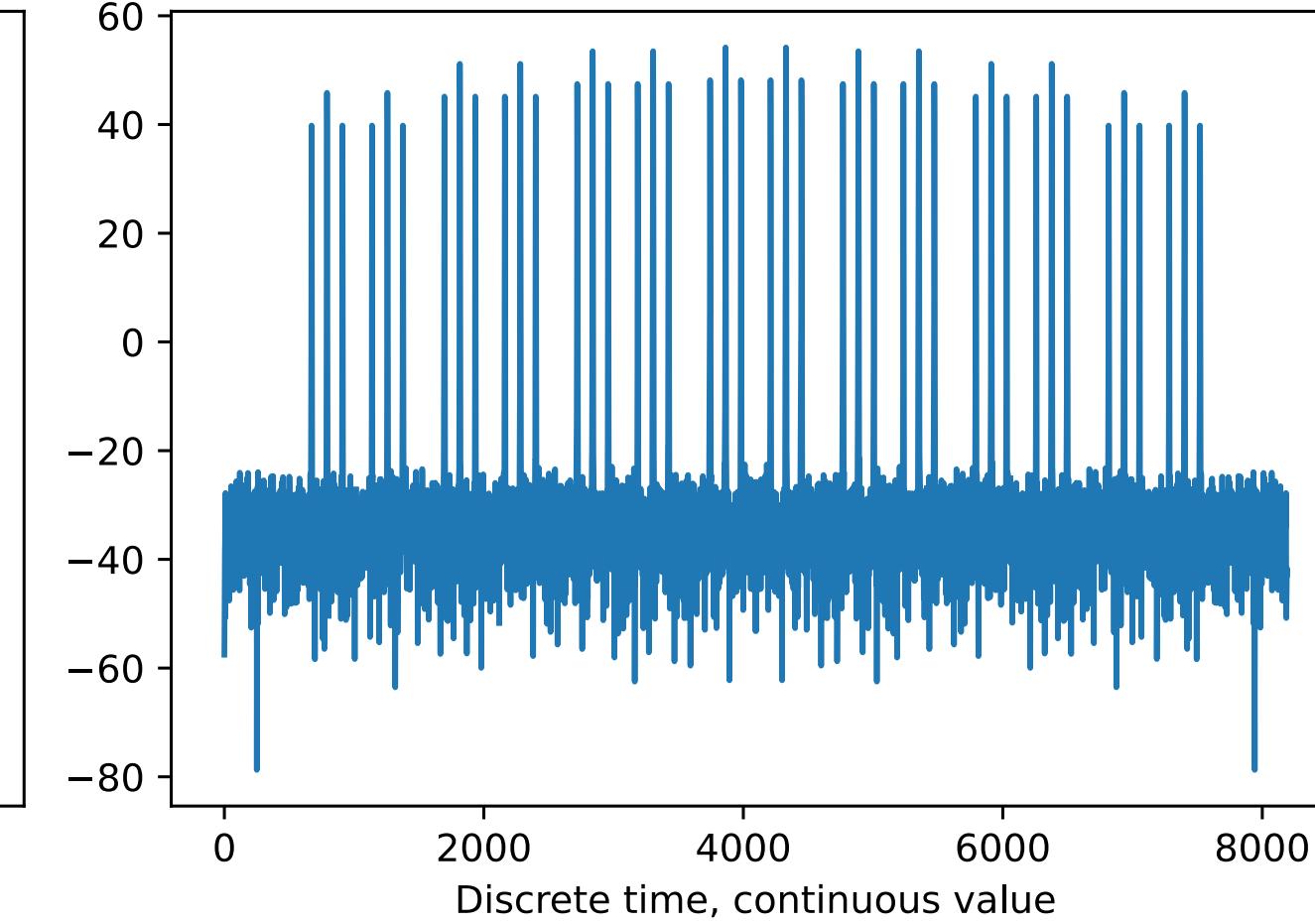
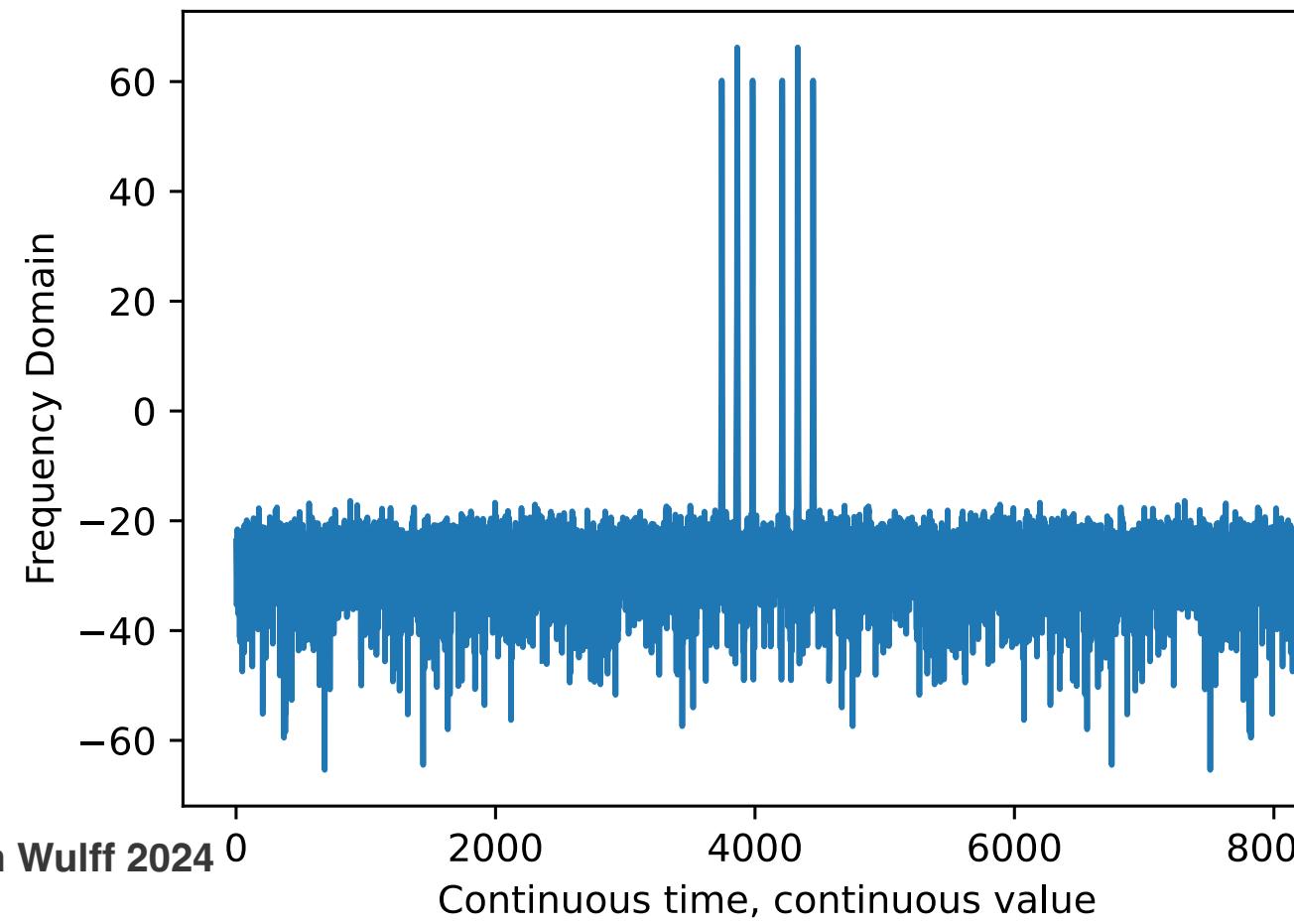
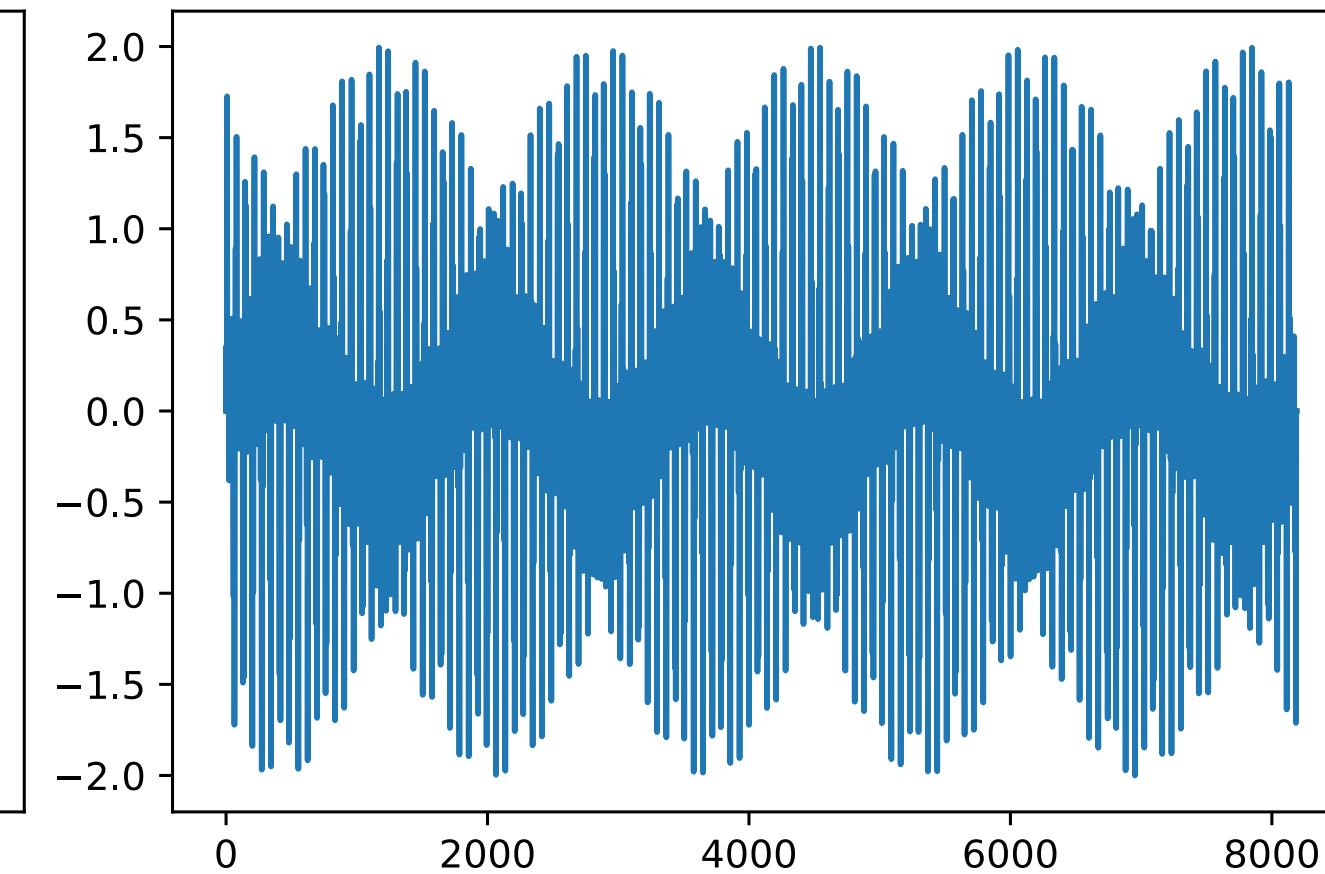
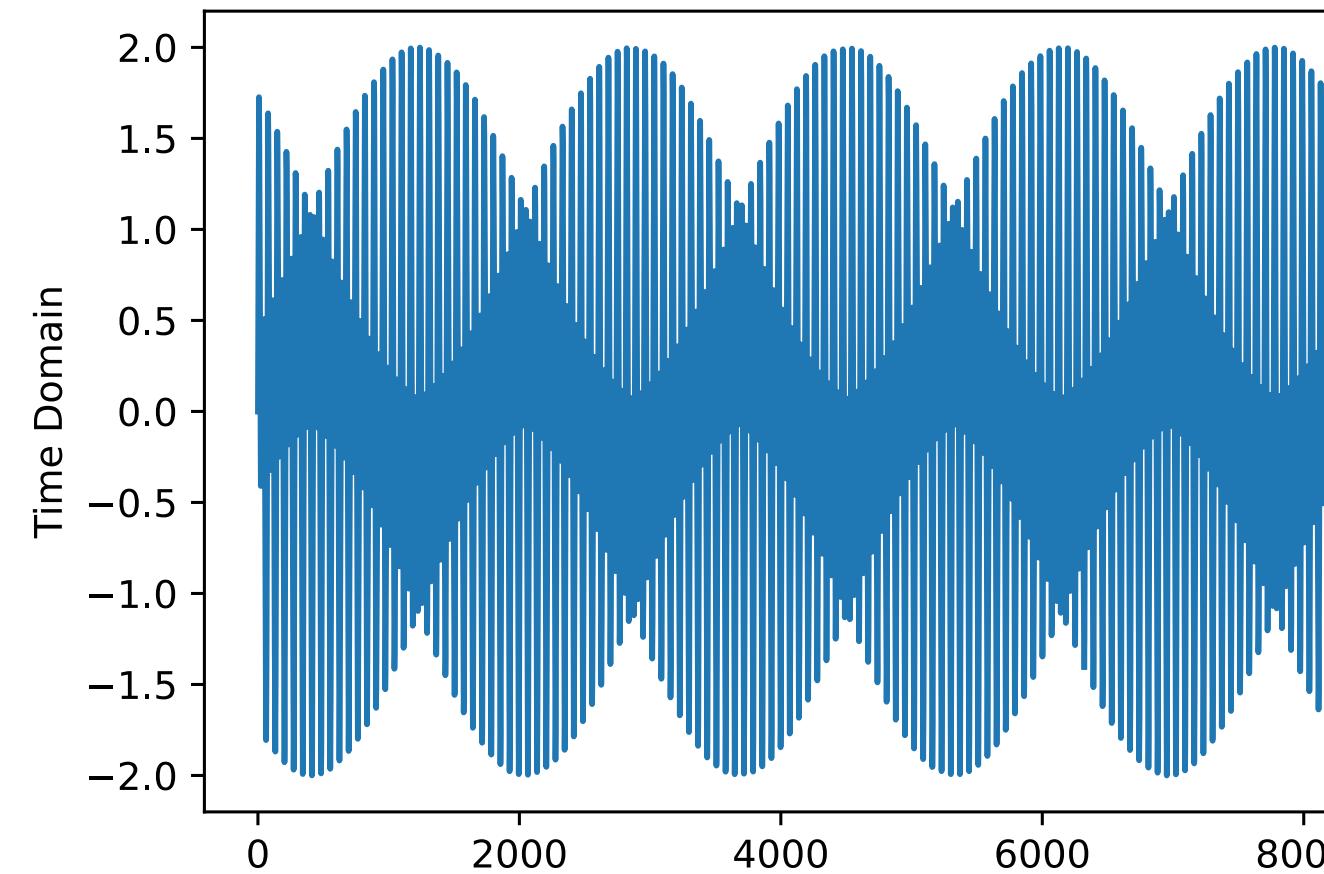
dt.py

```
#- Create a time vector
N = 2**13
t = np.linspace(0,N,N)

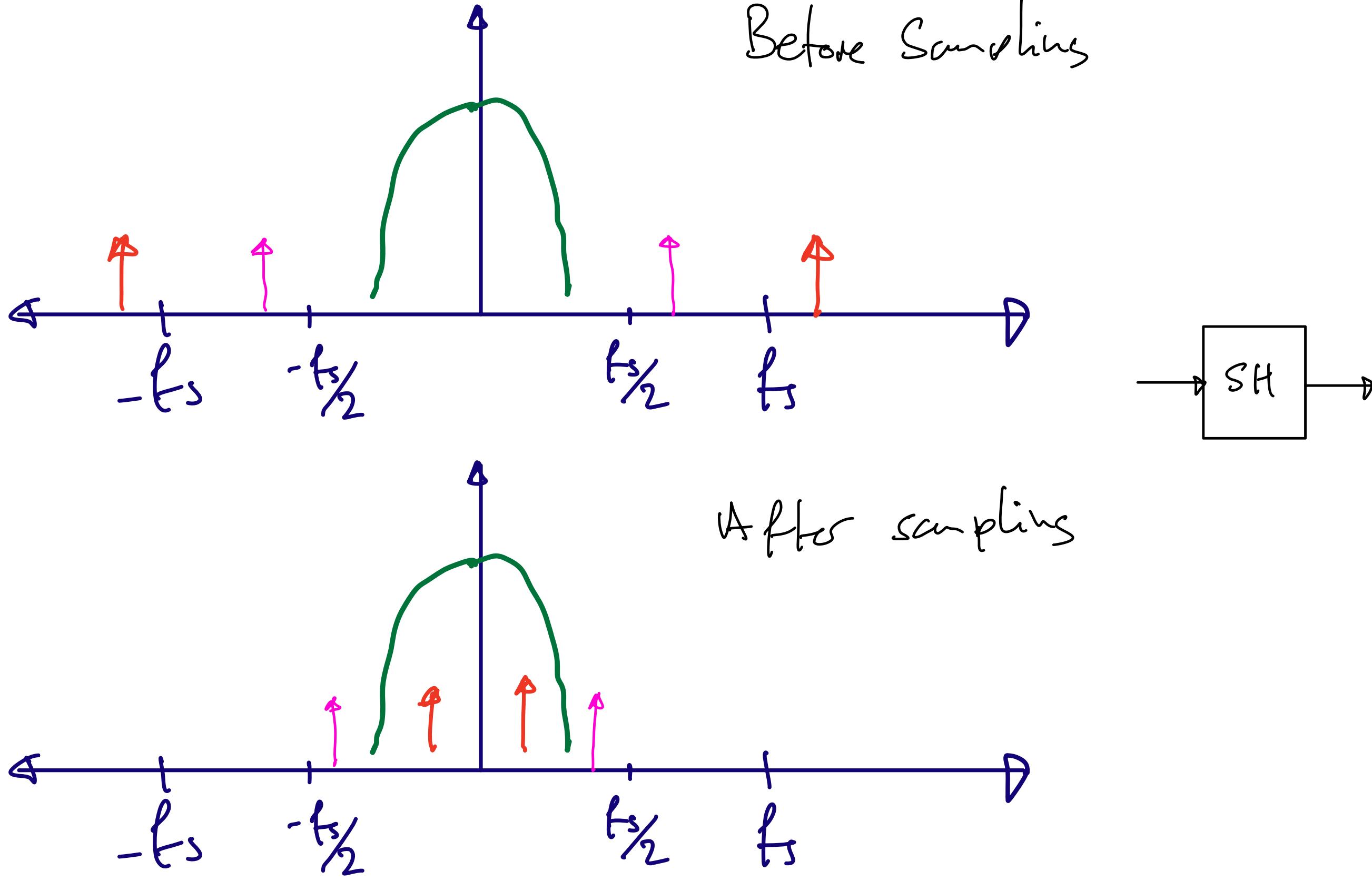
#- Create the "continuous time" signal with multiple
#- "sinusoidal signals and some noise
f1 = 233/N
fd = 1/N*119
x_s = np.sin(2*np.pi*f1*t) + 1/1024*np.random.randn(N) +
      0.5*np.sin(2*np.pi*(f1-fd)*t) + 0.5*np.sin(2*np.pi*(f1+fd)*t)

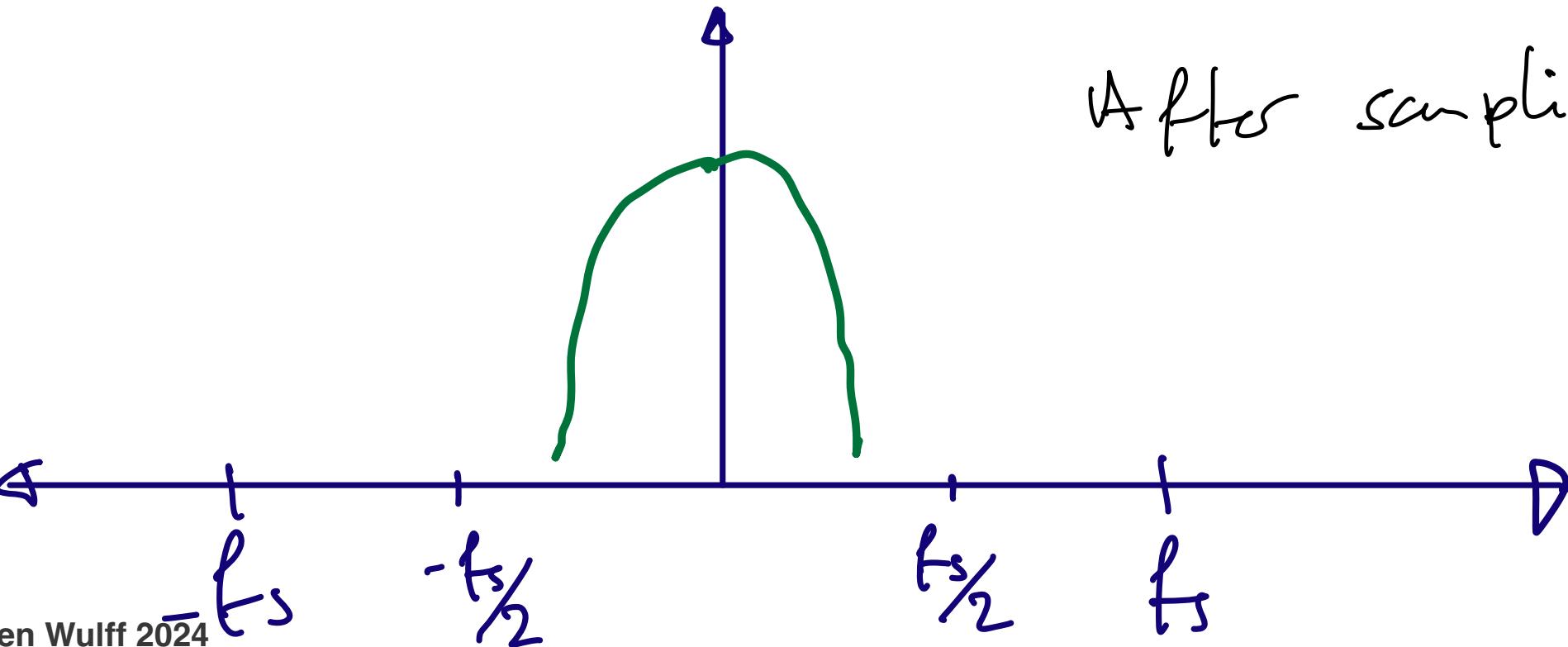
