Chapter 6

Inferences Based on a Single Sample
Tests of Hypothesis

6.2 The test statistic is used to decide whether or not to reject the null hypothesis in favor of the alternative hypothesis.

6.4 A Type I error is rejecting the null hypothesis when it is true.
A Type II error is accepting the null hypothesis when it is false.

α = the probability of committing a Type I error.
β = the probability of committing a Type II error.

6.6 We can compute a measure of reliability for rejecting the null hypothesis when it is true. This measure of reliability is the probability of rejecting the null hypothesis when it is true which is α. However, it is generally not possible to compute a measure of reliability for accepting the null hypothesis when it is false. We would have to compute the probability of accepting the null hypothesis when it is false, β, for **every** value of the parameter in the alternative hypothesis.

6.8 a. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

b. The rejection region requires $\alpha = .10$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.10} = 1.28$. The rejection region is $z > 1.28$.

c. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

d. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z > 1.96$ or $z < -1.96$.

6.10 Let μ = mean listening time of 16-month-old infants exposed to non-meaningful monosyllabic words. To see if the mean listening time of 16-month-old infants is different from 8 seconds, we test:

H_0: $\mu = 8$
H_a: $\mu \neq 8$

6.12 Let μ = mean calories in Virginia school lunches. To determine if the average caloric content of Virginia school lunches dropped from 863 calories, we test:

H_0: $\mu = 863$
H_a: $\mu < 863$

6.14 Let p = proportion of college presidents who believe that their online education courses are as good as or superior to courses that utilize traditional face-to-face instruction. To determine if the claim made by the Sloan Survey is correct, we test:

H_0: $p = .60$

6.16 a. Let μ = mean pain intensity reduction for trauma patients who receive normal analgesic care. To determine if the change in mean pain intensity for trauma patients who receive normal analgesic care is smaller than that for patients who received VRH training, we test:

H_0: $\mu = 10$
H_a: $\mu < 10$

b. A Type I error is rejecting the null hypothesis when it is true. For this test, a Type I error would be concluding the change in mean pain intensity for trauma patients who receive normal analgesic care is smaller than that for patients who received VRH training when, in fact, the change is not smaller.

c. A Type II error is accepting the null hypothesis when it is false. For this test, a Type II error would be concluding the change in mean pain intensity for trauma patients who receive normal analgesic care is not smaller than that for patients who received VRH training when, in fact, the change is smaller.

6.18 a. A Type I error is rejecting the null hypothesis when it is true. In a murder trial, we would be concluding that the accused is guilty when, in fact, he/she is innocent.

A Type II error is accepting the null hypothesis when it is false. In this case, we would be concluding that the accused is innocent when, in fact, he/she is guilty.

b. Both errors are bad. However, if an innocent person is found guilty of murder and is put to death, there is no way to correct the error. On the other hand, if a guilty person is set free, he/she could murder again.

c. In a jury trial, α is assumed to be smaller than β. The only way to convict the accused is for a unanimous decision of guilt. Thus, the probability of convicting an innocent person is set to be small.

d. In order to get a unanimous vote to convict, there has to be overwhelming evidence of guilt. The probability of getting a unanimous vote of guilt if the person is really innocent will be very small.

e. If a jury is prejudiced against a guilty verdict, the value of α will decrease. The probability of convicting an innocent person will be even smaller if the jury is prejudiced against a guilty verdict.

f. If a jury is prejudiced against a guilty verdict, the value of β will increase. The probability of declaring a guilty person innocent will be larger if the jury is prejudiced against a guilty verdict.

6.20 In a one-tailed test, the alternative hypothesis specifies that the population parameter is strictly greater than some value or strictly less than some value, but not both. In a two-tailed test, the alternative hypothesis specifies that the population parameter is either greater than or less than some value.

6.22 For values of the test statistic that fall in the rejection region, H_0 is rejected. The rejection region will include values of the test statistic that would be highly unusual if the null hypothesis were true.

For values of the test statistic that do not fall in the rejection region, H_0 would not be rejected. The values of the test statistic that would not fall in the rejection region would be those values that would not be unusual if the null hypothesis were true.

6.24 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{.323 - .36}{\frac{\sqrt{.034}}{\sqrt{64}}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part **a**).

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in the each tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

6.26 Let μ = mean student-driver response five months after a safe-driver presentation.

a. To determine if the true mean student-driver response five months after a safe-driver presentation is larger than 4.7, we test,

H_0: $\mu = 4.7$
H_a: $\mu > 4.7$

b. The test statistic is $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} \approx \dfrac{4.89 - 4.7}{\frac{1.62}{\sqrt{258}}} = 2.78.$

c. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

d. Since the observed value of the test statistic falls in the rejection region ($z = 2.78 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate the true mean student-driver response five months after a safe-driver presentation is greater than 4.7 at $\alpha = .05$.

e. Yes. We rejected H_0, that the mean response was equal to 4.7 in favor of H_a, that the mean response was greater than 4.7.

f. No. There were 258 responses. The Central Limit Theorem indicates that the distribution of $\bar{x}$ is approximately normal, regardless of the original distribution, as long as the sample size is sufficiently large.

6.28 a. The rejection region requires $\alpha = .01$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.01} = 2.33$. The rejection region is $z < -2.33$.

b. The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{19.3 - 20}{\frac{11.9}{\sqrt{46}}} = -.40$

c. Since the observed value of the test statistic does not fall in the rejection region ($z = -.40 \not< -2.33$), H_0 is not rejected. There is insufficient evidence to indicate the mean number of latex gloves used per week by hospital employees diagnosed with a latex allergy from exposure to the powder on latex gloves is less than 20 at $\alpha = .01$.

6.30 Let μ = mean heart rate during laughter.

a. To determine if the true mean heart rate during laughter exceeds 71 beats/minute, we test:

H_0: $\mu = 71$
H_a: $\mu > 71$

b. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

c. The test statistic is $z = \dfrac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} \approx \dfrac{73.5 - 71}{\frac{6}{\sqrt{90}}} = 3.95$

d. Since the observed value of the test statistic falls in the rejection region (z = 3.95 > 1.645). H_0 is rejected. There is sufficient evidence to indicate the true mean heart rate during laughter exceeds 71 beats/minute at $\alpha = .05$.

6.32 To determine if the area sampled is grassland, we test:

H_0: $\mu = 220$
H_a: $\mu \neq 220$

The test statistic is $z = \frac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} = \frac{225 - 220}{\frac{20}{\sqrt{100}}} = 2.50$

The rejection region requires $\alpha / 2 = .01/2 = .005$ in each tail of the z distribution. From Table III, Appendix A, $z_{.005}$ = 2.58. The rejection region is z > 2.58 or z < −2.58.

Since the observed value of the test statistic does not fall in the rejection region (z = 2.50 $\not>$ 2.58). H_0 is not rejected. There is insufficient evidence to indicate the true mean lacunarity measurements is different from 220 at $\alpha = .01$. There is insufficient evidence to indicate that the sampled area is not grassland.

6.34 a. Using MINITAB, some preliminary calculations are:

Descriptive Statistics: Heat

```
Variable   N   Mean  StDev  Minimum    Q1  Median     Q3  Maximum
Heat      67  11066   1595     8714  9918   10656  11842    16243
```

To determine if the mean heat rate of gas turbines augmented with high pressure inlet fogging exceeds 10,000kJ/kWh, we test:

H_0: $\mu = 10{,}000$
H_a: $\mu > 10{,}000$

The test statistic is $z = \frac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} = \frac{11{,}066 - 10{,}000}{\frac{1595}{\sqrt{67}}} = 5.47$

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05}$ = 1.645. The rejection region is z > 1.645.

