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Cooperation and popularity in spatial games

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HIGHLIGHTS

- In spatial games, neighbors strategies are preferentially selected based on their popularity.
- Popularity-driven selection mechanism can enhance the level of cooperation remarkably.
- The effectiveness of such a mechanism is verified by the snowdrift game and the different structures of networks.

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ABSTRACT

Selection of the competition opponent is crucial for the evolution of cooperation in evolutionary games. In this work, we introduce a simple rule, incorporating individual popularity via the single parameter α, to study how the selection of the potential strategy sources influences individual behavior traits. For positive α players with high popularity will be considered more likely, while for negative α the opposite holds. Setting α equal to zero returns the frequently adopted random selection of the opponent. We find that positive α (namely, adopting the strategy from a more popular player) promotes the emergence of cooperation, which is robust against different interaction networks and game classes. The essence of this boosting effect can be attributed to the fact: increasing α accelerates the microscopic organization of cooperator clusters to resist the exploitation of defectors. Moreover, we also demonstrated that the introduction of a new mechanism alters the impact of uncertainty by strategy adoption on the evolution of cooperation. We thus present a viable method of understanding the ubiquitous cooperative behaviors in nature and hope that it will inspire further studies to resolve social dilemmas.

1. Introduction

Understanding the emergence and stability of cooperation among unrelated individuals represents one of the major challenges of evolutionary biology and of behavioral sciences [1–3]. For elucidating this puzzle, researchers have traditionally adopted evolutionary game theory as the common formal framework to study the evolutionary dynamics of behavior traits [4,5]. A simple, paradigmatic model the prisoner's dilemma game in particular, illustrating the social conflict between cooperative and selfish behaviors, has attracted considerable attention both in theoretical as well as experimental fields

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[6–14]. In its basic version, two players simultaneously decide to adopt one of two strategies: cooperation (C) and defection (D). When a population of players has interaction in the well-mixed case, cooperation will disappear soon.

Over the past decades, a great number of scenarios have been identified that can offset the above unfavorable outcome and can lead to the evolution of cooperation [15–23]. Nowak attributed all these to five mechanisms: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [24], which can be somewhat related to the reduction of an opposing player's anonymity relative to the so-called well-mixed situation. Among these mechanisms, network reciprocity, where players are arranged on the spatially structured topology and interact only with their direct neighbors, has attracted the greatest interest (see Refs. [25–27] for a comprehensive review) because cooperators can survive by means of forming compact clusters, which minimizes the exploitation by defectors and protects those cooperators that are located in the interior of such clusters [28]. In line with this achievement, the effect of spatial topology, and its various underlying mechanisms have been explored extensively. For example, among the proposed scenarios of supporting the sustenance of cooperation, we have the presence of mobile agents [29–33], more complex interaction network [34–39], aspiration-driven ability [40–42], the influence of transferring capability [43,44] and differences in evolutionary time scales [19,20]. Moreover, individual heterogeneity, uncertainty or diversity [42,45,46], asymmetry payoff factor [47,48] and environmental influence [49,50] have also been considered as the potential mechanisms of enhancing cooperation. Other traits involve that coevolutionary selection of dynamical rules, where the strategy property of the population is allowed to evolve together with the network topology or the evolution rule, can have a positive impact on the evolution of cooperation [51–54].

Of particular renown, the influence of individual opinions on learning ability has attracted great attention recently. Looking at some examples more specifically, in a recent research work [55], where Szolnoki et al. assumed that individual learning ability was a function of collective opinion performance in groups, cooperator abundance on the spatial grid was supported due to the dynamical effect. In Ref. [56], it was shown that a conformity mechanism involving a tendency to copy the most frequent nearby strategy could lead to the long-term dominance of cooperation, even if the conditions did not necessarily favor the spreading of cooperators. Meanwhile, other scholars [57–59] reported that when the copying probability directly attenuated through the so-called "letting learning activity level decrease" the evolution of cooperation could also be guaranteed, especially for the co-evolution proposal suggested by Tanimoto et al. [60].

