Counting

Counting is one of the basic patterns that SQL developer learns after the basics: selection, projection, join, and subquery. While counting might look deceptively easy in a context of a single table, it becomes intellectually challenging as soon as we join two tables and apply grouping. The purpose of the first half of the chapter is to make the reader comfortable writing counting queries in a complex context.

The second half of the chapter studies *conditional summation*. This pattern is a beautiful combination of the two SQL constructs: the `case` operator, and aggregation. It has numerous applications, which we study for the rest of the chapter.

Counting Ordered Rows

Let's start with a basic counting problem. Suppose we are given a list of integers, for example:

x
2
3
4
6
9

and want to enumerate all of them sequentially like this:

x	#
2	1
3	2
4	3
6	4
9	5

Enumerating rows in the increasing order is the same as counting how many rows precede a given row.

SQL enjoys success unparalleled by any rival query language. Not the last reason for such popularity might be credited to its proximity to English[1]. Let examine the informal idea

Enumerating rows in increasing order is counting how many rows precede a given row.

carefully. Perhaps the most important is that we referred to the rows in the source table **twice**: first, to a **given** row, second, to a **preceding** row. Therefore, we need to join our number list with itself (fig 1.1).

Cartesian Product

Surprisingly, not many basic SQL tutorials, which are so abundant on the web today, mention Cartesian product. Cartesian product is a **join** operator with no join condition[2]

```
select A.*, B.* from A, B
```

[1] This proximity partially explains why newer relational languages had such a limited success. For for example, Tutorial D has succinct notation, better NULL handling, and pure set semantics. These are not breakthrough features however; a New and Improved query language had better be leaps and bounds ahead, not just simpler to type.

[2] Many DBAs would jump in pointing that queries with Cartesian product are inefficient, but the rule of thumb avoiding Cartesian product is just too simplistic.

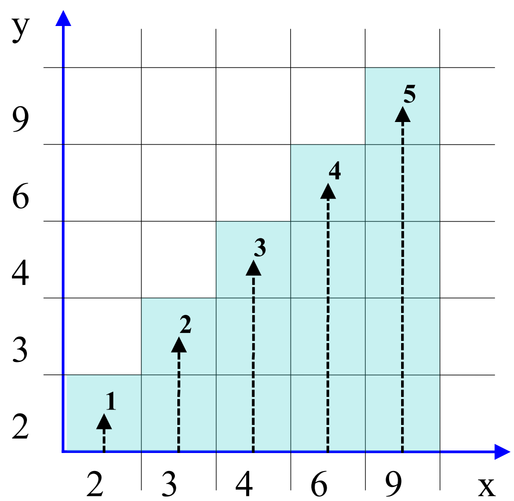


Figure 1.1: Cartesian product of the set A = {2,3,4,6,9} by itself. Counting all the elements x that are no greater than y produces the sequence number of y in the set A.

Carrying over this idea into formal SQL query is straightforward. As it is our first query in this book, let's do it step by step. The Cartesian product itself is

```
select t.x x, tt.x y[3]
from T t, T tt
```

Next, the triangle area below the main diagonal is

```
select t.x x, tt.x y
from T t, T tt
where tt.x <= t.x
```

Finally, we need only one column – `t.x` – which we group the previous result by and count

```
select t.x, count(*) seqNum
from T t, T tt
where tt.x <= t.x
group by t.x
```

[3] We use Oracle syntax and write `<column expr> <alias>` instead of ANSI SQL `<column expr> AS <alias>`. Ditto for table expressions.

Equivalence Relation and Group By

Almost any other SQL query uses `group by` operator. Why is `group by` operator so powerful? It is not among the fundamental relational algebra operators. A partial answer to this fascinating efficiency is that `group by` embodies an equivalence relation. Indeed, it partitions rows into equivalence classes of rows with identical values in a column or a group of columns, and calculates aggregate values per each equivalence class.

What if we modify the problem slightly and ask for a list of pairs where each number is coupled with its predecessor?

x	predecessor
2	
3	2
4	3
6	4
9	6

Let me provide a typical mathematician's answer, first -- it is remarkable in a certain way. Given that we already know how to number list elements successively, it might be tempted to reduce the current problem to the previous one:

Enumerate all the numbers in the increasing order and match each sequence number seq# with predecessor seq#-1. Next!

This attitude is, undoubtedly, the most economical way of thinking, although not necessarily producing the most efficient SQL. Therefore, let's revisit our original approach, as illustrated on fig 1.2.

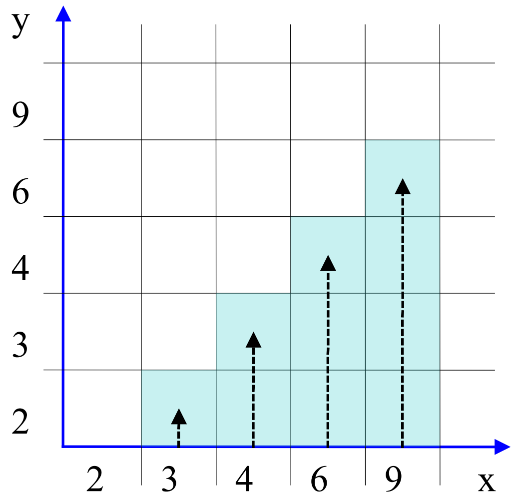


Figure 1.2: Cartesian product of the set A = {2,3,4,6,9} by itself. The predecessor of y is the maximal number in a set of x that are less than y. There is no predecessor for y = 2.

This translates into the following SQL query

```
select t.x, max(tt.x) predecessor
from T t, T tt
where tt.x < t.x
group by t.x
```

Both solutions are expressed in standard SQL leveraging join and grouping with aggregation. Alternatively, instead of joining and grouping why don't we calculate the `count` or `max` just in place as a *correlated scalar* subquery:

```
select t.x,
       (select count(*) from T tt where tt.x <= t.x) seq#
from T t
group by t.x
```

The subquery always returns a single value; this is why it is called scalar. The `tt.x <= t.x` predicate connects it to the outer query; this is why it is called correlated. Arguably, leveraging correlated scalar subqueries is one the most intuitive techniques to write SQL queries.

Is GROUP BY Redundant?

Chris Date[4] asserts that `group by` operator is redundant since

```
select deptno, avg(sal) from Emp
group by deptno
```

could be rewritten as

```
select distinct deptno,
       (select avg(sal) from Emp ee
        where e.deptno = ee.deptno)
from Emp e
```

Unlike Date, who exploits this fact as evidence of SQL deficiencies, we rather view it as yet another demonstration of the power of scalar subqueries.