Since the observed value of the test statistic falls in the rejection region (z = 5.47 > 1.645). H_0 is rejected. There is sufficient evidence to indicate the true mean heat rate of gas turbines augmented with high pressure inlet fogging exceeds 10,000kJ/kWh at $\alpha = .05$.

b. A Type I error is rejecting H_0 when H_0 is true. In this case, it would be concluding that the true mean heat rate of gas turbines augmented with high pressure inlet fogging exceeds 10,000kJ/kWh when, in fact, it does not.

A Type II error is accepting H_o when H_o is false. In this case, it would be concluding that the true mean heat rate of gas turbines augmented with high pressure inlet fogging does not exceed 10,000kJ/kWh when, in fact, it does.

6.36 Let μ = mean DOC value of all Wisconsin lakes. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: DOC

```
Variable   N   Mean  StDev  Minimum    Q1  Median     Q3  Maximum
DOC       25  14.52  12.96     2.40  4.15   13.20  19.10    56.90
```

To determine if the sample is representative of all Wisconsin's lakes for DOC, we test:

H_0: $\mu = 15$
H_a: $\mu \neq 15$

The test statistic is $z = \dfrac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} \approx \dfrac{14.52 - 15}{\frac{12.96}{\sqrt{25}}} = -0.19$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the z distribution. From Table III, Appendix A, $z_{.05}$ = 1.645. The rejection region is $z > 1.645$ or $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.19 \nless -1.645$). H_0 is not rejected. There is insufficient evidence to indicate the true mean DOC value is different from 15 at $\alpha = .10$. Thus, we can conclude that there is no evidence to indicate the sample is not representative of the entire population.

6.38 The observed significance level or p-value of a test is the probability of observing your test statistic or anything more unusual, given H_0 is true. The value of α is the significance level of a test. It is the probability of rejecting H_0 when H_0 is true.

6.40 We will reject H_0 if the p-value < α.

a. $.06 \nless .05$, do not reject H_0.

b. $.10 \nless .05$, do not reject H_0.

c. $.01 < .05$, reject H_0.

d. $.001 < .05$, reject H_0.

e. $.251 \nless .05$, do not reject H_0.

f. $.042 < .05$, reject H_0.

6.42 The smallest value of α for which the null hypothesis would be rejected is just greater than .06.

6.44 p-value = $P(z \geq 2.17) + P(z \leq -2.17) = (.5 - .4850)2 = .0300$ (using Table III, Appendix A)

6.46 First, find the value of the test statistic:

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{10.7 - 10}{3.1/\sqrt{50}} = 1.60$$

p-value = $P(z \leq -1.60$ or $z \geq 1.60) = 2P(z \geq 1.60) = 2(.5 - .4452) = 2(.0548) = .1096$

There is no evidence to reject H_0 for $\alpha \leq .10$.

6.48 a. From the printout, the p-value is $p = .003$. The p-value measures the probability of observing your test statistic or anything more unusual if H_0 is true.

b. Sine the p-value is less than α ($p = .003 < .05$), H_0 is rejected. There is sufficient evidence to indicate the true mean student-driver response five months after a safe-driver presentation is greater than 4.7 at $\alpha = .05$. This agrees with the answer to Exercise 6.26c.

6.50 a. From Exercise 6.28, $z = -.40$. The p-value is $p = P(z \leq -.40) = .5 - .1554 = .3446$ (using Table III, Appendix A).

b. The p-value is $p = .3446$. Since the p-value is greater than $\alpha = .01$, H_0 is not rejected. There is insufficient evidence to indicate the mean number of latex gloves used per week by hospital employees diagnosed with a latex allergy from exposure to the powder on latex gloves is less than 20 at $\alpha = .01$.

6.52 Let μ = mean emotional empathy score for females.

a. To test whether female college students score higher than 3.0 on the emotional empathy scale, we test:

H_0: $\mu = 3.0$
H_a: $\mu > 3.0$

b. The test statistic is $z = \dfrac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} \approx \dfrac{3.28 - 3.0}{\frac{.5}{\sqrt{30}}} = 3.07$

c. The p-value is $p = P(z \geq 3.07) = .5 - .4989 = .0011$.

d. Since the p-value is less than $\alpha = .01$ ($p = .0011 < .01$), H_0 is rejected. There is sufficient evidence to indicate female college students score higher than 3.0 on the emotional empathy scale at $\alpha = .01$.

e. The value of α can be just greater than .0011 and we could still reject H_0.

6.54 a. If chickens are more apt to peck at white string, then they are less apt to peck at blue string. Let μ = mean number of pecks at a blue string. To determine if chickens are more apt to peck at white string than blue string (or less apt to peck at blue string), we test:

H_0: $\mu = 7.5$
H_a: $\mu < 7.5$

The test statistic is $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{1.13 - 7.5}{\frac{2.21}{\sqrt{72}}} = -24.46$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = -24.46 < -1.645$), H_0 is rejected. There is sufficient evidence to indicate the chickens are less apt to peck at blue string at $\alpha = .05$.

b. In Exercise 5.21 b, we concluded that the birds were more apt to peck at white string. The mean number of pecks for white string is 7.5. Since 7.5 is not in the 99% confidence interval for the mean number of pecks at blue string, it is not a likely value for the true mean for blue string.

c. The p-value is $P(z \le -24.46) = .5 - .5 \approx 0$. Since the p-value is smaller than α ($p = 0 < .05$), H_0 is rejected. There is sufficient evidence to indicate the chickens are less apt to peck at blue string at $\alpha = .05$.

6.56 a. $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{52.3 - 51}{7.1/\sqrt{50}} = 1.29$

$$p\text{-value} = P(z \le -1.29) + P(z \ge 1.29) = (.5 - .4015) + (.5 - .4015)$$
$$= .0985 + .0985 = .1970$$

b. $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{52.3 - 51}{7.1/\sqrt{50}} = 1.29$

$$p\text{-value} = P(z \ge 1.29) = (.5 - .4015) = .0985$$

c. $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{52.3 - 51}{10.4/\sqrt{50}} = 0.88$

$$p\text{-value} = P(z \le -0.88) + P(z \ge 0.88) = (.5 - .3106) + (.5 - .3106)$$
$$= .1894 + .1894 = .3788$$

d. For part **a**, any value of α greater than .1970 would lead to the rejection of the null hypothesis.

For part **b**, any value of α greater than .0985 would lead to the rejection of the null hypothesis.

For part **c**, any value of α greater than .3788 would lead to the rejection of the null hypothesis.

e. For p-value of .01 and a one-tailed test, we need to find a z-value so that .01 is to the right of it. From Table III, Appendix A, $z_{.01} = 2.33$.

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} \Rightarrow 2.33 = \frac{52.3 - 51}{s/\sqrt{50}} \Rightarrow 2.33 \frac{s}{\sqrt{50}} = 1.3 \Rightarrow s = \frac{1.3(\sqrt{50})}{2.33} \Rightarrow s = 3.95$$

For any value of s less than or equal to 3.95, the p-value will be less than or equal to .01 for a one-tailed test.

6.58 We should use the t distribution in testing a hypothesis about a population mean if the sample size is small, the population being sampled from is normal, and the variance of the population is unknown.

6.60 $\alpha = P(\text{Type I error}) = P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$

a. $\alpha = P(t > 1.440)$ where df = 6
$= .10$ Table IV, Appendix A

b. $\alpha = P(t < -1.782)$ where df = 12
$= P(t > 1.782)$
$= .05$ Table IV, Appendix A

c. $\alpha = P(t < -2.060 \text{ or } t > 2.060)$ where df = 25
$= 2P(t > 2.060)$
$= 2(.025)$
$= .05$ Table IV, Appendix A

6.62 For this sample,

$$\bar{x} = \frac{\sum x}{n} = \frac{11}{6} = 1.8333 \qquad s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{41 - \frac{11^2}{6}}{6-1} = 4.1667$$

$$s = \sqrt{s^2} = 2.0412$$

a. H_0: $\mu = 3$
H_a: $\mu < 3$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{1.8333 - 3}{2.0412/\sqrt{6}} = -1.40$

The rejection region requires $\alpha = .05$ in the lower tail of the t distribution with df = $n - 1$ = $6 - 1 = 5$. From Table IV, Appendix A, $t_{.05} = 2.015$. The rejection region is $t < -2.015$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \not< -2.015$), H_0 is not rejected. There is insufficient evidence to indicate μ is less than 3 at $\alpha = .05$.

b. H_0: $\mu = 3$
H_a: $\mu \neq 3$

Test statistic: $t = -1.40$ (Refer to part **a**.)