However, while it is undisputable that the transferring capability of strategy is an effective approach to model the evolution of behavior traits, there may exist other potential ways of exploring the influence of individual opinion states. In our society, it is well-known that individuals having higher popularity usually obtain more attention than others. For example, the behaviors of movie or sport stars are often followed by their fans, while in the animal world, the leaders' behaviors are often imitated by subordinative animals. Inspired by these facts, an interesting question poses itself, which we aim to address in what follows. Namely, if the influence of popularity is evolved into the selection of strategy sources, how does cooperation fare? To answer this issue, the popularity can be regarded as a function of behavior opinions.

In the present work, we study the prisoner's dilemma game (and snowdrift game) with a preferential mechanism, where the selection of imitation object needs referring to its popularity. Differentiating from the previous research works [61,62], where an opponent is chosen uniformly at random, the neighbor possessing the higher popularity is more likely to be chosen as the potential strategy donor. Our main aim is to study the impacts of such a simple mechanism on the spreading of cooperation. Through numerical simulations, we unveil that this mechanism can allow for cooperative behavior to prevail even if the temptation to defect is large, irrespective of the potential interaction networks and games. Subsequently, we will show more details.

2. Evolution game model and dynamics

We consider an evolutionary prisoner's dilemma game that is characterized with the temptation to defect T (the highest payoff received by a defector if playing against a cooperator), reward for mutual cooperation $R = 1$, and both the punishment for mutual defection P as well as the sucker's payoff S (the lowest payoff received by a cooperator if playing against a defector) equaling 0. As a standard practice, $1 < T \leq 2$ ensures a proper payoff ranking ($T > R > P \geq S$) and captures the essential social dilemma between individual and common interests [28]. It is worth mentioning that though we choose a simple and weak version (namely, $S = 0$), our observations are robust and can be observed in the full parameterized space as well [63].

Throughout this work, each player x is initially designated either as a cooperator (C) or defector (D) with equal probability. With respect to the interaction network, we choose either the $L \times L$ regular lattice, the triangle network, or the honeycomb lattice with periodic boundary conditions. The game is iterated forward in accordance with the Monte Carlo simulation procedure comprising the following elementary steps. First, a randomly selected player x acquires its payoff P_x by playing the game with its nearest neighbors (the payoffs of all its neighbors are also evaluated in the same way). Then, it chooses a neighbor y according to the following probability:

$$\Omega_y = \frac{\exp(\alpha S_y)}{\sum_z \exp(\alpha S_z)} \tag{1}$$

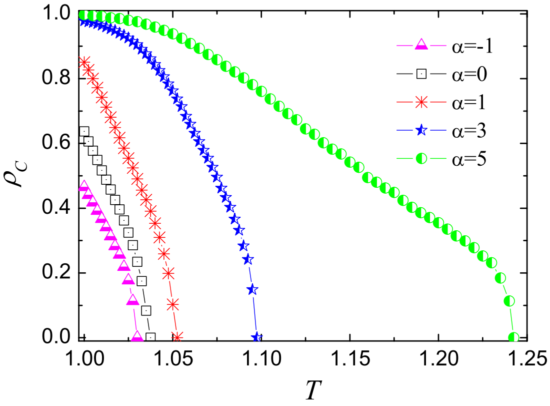

Fig. 1. (Color online) The fraction of cooperators ρ_C in dependence on the temptation to defect T for different values of α. Compared with traditional version (namely, $\alpha = 0$), it is obvious that the novel mechanism not only enables cooperators to reach their exclusive dominance, but also allows for cooperative behaviors to prevail at high temptation to defect. Depicted results are obtained for $K = 0.1$.

where α is the newly introduced evaluator factor, the sum runs over all the neighbors of player x and S_y represents the popularity of player y. Importantly, the popularity S_i of player i is defined as

$$S_i = \frac{n_i + 1}{k_i + 1} \tag{2}$$

n是成员i周围采取相同策略的邻居个数，k是i的度，对于格子网络，分母是常数

where n_i is the number of neighbors that have the same strategy with focal player i and k_i is the degree of player i. Evidently, from Eq. (1), $\alpha = 0$ returns to the frequently adopted case where the neighbor y is randomly chosen, while for $\alpha > 0$, it introduces a preference toward those neighbors that have higher popularity. However, for $\alpha < 0$, neighbors with lower probability are more likely to be selected as the potential strategy donors. On the other hand, we can regard this treatment as the influence of agents' opinion states (cooperation and defection corresponding to two attitudes), where for $\alpha > 0$ individuals trend to majority attitude [64] and conversely $\alpha < 0$ enforces individuals to transfer minority selection [65,66]. Since $\alpha > 0$ is more close to common situations in the society, our main aim is to explore the case of positive α, in which we expect that the popularity-driven selection can accelerate microscopic dynamics of cooperation behavior. Lastly, player x tries to adopt the strategy of the selected neighbor y with a probability depending on the payoff difference,