How about counting rows that are not necessarily distinct? This is where our method breaks. It is challenging to distinguish duplicate rows by purely logical means, so that various less "pure" counting methods were devised. They all, however, require extending the SQL syntactically, which was the beginning of slipping along the ever increasing language complexity slope.

Here is how analytic SQL extension counts rows

```
select x, rank() over(order by x) seq# from T; -- first problem
select x, lag() over(order by x) seq# from T; -- second problem
```

Many people suggest that it's not only more efficient, but more intuitive. The idea that "analytics rocks" can be challenged in many ways. The syntactic clarity has its cost: SQL programmer has to remember (or, at least, lookup) the list of analytic functions. The performance argument is not evident, since non-analytical queries are simpler construction from optimizer perspective. A shorter list of physical execution operators implies fewer query transformation rules, and less dramatic combinatorial explosion of the optimizer search space.

[4] C. J. Date, Relational Database Writings, 1994-1997, Addison Wesley.

It might even be argued that the syntax could be better. The `partition by` and `order by` clauses have similar functionality to the `group by` and `order by` clauses in the main query block. Yet one name was reused, and the other had been chosen to have a new name. Unlike other *scalar* expressions, which can be placed anywhere in SQL query where scalar values are accepted, the analytics clause lives in the scope of the `select` clause only. I have never been able to suppress an impression that analytic extension could be designed in more natural way.

Conditional Summation with CASE operator

The genesis of the conditional summation idiom is an equivalence between the `count(*)` and `sum(1)`. Formally,

```
select count(*) from emp
```

is the same as

```
select sum(1) from emp
```

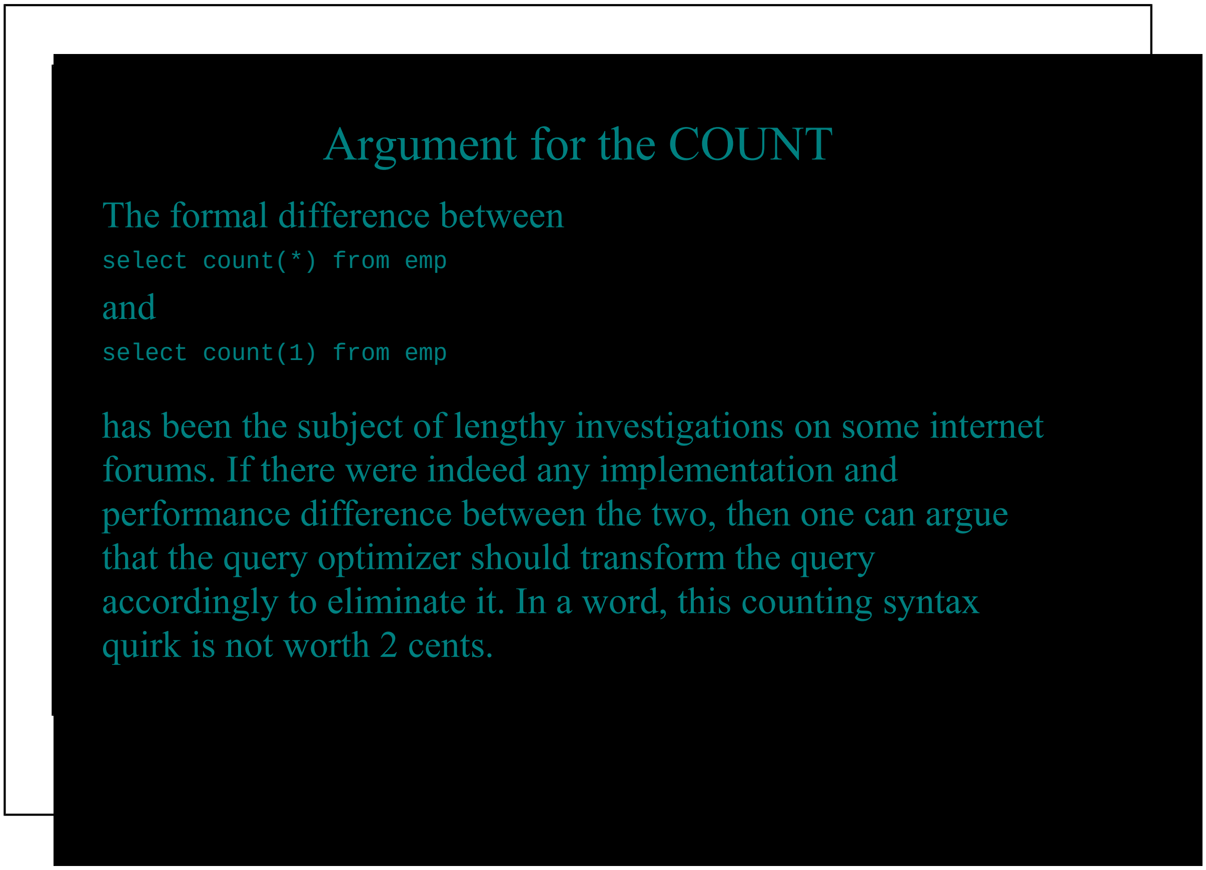
Before elevating this observation into the main topic of this section – the conditional summation pattern – let's clarify one peculiar detail about the syntax. It's just a misconception that `count` should have any arguments at all. First, for most practical purposes `count = sum(1)`, and there is no free variable parameter within the `sum(1)` expression. Second, think how the `count` function may be implemented on a low-level. A reasonable code must look like this

```
int count = 0;
for( int i = 0; i< 10; i++)
    count = count + 1;
```

The count variable is updated during each row processing with the unary increment operator `+1`. Unlike the `count`, any "normal" aggregation has to use a binary operation during each aggregate incremental computation step

```
int sum = 0;
for( int i = 0; i< 10; i++)
    sum = sum + element[i];
```

that is, `+` for `sum`, `∨` for `max`, `∧` for `min`, etc. Therefore, we need one argument for normal aggregates, and no arguments for the `count`.



OK, as far as simple counting is concerned, there doesn't appear to be any need for an argument. But what about

```
select count(ename) from emp
```

where only non-`null` values of the `ename` column are counted? The description of the `count(ename)` in the previous sentence translates directly into SQL

```
select count(*) from emp where ename is not null
```

We see that `count(ename)` is no more than a syntax shortcut.