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution with df = $n - 1 = 6 - 1 = 5$. From Table IV, Appendix A, $t_{.025} = 2.571$. The rejection region is $t < -2.571$ or $t > 2.571$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \not< -2.571$), H_0 is not rejected. There is insufficient evidence to indicate μ differs from 3 at $\alpha = .05$.

c. For part **a**: p-value = $P(t \leq -1.40)$

From Table IV, with df = 5, $P(t \leq -1.40) > .10$.
Using MINITAB, the p-value is $p = P(t \leq -1.40) = .110202$.

For part **b**: p-value = $P(t \leq -1.40) + P(t \geq 1.40)$

From Table IV, with df = 5, p-value = $2P(t \geq 1.40) > 2(.10) = .20$
Using MINITAB, the p-value is $p = 2P(t \leq -1.40) = 2(.110202) = .220404$.

6.64 a. We must assume that a random sample was drawn from a normal population.

b. The hypotheses are:

H_0: $\mu = 1000$
H_a: $\mu > 1000$

The test statistic is $t = 1.89$ and the p-value is .038.

There is evidence to reject H_0 for $\alpha > .038$. There is evidence to indicate the mean is greater than 1000 for $\alpha > .038$.

c. The hypotheses are:

H_0: $\mu = 1000$
H_a: $\mu \neq 1000$

The test statistic is $t = 1.89$ and the p-value is $p = 2(.038) = .076$.

There is evidence to reject H_0 for $\alpha > .076$. There is evidence to indicate the mean is different than 1000 for $\alpha > .076$.

6.66 a. To determine if the average number of books read by all students who participate in the extensive reading program exceeds 25, we test:

H_0: $\mu = 25$
H_a: $\mu > 25$

b. The rejection region requires $\alpha = .05$ in the upper tail of the t distribution with df = n – 1 = 14 – 1 = 13. From Table IV, Appendix A, $t_{.05} = 1.771$. The rejection region is $t > 1.771$.

c. The test statistic is $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{31.64 - 25}{\frac{10.49}{\sqrt{14}}} = 2.37$

d. Since the observed value of the test statistic falls in the rejection region ($t = 2.37 > 1.771$), H_0 is rejected. There is sufficient evidence to indicate that the average number of books read by all students who participate in the extensive reading program exceeds 25 at $\alpha = .05$.

e. The conditions required for this test are a random sample from the target population and the population from which the sample is selected is approximately normal.

f. From the printout, the p-value is $p = .017$. Since the p-value is smaller than $\alpha = .05$ ($p = .017 < .05$), H_0 is rejected. There is sufficient evidence to indicate that the average number of books read by all students who participate in the extensive reading program exceeds 25 at $\alpha = .05$.

6.68 a. The parameter of interest is μ = mean chromatic contrast of crab-spiders on daisies.

b. To determine if the mean chromatic contrast of crab-spiders on daisies is less than 70, we test:

H_0: $\mu = 70$
H_a: $\mu < 70$

c. $\bar{x} = \dfrac{\sum x}{n} = \dfrac{575}{10} = 57.5$

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{42,649 - \frac{575^2}{10}}{10-1} = \frac{9,586.5}{9} = 1,065.1667$$

$s = \sqrt{1,065.1667} = 32.6369$

The test statistic is $t = \frac{\bar{x} - \mu_o}{s/\sqrt{n}} = \frac{57.5 - 70}{\frac{32.6369}{\sqrt{10}}} = -1.21$

d. The rejection region requires $\alpha = .10$ in the lower tail of the t-distribution with $df = n - 1 = 10 - 1 = 9$. From Table IV, Appendix A, $t_{.10} = 1.383$. The rejection region is $t < -1.383$.

e. Since the observed value of the test statistic does not fall in the rejection region ($t = -1.21 \nless -1.383$), H_0 is not rejected. There is insufficient evidence to indicate that the mean chromatic contrast of crab-spiders on daisies is less than 70 at $\alpha = .10$.

6.70 Let μ = mean score difference between the first trial and the second trial.

a. To determine if the true mean score difference exceeds 0, we test:

H_0: $\mu = 0$
H_a: $\mu > 0$

b. The test statistic is $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{.11 - 0}{\frac{.19}{\sqrt{17}}} = 2.387$.

The rejection region requires $\alpha = .05$ in the upper tail of the t distribution, with df = $n - 1 = 17 - 1 = 16$. From Table IV, Appendix A, $t_{.05} = 1.746$. The rejection region is $t > 1.746$.

Since the observed value of the test statistic falls in the rejection region ($t = 2.387 > 1.746$), H_0 is rejected. There is sufficient evidence to indicate that the true mean score difference exceeds 0 at $\alpha = .05$.

6.72 Let μ = mean breaking strength of the new bonding adhesive.

To determine if the mean breaking strength of the new bonding adhesive is less than 5.70 Mpa, we test:

H_0: $\mu = 5.70$
H_a: $\mu < 5.70$

The test statistic is $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{5.07 - 5.70}{\frac{.46}{\sqrt{10}}} = -4.33$.

The rejection region requires $\alpha = .01$ in the lower tail of the t distribution with df = $n - 1 = 10 - 1 = 9$. From Table IV, Appendix A, $t_{.01} = 2.821$. The rejection region is $t < -2.821$.

Since the observed value of the test statistic falls in the rejection region ($t = -4.33 < -2.821$), H_o is rejected. There is sufficient evidence to indicate that the mean breaking strength of the new bonding adhesive is less than 5.70 Mpa at $\alpha = .01$.

6.74 Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Species

```
Variable   N  Mean  StDev  Minimum    Q1  Median    Q3  Maximum
Species   11 12.82  18.68     3.00  4.00    5.00  7.00    52.00
```

To determine if the average number of ant species at Mongolian desert sites differs from 5 species, we test:

H_0: $\mu = 5$
H_a: $\mu \neq 5$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{12.82 - 5}{18.68/\sqrt{11}} = 1.391$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution with df = $n - 1 = 11 - 1 = 10$. From Table IV, Appendix A, $t_{.025} = 2.228$. The rejection region is $t < -2.228$ or $t > 2.228$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.391 \ngtr 2.228$), H_0 is not rejected. There is insufficient evidence to indicate the average number of ant species at Mongolian desert sites differs from 5 species at $\alpha = .05$.

Using MINITAB, the stem-and-leaf display for the data is:

Stem-and-Leaf Display: Species

```
Stem-and-leaf of Species  N  = 11
Leaf Unit = 1.0

(9)  0  334445557
 2   1
 2   2
 2   3
 2   4  9
 1   5  2
```

One of the conditions for the above test is that the sample comes from a normal distribution. From the above stem-and-leaf display, the data do not look mound-shaped. This condition is probably not met.

6.76 Some preliminary calculations:

$$\bar{x} = \frac{\sum x}{n} = \frac{66}{3} = 22$$

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{1460 - \frac{66^2}{3}}{3-1} = 4 \qquad s = \sqrt{4} = 2$$

To determine if the mean length of great white sharks off the Bermuda coast exceeds 21 feet, we test:

H_0: $\mu = 21$
H_a: $\mu > 21$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{22-21}{2/\sqrt{3}} = 0.87$

The rejection region requires $\alpha = .10$ in the upper tail of the t distribution with df = n − 1 = 3 − 1 = 2. From Table IV, Appendix A, $t_{.10}$ = 1.886. The rejection region is z > 1.886.