$$W = \frac{1}{1 + \exp[(P_x - P_y)/K]}, \tag{3}$$

where K denotes the amplitude of noise or its inverse ($1/K$), the so-called intensity of selection [61,67]. During one full Monte Carlo step (MCS) each player has a chance to adopt one of the neighboring strategies once on average.

The results of Monte Carlo simulations presented below were obtained on 200×200 lattices. The key quantity the fraction of cooperators ρ_C was determined within the last 10^4 full MCS over the total 2×10^5 steps. Moreover, since the popularity-driven selection process may introduce additional disturbances, the final results were averaged over up to 100 independent realizations for each set of parameter values in order to assure suitable accuracy.

3. Results and analysis

We start by examining the influence of the popularity-driven selection mechanism on the sustainability of cooperation. Fig. 1 shows how ρ_C varies in dependence on the temptation to defect T for different values of α. We can observe, for $\alpha = 0$ (namely, the classical version), that cooperators will be decimated soon even if the temptation is not very large [61,62]. Evidently, for negative values of α, the survival environment of cooperators becomes more difficult, thus yielding the exclusive dominance of defectors. For positive values of α, however, cooperators start to mushroom: cooperators are not only able to reach complete dominance, but even prevail over a larger interval of T. Interestingly, the larger the value of α, the more effective the spreading and survivability of cooperation. These results suggest that the consideration of such a simple mechanism can significantly sustain the emergence and evolution of cooperation.

Moreover, it is worthwhile to explore how the threshold values, marking the extinction of cooperators (defectors), vary in dependence on the evaluation parameter α. We present in Fig. 2 full $T - \alpha$ phase diagrams. Obviously, across the whole span of α, not just the mixed $C + D$ phase region enhances, but also the extent of full C phase becomes larger. For large α (such as, $\alpha = 5$), the emergence of full D phase needs extremely large T, which means that cooperation is substantially improved. In what follows we will mainly focus on the effect of positive α.

In order to validate the promotion of cooperation induced by the evaluation factor α, we apply the dynamical mean-field rate equation to investigate the evolution of the frequency of cooperators ρ_C. If $W_{C\to D}$ ($W_{D\to C}$) denotes the transition probability of cooperators (defectors) changing into defectors (cooperators), then the time variation of ρ_C can be approximatively

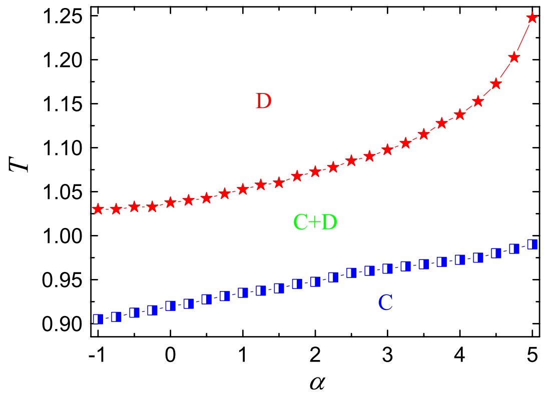

Fig. 2. (Color online) Full $T - \alpha$ phase diagram shown with the pure C, D and mixed $C+D$ phases. As α increases, there is a fast increase in both cooperators dominated phase (namely, pure C phase) and the coexistence one. Depicted results are obtained for $K = 0.1$.