Well, how about

```
select count(distinct ename) from emp
```

where the `count` aggregate function accepts a column expression with a keyword? This is just yet another shortcut

```
select count(*) from (
   select distinct empno from emp
)
```

Next, what if we want to count two different values at the same time like this

```
select count(ename), count(*) from emp
```

Even though it looks like SQL has a dedicated syntax shortcut for every imaginable task, at this point it is easy to argue that these extensions are nifty at least in some practical cases.

Enter the *conditional summation* pattern. Whenever we count rows satisfying a certain criteria, e.g.

```
select count(*) from emp
where sal < 1000
```

and feel that the `where` clause is an obstacle for the query to evolve to a more sophisticated form, we rewrite it without `where` clause

```
select sum(case when sal < 1000 then 1 else 0 end)
from emp
```

Conditional summation queries scale up nicely to accommodate more complex requirements. In the example with the familiar `Emp` table

DEPTNO	ENAME	SAL
10	MILLER	1300
10	CLARK	2450
10	KING	5000
20	SMITH	800
20	ADAMS	1100
20	JONES	2975
20	SCOTT	3000
20	FORD	3000
30	JAMES	950
30	MARTIN	1250
30	WARD	1250
30	TURNER	1500
30	ALLEN	1600
30	BLAKE	2850

we transform the previous query to count small salaries **per each department** by amending it with `group by`:

```
select deptno,
       sum(case when sal < 1000 then 1 else 0 end) cnt
from emp
group by deptno
```

DEPTNO	CNT
30	1
20	1

Aggregation without Grouping

An aggregation with no grouping is, in fact, an aggregation within a single group. If SQL syntax allowed grouping by the empty set of columns ∅, then a simple aggregate

```
select count(*) from T
```

could be also represented as

```
select count(*) from T
group by ∅
```

Without the empty set syntax, we still can write

```
select count(*) from T
group by 0
```

The `0` pseudo column is a constant expression, so that the table T is partitioned into a single group effectively the same way as with the empty set.

10	0

The subtle novelty here is that the conditional summation query is no longer equivalent to the former attempt restricting condition in the `where` clause

```
select deptno,count(*) from emp
where sal < 1000
group by deptno
```

DEPTNO	COUNT(*)
30	1
20	1

Zero counts were perfectly legal in the aggregation without grouping case. Disappearing zeros are certainly a sign of (yet another) SQL inconsistency.

Perhaps the most important rationalization for the conditional summation idiom is counting by different criteria. Without conditional summation we would have to count by each individual condition in a dedicated query block, and combine those counts with a join. The `pivot` operator, which would be studied in the chapter 3, is a typical showcase of this idea.

Before the `case` operator became widely available in the off-the-shelf RDBMS systems, much more ingenious counting method with indicator function was employed.

Indicator and Step Functions

An indicator function[5] $1_A(x)$ maps every element `x` of a set `A` into `1`, and any element which is not in `A` into `0`. Formally,

$$1_A(x) := \text{if } x \in A \text{ then } 1 \text{ else } 0 \text{ endif}$$

Set operations can be expressed in terms of indicator functions. Intersection is as simple as multiplication

$$1_{A\cap B}(x) = 1_A(x)\ 1_B(x)$$

Unlike set theory where union is dual to intersection, there is no duality between multiplication and addition of indicator functions. Therefore, union has to be expressed via *inclusion-exclusion principle* as

$$1_{A\cup B}(x) = 1_A(x) + 1_B(x) - 1_A(x)\ 1_B(x)$$

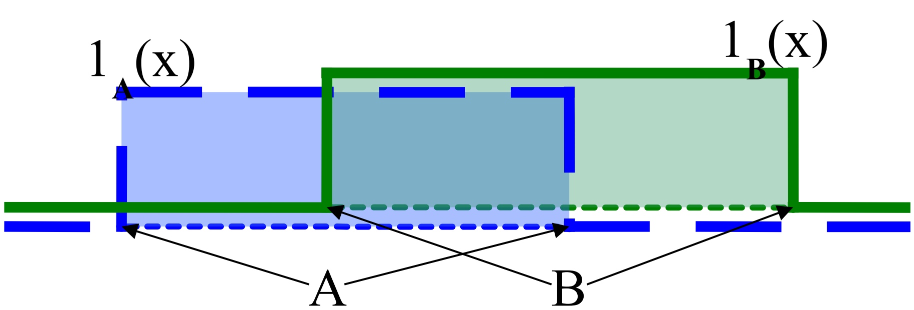


Figure 1.3: Indicator functions $1_A(x)$ and $1_B(x)$.

[5] The indicator function is sometimes also called characteristic function, although this usage is much less frequent now.

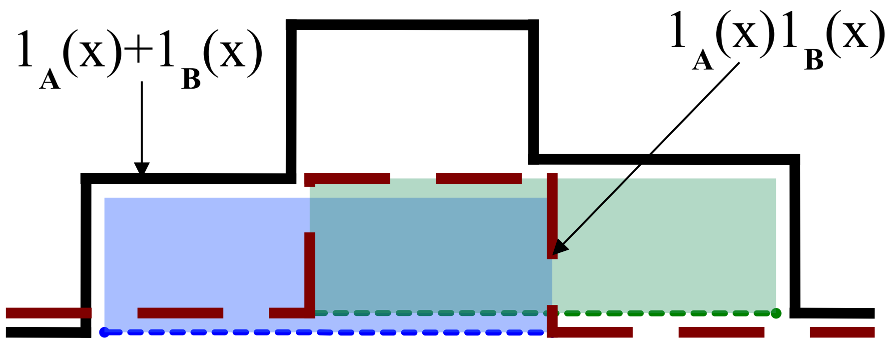


Figure 1.4: $1_A(x)+1_B(x)$ is not an indicator function, because it is equal to 2 for any $x \in A \cap B$. By inclusion-exclusion principle we have to subtract $1_A(x)1_B(x)$.

In SQL context, the indicator function's domain is a set of numbers. An indicator function for a set $x \in [0,\infty)$ is a very important special case. It is called (a primitive) *step* function, and has a designated notation

$$1_x := 1_{[0,\infty)}(x)$$

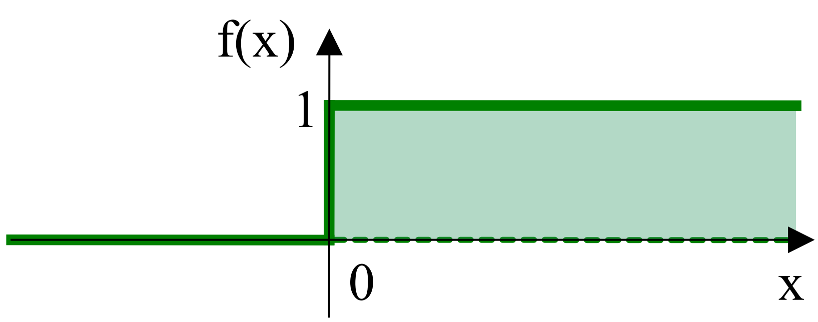


Figure 1.5: Step function $f(x)=1_x$.