Since the observed value of the test statistic does not fall in the rejection region ($t = 0.87 \not> 1.886$), H_0 is not rejected. There is insufficient evidence to indicate that the mean length of the great white sharks off the Bermuda coast exceeds 21 feet at $\alpha = .10$.

6.78 The conditions required for a valid large-sample test for p are a random sample from a binomial population and a large sample size n. The sample size is considered large if both np_o and nq_o are at least 15.

6.80 a. Because $\hat{p} = .69$ is less than the hypothesized value of .75, intuition tells us that this does contradict the null hypothesis that p = .75.

b. In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

np_o = 100(.75) = 75 and nq_o = 100(1 − .75) = 100(.25) = 25. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

H_0: p = .75
H_a: p < .75

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.69 - .75}{\sqrt{\dfrac{.75(.25)}{100}}} = -1.39$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.05}$ = 1.645. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($-1.39 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .75 at $\alpha = .05$.

c. p-value = $P(z \le -1.39) = .5 - .4177 = .0823$. Since the p-value is not less than α ($p = .0823 \not< .05$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .75 at $\alpha = .05$.

6.82 a. H_0: $p = .65$
H_a: $p > .65$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.74 - .65}{\sqrt{\dfrac{.65(.35)}{100}}} = 1.89$

The rejection region requires $\alpha = .01$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic does not fall in the rejection region ($z = 1.89 \not> 2.33$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is greater than .65 at $\alpha = .01$.

b. H_0: $p = .65$
H_a: $p > .65$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.74 - .65}{\sqrt{\dfrac{.65(.35)}{100}}} = 1.89$

The rejection region requires $\alpha = .10$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.10} = 1.28$. The rejection region is $z > 1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = 1.89 > 1.28$), H_0 is rejected. There is sufficient evidence to indicate that the proportion is greater than .65 at $\alpha = .10$.

c. H_0: $p = .90$
H_a: $p \neq .90$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.74 - .90}{\sqrt{\dfrac{.90(.10)}{100}}} = -5.33$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic falls in the rejection region ($z = -5.33 < -1.96$), H_0 is rejected. There is sufficient evidence to indicate that the proportion is different from .90 at $\alpha = .05$.

d. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, Appendix A, $z_{.025}$ = 1.96. The confidence interval is:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .74 \pm 1.96\sqrt{\frac{(.74)(.26)}{100}} \Rightarrow .74 \pm .09 \Rightarrow (.65, .83)$$

e. For confidence coefficient .99, $\alpha = .01$ and $\alpha / 2 = .01 / 2 = .005$. From Table III, Appendix A, $z_{.005}$ = 2.58. The confidence interval is:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .74 \pm 2.58\sqrt{\frac{(.74)(.26)}{100}} \Rightarrow .74 \pm .11 \Rightarrow (.63, .85)$$

6.84 a. Let p = proportion of satellite radio subscribers who have a satellite receiver in their car.

b. To determine if the true proportion of satellite radio subscribers who have satellite radio receivers in their car is too high, we test:

H_0: $p = .80$

c. The alternative hypothesis would be:

H_a: $p < .80$

d. The sample proportion is $\hat{p} = \frac{396}{501} = .79$. The test statistic is:

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.79 - .80}{\sqrt{\frac{.8(1-.8)}{501}}} = -.56$$

e. The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.10}$ = 1.28. The rejection region is z < -1.28.

f. The p-value is $p = P(z \le -.56) = P(z \ge .56) = (.5 - .2123) = .2877$. (From Table III, Appendix A.)

g. Since the p-value is greater than $\alpha = .10$ (p = .2877>.10), H_0 is not rejected. There is insufficient evidence to indicate the true proportion of satellite radio subscribers who have satellite radio receivers in their car is less than .80 at $\alpha = .10$.

Since the observed value of the test statistic does not fall in the rejection region (z = -.56 $\nless$ -1.28), H_0 is not rejected. There is insufficient evidence to indicate the true proportion of satellite radio subscribers who have satellite radio receivers in their car is less than .80 at $\alpha = .10$.

6.86 a. Let p = true proportion of students who correctly identify the color. If there is no relationship between color and flavor, then p = .5.

b. To determine if color and flavor are related, we test:

H_0: $p = .5$
H_a: $p \neq .5$

c. In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 121(.5) = 60.5$ and $nq_o = 121(1 - .5) = 121(.5) = 60.5$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

The sample proportion is $\hat{p} = \frac{97}{121} = .80$. The test statistic is:

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.80 - .5}{\sqrt{\frac{.5(1-.5)}{121}}} = 6.60$$

The p-value is $p = P(z \geq 6.60) + P(z \leq -6.60) \approx (.5 - .5) + (.5 - .5) = 0$. Since the p-value is less than α ($p = 0 < .01$), H_0 is rejected. There is sufficient evidence to indicate color and flavor are related at $\alpha = .01$.

6.88 Let p = true proportion of all U.S. teenagers who have used at least one informal element in school writing assignments.

The sample proportion is $\hat{p} = \frac{448}{700} = .64$.

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 700(.65) = 455$ and $nq_o = 700(1 - .65) = 700(.35) = 245$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if less than 65% of all U.S. teenagers have used at least one informal element in school writing assignments, we test:

H_0: $p = .65$
H_a: $p < .65$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.64 - .65}{\sqrt{\frac{.65(1-.65)}{700}}} = -.55$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.55 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that less than 65% of all U.S. teenagers have used at least one informal element in school writing assignments at $\alpha = .05$.

6.90 a. $\hat{p} = \frac{x}{n} = \frac{315}{500} = .63$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 500(.60) = 300$ and $nq_o = 500(1 - .60) = 500(.40) = 200$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the GSR for all scholarship athletes at Division I institutions differs from 60%, we test:

H_0: $p = .60$
H_a: $p \neq .60$

The test statistic is $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{.63 - .60}{\sqrt{\frac{.60(.40)}{500}}} = 1.37$

The rejection region requires $\alpha / 2 = .01/2 = .005$ in each tail of the z-distribution. From Table III, Appendix A, $z_{.005} = 2.58$. The rejection region is $z > 2.58$ or $z < -2.58$.

Since the observed value of the test statistic does not fall in the rejection region ($z = 1.37 \not> 2.58$), H_0 is not rejected. There is insufficient evidence to indicate the GSR for all scholarship athletes at Division I institutions differs from 60% at $\alpha = .01$.

b. $\hat{p} = \frac{x}{n} = \frac{84}{200} = .42$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 200(.58) = 116$ and $nq_o = 200(1 - .58) = 200(.42) = 84$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the GSR for all male basketball players at Division I institutions differs from 58%, we test:

H_o: $p = .58$
H_a: $p \neq .58$

The test statistic is $z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}} = \dfrac{.42 - .58}{\sqrt{\dfrac{.58(.42)}{200}}} = -4.58$

The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in each tail of the z-distribution. From Table III, Appendix A, $z_{.005} = 2.58$. The rejection region is $z > 2.58$ or $z < -2.58$.

Since the observed value of the test statistic falls in the rejection region ($z = -4.58 < -2.58$), H_0 is rejected. There is sufficient evidence to indicate the GSR for all male basketball players at Division I institutions differs from 58% at $\alpha = .01$.

6.92 **For the Top of the Core:**

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 84(.50) = 42$ and $nq_o = 84(1 - .50) = 84(.50) = 42$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

$$\hat{p} = \frac{x}{n} = \frac{64}{84} = .762$$

To determine if the coat index exceeds .5, we test:

H_0: $p = .5$
H_a: $p > .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.762 - .5}{\sqrt{\dfrac{.5(.5)}{84}}} = 4.80$

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = 4.80 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the coat index exceeds .5 at $\alpha = .05$.