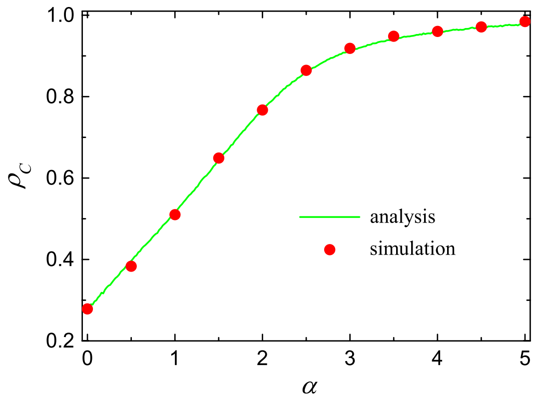

Fig. 3. (Color online) Fraction of cooperators ρ_C in dependence on the parameter α for the analytical and numerical results. Note that although there exists tiny error between them, the theoretical approach correctly predicts the facilitation trend of cooperation. Depicted results are obtained for $T = 1.028$ and $K = 0.1$.

characterized in the following expression [25],

$$\frac{\partial \rho_C}{\partial t} = (1-\rho_C)W_{D\to C} - \rho_C W_{C\to D}. \tag{4}$$

Interestingly, when the system reaches the steady state, the fraction of cooperators does not change over time, *i.e.*, $\frac{\partial \rho_C}{\partial t} = 0$. According to the above expression, the fraction of cooperators in steady states can be deduced as

$$\rho_C = \frac{W_{D\to C}}{W_{D\to C} + W_{C\to D}} = \frac{1}{1 + W_{C\to D}/W_{D\to C}}. \tag{5}$$

Fig. 3 depicts the numerical mean-field approximation and simulation results when the influence of the parameter α is taken into account. We find that numerically analytical results are consistent with simulations: in spite of tiny error, increasing α can greatly elevate the survivability of cooperators. For large α, an exclusive dominance of defectors is even guaranteed. We thus confirm that popularity-driven selection can improve the survivability of cooperation remarkably.

Subsequently, it is interesting to analyze typical spatial configurations of cooperators and defectors for different values of α, as shown in Fig. 4. For $\alpha = 0$, the applied temptation value does not sustain cooperation. As α enhances ($\alpha = 1$), a small fraction of cooperators can survive on the lattice by means of forming clusters, thereby protecting themselves against the exploitation by defectors. Moreover, it is worth mentioning that these clusters are usually small and detached; the distance among them is much larger than the size of clusters. With the continuous increment of α ($\alpha = 3$ and $\alpha = 5$), cooperators prevail and even reach their undisputed dominance, whereby clustering remains their mechanism of spreading and survivability. One can find, compared with the case of $\alpha = 1$, that the clusters of cooperators become larger and more compact, which further results in less space left for defectors. Thus, these illustrative snapshots attest to the fact that: when individual popularity is involved into the strategy updating, the sustainability of cooperation can be improved via expanding the clusters, which goes beyond the content of traditional spatial reciprocity [24,25].

Now that the popularity-driven selection rule enables the formation of extremely robust clusters of cooperators, it is significant to elucidate its potential mechanism. To answer this question, Fig. 5 shows the temporal traits of the fraction

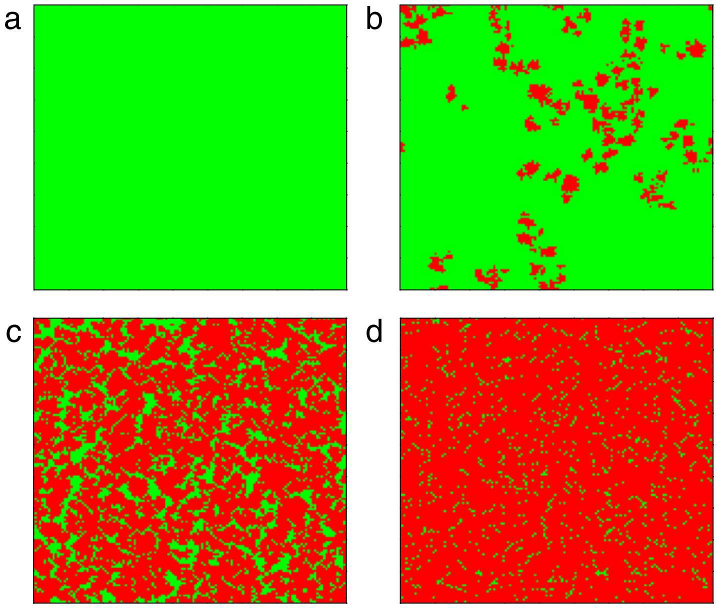

Fig. 4. Characteristic snapshots of cooperators (red) and defectors (green) for different values of the selection parameter α after 10^5 MCS. From (a) to (d), the values of α are 0, 1, 3 and 5, respectively. Noticeably, increasing α supports the formation of extremely robust clusters of cooperators. Depicted results are obtained for $T = 1.05$ and $K = 0.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