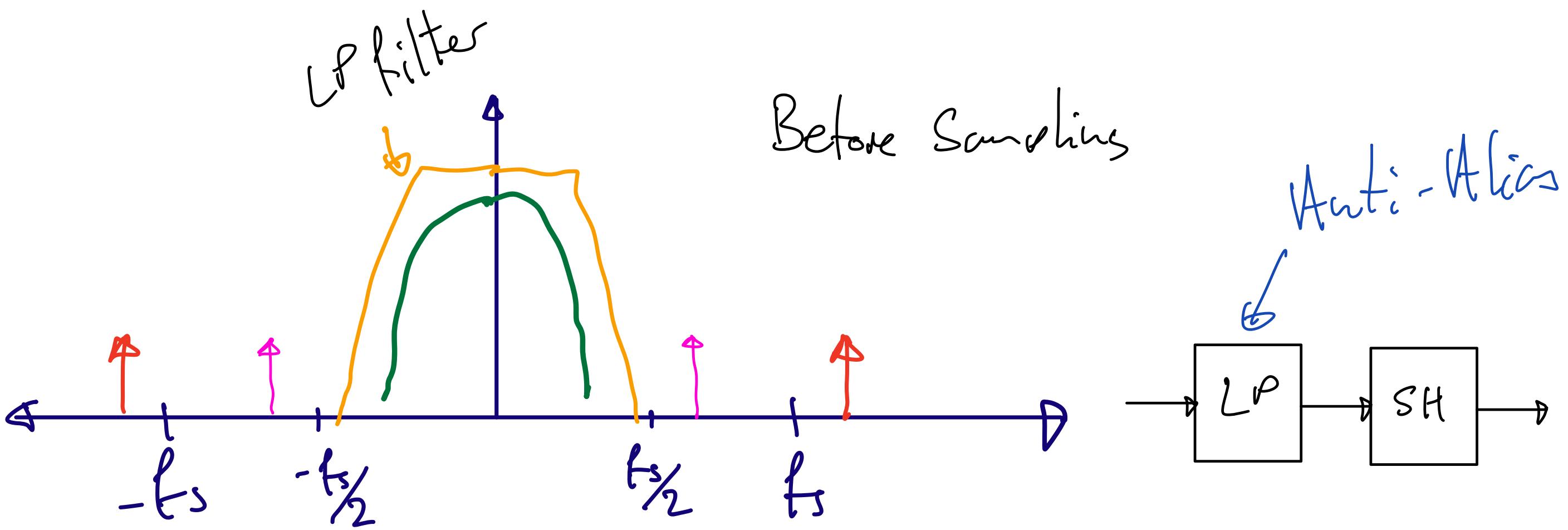
#- Create the sampling vector, and the sampled signal
t_s_unit = [1,1,0,0,0,0,0,0]
t_s = np.tile(t_s_unit,int(N/len(t_s_unit)))
x_sn = x_s*t_s

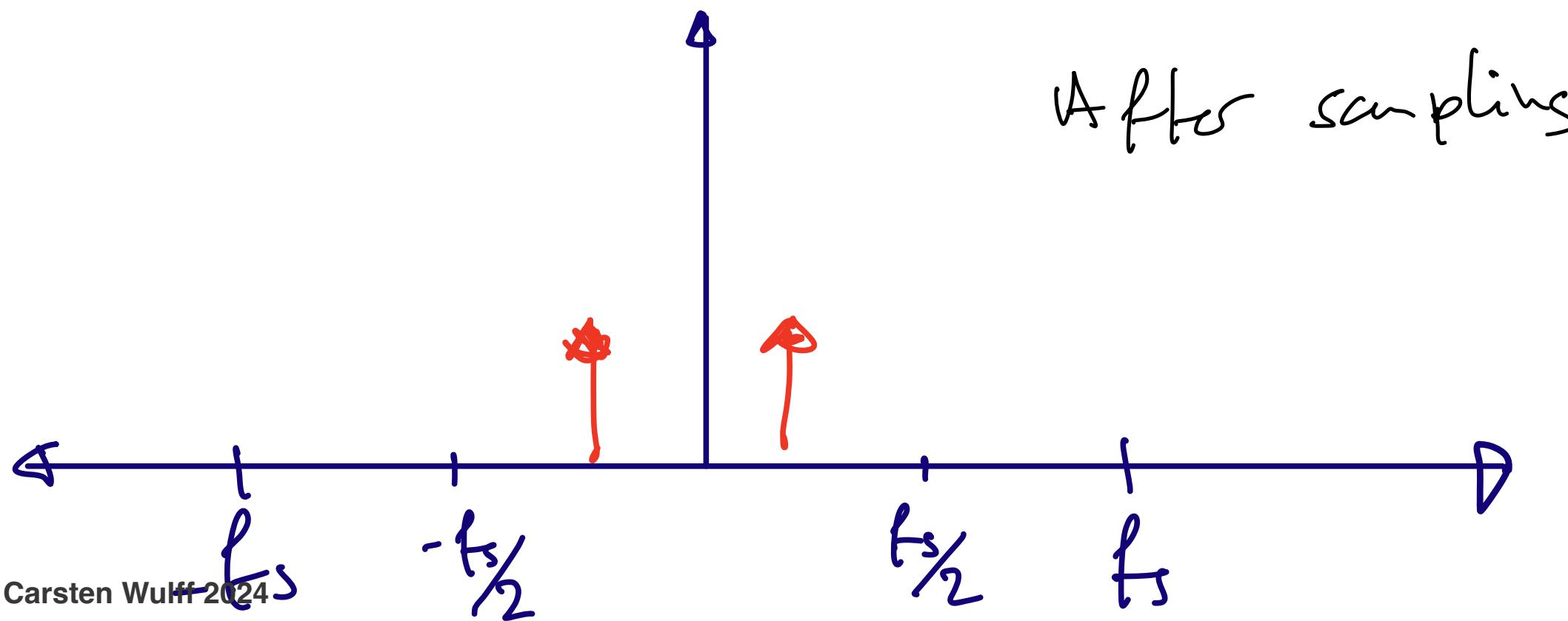
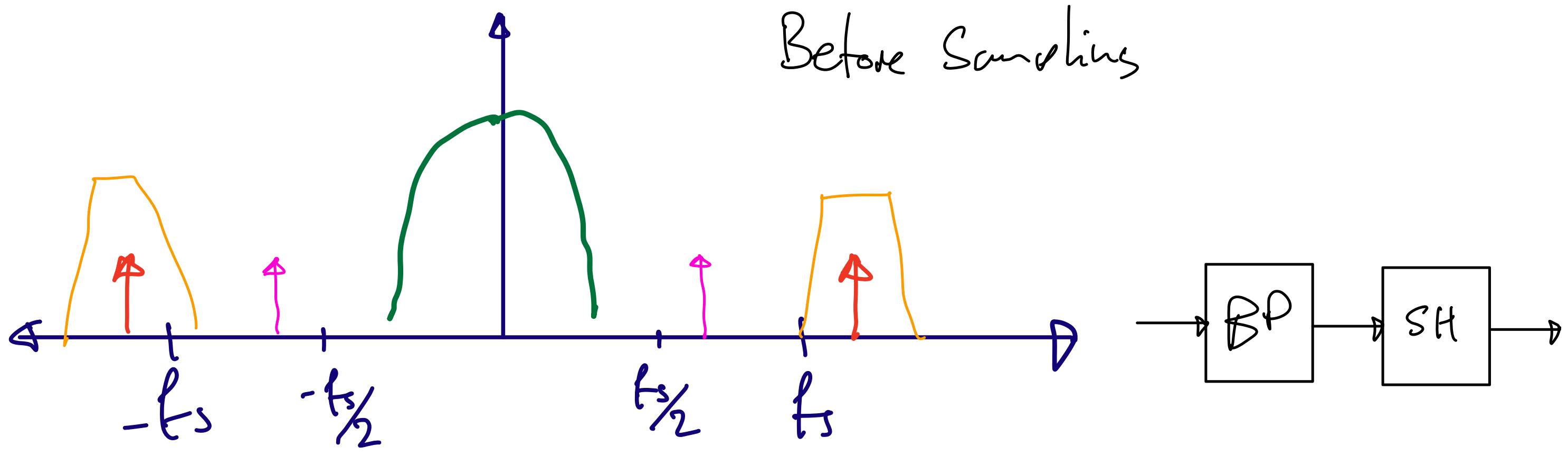
#- Convert to frequency domain with a hanning window to avoid FFT bin
#- energy spread
Hann = True
if(Hann):
    w = np.hanning(N+1)
else:
    w = np.ones(N+1)
X_s = np.fft.fftshift(np.fft.fft(np.multiply(w[0:N],x_s)))
X_sn = np.fft.fftshift(np.fft.fft(np.multiply(w[0:N],x_sn)))
```