For the Middle of the Core:

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 73(.50) = 36.5$ and $nq_o = 73(1 - .50) = 73(.50) = 36.5$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

$$\hat{p} = \frac{x}{n} = \frac{35}{73} = .479$$

To determine if the coat index differs from .5, we test:

H_0: $p = .5$
H_a: $p \neq .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.479 - .5}{\sqrt{\dfrac{.5(.5)}{73}}} = -.36$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.36 \not< -1.96$), H_0 is not rejected. There is insufficient evidence to indicate that the coat index differs from .5 at $\alpha = .05$.

For the Bottom of the Core:

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 81(.50) = 40.5$ and $nq_o = 81(1 - .50) = 81(.50) = 40.5$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

$$\hat{p} = \frac{x}{n} = \frac{29}{81} = .358$$

To determine if the coat index is less than .5, we test:

H_0: $p = .5$
H_a: $p < .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.358 - .5}{\sqrt{\dfrac{.5(.5)}{81}}} = -2.56$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = -2.56 < -1.645$), H_0 is rejected. There is sufficient evidence to indicate that the coat index is less than .5 at $\alpha = .05$.

6.94 Let p = true mortality rate for weevils exposed to nitrogen.

The point estimate for the population parameter is

$$\hat{p} = \frac{x}{n} = \frac{31{,}386}{31{,}386 + 35} = \frac{31{,}386}{31{,}421} = .9989.$$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 31{,}421(.99) = 31{,}106.79$ and $nq_o = 31{,}421(1 - .99) = 31{,}421(.01) = 314.21$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the true mortality rate for weevils exposed to nitrogen is higher than 99%, we test:

H_0: $p = .99$
H_a: $p > .99$

The test statistic is $z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}} = \dfrac{.9989 - .99}{\sqrt{\dfrac{.99(.01)}{31{,}421}}} = 15.86$

The p-value for the test is $p = P(z \geq 15.86) \approx .5 - .5 = 0$. (From Table III, Appendix A.) Since the p-value is so small, we would reject H_0 for any reasonable value of α. There is sufficient evidence to indicate the true mortality rate for weevils exposed to nitrogen is higher than 99%.

From the study, the entomologists could conclude that the mortality rate for nitrogen after 24 hours is higher than that for carbon dioxide after 4 days.

6.96 The sampling distribution used for making inferences about σ^2 is the chi-squared distribution.

6.98 The statement "The null hypotheses, H_0: $\sigma^2 = 25$ and H_0: $\sigma = 5$, are equivalent" is true. We know that the standard deviation (σ) can only be the positive square root of the variance (σ^2).

6.100 Using Table V, Appendix A:

a. For $n = 12$, df $= n - 1 = 12 - 1 = 11$

$P(\chi^2 > \chi_0^2) = .10 \Rightarrow \chi_0^2 = 17.2750$

b. For $n = 9$, df $= n - 1 = 9 - 1 = 8$

$P(\chi^2 > \chi_0^2) = .05 \Rightarrow \chi_0^2 = 15.5073$

c. For $n = 5$, df $= n - 1 = 5 - 1 = 4$

$P(\chi^2 > \chi_0^2) = .025 \Rightarrow \chi_0^2 = 11.1433$

6.102 a. It would be necessary to assume that the population has a normal distribution.

b. H_0: $\sigma^2 = 1$
H_a: $\sigma^2 > 1$

The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(7-1)(1.84)}{1} = 11.04$

The rejection region requires $\alpha = .05$ in the upper tail of the χ^2 distribution with df = n − 1 = 7 − 1 = 6. From Table V, Appendix A, $\chi^2_{.05} = 12.5916$. The rejection region is $\chi^2 > 12.5916$.

Since the observed value of the test statistic does not fall in the rejection region $(\chi^2 = 11.04 \not> 12.5916)$, H_0 not is rejected. There is insufficient evidence to indicate that the variance is greater than 1 at $\alpha = .05$.

c. Using Table V, with df = n – 1 = 7 – 1 = 6, the p-value would be $p = P(\chi^2 \geq 14.45) = .025$.

d. H_0: $\sigma^2 = 1$
H_a: $\sigma^2 \neq 1$

The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(7-1)1.84}{1} = 11.04$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the χ^2 distribution with df = n − 1 = 7 − 1 = 6. From Table V, Appendix A, $\chi^2_{.975} = 1.237347$ and $\chi^2_{.025} = 14.4494$. The rejection region is $\chi^2 < 1.237347$ or $\chi^2 > 14.4494$.

Since the observed value of the test statistic does not fall in the rejection region $(\chi^2 = 11.04 \not> 14.4494$ and $\chi^2 = 11.04 \not< 1.237347)$, H_0 is not rejected. There is insufficient evidence to indicate that the variance is not equal to 1 at $\alpha = .05$.

6.104 Some preliminary calculations are:

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{176 - \frac{30^2}{7}}{7-1} = 7.9048$$

To determine if $\sigma^2 < 2$, we test:

H_0: $\sigma^2 = 2$
H_a: $\sigma^2 < 2$

The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(7-1)7.9048}{2} = 23.71$

The rejection region requires $\alpha = .05$ in the lower tail of the χ^2 distribution with df $= n - 1 = 7 - 1 = 6$. From Table V, Appendix A, $\chi^2_{.95} < 1.63539$. The rejection region is $\chi^2 < 1.63539$.

Since the observed value of the test statistic does not fall in the rejection region $\left(\chi^2 = 23.71 \nless 1.63539\right)$, H_0 is not rejected. There is insufficient evidence to indicate the variance is less than 1 at $\alpha = .05$.

6.106 a. To determine if the breaking strength variance differs from .5 Mpa, We test:

H_0: $\sigma^2 = .5$
H_a: $\sigma^2 \neq .5$

b. The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in the upper tail of the χ^2 distribution with df = $n - 1 = 10 - 1 = 9$. From Table V, Appendix A, $\chi^2_{.995} = 1.734926$ and $\chi^2_{.005} = 23.5893$. The rejection region is $\chi^2 < 1.734926$ or $\chi^2 > 25.5893$.

c. The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(10-1)(.46)^2}{.5} = 3.809$.

d. Since the observed value of the test statistic does not fall in the rejection region ($\chi^2 = 3.809 \nless 1.734926$ and $\chi^2 = 3.809 \ngtr 25.5893$), H_0 is not rejected. There is insufficient evidence to indicate the breaking strength variance differs from .5 Mpa at $\alpha = .01$.

e. The conditions required for the test results to be valid are:

1. A random sample is selected from the target population.
2. The population from which the sample is selected has a distribution that is approximately normal.

6.108 a. If the standard deviation is less than 22, then the variance is less than $22^2 = 484$. To determine if the standard deviation of all grassland pixels is less than 22, we test:

H_0: $\sigma^2 = 484$
H_a: $\sigma^2 < 484$

b. The p-value of the test is $p = .105$. Since the p-value is not less than $\alpha = .10$ ($p = .105 \nless .10$), H_o is not rejected. There is insufficient evidence to indicate the standard deviation of all grassland pixels is less than 22 at $\alpha = .10$.

6.110 Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Species

```
Variable   N  Mean  StDev  Minimum    Q1  Median    Q3  Maximum
Species   11 12.82  18.68     3.00  4.00    5.00  7.00    52.00
```

To determine if the standard deviation of the number of species at all Mongolian desert sites exceeds 15 species (variance exceeds $15^2 = 225$), we test:

H_0: $\sigma^2 = 225$
H_a: $\sigma^2 > 225$

The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{(11-1)(18.68^2)}{225} = 15.51$.

The rejection region requires $\alpha = .05$ in the upper tail of the χ^2 distribution with df = n – 1 = 11 – 1 = 10. From Table V, Appendix A, $\chi^2_{.05} = 18.3070$. The rejection region is $\chi^2 > 18.3070$.