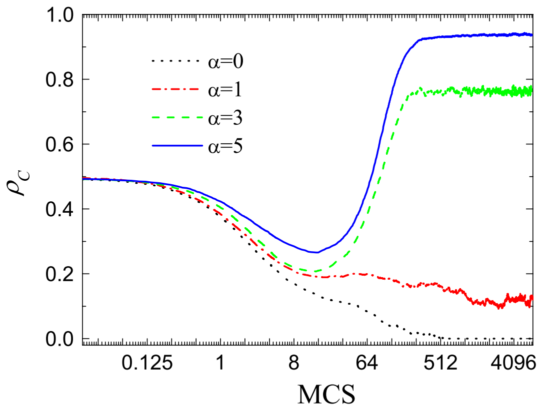

Fig. 5. (Color online) Time courses of the fraction of cooperation ρ_C for different values of the selection parameter α. Since the initial state is random and cooperators are not yet organized in compact groups, many of them are easy prey for defectors and there always is an initial decrease in their number. However, the increment of α reverts this trend to the undisputed dominance of cooperators. Depicted results are obtained for $T = 1.05$ and $K = 0.1$.

of cooperation ρ_c for different values of the selection parameter α. What first attracts our attention are the early stages of the evolution process, in which the performance of defectors is better than cooperators, irrespective of the α values. In fact, this is consistent with what one would expect, given that in mixed environment defectors are, as individuals, more successful than cooperators and will thus be chosen more likely as potential strategy donors. However, after this stage, the evolution destiny becomes completely different. For $\alpha = 0$ (namely, the classical version), the decimation of cooperators cannot halt and cooperative behavior dies out soon. As α increases ($\alpha = 1$), we can observe that the exploitation of defectors is effectively restrained and the spreading of cooperators is amplified. Strikingly, the larger the value of α, the more evident the reversal of cooperation. For large α ($\alpha = 5$), the initial decay of cooperation is restrained earlier, and finally cooperation reaches higher level (more than 90%). Based on these observations, we argue that the introduction of the selection parameter α accelerates the microscopic dynamics of cooperator clusters. In the early stages, the traditional version cannot organize effective clusters to resist the aggression of defectors. With the increase of α ($\alpha = 1$), cooperators try to organize clusters, but which are small and can only lead to a slight fraction of cooperation. Great promotion of cooperation is that for large α ($\alpha = 3$ and $\alpha = 5$), cooperators situated at the boundaries of clusters become imperious to defector attacks and even transfer the weakened defectors into their own compartments, which finally causes the undisputed dominance of cooperation.