Carsten Wulff 2024
Continuous time, continuous value







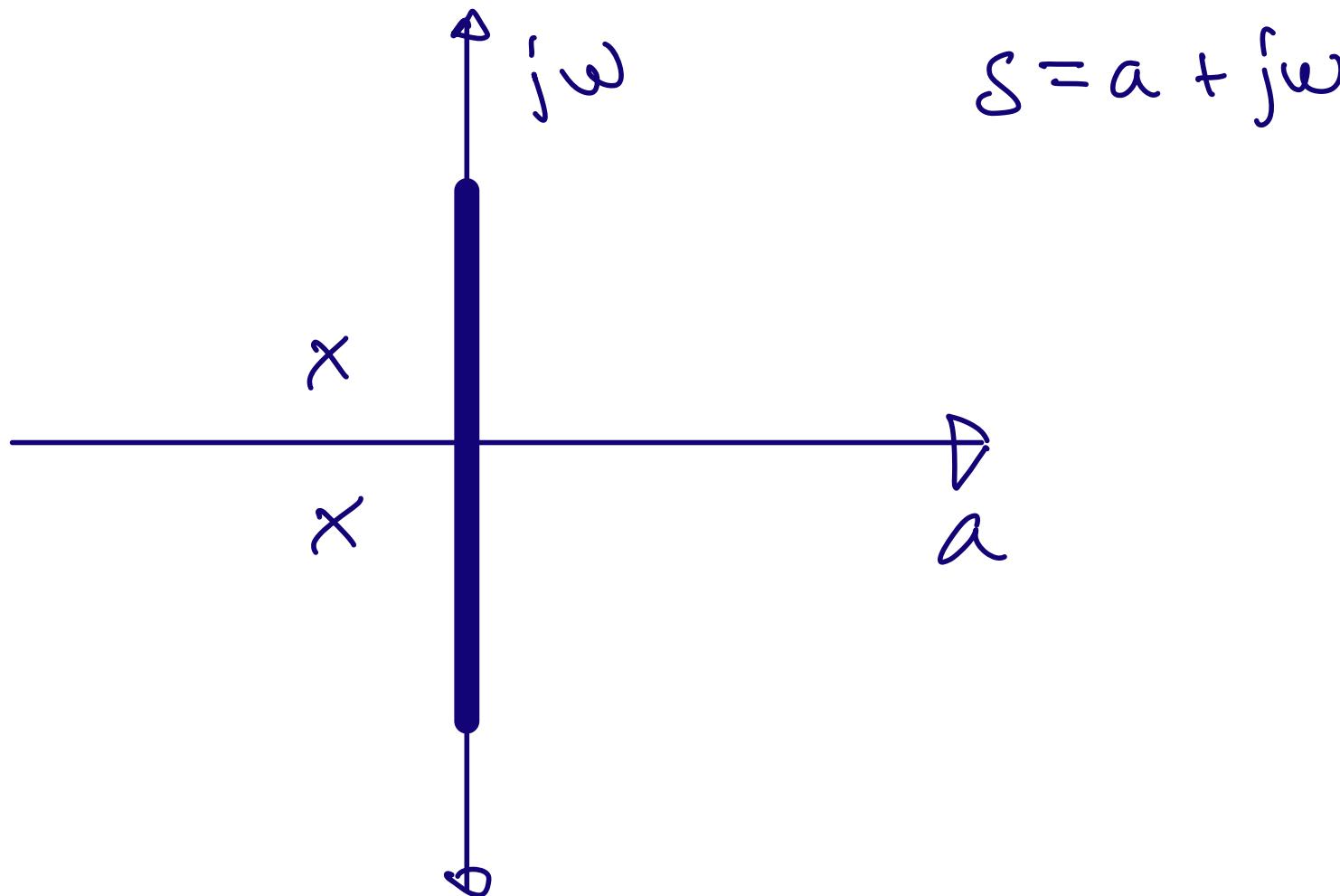
$$X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$X_s(z) = \sum_{n=-\infty}^{\infty} x_c[n] z^{-n}$$

For discrete time signal processing we use Z-transform

If you're unfamiliar with the Z-transform, read the book or search <https://en.wikipedia.org/wiki/Z-transform>

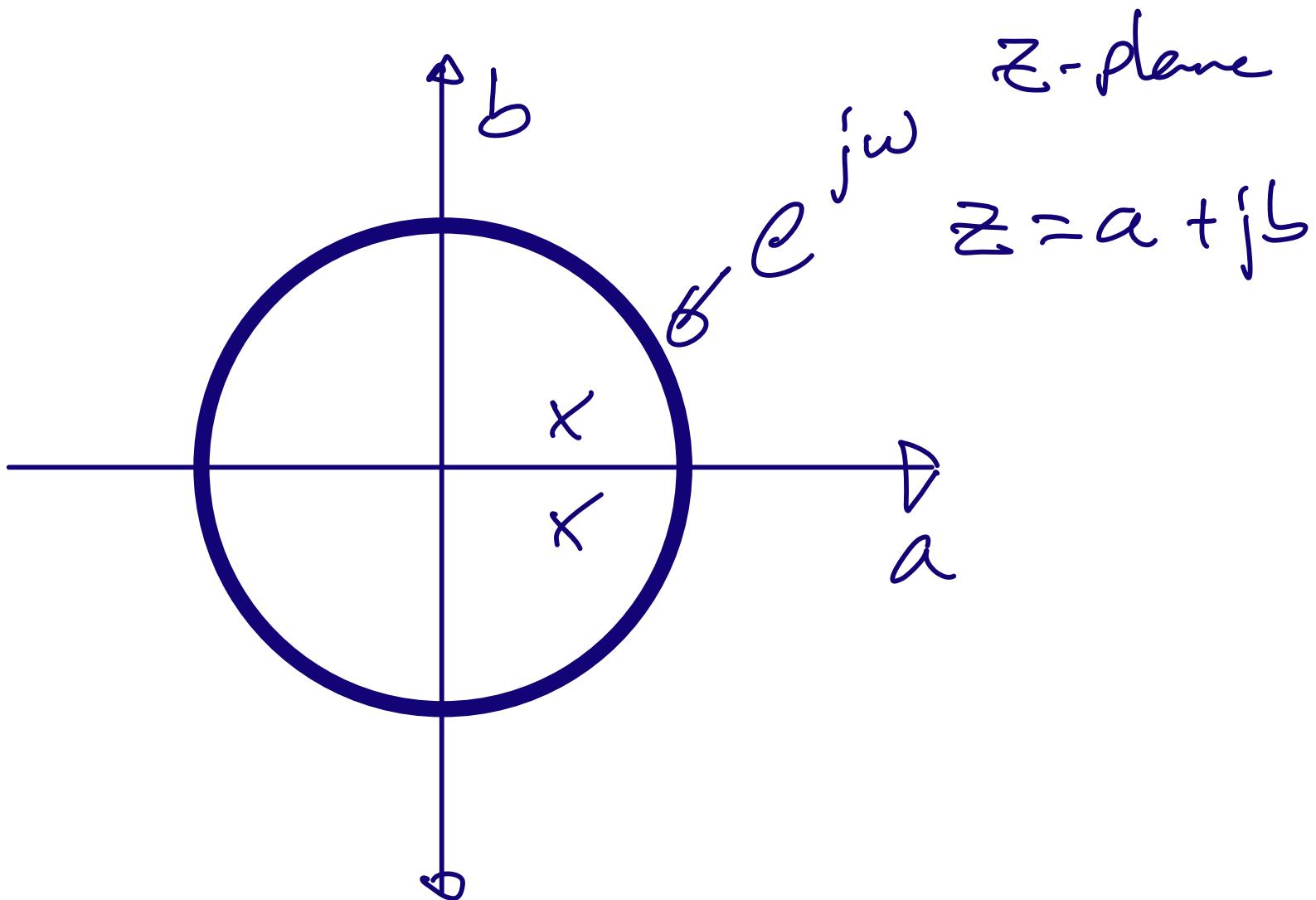
Pole-Zero plots



If you're not comfortable with pole/zero plots, have a look at

[What does the Laplace Transform really tell us](#)

Z-domain



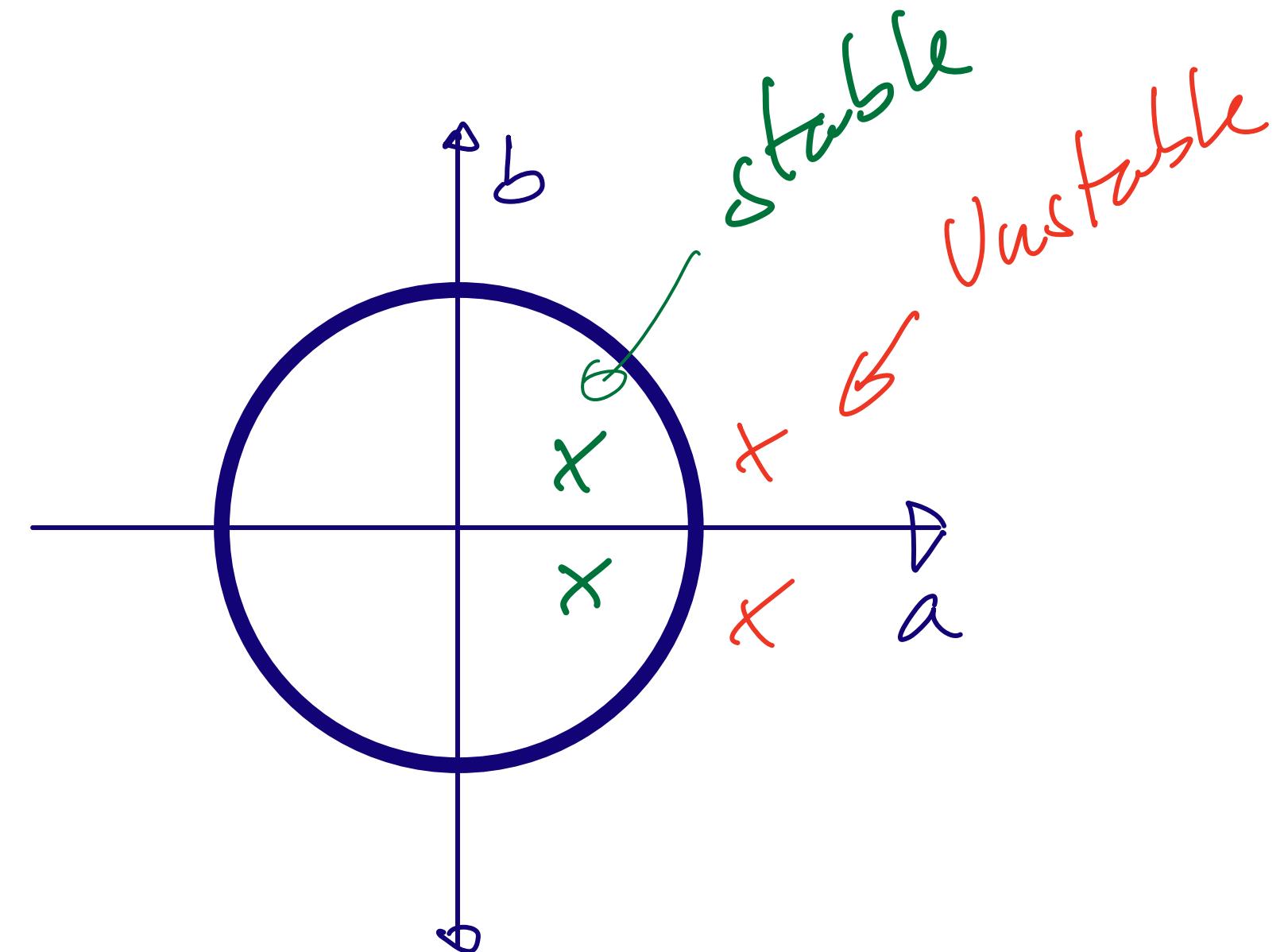
Spectra repeat every 2π

Bi-linear transform

$$s = \frac{z - 1}{z + 1}$$

Warning: First-order approximation https://en.wikipedia.org/wiki/Bilinear_transform