Let's also write the primitive step function definition in pseudo programming notation, which we used for the indicator function in the very beginning of the session

$$1_x := \text{if } x \geq 0 \text{ then } 1 \text{ else } 0 \text{ endif}$$

If we raise the abstraction level and convert these hardcoded constants `0`, `1` and yet another `0`, into variables

$$\text{if } x \geq x_0 \text{ then } \alpha \text{ else } \beta \text{ endif}$$

then, our pseudo code starts looking similar to the `case` operator. This expression is a generic step function (although it doesn't have abbreviated notation).

Generic step function can be expressed via primitive step function

$$\text{if } x \geq x_0 \text{ then } \alpha \text{ else } \beta \text{ endif} = \alpha\, 1_{x-x0} + \beta\, 1_{x0-x}$$

So far, so good: we were able to handle simple case operators. What about nested `case` expressions? Consider

$$\text{if } x \geq 0 \text{ then (if } y \geq 0 \text{ then } 1 \text{ else } 0 \text{ endif) else } 0 \text{ endif}$$

Easy: the above formula for general step function should handle the case where α is an expression, rather than constant. By substitution we have

$$\text{if } x \geq 0 \text{ then (if } y \geq 0 \text{ then } 1 \text{ else } 0 \text{ endif) else } 0 \text{ endif} = 1_y\, 1_x$$

as if we had just an intersection of the $x \geq 0$ and $y \geq 0$ sets!

Finally, what is still missing is a way to express the step function in SQL. True, we can trivially utilize the `case` operator, but then we can hardly justify learning the step and indicator functions. The key formula is expressing the step function via standard numeric `sign` function

```
step(x)  := sign(1+sign(x))
```

Now we can write queries with conditional expressions in a quite peculiar way

```
select ename, sal,
       sign(1+sign(sal-2000)) SALgt2000 -- i.e. step(x-2000)
from emp
```

ENAME	SAL	SALgt2000
SMITH	800	0
ALLEN	1600	0
WARD	1250	0
JONES	2975	1
MARTIN	1250	0
BLAKE	2850	1
CLARK	2450	1

SCOTT	3000	1
KING	5000	1
TURNER	1500	0
ADAMS	1100	0
JAMES	950	0
FORD	3000	1
MILLER	1300	0

Step functions method was the only game in town long time ago, when the `case` operator wasn't part of the SQL standard yet[6]. Nowadays a seasoned SQL programmer writes a `case` expression without giving it a second thought. Yet, there are rare cases when clever application of indicator function is still a contender. Consider the following data

```
select * from Transactions
```

Date	Amount	Type
01-01-2005	800	debit
01-01-2005	1600	credit
01-01-2005	1400	credit
01-01-2005	200	debit
01-02-2005	250	debit
01-02-2005	150	debit
01-02-2005	850	credit

You are asked to sum transaction quantities grouped by `Date` to produce the output like this

Date	debit	credit
01-01-2005	1000	3000
01-02-2005	400	850

The first step towards solution is recognizing indicator function hidden in the above data. The character `Type` column is very inconvenient to deal with; can it be transformed into numeric? The two value column certainly can be coded with numbers, but that is not what we are looking for. It is the `instr(Type, 'debit')` and `instr(Type, 'credit')` expressions that convert the character column values into indicator functions. We proceed by just summarizing the `Amount` weighted by indicator functions:

```
select Date,
       sum(Amount*instr(Type,'debit')) debit,
```

[6] SQL technique of indicator and step functions is credited to David Rozenshtein, Anatoly Abramovich, and Eugene Birger

```
       sum(Amount*instr(Type,'credit')) credit
from Transactions
group by Date
```

This nice solution is due to Laurent Schneider. Compare it to conditional summation with `case` operator[7]:

```
select Date,
       sum(case when Type='debit' then 1 else 0 end) debit,
       sum(case when Type='credit' then 1 else 0 end) credit
from Transactions
group by Date
```

A Case for the CASE operator

In this section we embark upon a little bit longer journey. We have already mentioned SQL proximity to English. Consider the following query

> For each customer, show the number of calls made during the first 6 months that exceeded the average length of all calls made during the year, and the number of calls made during the second 6 months that exceeded the same average length.[8]

True, you have to read this long sentence more than once. Once you examine it carefully, though, you'll easily convince yourself that the apparent complexity is illusory. The query can be translated into SQL in small increments. But first, we need a schema to anchor SQL query to:

```
table Calls (
    FromPh      integer(10),
    ToPh        integer(10),
    ConnectedAt date,
    Length      integer
);
```

The `Calls` table stores the calls placed on a telephone network over a period of one year. Each `FromPh` number identifies a customer.

Without further ado we begin by translating

> For each customer …

into

```
select FromPh, …
from Calls
group by FromPh
```

We intentionally marked with ellipsis the missing part that will be gradually developed in the next steps.

---

[7] In chapter 3 we'll learn the conventional name for this problem: the *pivot* pattern.

[8] Adopted from paper by Damianos Chantziantoniou and Kenneth Ross: Querying Multiple Features of Groups in Relational Databases. http://www.dmst.aueb.gr/damianos/vldb96-acc.ps

At this moment a reader may wonder if

```
select distinct FromPh
from Calls
```

is an easier way to list all the customers. It certainly is, but now what? This query is complete, it answers the question partially, but we can't expand it to answer the remainder. The `group by` clause, on the other hand, is one of the most powerful SQL constructs.