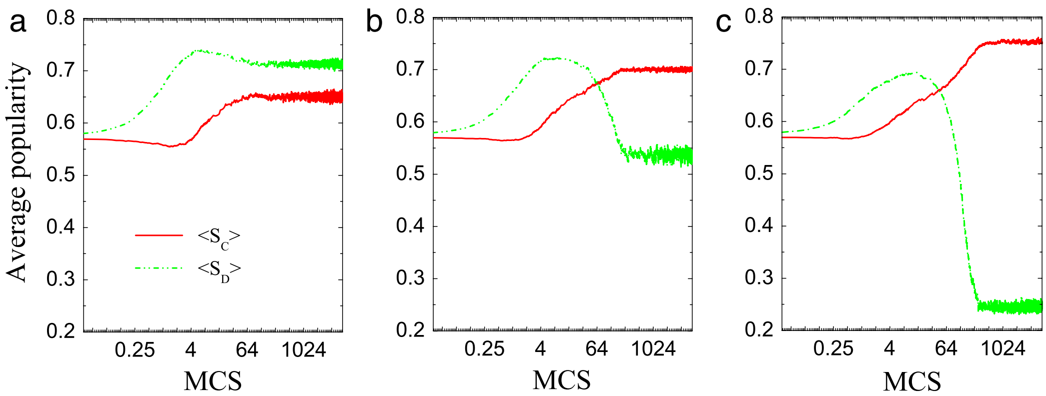

Fig. 6. (Color online) Time courses depicting the evolution of average popularity along the boundary of clusters $\langle S_C \rangle$ and $\langle S_D \rangle$ for different values of α. From (a) to (c), the values of α are 1, 3 and 5, respectively. (For $\alpha = 0$, cooperators die out fast; therefore the values of $\langle S_C \rangle$ and $\langle S_D \rangle$ make no sense, and we do not show this case.) As α increases, we can see that $\langle S_C \rangle$ suppresses the development of $\langle S_D \rangle$, which promotes the formation of extremely robust cooperator clusters. Depicted results are obtained for $T = 1.05$ and $K = 0.1$.

Along the above comment, we also investigate the time courses of two new statistical parameters defined as follows,

$$\langle S_C \rangle = \frac{\sum_i S_i^C}{N_C} \tag{6}$$

$$\langle S_D \rangle = \frac{\sum_i S_i^D}{N_D} \tag{7}$$

where S_i^C (S_i^D) is the popularity of agent i located at the boundary of cooperator (defector) clusters; N_C (N_D) denotes the total number of cooperators (defectors) along the boundary; and $\langle S_C \rangle$ ($\langle S_D \rangle$) represents the average popularity of these cooperators (defectors). Fig. 6 features the time courses of $\langle S_C \rangle$ and $\langle S_D \rangle$ for different values of α. One can find that in the early stages $\langle S_D \rangle$ monotonously increases ($\langle S_C \rangle$ decreases) due to the plunder of defectors. For $\alpha = 0$, cooperators die out fast; therefore there will be no agents along the boundary of clusters (correspondingly, the values of $\langle S_C \rangle$ and $\langle S_D \rangle$ make no sense, and here we do not show this case). As α increases ($\alpha = 1$), $\langle S_C \rangle$ reverts its decay and expands to a stable value, but which is still lower than $\langle S_D \rangle$. Importantly, the slight discrepancy between $\langle S_C \rangle$ and $\langle S_D \rangle$ can merely support small clusters of cooperators. However, for large α ($\alpha = 3$ and $\alpha = 5$), a different scenario takes place: enlarging $\langle S_C \rangle$ effectively suppresses the spreading of $\langle S_D \rangle$ and merely leaves limited space for $\langle S_D \rangle$, which in turn accelerates the expanding of cooperator clusters. Thus, we argue that this newly identified mechanism accelerates the microscopic organization of cooperator clusters, which ultimately leads to widespread cooperation.

Aiming to further support the explanation that the consideration of the selection parameter α accelerates the microscopic dynamics of cooperator clusters, it is instructive to examine how the cooperator cluster expands under the prepared initial state: namely, there exists a small cooperator domain in the center of the whole system (similar to the case of perfect cooperator cluster in a recent research [68]). On one hand, this setup is consistent with the treatment in many biological scenarios. On the other hand, the process of microscopic expansion of cooperator cluster could be shown more explicitly. Fig. 7(a) features the time courses of different α values for this special initial state. It is clear that, for large α ($\alpha = 3$ and 5), the cluster starts to expand at earlier stages and reaches the exclusive dominance state faster (namely, the expansion process becomes more effective and faster). Then in the bottom panel of Fig. 7, we present the typical evolution snapshots of this cluster. After the early interactions, this cluster becomes impervious to defector attacks and even attracts more agents penetrating into the cluster [69], which in turn amplifies the effective expansion of the cooperator domain. This newly identified mechanism ultimately results in widespread cooperation.