First order filter



$$y[n + 1] = bx[n] + ay[n] \Rightarrow Yz = bX + aY$$

$$y[n] = bx[n - 1] + ay[n - 1] \Rightarrow Y = bXz^{-1} + aYz^{-1}$$

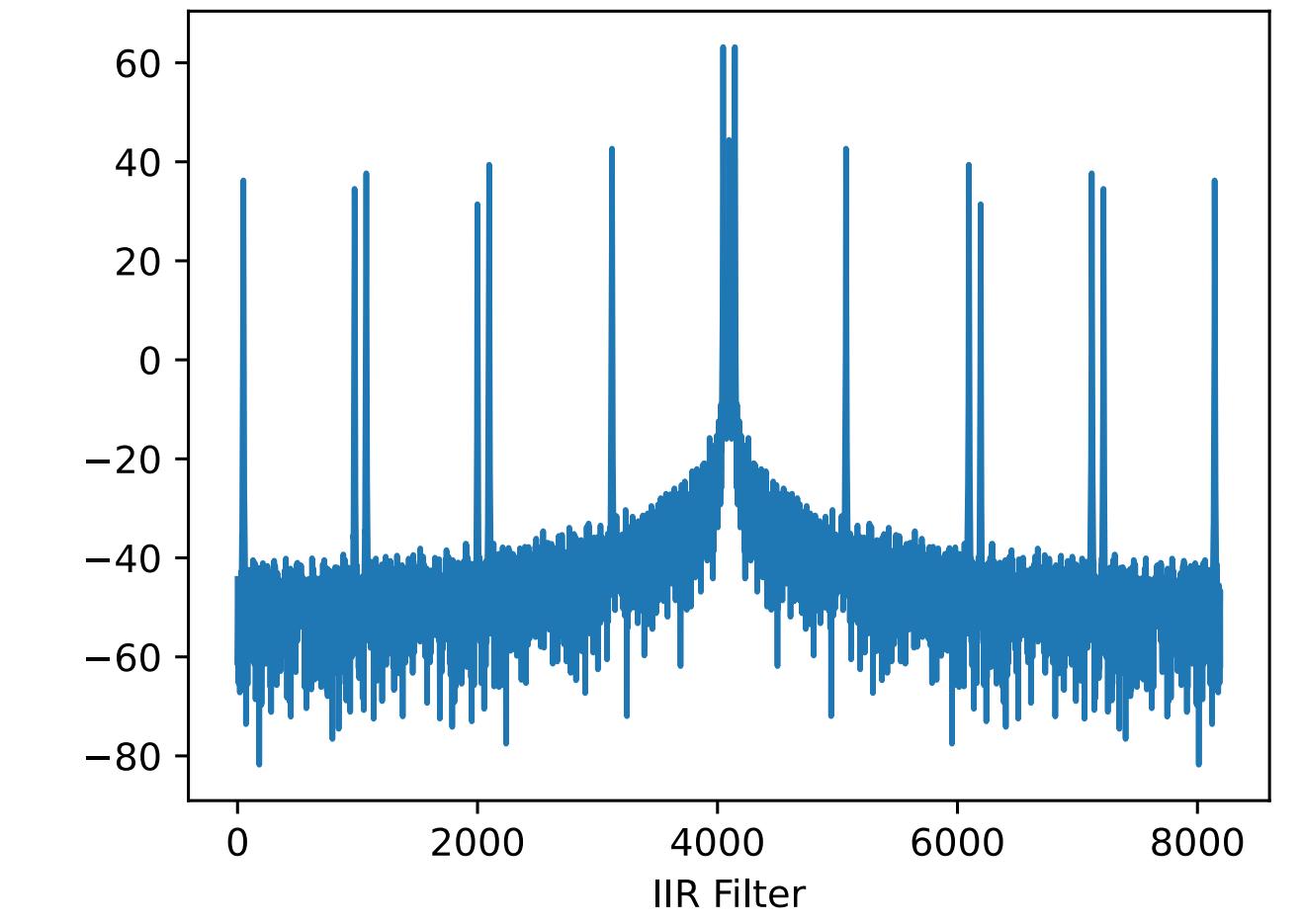
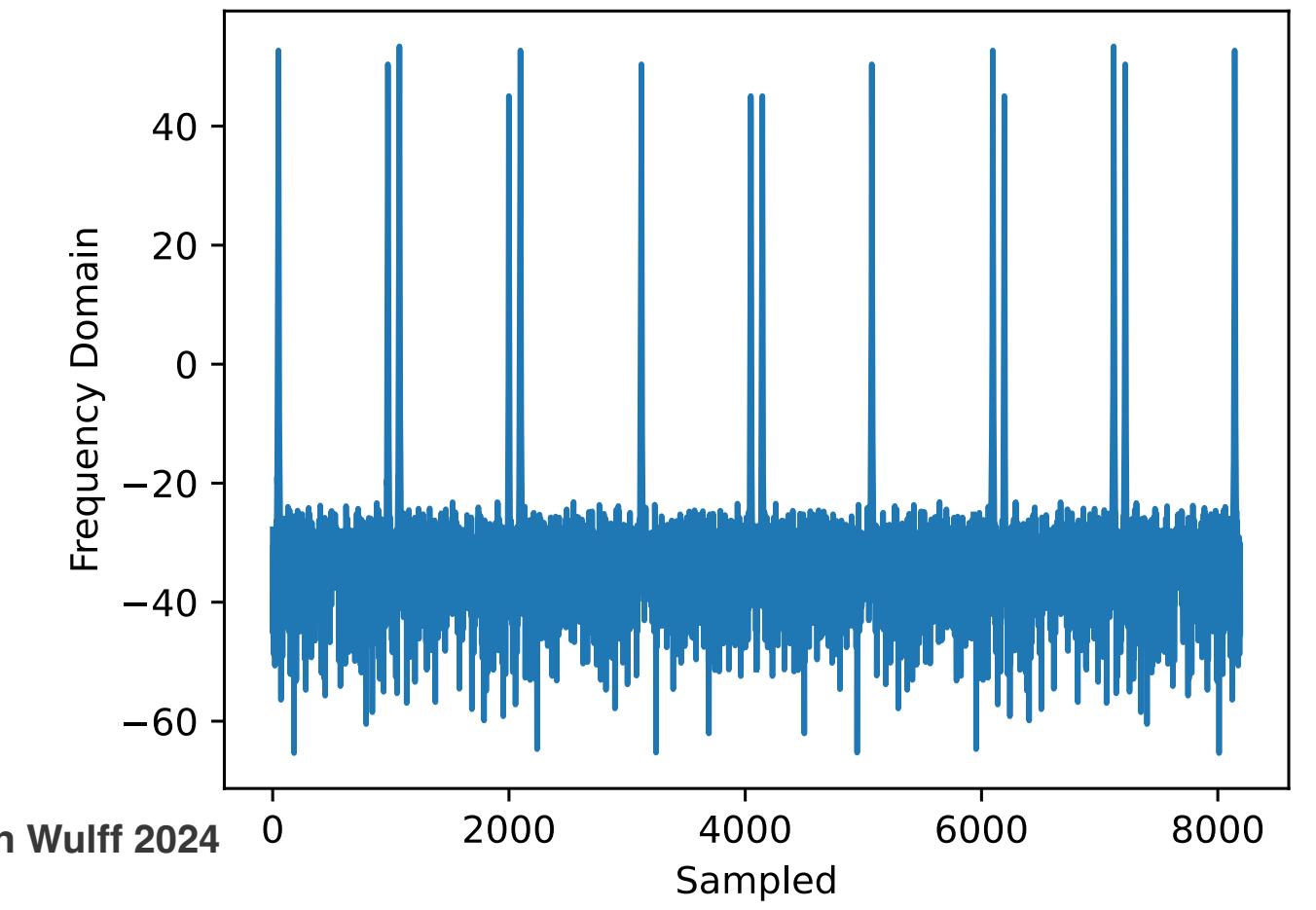
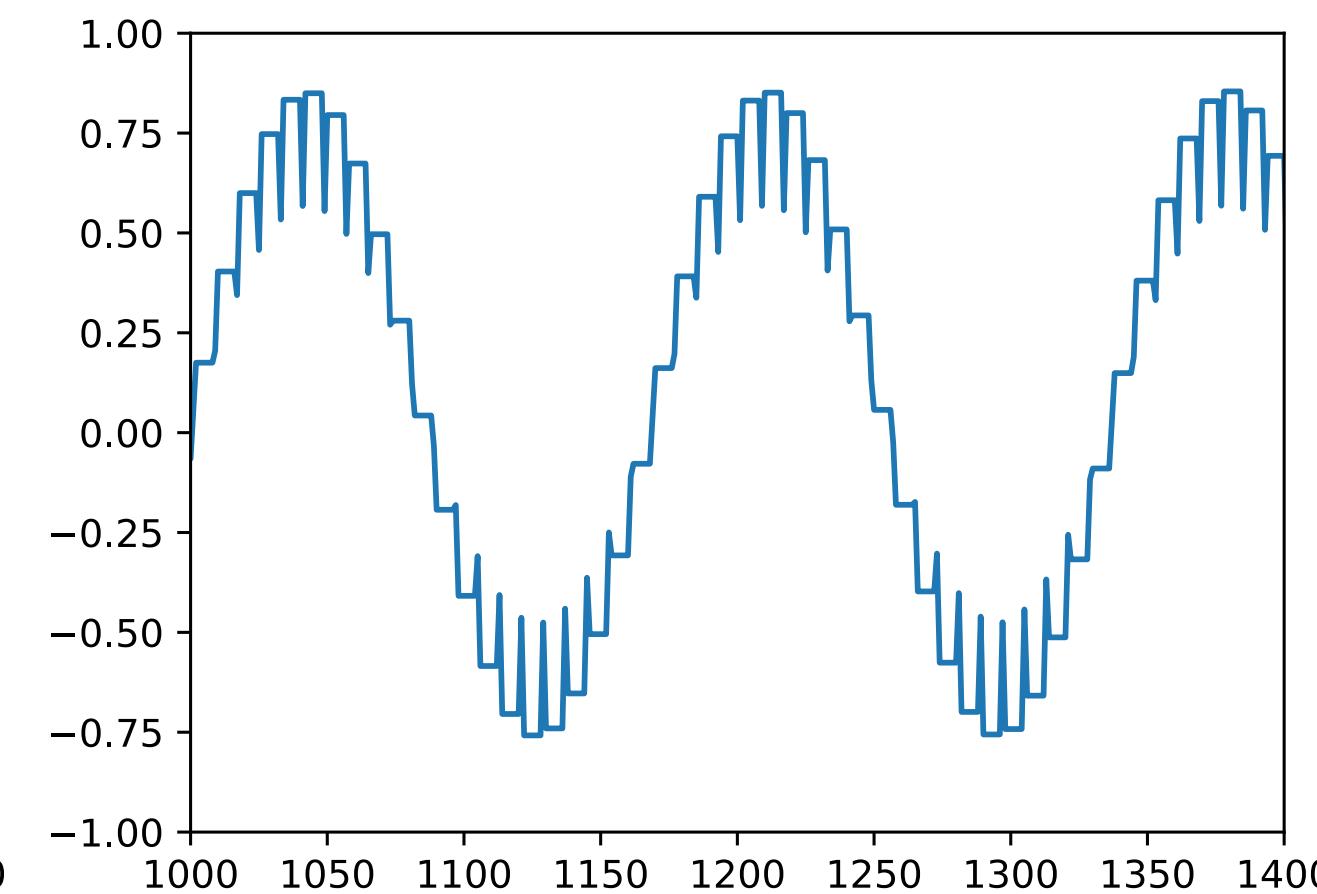
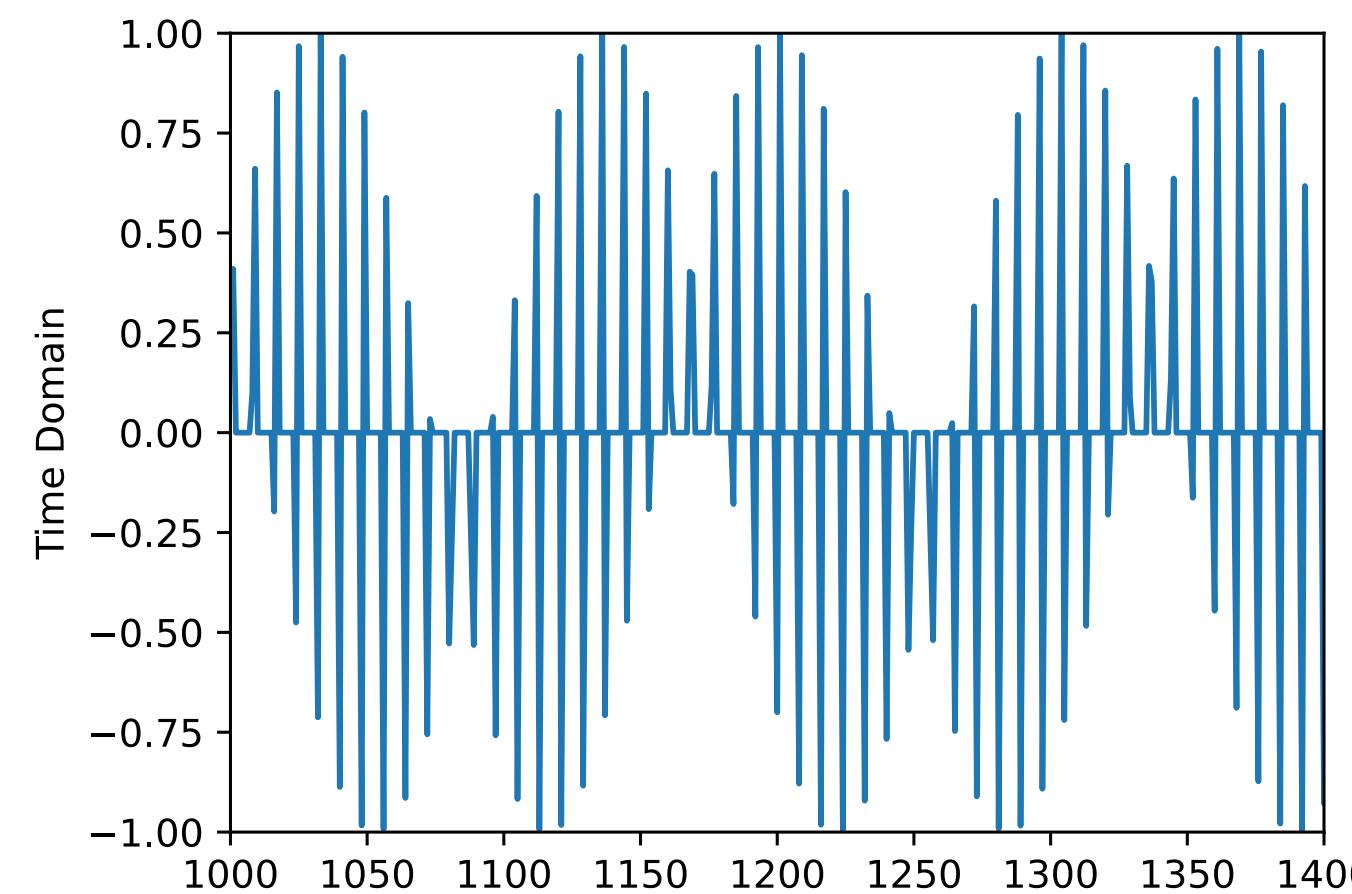
$$H(z) = \frac{b}{z - a}$$

Infinite-impulse response (IIR)

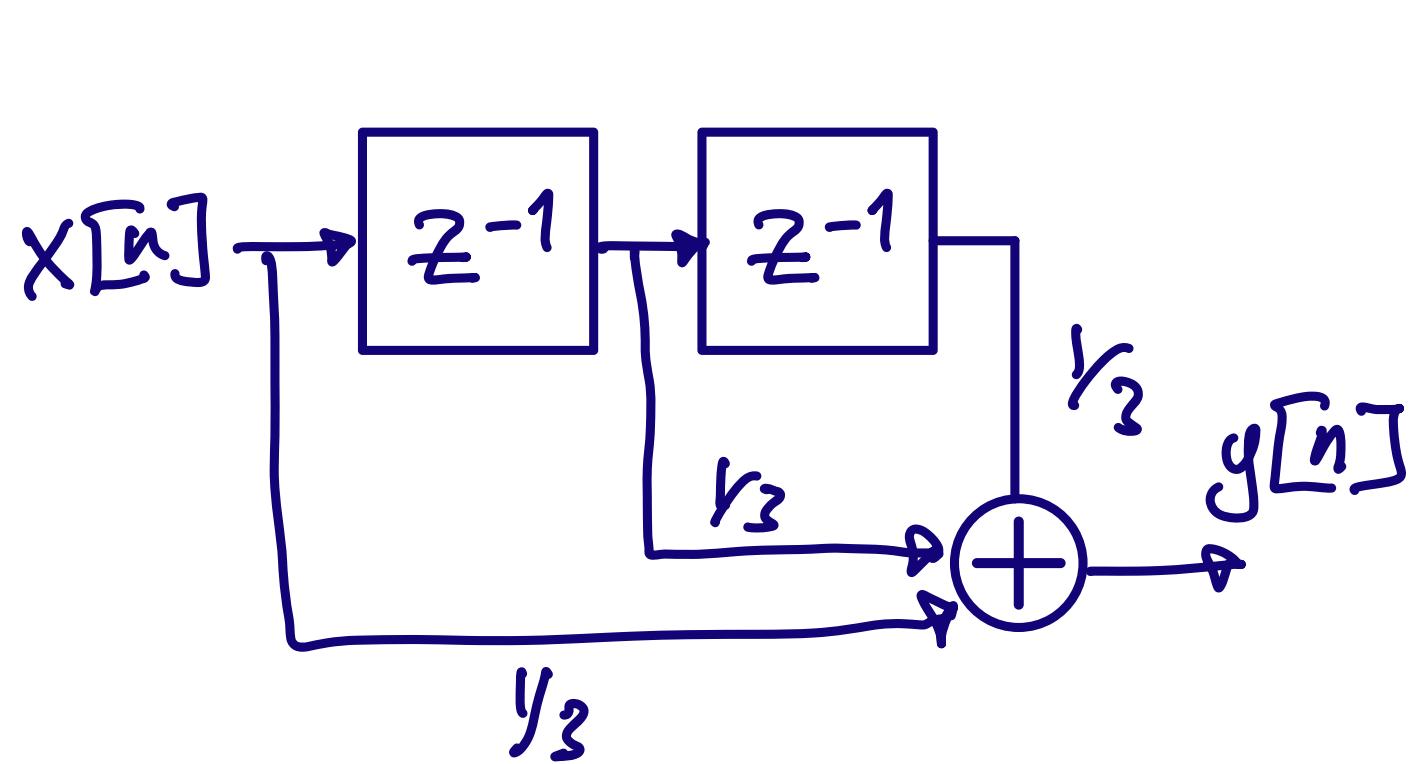
$$h[n] = \begin{cases} k & \text{if } n < 1 \\ a^{n-1}b + a^n k & \text{if } n \geq 1 \end{cases}$$

Head's up: Fig 13.12 in AIC is wrong

[iir.py](#)