DISTINCT operator is redundant

Technically, the `distinct` operator is a special case of `group by`. For any table (or view) T

```
select distinct x, y from T
```

is equivalent to

```
select x, y from T
group by x, y
```

The next clause

> …, show the number of calls made during the first 6 months that exceeded the average length of all calls made during the year, …

leaves us a choice. We can place the condition into the `where` clause, but then we might face some difficulty assembling the query pieces together. A better alternative is leveraging a familiar conditional summation pattern

```
select FromPh,
       sum( case when … then 1 else 0 end ),
       …
from Calls
group by FromPh
```

Ellipsis means that we still have to interpret the condition

… during the first 6 months that exceeded the average length of all calls made during the year …

This is still a relatively complex sentence. We may notice that the two variables -- `ConnectedAt` and `Length` – are involved. The condition begins with

… during the first 6 months …

which is easily translated into `ConnectedAt < '1-July-2005'`. Next fragment

… the average length of all calls made during the year …

is a little bit trickier. First, the query is ambiguous. Did the author mean the average length of all the calls in the system, or the average length for each customer? Both interpretations are perfectly reasonable. The average length of the call is just

```
select avg(Length)
from Calls
```

while the average length of the call per each customer is

```
select FromPh,
       avg(Length)
from Calls
group by FromPh
```

The first interpretation is easier to implement than the second one, therefore, we leave it as an exercise to the reader.

So, given the query we have developed so far

```
select FromPh,
       sum( case when … then 1 else 0 end ),
       …
from Calls
group by FromPh
```

where does the relation

```
select FromPh,
       avg(Length)
from Calls
group by FromPh
```

fit in? The only place that admits arbitrary relations is the `from` clause.

Relational Closure

SQL query block inside the `from` clause is called *inline view*. From logical perspective there is no difference if a relation within the `from` clause (or anywhere in the SQL statement, for that matter) is a table or a view. It is a manifestation of fundamental property of the Relational Model – Relational Closure. It is common to organize a query in a chain of inline views so that every step is small and easily comprehendible.

Let's nest the second query into the first as inline view:

```
select c1.FromPh,
       sum( case when … then 1 else 0 end ),
       …
from Calls /*as*/ c1, (
    select FromPh,
           avg(Length) /*as*/ av
    from Calls
    group by FromPh
) c2
group by FromPh
```

We introduced aliases `c1`, `c2`, and `av`, along the way, which might be helpful for further development. The `c1`, in fact, is required to disambiguate the `FromPh` column name in the select clause.

We are just a couple of steps away from finishing our translation of the informal query into SQL . First, the relations `c1` and `c2` are naturally joined by the customer id, that is `FromPh`. Second, the `av` alias is the average length of the call per each customer that was required to complete the predicate inside the case operator. Thus we have

```
select c1.FromPh,
       sum(case when ConnectedAt < '1-July-2005'
                and Length < av
```

```
            then 1 else 0 end),
       …
from Calls /*as*/ c1, (
    select FromPh,
           avg(Length) /*as*/ av
    from Calls
    group by FromPh
) c2
where c1.FromPh = c2.FromPh
group by FromPh
```

The final clause of our informal query

…and the number of calls made during the second 6 months that exceeded the same average length.

is very similar to the clause that we just analyzed. It's no challenge for the reader who managed to follow me so far.

Let's explore a slightly different path. Instead of introducing an inline view `c2`, why don't we calculate average length for the customer just in place as a correlated scalar subquery:

```
select c1.FromPh,
       sum(case when ConnectedAt < '1-July-2005'
                and Length < (
                        select avg(Length)
                        from Calls c2
                        where c1.FromPh = c2.FromPh
                )
            then 1 else 0 end
       ),
       …
from Calls c1
group by FromPh
```

Which of the two queries, the one with inline view, or the one with scalar subquery performs better? Well, they are logically **equivalent**, aren't they? The SQL engine reserves the right to transform a query to a logically equivalent one. A curious reader might want to check if both queries have the same execution plans on the RDBMS of his choice.

Let's pause and reflect back a little. The genesis of our solution is the `case` operator inside the `sum` aggregate. It is possible to express this query in SQL without it. Chantziantoniou et al (the authors of the article where I borrowed the problem from) followed that route and introduced a chain of named intermediate views[9] as follows

```
create view AvgCallLengthPerCust as
select FromPh, avg(Length) /*as*/ avgL
```

[9] Instead of piling inline views inside of single, but big and messy SQL query

```
from Calls
group by FromPh

create view ExcAvgDuring1stHalfYear as
select C.FromPh, count(*) /*as*/ count
from Calls /*as*/ C, AvgCallLengthPerCust /*as*/ V
where C.FromPh = V.FromPh AND
Length > avgL AND Date<'1-July-2005'
group by C.FromPh

create view ExcAvgDuring2ndHalfYear as
select C.FromPh, count(*) /*as*/ count
from Calls /*as*/ C, AvgCallLengthPerCust /*as*/ V
where C.FromPh = V.FromPh AND
Length > avgL AND Date>='1-July-2005'
group by C.FromPh

select a1.FromPh, a1.cnt, a2.cnt
from ExcAvgDuring1stHalfYear /*as*/ a1,
     ExcAvgDuring2ndHalfYear /*as*/ a2
where a1.FromPh=a2.FromPh
```

Based on this example, Chantziantoniou et al proposed extending SQL language in such a way that would make writing queries involving multiple features of the same group easier. As we have seen, a solution leveraging the `case` operator makes this argument less convincing.

Summarizing by more than one Relation

In the previous section we already mentioned that

```
select deptno, count(*) from Emp
group by deptno
```

could be rewritten into an equivalent form leveraging correlated scalar subquery

```
select distinct deptno,
       (select count(*) from Emp ee
        where ee.deptno = e.deptno)
from Emp e
```

Both queries project the `Emp` relation onto the `deptno` column, and extend the result with one extra column that counts the number of rows in each group in the original relation. What about those `deptno` values that are missing in the `Emp` table, shouldn't they be listed with count 0? Suppose `deptno`, say, 40 is a valid department on the system, how do we change the query to show it with the count 0? Well, if `deptno = 40` is a valid department, then it should be in some table -- `Dept`, for example, where it is most likely a (primary) key. Then, why don't we use this table in the outer query:

```
select deptno,
       (select count(*) from Emp e
        where e.deptno = d.deptno)
```

Hugh Darwen's Summarize

Hugh Darwen argued that `group by` with aggregation is an operator that requires **two** tables as the arguments, in general. The idea of introducing such an operator in SQL never caught on. Yet, in each practical situation it might be useful to double check if writing `group by` clause as a one- or two- argument operator is more appropriate.