It remains of interest to explore how the sustainability of cooperation depends on the uncertainty by strategy adoptions. The levels of uncertainty can be tuned by K (see Eq. (3)), which acts as a temperature parameter in the employed Fermi strategy adoption function [61]. $K = 0$ and $K \to \infty$ denote the completely deterministic and completely random selections of the neighbor's strategy, whereas, for any finite positive values, K incorporates the uncertainties in the strategy. Fig. 8 shows the full $b - K$ phase diagrams for different values of α. The phase diagram in Fig. 8(a) ($\alpha = 0$), except for the monotonous increasing border between the pure C and mixed $C + D$ phases, features a bell shaped phase boundary separating the pure D and mixed $C + D$ phases, implying the existence of an optimal level of uncertainty ($K \approx 0.3$) for the evolution of cooperation. This phenomenon can be interpreted as an evolutionary resonance [70]. As α increases, this scenario changes drastically. The larger the value of α, the wider the space of both mixed $C + D$ and pure C phases. This phenomenon proves the fact once again that the introduction of the selection parameter α supports the presence of cooperation. Moreover, another interesting point is that increasing α gradually eradicates the existence of an optimal

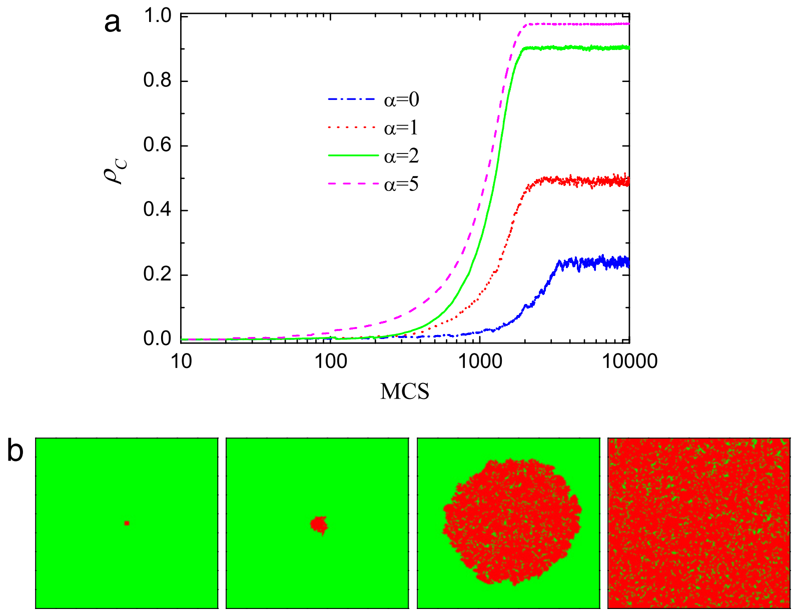

Fig. 7. (a) Time courses of the fraction of cooperation ρ_C for different values of α under the special initial state: only a small cooperator cluster in the center. It is obvious that the quantitatively identical values are obtained with Fig. 1. (b) Snapshots of the distribution of cooperators (red) and defectors (green) for $\alpha = 0.3$ at 0, 300, 1000, 5000 MCS from left to right. Depicted results are obtained for $T = 1.03$ and $K = 0.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

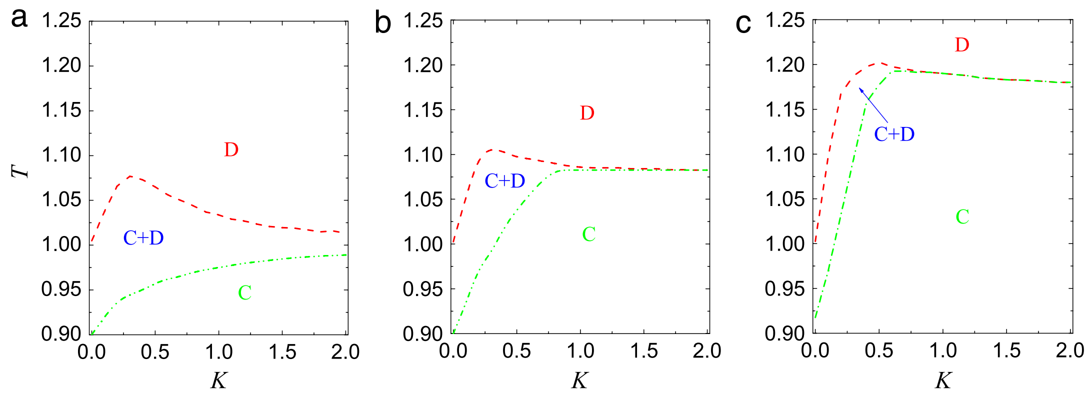

Fig. 8. (Color online) Full $T - K$ phase diagrams for different values of α. From (a) to (c), the values of α are 0, 1, and 3, respectively (since the phase diagram of $\alpha = 5$ is similar to the case of $\alpha = 3$, we do not show it here). As α increases, the space of mixed $C + D$ and pure C phases becomes larger, but the optimal uncertainty is less and less evident.