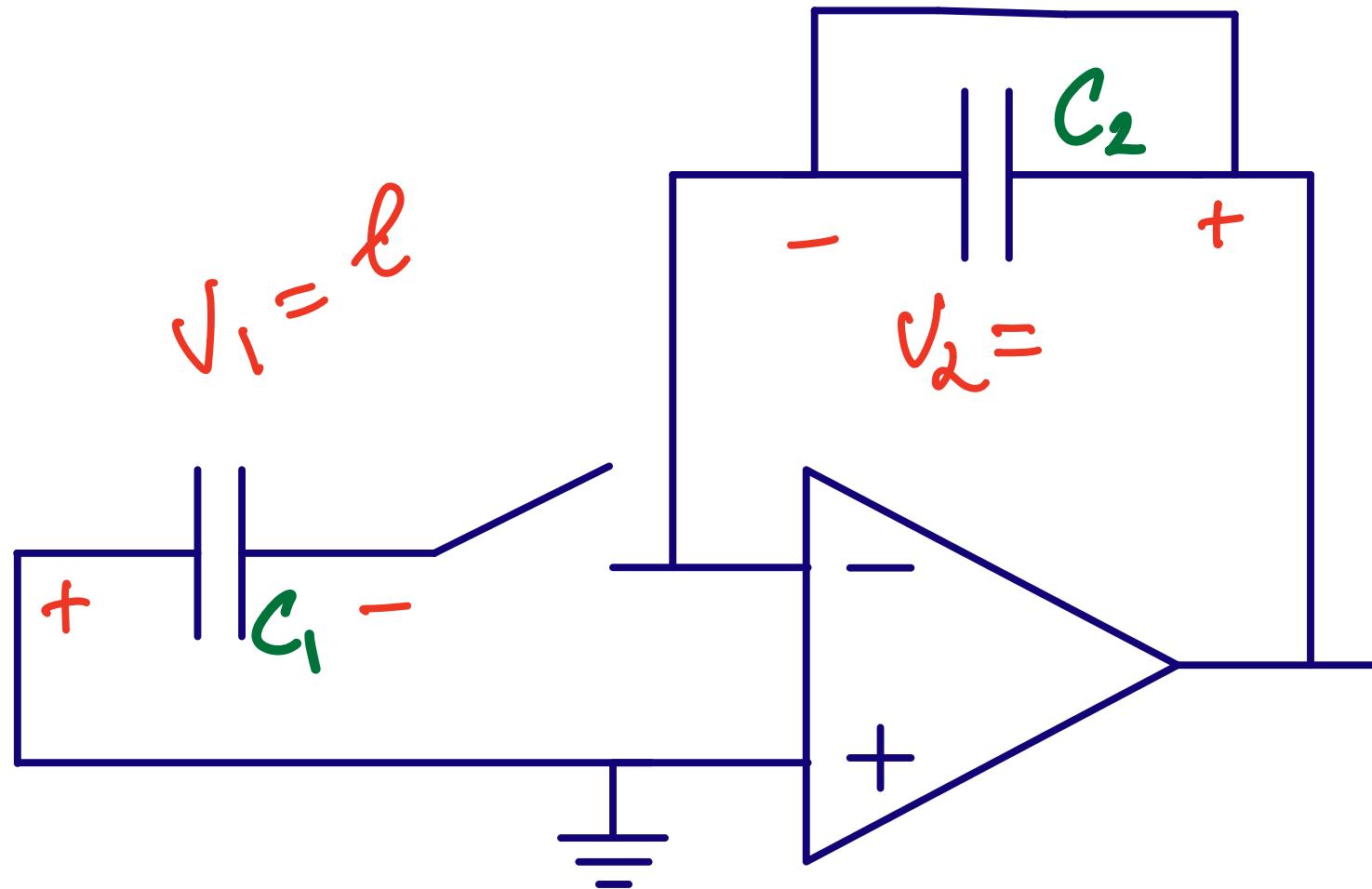


Finite-impulse response(FIR)



$$H(z) = \frac{1}{3} \sum_{i=0}^2 z^{-i}$$

Switched-Capacitor



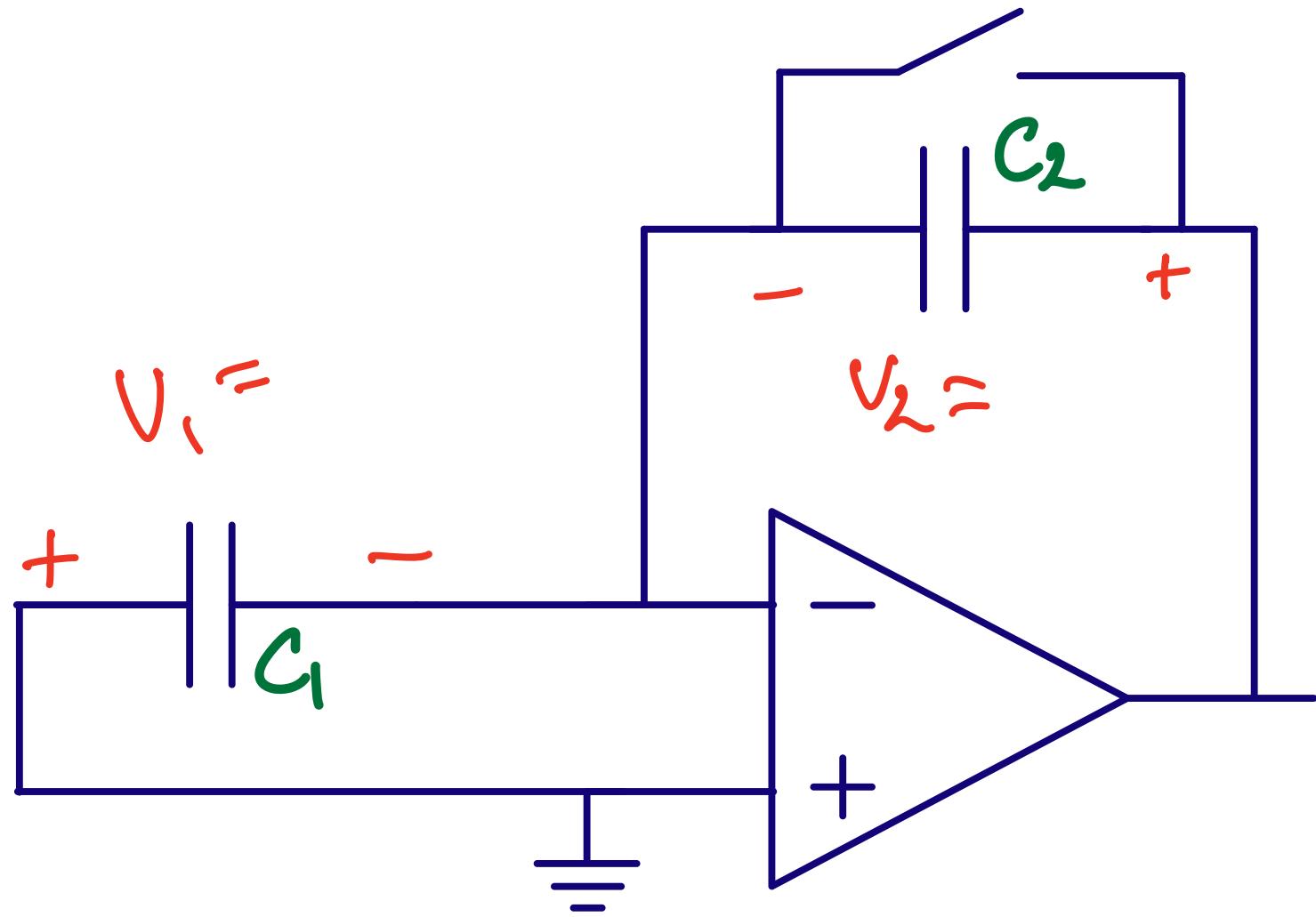
$$V_1 = \ell$$

$$V_2 =$$

C_2

$$Q_{1\phi_1\$} = C_1 V_1$$

$$Q_{2\phi_1\$} = 0$$



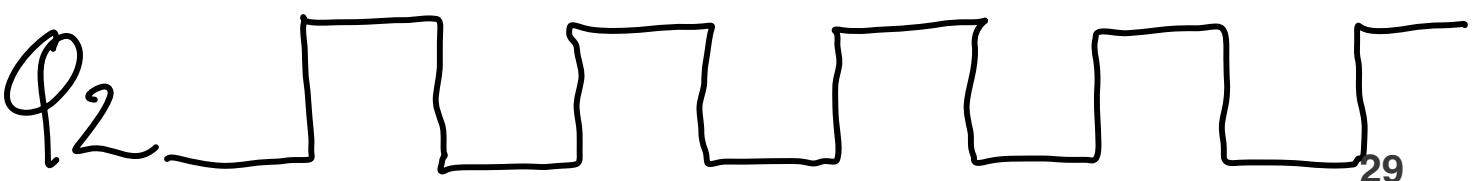
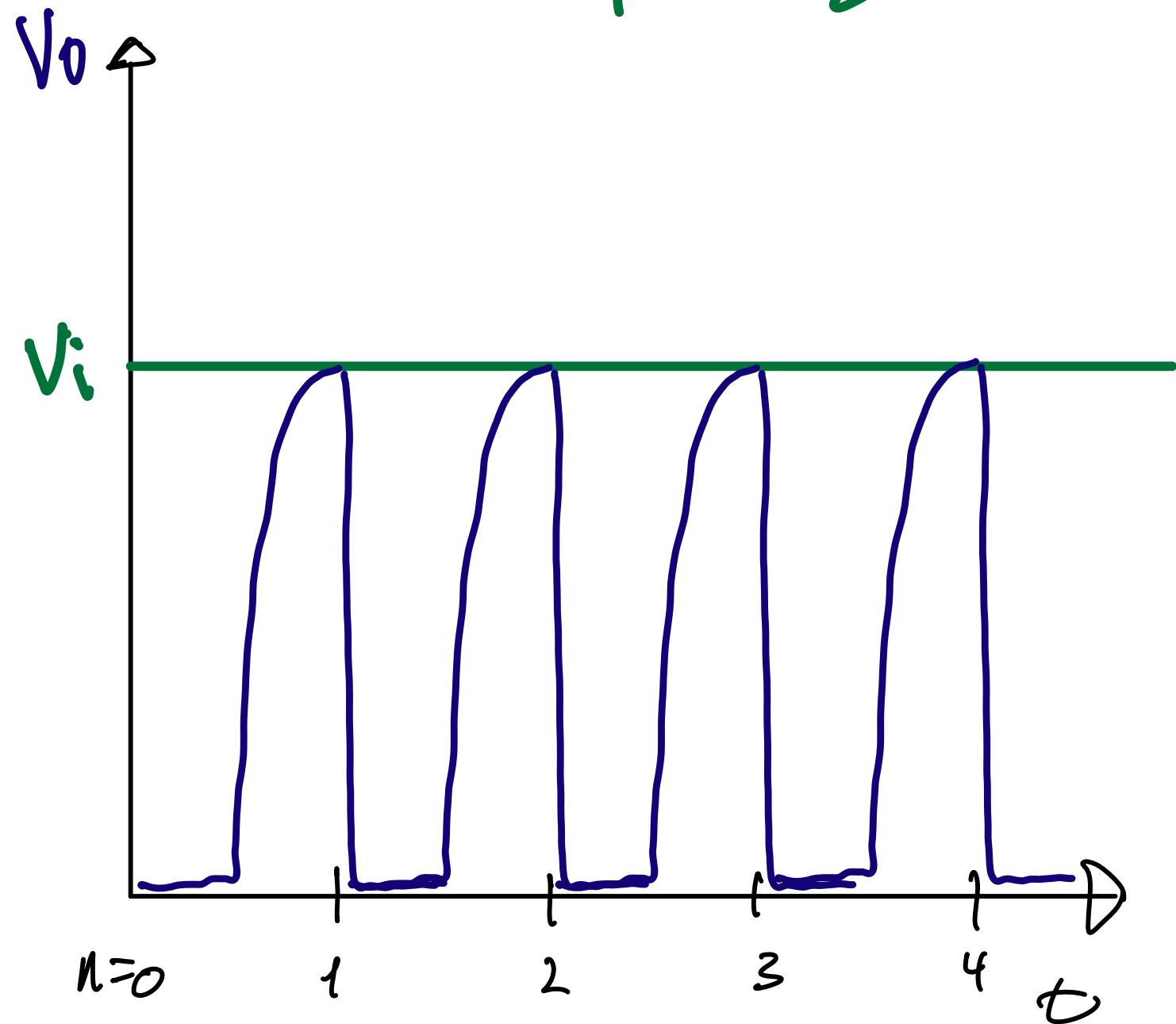
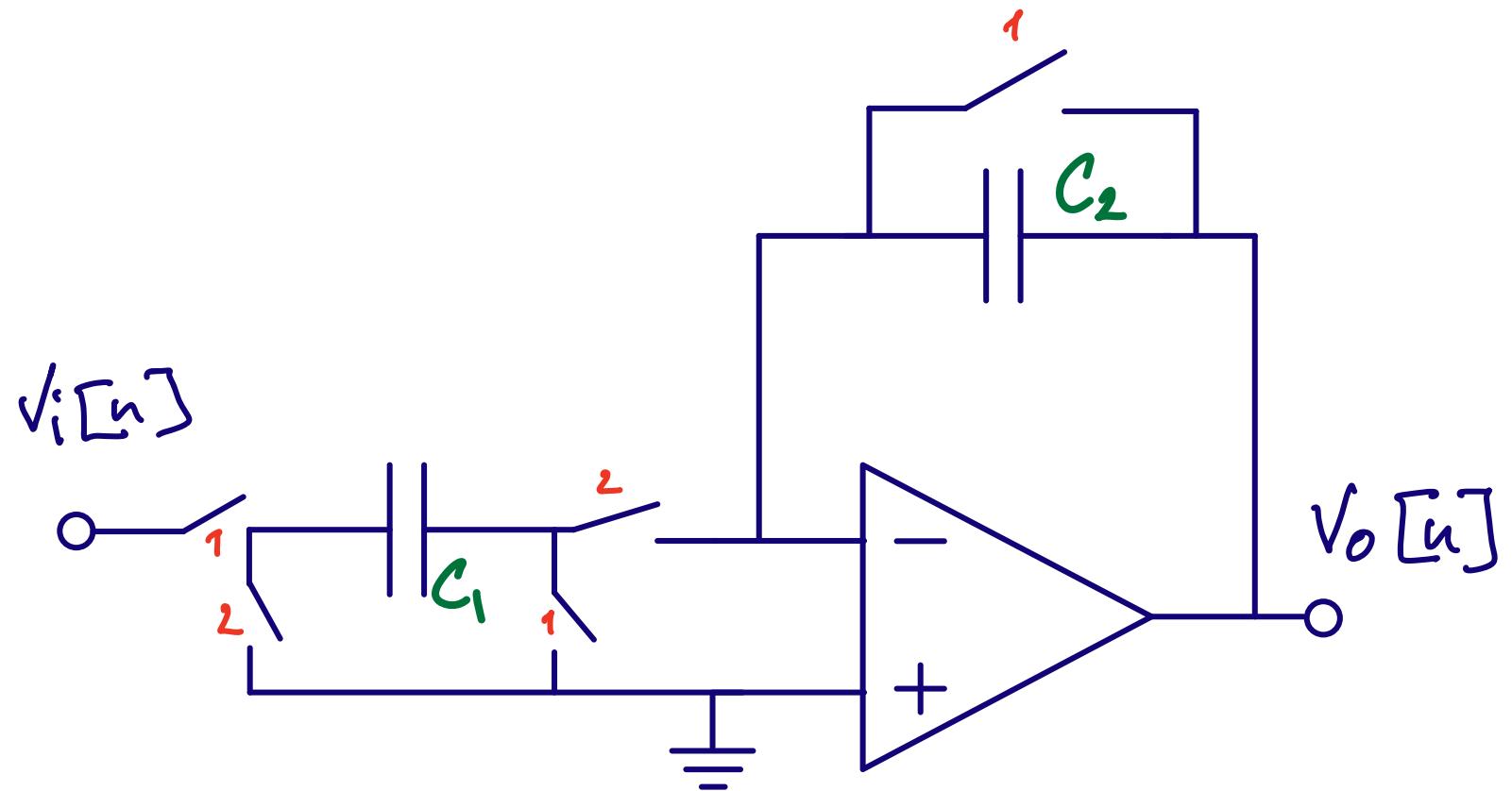
$$Q_{1\phi_2\$} = 0$$