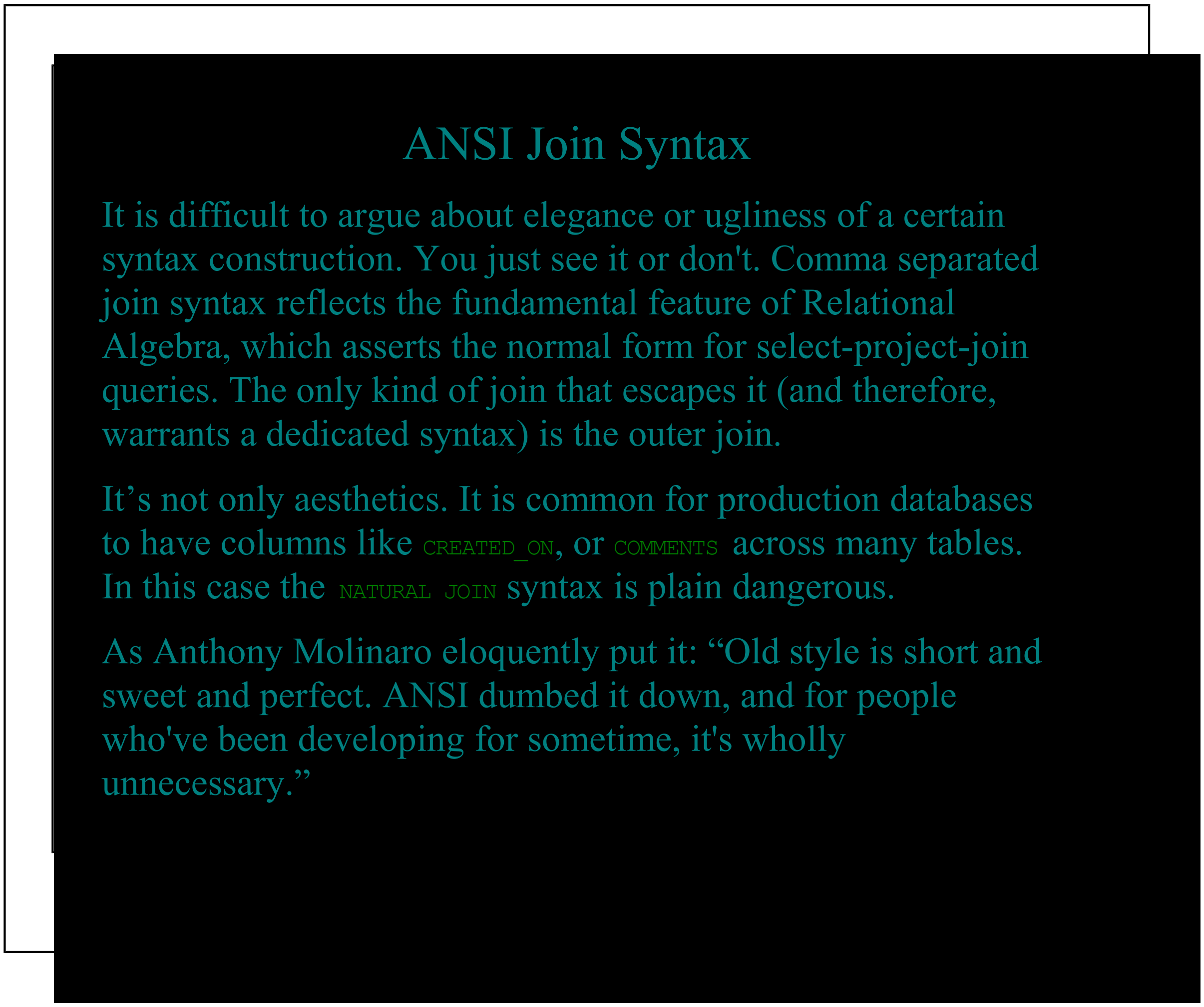
```
from Dept d
```

An added bonus of having two tables in the query is that the `distinct` qualifier is no longer required.

SQL is notorious for allowing multiple ways to express the same query. Listing all the departments with the employee counts could be also rewritten via the outer join

```
select d.deptno, d.dname,
       sum(case when e.deptno is not null then 1 else 0 end)
from Emp e right outer join Dept d
where d.deptno = e.deptno
group by d.deptno, d.dname
```

If we reduce the conditional summation pattern to a simple `count(*)`, then the departments with no employees will count 1 employee instead of 0.



Which form, scalar subquery or outer join, is more performant? Surely, the answer differs between the vendors. Oracle, for example, is better at outer join optimization than unnesting scalar subqueries in the `select` clause[10]. Outer join from the optimizer's perspective has almost the same rights as normal join. It can be permuted with the other joins in the join order, it is costed similarly, etc. If the summarizing query is a part of the other query, chances are the optimizer may find the better plan when the query is written via outer join.

[10] This may certainly change as soon as Oracle implement scalar subquery unnesting.

Interval Coalesce

Normally, the shorter the problem statement is, the simpler its formal expression in SQL. There are notable exceptions, and *interval coalesce*[11] is one of them.

Given a set of intervals, return the smallest set of intervals that *cover* them.

This deceptively simple formulation, however, leaves a pile of questions. First, what is the intervals table? This one is easy

```
table Intervals (
   x integer, -- start of the interval
   y integer  -- end   of the interval
);

ALTER TABLE Intervals
ADD CONSTRAINT ends_ordering CHECK( x < y );
```

Perhaps, the `from` and `to`, or maybe, `start` and `end` column names may sound more appropriate. Unfortunately, they are SQL keywords.

Next, what is the concept of one interval covering the other? Easy: interval is a set of points between the interval endpoints `x` and `y`. What about the endpoints, are they included into interval or not? For our purpose, let's agree that both `x` and `y` belong to interval set of points, in other words, intervals are closed from both ends. Open and half-open intervals require careful reexamination of all the inequality predicates in the query which we are going to write. Therefore, formally: $A = \{p|x \le p \le y\}$. Finally, interval A covers interval B when B is subset of A: $B \subseteq A$.

Armed with these definitions, we are ready to write the query. The `Intervals` table is the only candidate to be placed into the query `from` clause, but there is a catch. If we just select some intervals out of the set we have, we won't get the answer. Think about an easy case of two overlapping intervals. The answer to the query is an interval whose endpoints are combined from different records. Therefore, we need a *selfjoin*:

```
select fst.x, lst.y
from Intervals fst, Intervals lst
where fst.x < lst.y
```

[11] There are two terms in the literature. "Developing Time-Oriented Database Applications in SQL" by R. Snodgrass uses *interval coalesce*, while "Temporal Data and The Relational Model" by C. Date et al calls it *interval packing*.

```
and …
```

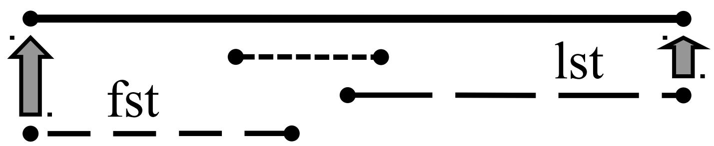


Figure 1.6: Endpoints of the covering interval are constructed from the endpoints of the first and last intervals in a chain of intervals with no gaps.