K on the $D \leftrightarrow C + D$ transition line. It has been conjectured that an optimal uncertainty can only occur on interaction graphs lacking a percolating cluster of overlapping triangles [71,72]. Therefore, if the conjecture proves to be valid, the above phase diagrams seem to indicate that with increasing α, there is a change in the effective interaction topology: previously disconnected triplets (triangles are absent in the square lattice) may now become effective triangles. A similar phenomenon was observed recently in public goods games as well [72].

Lastly, an important question is to examine the generality of this newly introduced mechanism on promoting cooperation. Due to the well-known claim that spatial interaction may inhibit the cooperation in the snowdrift game, the snowdrift game naturally becomes an appropriate candidate for this task [73]. While for the snowdrift game, payoffs satisfy the ranking $T > R > S > P$. Fig. 9 shows how cooperators fare on different interaction networks and snowdrift game. Similarly as in Fig. 1, a virtually identical impact can be observed: positive value of α promotes cooperation, while negative α restrains the emergence of cooperation. This agreement thus suggests that introducing popularity into strategy selection is a universally effective way in promoting cooperation and aiding the resolution of social dilemmas.

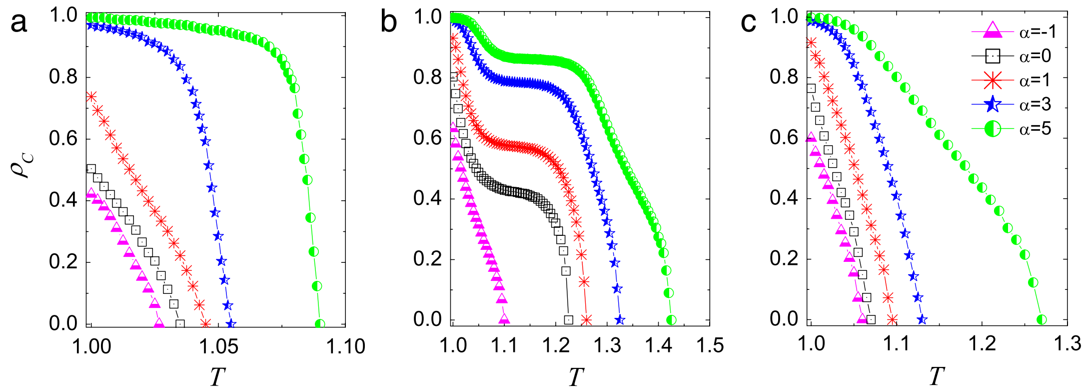

Fig. 9. (Color online) Fraction of cooperators ρ_C in dependence on the temptation to defect T for different values of α on honeycomb lattice (a), triangle lattice (b) and snowdrift game. For the snowdrift game, we choose $S = 0.03$ to guarantee the payoff ranking $T > R > S > P$. Similar to Fig. 1, increasing α promotes the evolution of cooperation, which proves the robustness of this newly introduced mechanism. Depicted results are obtained for $K = 0.1$.

4. Conclusion

Summarizing the results, we have shown that popularity-driven selection promotes the sustainability of cooperation in the spatial games, regardless of the underlying interaction networks. Through numerical calculation, the effect of this mechanism can be attributed to the formation of extremely robust cooperator clusters. Notably, enhancing the value of the selection parameter can help to accelerate microscopic organization of cooperator clusters, which become impervious to defector attacks even at high temptation. In such a case, the initial decay of cooperation will be reverted to undisputed dominance of cooperators. In addition, with the introduction of this new mechanism, the impact of uncertainty K on the evolution of cooperation changes as well. Although the survival space of cooperation (namely, mixed $C + D$ and pure C phases) broadens as the selection parameter increases, the optimal level of uncertainty seems less and less evident. According to previous studies [71,72], we conjecture that the newly introduced mechanism may cause the change of effective interaction topology. Since popularity-related phenomena are ubiquitous, we hope that this work can inspire more studies for resolving the social dilemmas, especially combining with more realistic models of populations.

Acknowledgments

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