Since the observed value of the test statistic does not fall in the rejection region ($\chi^2 = 15.51 \not> 18.3070$), H_0 is not rejected. There is insufficient evidence to indicate the standard deviation of the number of species at all Mongolian desert sites exceeds 15 species (variance exceeds $15^2 = 225$) at $\alpha = .05$.

The conditions required for the test results to be valid are:

1. A random sample is selected from the target population.
2. The population from which the sample is selected has a distribution that is approximately normal.

A stem-and-leaf display of the data is:

Stem-and-Leaf Display: Species

```
Stem-and-leaf of Species  N  = 11
Leaf Unit = 1.0

(9)  0  334445557
 2   1
 2   2
 2   3
 2   4  9
 1   5  2
```

The condition that the data come from a normal distribution does not appear to be met.

6.112 From Exercise 6.34, s = 1,595. To determine if the heat rates of the augmented gas turbine engine are more variable than the heat rates of the standard gas turbine engine, we test:

H_0: $\sigma^2 = 1,500^2 = 2,250,000$
H_a: $\sigma^2 > 2,250,000$

The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma_o^2} = \dfrac{(67-1)1,595^2}{1,500^2} = 74.6247$

The rejection region requires $\alpha = .05$ in the upper tail of the χ^2 distribution with df = n – 1 = 67 – 1 = 66. From Table V, Appendix A, $\chi^2_{.05} \approx 79.0819$. The rejection region is $\chi^2 > 79.0819$.

Since the observed value of the test statistic does not fall in the rejection region $\chi^2 = 74.6247 \not> 79.0819$), H_0 is not rejected. There is insufficient evidence to indicate the heat rates of the augmented gas turbine engine are more variable than the heat rates of the standard gas turbine engine at $\alpha = .05$.

6.114 The psychologist claims that the range of WR scores is 42. If the WR scores are normally distributed, then approximately 95% of the scores fall within 2 standard deviations of the mean and approximately 99.7% of the scores fall within 3 standard deviations of the mean. Thus, the range will cover between 4 and 6 standard deviations. We will use the 6 standard deviations. The estimate of the standard deviation is the range divided by 6 or 42/6 = 7. To determine if the psychologists claim is correct, we test:

H_0: $\sigma^2 = 7^2 = 49$
H_a: $\sigma^2 \neq 49$

The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma_o^2} = \dfrac{(100-1)6^2}{7^2} = 72.735$

Since no α is given, we will use $\alpha = .05$. The rejection region requires $\alpha / 2 = .05/2 = .025$ in the each tail of the χ^2 distribution with df = n – 1 = 100 – 1 = 99. From Table V, Appendix A, $\chi^2_{.975} \approx 74.2219$ and $\chi^2_{.025} \approx 129.561$. The rejection region is $\chi^2 < 74.2219$ or $\chi^2 > 129.561$.

Since the observed value of the test statistic falls in the rejection region $(\chi^2 = 72.735 < 74.2219)$, H_0 is rejected. There is sufficient evidence to indicate the psychologist's claim is incorrect at $\alpha = .05$.

6.116 a. Since the normal distribution is symmetric, the probability that a randomly selected observation exceeds the mean of a normal distribution is .5.

b. By the definition of "median," the probability that a randomly selected observation exceeds the median of a normal distribution is .5.

c. If the distribution is not normal, the probability that a randomly selected observation exceeds the mean depends on the distribution. With the information given, the probability cannot be determined.

d. By definition of "median," the probability that a randomly selected observation exceeds the median of a non-normal distribution is .5.

6.118 a.

H_0: $\eta = 9$
H_a: $\eta > 9$

The test statistic is S = {Number of observations greater than 9} = 7.

The p-value = $P(x \geq 7)$ where x is a binomial random variable with n = 10 and p = .5. From Table II,

p-value = $P(x \geq 7) = 1 - P(x \leq 6) = 1 - .828 = .172$

Since the p-value = $.172 > \alpha = .05$, H_0 is not rejected. There is insufficient evidence to indicate the median is greater than 9 at $\alpha = .05$.

b.

H_0: $\eta = 9$
H_a: $\eta \neq 9$

S_1 = {Number of observations less than 9} = 3 and
S_2 = {Number of observations greater than 9} = 7

The test statistic is S = larger of S_1 and S_2 = 7.

The p-value = $2P(x \geq 7)$ where x is a binomial random variable with n = 10 and p = .5. From Table II,

p-value = $2P(x \geq 7) = 2(1 - P(x \leq 6)) = 2(1 - .828) = .344$

Since the p-value = $.344 > \alpha = .05$, H_0 is not rejected. There is insufficient evidence to indicate the median is different than 9 at $\alpha = .05$.

c.

H_0: $\eta = 20$
H_a: $\eta < 20$

The test statistic is S = {Number of observations less than 20} = 9

The p-value = $P(x \geq 9)$ where x is a binomial random variable with n = 10 and p = .5. From Table II,

p-value = $P(x \geq 9) = 1 - P(x \leq 8) = 1 - .989 = .011$

Since the p-value = $.011 < \alpha = .05$, H_0 is rejected. There is sufficient evidence to indicate the median is less than 20 at $\alpha = .05$.

d.

H_0: $\eta = 20$
H_a: $\eta \neq 20$

S_1 = {Number of observations less than 20} = 9 and
S_2 = {Number of observations greater than 20} = 1

The test statistic is S = larger of S_1 and S_2 = 9.

The p-value = $2P(x \geq 9)$ where x is a binomial random variable with $n = 10$ and $p = .5$. From Table II,

$$p\text{-value} = 2P(x \geq 9) = 2(1 - P(x \leq 8)) = 2(1 - .989) = .022$$

Since the p-value = $.022 < \alpha = .05$, H_0 is rejected. There is sufficient evidence to indicate the median is different than 20 at $\alpha = .05$.

e. For all parts, $\mu = np = 10(.5) = 5$ and $\sigma = \sqrt{npq} = \sqrt{10(.5)(.5)} = 1.581$.

For part **a**, $P(x \geq 7) \approx P\left(z \geq \frac{(7-.5)-5}{1.581}\right) = P(z \geq .95) = .5 - .3289 = .1711$

This is close to the probability .172 in part **a**. The conclusion is the same.

For part **b**, $2P(x \geq 7) \approx 2P\left(z \geq \frac{(7-.5)-5}{1.581}\right) = 2P(z \geq .95) = 2(.5 - .3289) = .3422$

This is close to the probability .344 in part **b**. The conclusion is the same.

For part **c**, $P(x \geq 9) \approx P\left(z \geq \frac{(9-.5)-5}{1.581}\right) = P(z \geq 2.21) = .5 - .4864 = .0136$

This is close to the probability .011 in part **c**. The conclusion is the same.

For part **d**, $2P(x \geq 9) \approx 2P\left(z \geq \frac{(9-.5)-5}{1.581}\right) = 2P(z \geq 2.21) = 2(.5 - .4864) = .0272$

This is close to the probability .022 in part **d**. The conclusion is the same.

f. We must assume only that the sample is selected randomly from a continuous probability distribution.

6.120 a. To determine if whether the median amount of caffeine in Breakfast Blend coffee exceeds 300 milligrams, we test:

H_0: $\eta = 300$
H_a: $\eta > 300$

b. S = {number of observations greater than 300} = 4.

c. The p-value = $P(x \geq 4)$ where x is a binomial random variable with $n = 5$ and $p = .5$. (One observation is eliminated because it is equal to the hypothesized value.) From Table II, Appendix A,

$$p\text{-value} = P(x \geq 4) = 1 - P(x \leq 3) = 1 - .812 = .188$$

d. Since the p-value = $.188 > \alpha = .05$, H_0 is not rejected. There is insufficient evidence to indicate the median amount of caffeine in Breakfast Blend coffee exceeds 300 milligrams at $\alpha = .05$.