$$Q_{2\phi_2\$} = Q_{1\phi_1\$} = C_1 V_1 = C_2 V_2$$

$$\frac{V_2}{V_1} = \frac{C_1}{C_2}$$

Switched capacitor gain circuit

$$C_1 = C_2$$

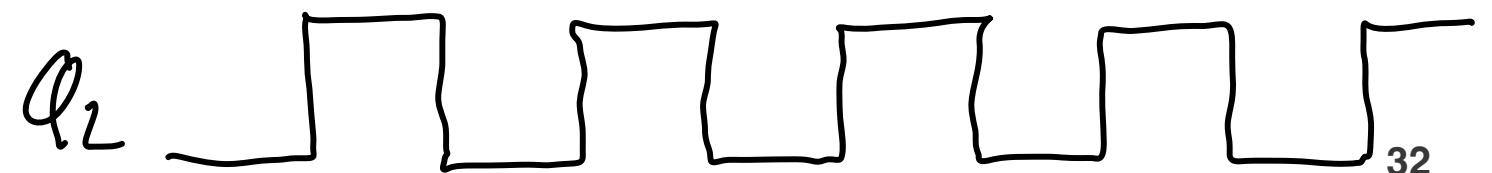
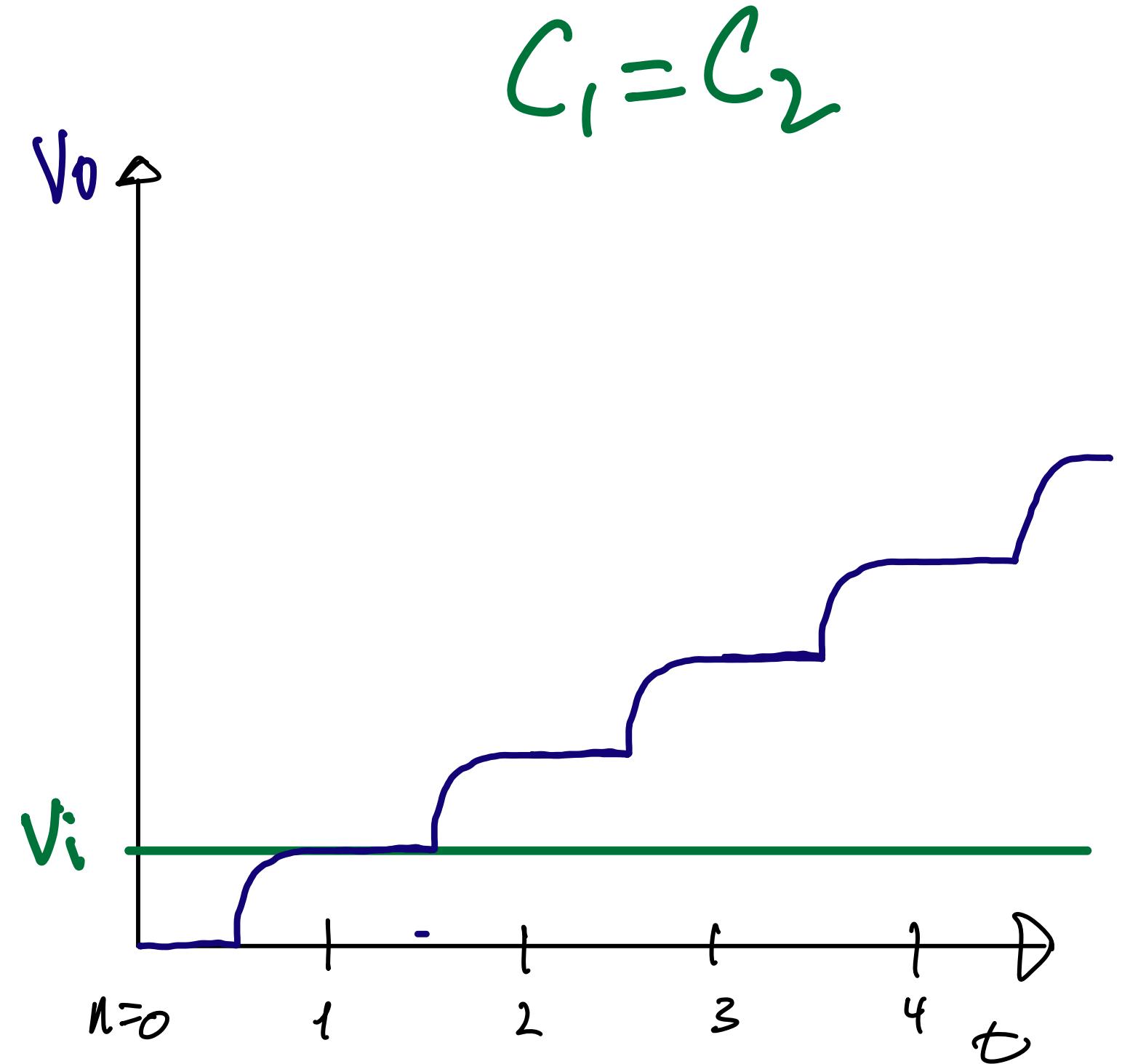
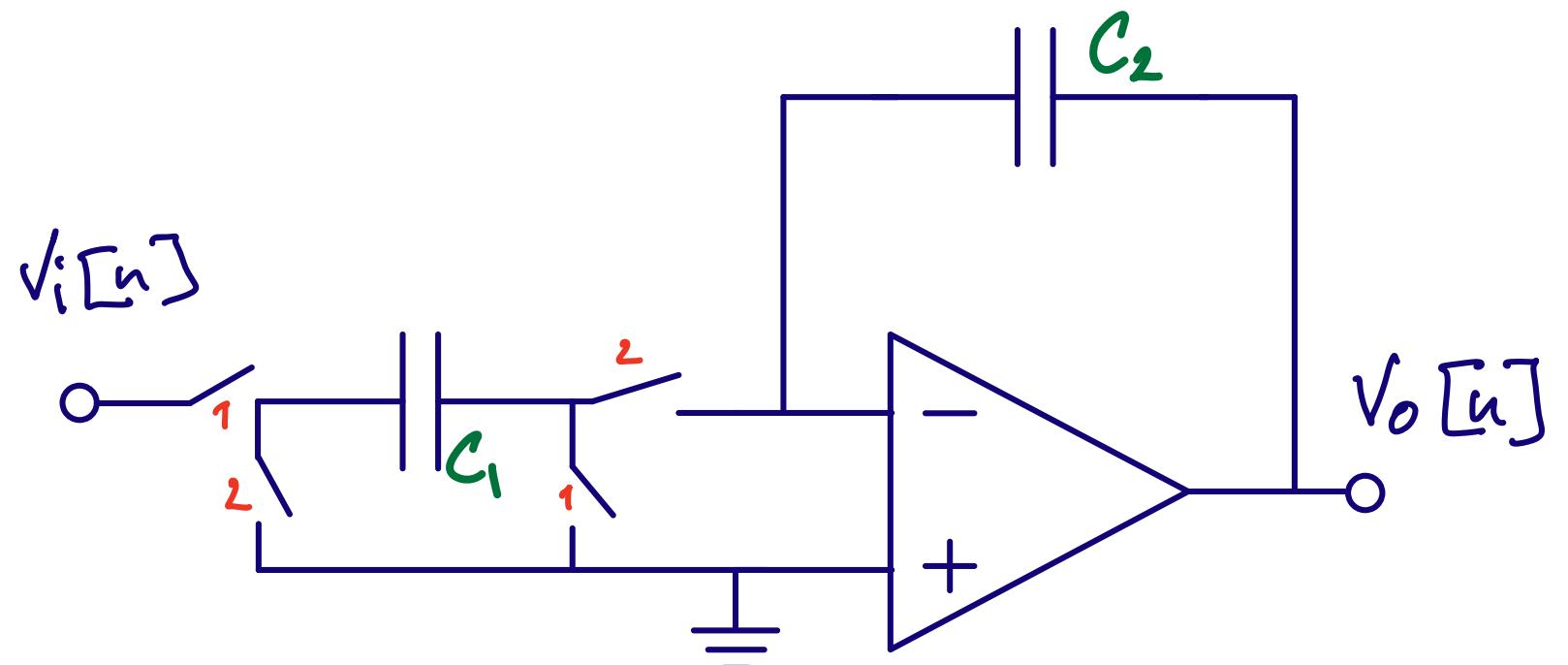


$$V_o[n+1] = \frac{C_1}{C_2} V_i[n]$$

$$V_o z = \frac{C_1}{C_2} V_i$$

$$\frac{V_o}{V_i} = H(z) = \frac{C_1}{C_2} z^{-1}$$

Switched capacitor integrator



$$V_o[n] = V_o[n - 1] + \frac{C_1}{C_2} V_i[n - 1]$$

$$V_o - z^{-1} V_o = \frac{C_1}{C_2} z^{-1} V_i$$

$$H(z) = \frac{C_1}{C_2} \frac{z^{-1}}{1 - z^{-1}} = \frac{C_1}{C_2} \frac{1}{z - 1}$$

Both phases add noise, $V_n^2 > \frac{2kT}{C}$

Mean

$$\overline{x(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t) dt$$

Mean Square

$$\overline{x^2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt$$

Variance

$$\sigma^2 = \overline{x^2(t)} - \overline{x(t)}^2$$

where σ is the standard deviation.

If mean is removed, or is zero, then

$$\sigma^2 = \overline{x^2(t)}$$

Assume two random processes, $x_1(t)$ and $x_2(t)$ with mean of zero (or removed).

$$x_{tot}(t) = x_1(t) + x_2(t)$$

$$x_{tot}^2(t) = x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t)$$

Variance (assuming mean of zero)

$$\sigma_{tot}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_{tot}^2(t) dt$$

$$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2 + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt$$

**Assuming uncorrelated processes
(covariance is zero), then**

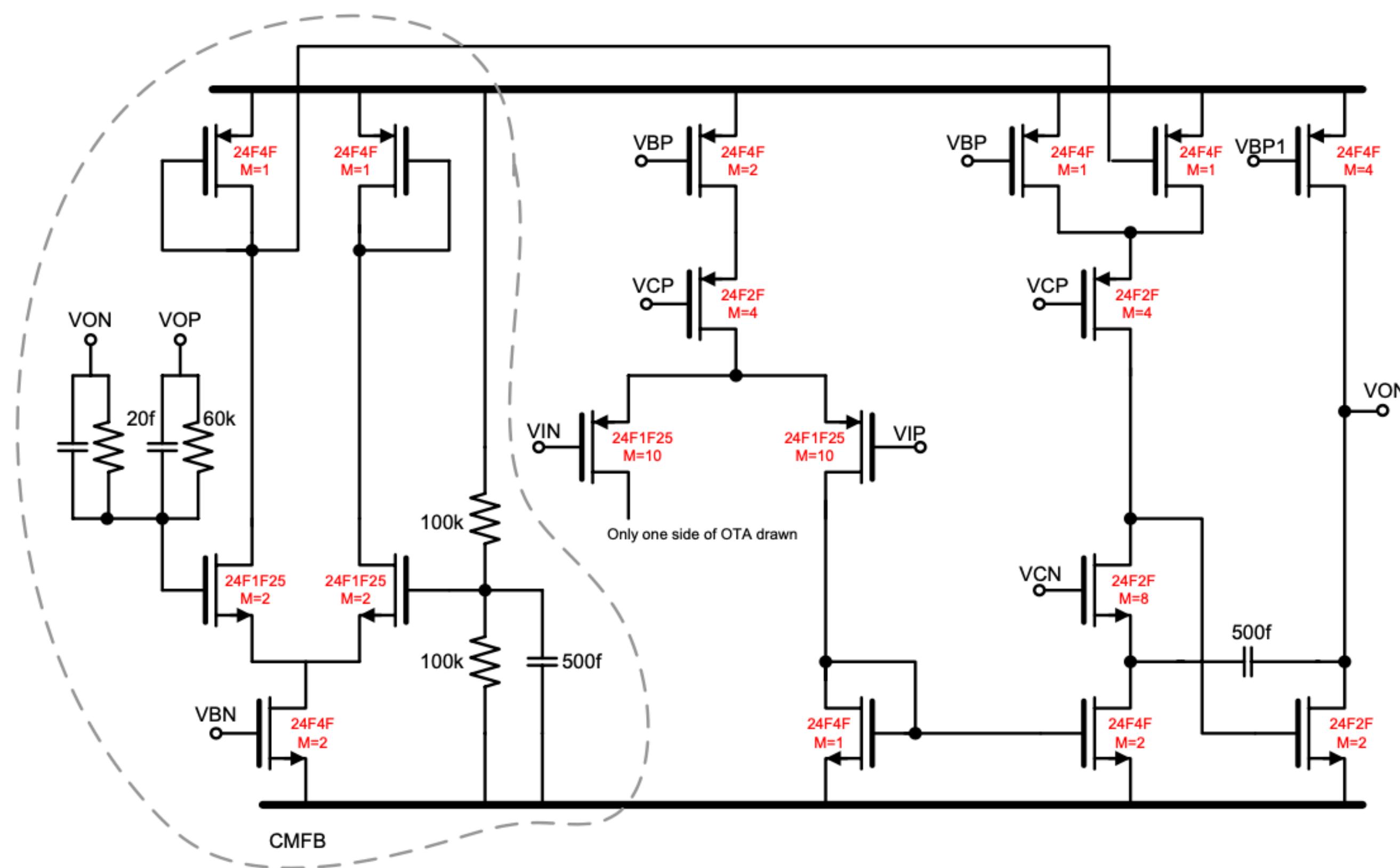
$$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2$$