Next, we take onto "no gaps" condition. Consider any interval endpoint between `fst.x` and `fst.y`. It has to be covered by some interval!

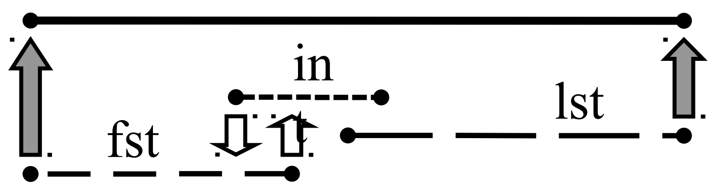


Figure 1.7: In the chain of intervals fst, int, lst the endpoint int.x is covered by the fst interval. Likewise, the fst.y is covered by the int.

Actually, if there is a gap in a chain of intervals, then both ends of the gaps would be exposed (that is not covered by other interval). This observation allows a small optimization, so that we have to check the `x` endpoints only.

Unfortunately, the "no gaps" condition can't be naturally translated into SQL, since it doesn't support *universal quantifiers*. The standard approach is converting the clause into existential quantifiers, which we'll explain in a little bit more detail in the section on Relational Division. Here let's take a little jump, and write the whole condition at once:

```
select fst.x, lst.y
from Intervals fst, Intervals lst
where fst.x < lst.y
and not exists (
   select * from Intervals int
   where int.x > fst.x and int.x < lst.y
   and not exists (
      select * from Intervals cov
      where int.x >= cov.x and int.x =< cov.y
```

```
   )
) and …
```

That's not all. The chain of intervals has to be *maximal*, in other words, it can't be extended at the either end.

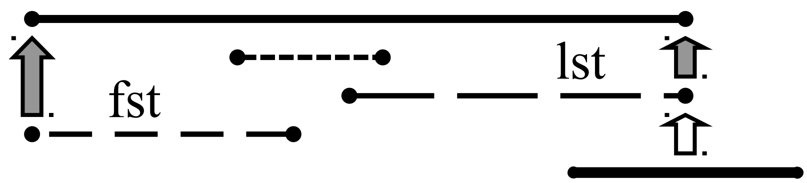


Figure 1.8: The chain starting with the fst interval and ending with lst is not maximal, since the lst.y is covered by some other interval.

This condition translates into one extra subquery:

```
select fst.x, lst.y
from Intervals fst, Intervals lst
where fst.x < lst.y
and not exists (
   select * from Intervals int
   where int.x > fst.x and int.x < lst.y
   and not exists (
      select * from Intervals cov
      where int.x >= cov.x and int.x =< cov.y
   )
) and not exists (
   select * from Intervals cov
   where cov.x < fst.x and fst.x <= cov.y
      or cov.x <= lst.y and lst.y < cov.y
)
```

The final query is surprisingly verbose. Terser expression would involve converting NOT EXISTS subqueries into counting.

Is (NOT) EXISTS or (NOT) IN faster?

This is the wrong question. The EXISTS and IN subqueries are equivalent; in fact, the optimizer uses the term *semijoin* for both. The NOT EXISTS and NOT IN subqueries are different as far as NULL semantics is concerned, but still they are very similar so that the SQL execution engine uses the term *antijoin* for both. It might be argued that SQL standard should have defined them identically, leaving explicit control of NULL semantics to the end user (by means of IS (NOT) NULL predicate). History proved any attempt to get UNKNOWN value semantics right as futile, therefore it is better just to forget about NULLs and aim for elementary consistency.

Which `NOT EXISTS` clause, because we have two? Let's revisit both conditions

- A chain of intervals must have no gaps (fig.1.7)
- A chain is maximal if it has uncovered ends (fig.1.8)

These conditions are not entirely independent, since every maximal chain of intervals is delimited by gaps on both ends. What we need is a change of perspective.

Consider any pair of intervals `fst` and `lst`. Suppose `fst.x` and `lst.y` are not covered by other intervals. Are the `fst` and `lst` the beginning and the end of a chain? Not necessarily, because there might be gaps between them. Let's shift our focus onto a **set** of `fst`, `lst` pairs such that `fst.x` and `lst.y` are not covered by other intervals. If there is a gap between some pair of intervals fst_1, lst_1 then, it follows that there has to be another pair fst_2, lst_2 with the same first element $fst_2 = fst_1$ and some other second element lst_2 such that $lst_1.y < lst_2.y$

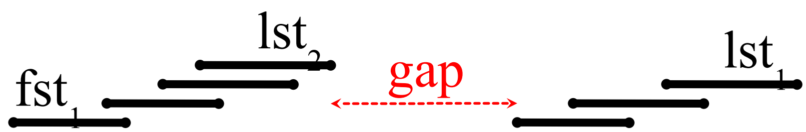


Figure 1.9: A chain of intervals beginning with fst_1 and ending with lst_1 has a gap. Therefore, there is an element lst_2, such that $lst_2.y$ is not covered by any other interval.

In other words, among all the `fst`, `lst` pairs with the same `fst` element, it is the pair with the minimal `lst` element that defines a chain without gaps.

Let's formalize these ideas. We have already written a `NOT EXISTS` subquery that defined a set of `fst`, `lst` pairs such that `fst.x` and `lst.y` are not covered by other intervals. Alternatively, we could have applied the conditional summation pattern. Indeed

```
select fst.x, lst.y, sum(
      case when cover.x < fst.x and fst.x <= cover.y
             or cover.x <= lst.y and lst.y < cover.y
           then 1 else 0 end
)
from intervals fst, intervals lst, intervals cover
where fst.y <= lst.y
group by fst.x, lst.y
```

counts if either `fst.x` or `lst.y` is covered by some interval. If the count is 0, then the `fst`, `lst` pair defines a maximal chain (possibly with gaps). Therefore, a formal query returning all the maximal chains is:

```
select fst.x, lst.y
from intervals fst, intervals lst, intervals cover
where fst.y <= lst.y
group by fst.x, lst.y
having sum(
      case when cover.x < fst.x and fst.x <= cover.y
             or cover.x <= lst.y and lst.y < cover.y
           then 1 else 0 end
) = 0
```

The final step is selecting all the pairs with the minimal `lst` element. We were considering all the pairs with the fixed `fst` element, which translates into `group by`:

```
select x, min(y) from (
   select fst.x, lst.y
   from intervals fst, intervals lst, intervals cover
   where fst.y <= lst.y
   group by fst.x, lst.y
   having sum(
         case when cover.x < fst.x and fst.x <= cover.y
                or cover.x <= lst.y and lst.y < cover.y
              then 1 else 0 end
   ) = 0
)
group by x
```

There is an important special case of the interval coalesce problem.