6.122 a. To determine if the cohesiveness will deteriorate after storage, we test:

H_0: $\eta = 0$
H_a: $\eta > 0$

b. S = number of measurements greater than 0 = 13. The p-value = $P(x \geq 13)$ where x is a binomial random variable with $n = 20$ and $p = .5$. From Table II, Appendix A,

p-value = $P(x \geq 13) = 1 - P(x \leq 12) = 1 - .868 = .132$.

c. Since the p-value $= .132 > \alpha = .05$, H_0 is not rejected. There is insufficient evidence to indicate that cohesiveness will deteriorate after storage for any value of $\alpha < .132$.

6.124 a. To determine if the population median chromatic contrast of spiders on flowers is less than 70, we test:

H_0: $\eta = 70$
H_a: $\eta < 70$

b. The test statistic is S = number of measurements less than 70 = 7.

c. The p-value = $P(x \geq 7)$ where x is a binomial random variable with $n = 10$ and $p = .5$. From Table II, Appendix A,

$P(x \geq 7) = 1 - P(x \leq 6) = 1 - .828 = .172$

d. Since the p-value is greater than $\alpha = .10$ ($p = .172 > .10$), H_0 is not rejected. There is insufficient evidence to indicate the population median chromatic contrast of spiders on flowers is less than 70 at $\alpha = .10$.

6.126 To determine if the median rebound length exceeds 10 meters, we test:

H_0: $\eta = 10$
H_a: $\eta > 10$

The test statistic is S = {number of observations greater than 10} = 7.

The p-value = $P(x \geq 7)$ where x is a binomial random variable with $n = 13$ and $p = .5$. Using MINITAB,

p-value = $P(x \geq 7) = 1 - P(x \leq 6) = 1 - .5 = .5$

Since the p-value is not less than α ($p = .5 \nless .10$), H_0 is not rejected. There is insufficient evidence to indicate the median rebound length exceeds 10 meters at $\alpha = .10$.

6.128 To determine whether the median is less than 40, we test:

H_0: $\eta = 40$
H_a: $\eta < 40$

The test statistic is $z = \frac{(S - .5) - .5n}{.5\sqrt{n}} = \frac{(25 - .5) - .5(30)}{.5\sqrt{30}} = 3.47$
(where S = number of observations less than 40 = 25)

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = 3.47 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the median number of fish remaining in the left compartment is less than 40 at $\alpha = .05$.

6.130 For a large sample test of hypothesis about a population mean, no assumptions are necessary because the Central Limit Theorem assures that the test statistic will be approximately normally distributed. For a small sample test of hypothesis about a population mean, we must assume that the population being sampled from is normal. For both the large and small sample tests we must assume that we have a random sample. The test statistic for the large sample test is the z statistic, and the test statistic for the small sample test is the t statistic.

6.132 The elements of the test of hypothesis that should be specified prior to analyzing the data are: null hypothesis, alternative hypothesis, and significance level.

6.134 a. To determine if the population median exceeds 150, we test:

H_0: $\eta = 150$
H_a: $\eta > 150$

The test statistic is S = {Number of observations greater than 150} = 7.

The p-value = $P(x \geq 7)$ where x is a binomial random variable with $n = 10$ and $p = .5$. From Table II,

p-value = $P(x \geq 7) = 1 - P(x \leq 6) = 1 - .828 = .172$

b. Since the p-value is not less than $\alpha = .05$, $(p = .172 \nless .05)$, H_0 is not rejected. There is insufficient evidence to indicate the median of the population is greater than 150 at $\alpha = .05$.

6.136 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \Rightarrow 8.2 \pm 1.96 \frac{.79}{\sqrt{175}} \Rightarrow 8.2 \pm .12 \Rightarrow (8.08, 8.32)$$

We are 95% confident the mean is between 8.08 and 8.32.

b. H_0: $\mu = 8.3$
H_a: $\mu \neq 8.3$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{8.2 - 8.3}{.79 / \sqrt{175}} = -1.67$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.67 \nless -1.96$), H_0 is not rejected. There is insufficient evidence to indicate that the mean is different from 8.3 at $\alpha = .05$.

c. H_0: $\mu = 8.4$
H_a: $\mu \neq 8.4$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{8.2 - 8.4}{.79 / \sqrt{175}} = -3.35$

The rejection region is the same as part **b**, $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic falls in the rejection region ($z = -3.35 < -1.96$), H_0 is rejected. There is sufficient evidence to indicate that the mean is different from 8.4 at $\alpha = .05$.

6.138 a. The p-value = .1288 = $P(t \geq 1.174)$. Since the p-value is not very small, there is no evidence to reject H_0 for $\alpha \leq .10$. There is no evidence to indicate the mean is greater than 10.

b. We must assume that we have a random sample from a population that is normally distributed.

c. For the alternative hypothesis H_a: $\mu \neq 10$, the p-value is 2 times the p-value for the one-tailed test. The p-value = 2(.1288) = .2576. There is no evidence to reject H_0 for $\alpha \leq .10$. There is no evidence to indicate the mean is different from 10.

6.140 a. Since the company must give proof the drug is safe, the null hypothesis would be the drug is unsafe. The alternative hypothesis would be the drug is safe.

b. A Type I error would be concluding the drug is safe when it is not safe. A Type II error would be concluding the drug is not safe when it is. α is the probability of concluding the drug is safe when it is not. β is the probability of concluding the drug is not safe when it is.

c. In this problem, it would be more important for α to be small. We would want the probability of concluding the drug is safe when it is not to be as small as possible.

6.142 a. The point estimate for p is $\hat{p} = \frac{x}{n} = \frac{35}{1,165} = .030$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 1,165(.02) = 23.3$ and $nq_o = 1,165(1 - .02) = 1,165(.98) = 1,141.7$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the true driver cell phone use rate differs from .02, we test:

H_0: $p = .02$
H_a: $p \neq .02$

The test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.030 - .02}{\sqrt{\frac{.02(.98)}{1,165}}} = 2.44$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic falls in the rejection region ($z = 2.44 > 1.96$), H_0 is rejected. There is sufficient evidence to indicate the true driver cell phone use rate differs from .02 at $\alpha = .05$.

b. In Exercise 5.119, the confidence interval for p is (.020, .040). Since .02 is contained in this interval, there is no evidence to reject H_0. This does not agree with the conclusion for this test. However, if the calculations were carried out to 4 decimal places, the interval would be (.0202, .0398). Using this interval, .02 is not contained in the interval. This agrees with the test in part **a**.

6.144 In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 100(.5) = 50$ and $nq_o = 100(1 - .5) = 100(.5) = 50$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

$$\hat{p} = \frac{x}{n} = \frac{56}{100} = .56$$

To determine if more than half of all Diet Coke drinkers prefer Diet Pepsi, we test:

H_0: $p = .5$
H_a: $p > .5$

The test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.56 - .5}{\sqrt{\frac{.5(.5)}{100}}} = 1.20$

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05}$ = 1.645. The rejection region is $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = 1.20 \not> 1.645$), H_0 is not rejected. There is insufficient evidence to indicate that more than half of all Diet Coke drinkers prefer Diet Pepsi at $\alpha = .05$.

Since H_0 was not rejected, there is no evidence that Diet Coke drinkers prefer Diet Pepsi.

6.146 To determine if the variance of the solution times differs from 2, we test:

H_0: $\sigma^2 = 2$
H_a: $\sigma^2 \neq 2$

The test statistic is $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2} = \dfrac{(52-1)2.2643}{2} = 57.74$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the χ^2 distribution with df = $n - 1 = 52 - 1 = 51$. From Table V, Appendix A, $\chi^2_{.025} \approx 71.4202$ and $\chi^2_{.975} \approx 32.3574$. The rejection region is $\chi^2 < 32.3574$ or $\chi^2 > 71.4202$.