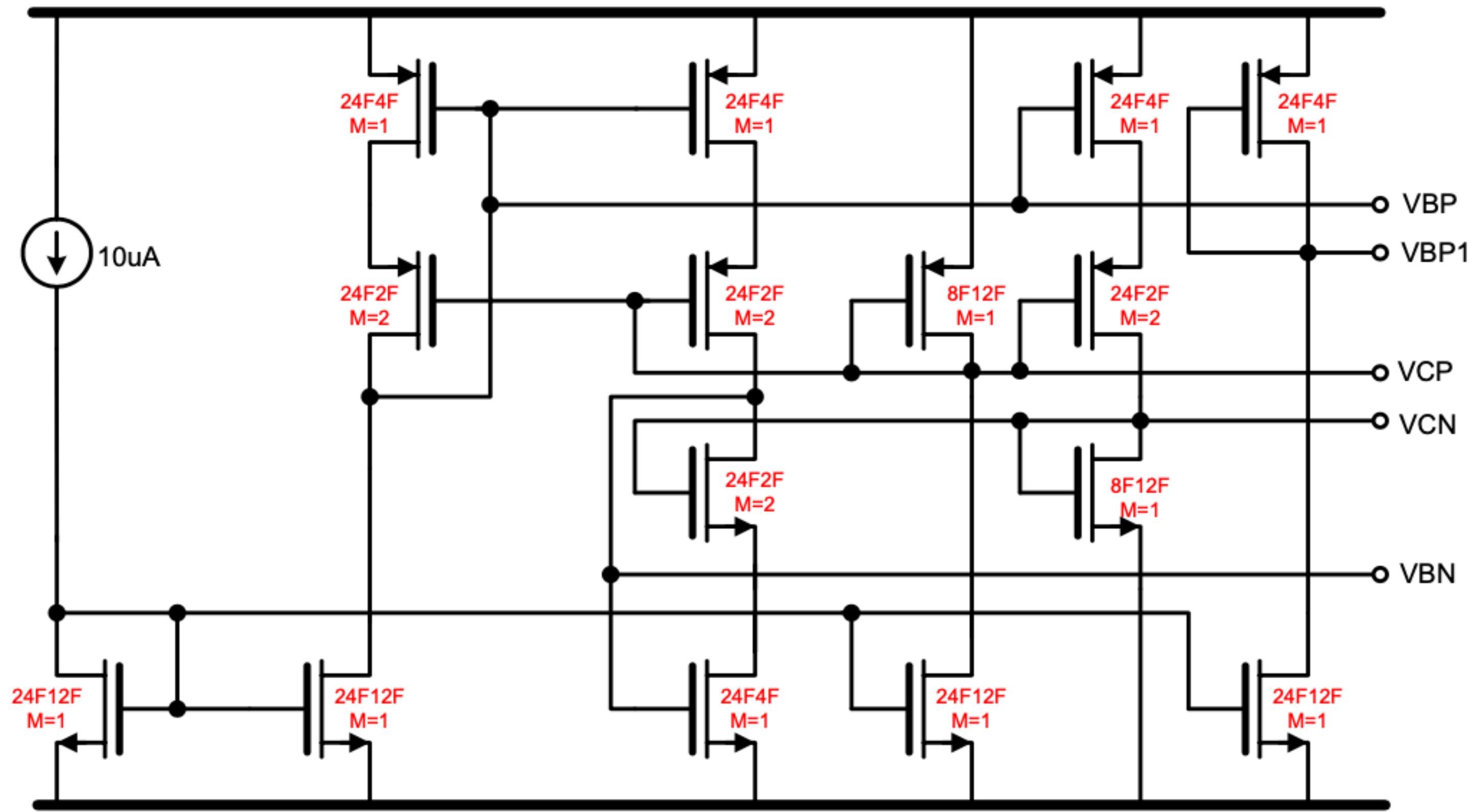
Sub-circuits for SC-circuits

O

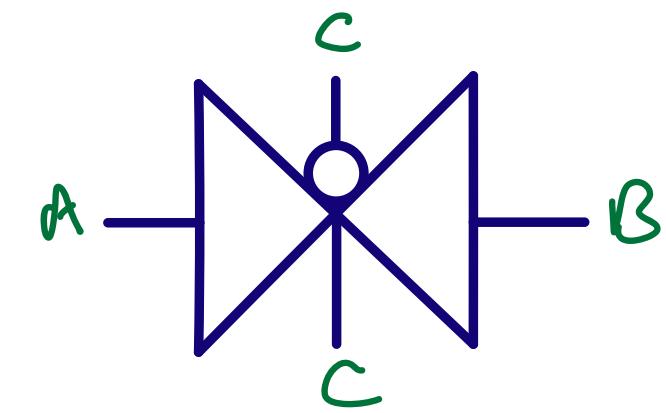
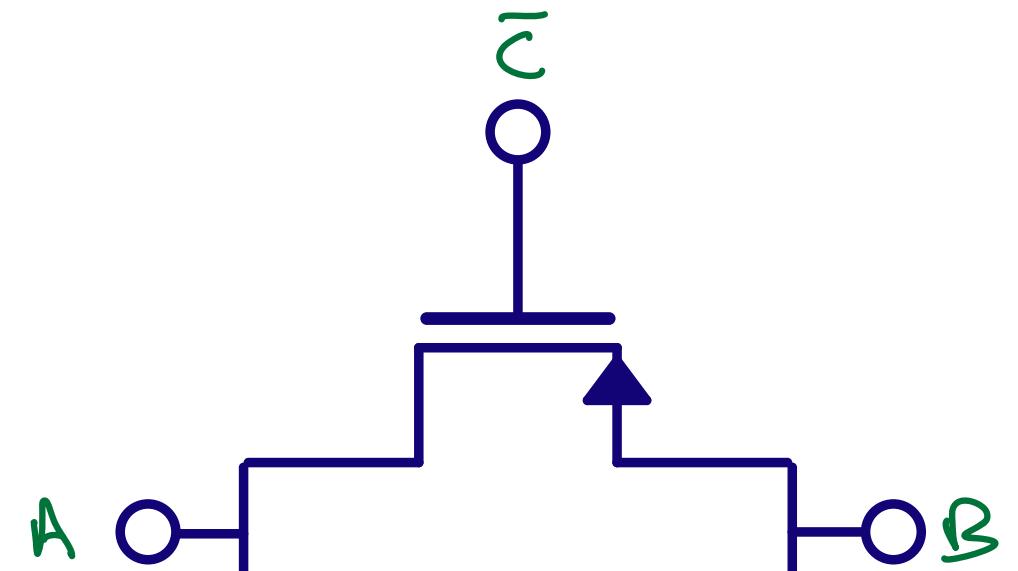
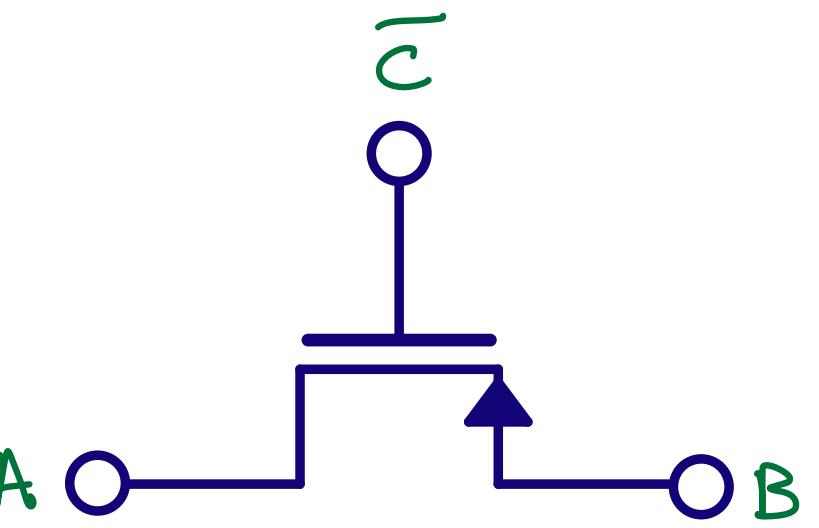
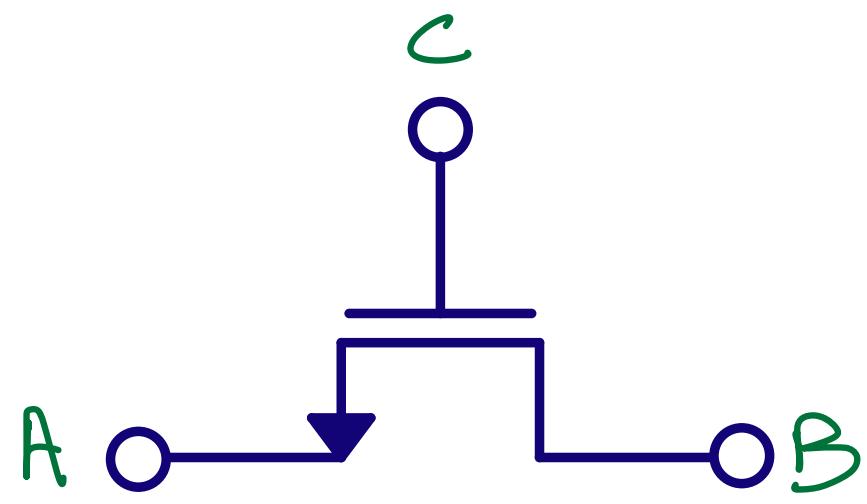
IT

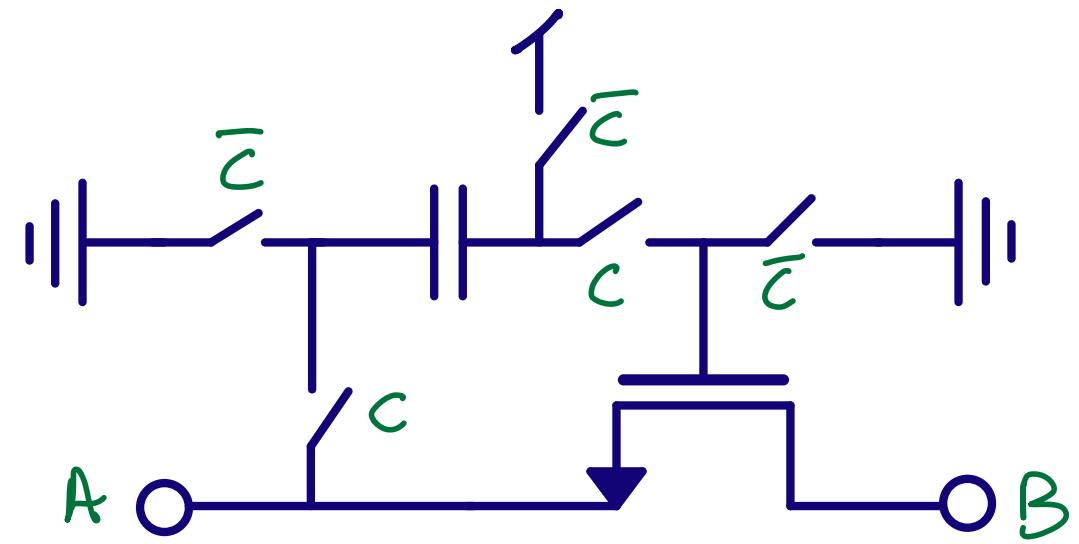
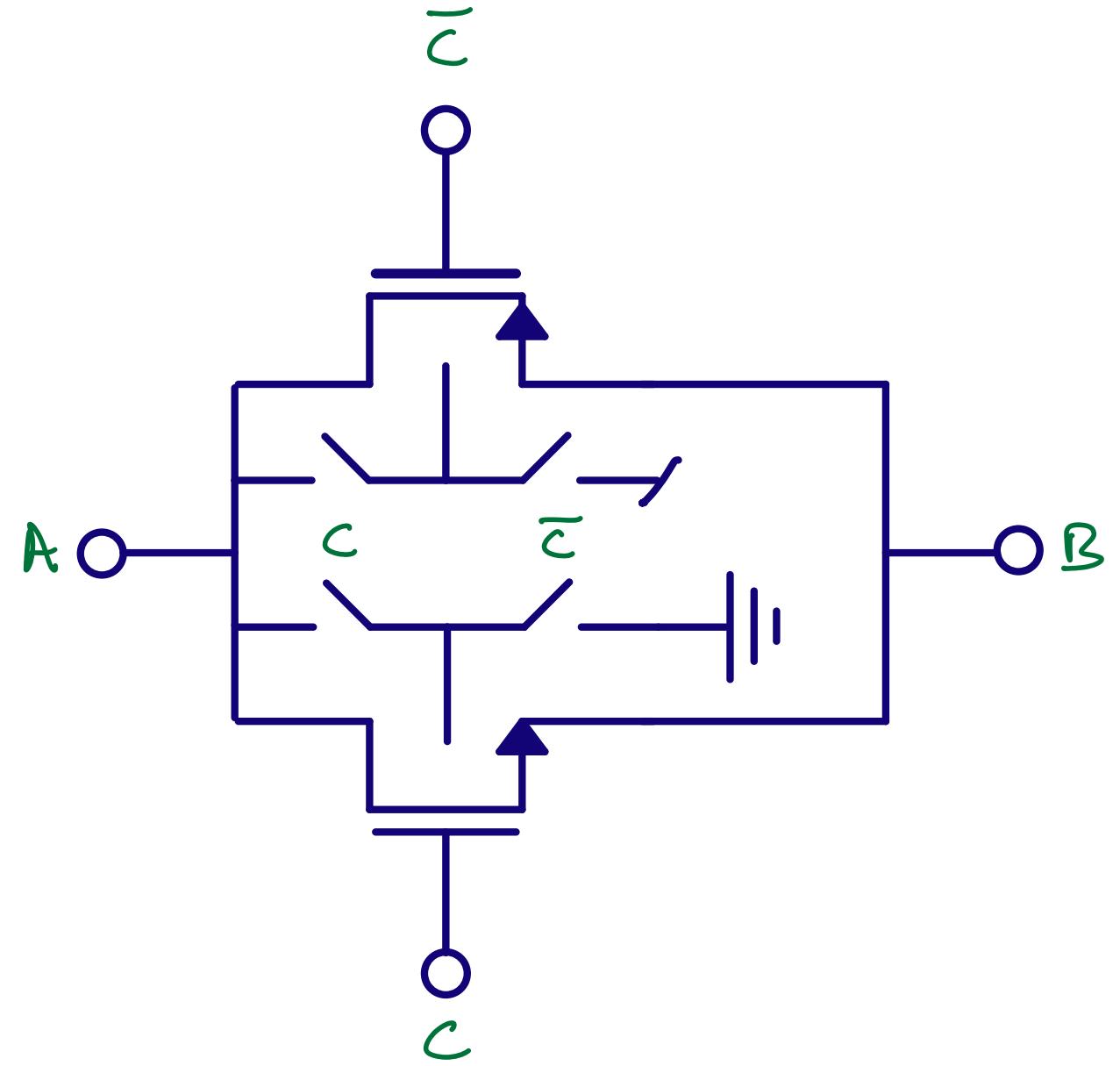
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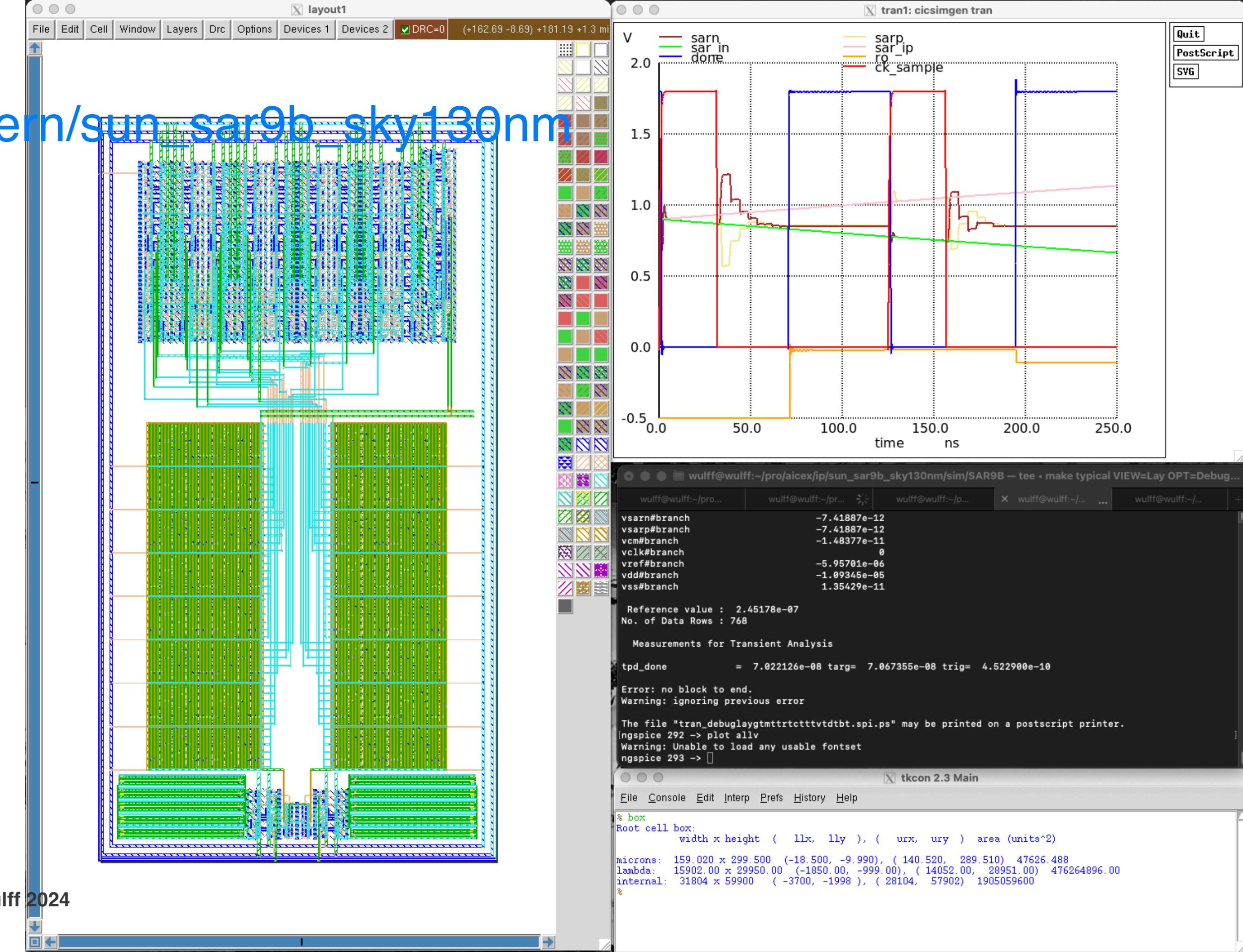