Given a set of integers, partition them into ranges of successive numbers.

This problem can also be called as *interval packing*, and is the reverse of the *discrete interval sampling*, that we we'll study in the next chapter.

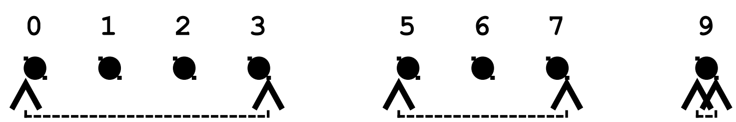


Figure 1.10: Interval Packing. A set of integers 0,1,2,3,5,6,7,9 is packed into the intervals [0,3], [5,7], and [9,9].

If we represent each integer `x` as a (closed) interval `[x,x+1]` then, the problem reduces to interval coalesce. Rod West suggested much more elegant solution[12], however. His key insight was an expression that groups the numbers within the same range. Then, if we know how to group integers, then the ranges are defined by taking minimum and maximum inside each group. What criterion identifies each group?

In the section on counting we learned how to enumerate rows in the increasing order:

```
select t.x, count(*) seq#
from T t, T tt
where tt.x <= t.x
group by t.x
```

[12] Perhaps, I'm little unfair by giving credits to people mostly from oracle community in this book. Unfortunately, the "big 3" database communities have grown apart, so that the same problem solution is rediscovered by different people. Joe Celko, who apparently lives in SQL Server world, gives the credit for a similar problem to Steve Kaas.

It is the `x - seq#` expression that remains constant within each group!

The rest is straightforward. We `group by` this expression and calculate `min` and `max` aggregate values, which are demarcating the beginning and the end of each interval

```
select min(x), max(x) from (
   select x, rank() over(order by x) seq# from T
) group by x-seq
```

Predictably, this problem might occur in a slightly more complicated context. Our input relation can have one more column, say `name`, so that integers are grouped by the `name` values. Rod's solution scales up naturally to the new requirements. The additional grouping column just emerges in the appropriate places

```
select name, min(x), max(x) from (
   select x, rank() over(partition by name order by x) seq#
   from T
) group by name, x-seq
```

Summary

- A (naturally ordered) list of values can be enumerated via join and grouping with aggregation, or SQL analytics query.
- Use `CASE` operator inside the `SUM` aggregate function for counting. Leveraging the `CASE` operator is a pragmatic alternative to counting with Indicator and Step functions.
- Write complex queries as a chain of inner views nested inside each other.
- `DISTINCT` operator can be expressed via `GROUP BY`.
- `COUNT` operator doesn't have any arguments.
- `GROUP BY` operator summarizes over two tables.
- Use comma join syntax where possible.

Exercises

1. Suppose we have the two queries
   ```
   select deptno, count(*) from emp;
   select deptno, count(*) from emp
   where sal < 1000;
   ```
   Can they be combined into one?
2. The query
   ```
   select count(distinct ename), count(*) from emp
   ```
   refers to the `count` function, which accepts a funny argument with the `distinct` keyword. Rewrite the query in such a way that it would utilize only “politically correct” aggregate function – the `count(*)` with no arguments.
3. With column expression as an argument for `count()` function all `NULL` values are ignored. Write an equivalent query for
   ```
   select count(comm) from emp
   ```
   via conditional summation.
4. There is no shorthand syntax for counting distinct column combinations. Write down a query that counts the distinct number of the `first_name`, `last_name` in the `Customers` table by separating `count`ing and `distinct`ion in different query blocks.
5. The `having` clause is redundant. Transform
   ```
   select deptno from emp
   group by deptno
   having min(sal) < 1000
   ```
   into an equivalent form leveraging inner subquery.
6. Correlated scalar subquery can be even used within the `order by` clause. Explain the purpose of the following query
   ```
   select * from emp
   order by (select dname from dept where emp.deptno=dept.deptno);
   ```
7. Express the indicator function 1_x via standard numeric functions `abs(x)` and `sign(x)`.
8. Figure 1.6 demonstrates one way to implement the maximal chain of intervals condition. Alternatively, we could have defined the maximal chain as the one that maximizes the distance `lst.y - fst.x`. Write the query that formalizes that idea.
9. Write a set intersection query. Given a collection of sets, e.g.
   ```
   set1 = {1,3,5,7,9}
   ```

```
set2 = {1,2,3,4,5}
set3 = {4,5,6,7}
```

stored as a relation `Sets`

ID	ELEMENT
1	1
1	3
1	5
1	7
1	9
2	1
2	2
2	3
2	4
2	5
3	4
3	5
3	6
3	7

your query should return intersection of all the sets listed in the relation `Sets`; i.e. `{5}` in our example. Hint: group by the `element` column and count.

10. Counting words. Suppose you have a table of sentences

```
table Sentences {
    text  varchar(4000);
}
```

The SQL function `instr(string, substring, startPosition)` returns the position of the first occurrence of the `substring` in the `string` beginning with `startPosition`. Write a SQL query that counts the number of sentences where a given word is occurring once, twice, and so on.

11. This is an example[13] where nothing more than correct identification of a problem is required. You are challenged to write a query, which tells when an `id` has been a certain value for `x` consecutive years. Given

ID	YR	VALUE
100	1998	0
100	1999	0
100	2000	0
100	2001	0
100	2002	1

[13] Adopted from microsoft.public.sqlserver.programming

100	2003	0
100	2004	0
100	2005	0
100	2006	0
100	2007	0
100	2008	1
200	1999	0
200	2001	0
200	2002	0
300	2001	0
300	2002	0
300	2003	0
300	2004	0
300	2005	0

find `id`, `startdate`, `enddate` when value is `0` for `4` or more consecutive years. The expected output is

ID	STARTDATE	ENDDATE	YEARS
100	1998	2001	4
100	2003	2007	5
300	2001	2005	5