Since the observed value of the test statistic does not fall in the rejection region ($\chi^2 = 57.740 \not> 71.4202$ and $\chi^2 = 57.740 \not< 32.3574$), H_0 is not rejected. There is insufficient evidence to indicate the variance of the solution times differ from 2 at $\alpha = .05$.

6.148 a. The hypotheses would be:

H_0: Individual does not have the disease
H_a: Individual does have the disease

b. A Type I error would be: Conclude the individual has the disease when in fact he/she does not. This would be a false positive test.
A Type II error would be: Conclude the individual does not have the disease when in fact he/she does. This would be a false negative test.

c. If the disease is serious, either error would be grave. Arguments could be made for either error being graver. However, I believe a Type II error would be more grave: Concluding the individual does not have the disease when he/she does. This person would not receive critical treatment, and may suffer very serious consequences. Thus, it is more important to minimize β.

6.150 a. To determine if the true mean crack intensity of the Mississippi highway exceeds the AASHTO recommended maximum, we test:

H_0: $\mu = 100$
H_a: $\mu > 100$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{.210 - .100}{\sqrt{.011}/\sqrt{8}} = 2.97$

The rejection region requires $\alpha = .01$ in the upper tail of the t distribution with df = $n - 1 = 8 - 1 = 7$. From Table IV, Appendix A, $t_{.01} = 2.998$. The rejection region is $t > 2.998$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 2.97 \not> 2.998$), H_0 is not rejected. There is insufficient evidence to indicate that the true mean crack intensity of the Mississippi highway exceeds the AASHTO recommended maximum at $\alpha = .01$.

b. A Type I error is rejecting H_0 when H_0 is true. In this case, it would be concluding that the true mean crack intensity of the Mississippi highway exceeds the AASHTO recommended maximum when, in fact, it does not.

A Type II error is accepting H_0 when H_0 is false. In this case, it would be concluding that the true mean crack intensity of the Mississippi highway does not exceed the AASHTO recommended maximum when, in fact, it does.

6.152 a. To determine if the variance of the weights of parrotfish is less than 4, we test:

H_0: $\sigma^2 = 4$
H_a: $\sigma^2 < 4$

The test statistic is $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(10-1)1.4^2}{4} = 4.41$

The rejection region requires $\alpha = .05$ in the lower tail of the χ^2 distribution with df = $n - 1 = 10 - 1 = 9$. From Table V, Appendix A, $\chi^2_{.95} = 3.32511$. The rejection region is $\chi^2 < 3.32511$.

Since the observed value of the test statistic does not fall in the rejection region ($\chi^2 = 4.41 \not< 3.32511$), H_0 is not rejected. There is insufficient evidence to indicate that the variance of the weights of parrotfish is less than 4 at $\alpha = .05$.

b. We must assume that the weights of the parrotfish are normally distributed.

6.154 a. To determine if the median level differs from the target, we test:

H_0: $\eta = .75$
H_a: $\eta \neq .75$

b. S_1 = number of observations less than .75 and S_2 = number of observations greater than .75.

The test statistic is S = larger of S_1 and S_2.

The p-value = $2P(x \geq S)$ where x is a binomial random variable with $n = 25$ and $p = .5$. If the p-value is less than $\alpha = .10$, reject H_0.

c. A Type I error would be concluding the median level is not .75 when it is. If a Type I error were committed, the supervisor would correct the fluoridation process when it was not necessary. A Type II error would be concluding the median level is .75 when it is not. If a Type II error were committed, the supervisor would not correct the fluoridation process when it was necessary.

d. S_1 = {number of observations less than .75} = 7 and S_2 = {number of observations greater than .75} = 18.

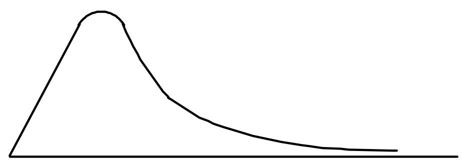
The test statistic is S = larger of S_1 and S_2 = 18.

The p-value = $2P(x \geq 18)$ where x is a binomial random variable with $n = 25$ and $p = .5$. From Table II,

$$p\text{-value} = 2P(x \geq 18) = 2(1 - P(x \leq 17)) = 2(1 - .978) = 2(.022) = .044$$

Since the p-value $= .044 < \alpha = .10$, H_0 is rejected. There is sufficient evidence to indicate the median level of fluoridation differs from the target of .75 at $\alpha = .10$.

e. A distribution heavily skewed to the right might look something like the following:



One assumption necessary for the t test is that the distribution from which the sample is drawn is normal. A distribution which is heavily skewed in one direction is not normal. Thus, the sign test would be preferred.

6.156 The point estimate for p is $\hat{p} = \dfrac{x}{n} = \dfrac{71+68}{678} = \dfrac{139}{678} = .205$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 678(.25) = 169.5$ and $nq_o = 678(1 - .25) = 678(.75) = 508.5$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the true percentage of appealed civil cases that are actually reversed is less than 25%, we test:

H_0: $p = .25$
H_a: $p < .25$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.205 - .25}{\sqrt{\dfrac{.25(.75)}{678}}} = -2.71$

The rejection region requires $\alpha = .01$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.01} = 2.33$. The rejection region is $z < -2.33$.

Since the observed value of the test statistic falls in the rejection region ($z = -2.71 < -2.33$), H_0 is rejected. There is sufficient evidence to indicate the true percentage of appealed civil cases that are actually reversed is less than 25% at $\alpha = .01$.

6.158 To refute the claim that 60% of parents with young children condone spanking their children as a regular form of punishment, we test:

H_0: $p = .6$
H_a: $p \neq .6$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{\hat{p} - .6}{\sqrt{\dfrac{.6(.4)}{100}}} = \dfrac{\hat{p} - .6}{.049}$

No value was given for α. We will use $\alpha = .05$. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025}$ = 1.96. The rejection region is $z < -1.96$ or $z > 1.96$.

To find the number of parents who would need to say they condone spanking to refute the claim, we will set the value of the test statistic equal to the z-values associated with the rejection region.

$$\frac{\hat{p} - .6}{.049} \leq -1.96 \Rightarrow \hat{p} - .6 \leq -.096 \Rightarrow \hat{p} \leq .504 \Rightarrow \frac{x}{100} \leq .504 \Rightarrow x \leq 50$$

and

$$\frac{\hat{p} - .6}{.049} \geq 1.96 \Rightarrow \hat{p} - .6 \geq .096 \Rightarrow \hat{p} \geq .696 \Rightarrow \frac{x}{100} \geq .696 \Rightarrow x \geq 70$$

Thus, to refute the claim, we would need either 50 or fewer parents to say they condone spanking or 70 or more parents to say they condone spanking to refute the claim at $\alpha = .05$.

6.160 Some preliminary calculations are:

$$\bar{x} = \frac{\sum x}{n} = \frac{110}{5} = 22 \qquad s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{2{,}436 - \frac{110^2}{5}}{5-1} = 4$$

$s = \sqrt{s^2} = \sqrt{4} = 2$

To determine if the data collected were fabricated, we test:

H_0: $\mu = 15$
H_a: $\mu \neq 15$

The test statistic is $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{22 - 15}{2 / \sqrt{5}} = 7.83$

If we want to choose a level of significance to benefit the students, we would choose a small value for α. Suppose we use $\alpha = .01$. The rejection region requires $\alpha / 2 = .01 / 2 = .005$ in each tail of the t distribution with df = n – 1 = 5 – 1 = 4. From Table IV, Appendix A, $t_{.005} = 4.604$. The rejection region is $t < -4.604$ or $t > 4.604$.

Since the observed value of the test statistic falls in the rejection region ($t = 7.83 > 4.604$), H_0 is rejected. There is sufficient evidence to indicate the mean data collected were fabricated at $\alpha = .01$.