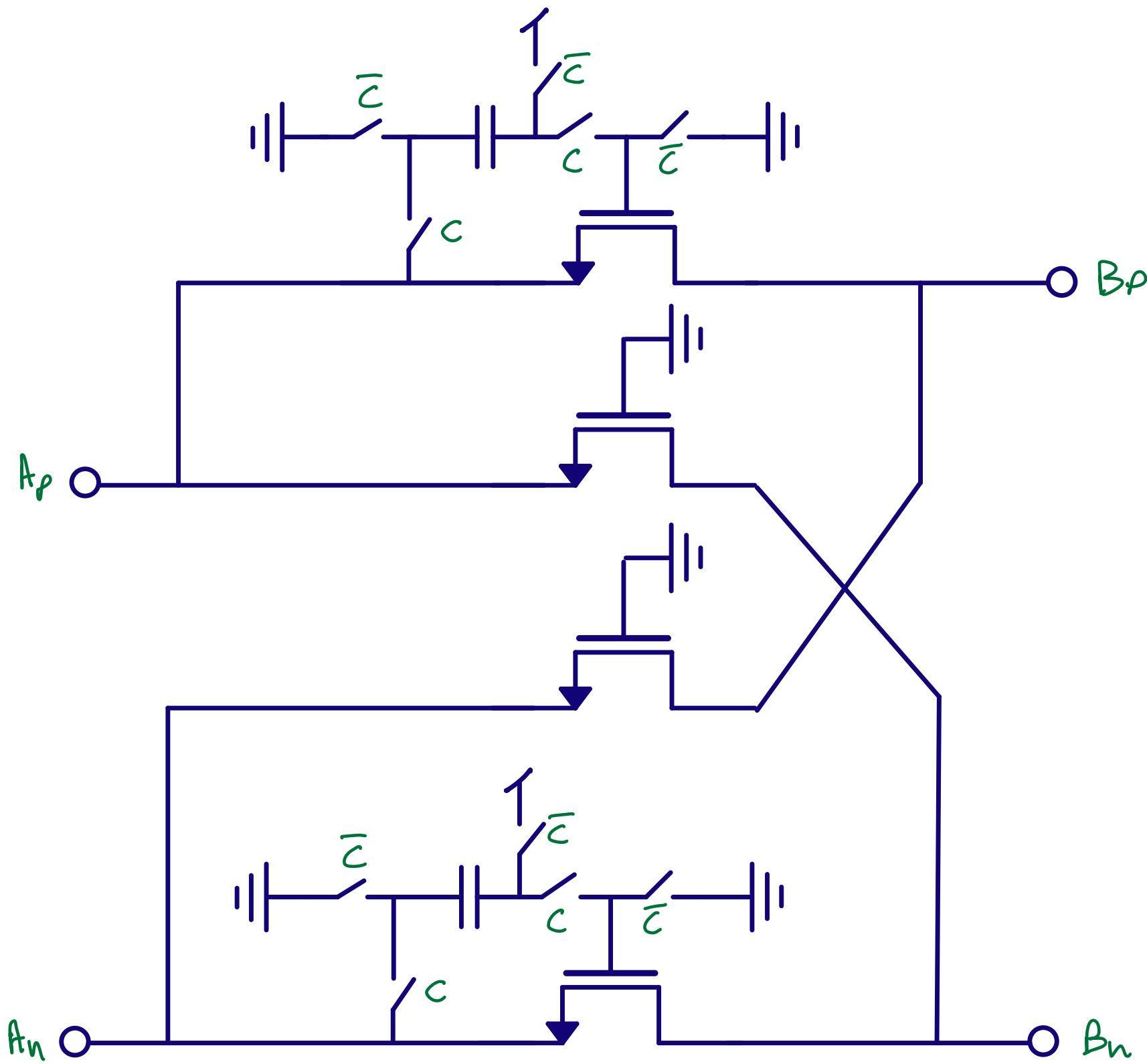
Switches



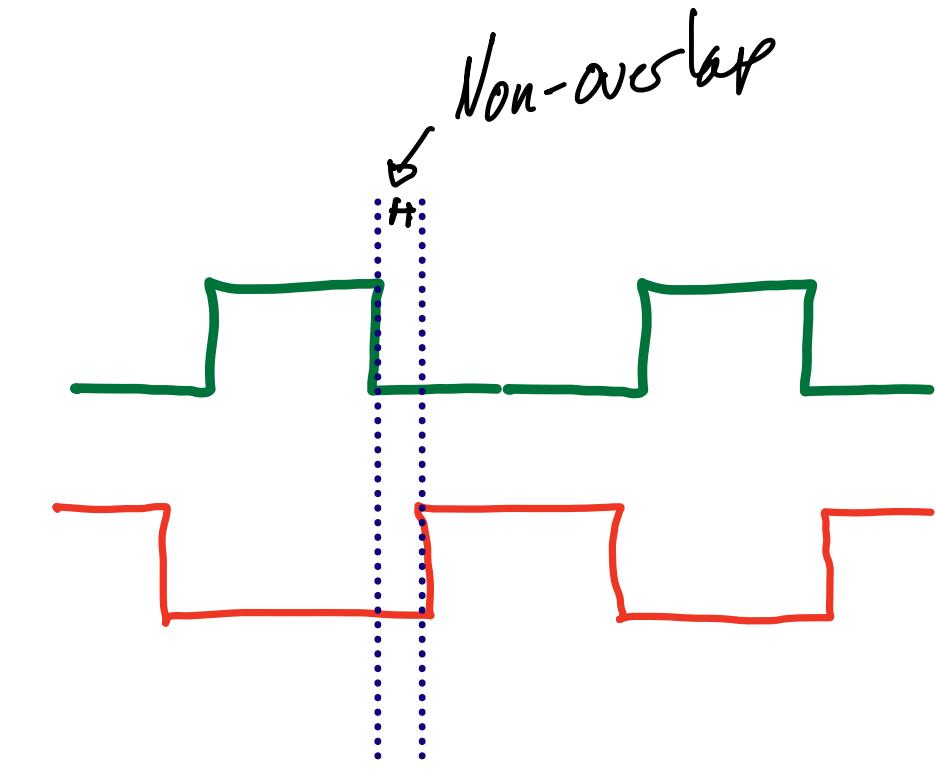
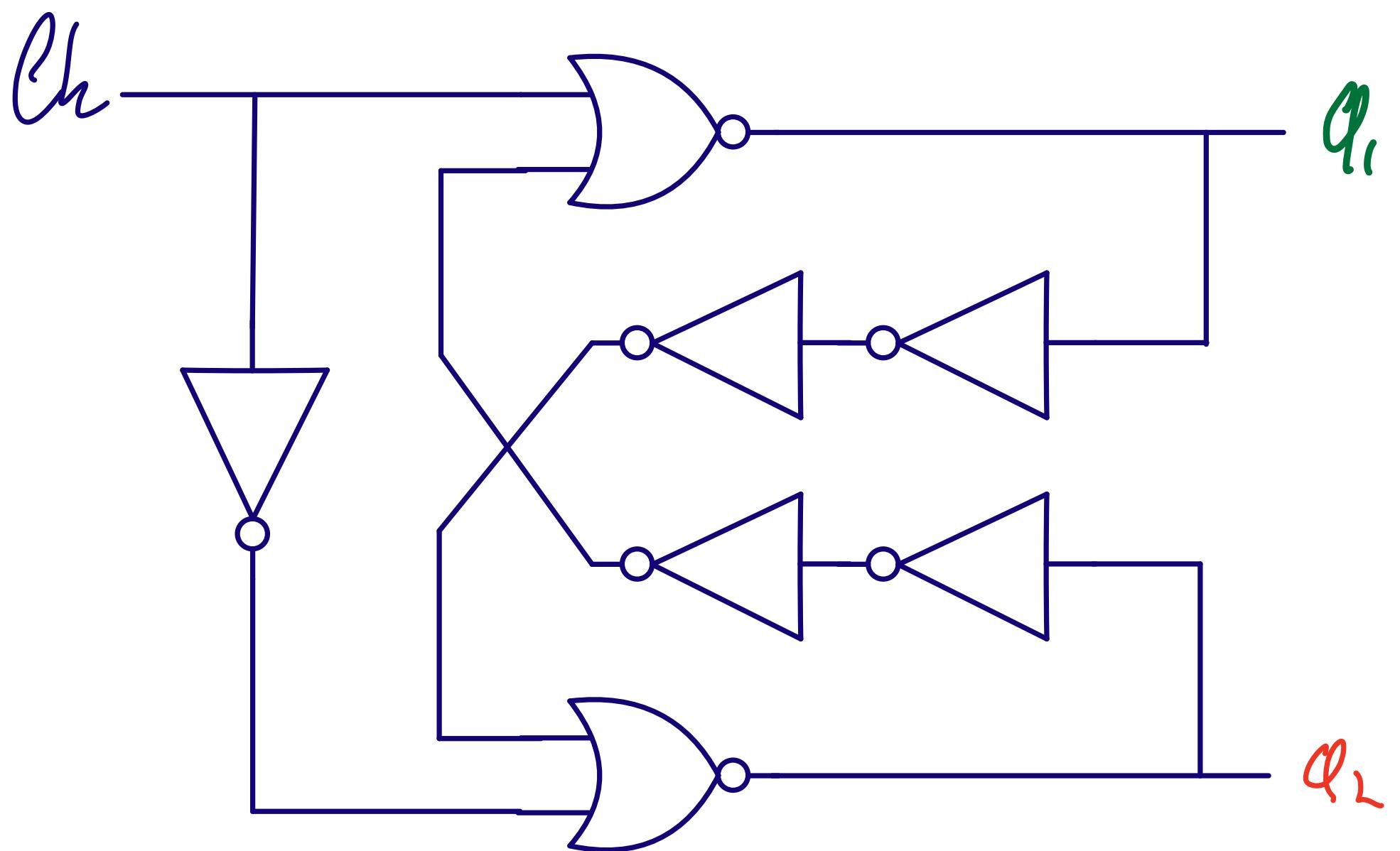


wulffern/sun_sar9b_sky130nm

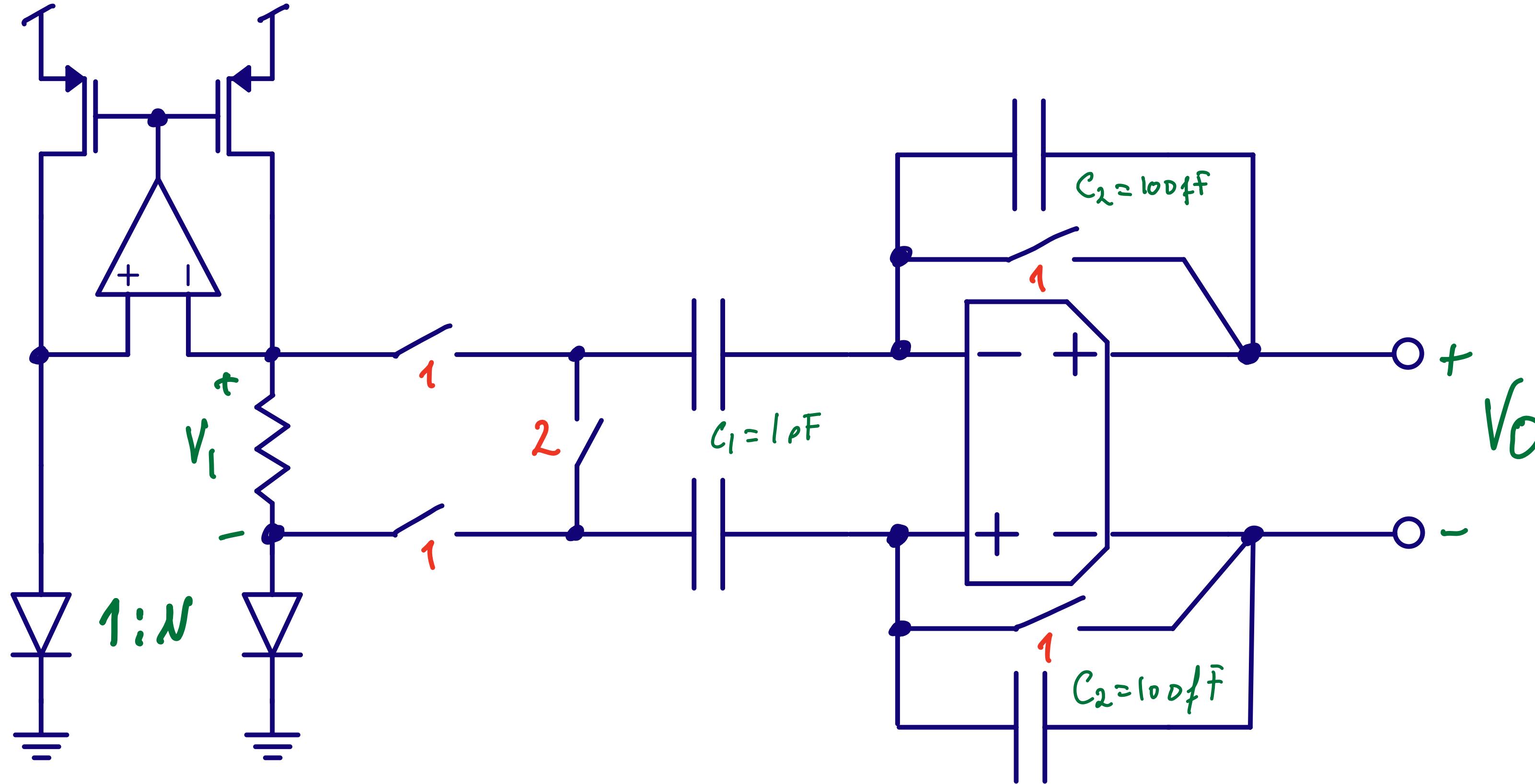




Non-overlapping clocks



E-xample



Thanks!

