Comparing Population Means

Chapter 7

7.2 Assumptions about the two populations:

1. Both sampled populations have relative frequency distributions that are approximately normal.
2. The population variances are equal.

Assumptions about the two samples:

The samples are randomly and independently selected from the population.

7.4 The confidence interval for $(\mu_1 - \mu_2)$ is (–10, 4). The correct inference is **d** –no significant difference between means. Since 0 is contained in the interval, it is a likely value for $(\mu_1 - \mu_2)$. Thus, we cannot say that the 2 means are different.

7.6 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05$. From Table III, Appendix A, $z_{.025}$ = 1.96. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (5,275 - 5,240) \pm 1.96\sqrt{\frac{150^2}{400} + \frac{200^2}{400}}$$

$$\Rightarrow 35 \pm 24.5 \Rightarrow (10.5,\ 59.5)$$

We are 95% confident that the difference between the population means is between 10.5 and 59.5.

b. The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(5275 - 5240) - 0}{\sqrt{\dfrac{150^2}{400} + \dfrac{200^2}{400}}} = 2.8$

The p-value of the test is $p = P(z \le -2.8) + P(z \ge 2.8) = 2P(z \ge 2.8) = 2(.5 - .4974)$
$= 2(.0026) = .0052$

Since the p-value is so small, there is evidence to reject H_0. There is evidence to indicate the two population means are different for $\alpha > .0052$.

c. The p-value would be half of the p-value in part **b**. The p-value = $P(z \ge 2.8)$ = $.5 - .4974 = .0026$. Since the p-value is so small, there is evidence to reject H_0. There is evidence to indicate the mean for population 1 is larger than the mean for population 2 for $\alpha > .0026$.

d. The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \dfrac{(5275 - 5240) - 25}{\sqrt{\dfrac{150^2}{400} + \dfrac{200^2}{400}}} = .8$

The p-value of the test is $p = P(z \le -.8) + P(z \ge .8) = 2P(z \ge .8) = 2(.5 - .2881)$
$= 2(.2119) = .4238$

Since the p-value is so large, there is no evidence to reject H_0. There is no evidence to indicate that the difference in the 2 population means is different from 25 for $\alpha \le .10$.

e. We must assume that we have two independent random samples.

7.8 a. $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \dfrac{(25-1)200+(25-1)180}{25+25-2} = \dfrac{9120}{48} = 190$

b. $s_p^2 = \dfrac{(20-1)25+(10-1)40}{20+10-2} = \dfrac{835}{28} = 29.8214$

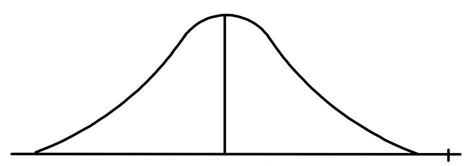
c. $s_p^2 = \dfrac{(8-1).20+(12-1).30}{8+12-2} = \dfrac{4.7}{18} = .2611$

d. $s_p^2 = \dfrac{(16-1)2500+(17-1)1800}{16+17-2} = \dfrac{66,300}{31} = 2138.7097$

e. s_p^2 falls nearer the variance with the larger sample size.

7.10 a. $\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}} = \sqrt{\dfrac{100}{100}+\dfrac{64}{100}} = \sqrt{1.64} = 1.2806$

b. The sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal by the Central Limit Theorem since $n_1 \ge 30$ and $n_2 \ge 30$.



c. $\bar{x}_1 - \bar{x}_2 = 70 - 50 = 20$

Yes, it appears that $\bar{x}_1 - \bar{x}_2 = 20$ contradicts the null hypothesis H_0: $\mu_1 - \mu_2 = 5$.

d. The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

e. H_0: $\mu_1 - \mu_2 = 5$
H_a: $\mu_1 - \mu_2 \ne 5$

The test statistic is $z = \dfrac{(\bar{x}_1-\bar{x}_2)-5}{\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}} = \dfrac{(70-50)-5}{1.2806} = 11.71$

The rejection region is $z < -1.96$ or $z > 1.96$. (Refer to part **d**.)

Since the observed value of the test statistic falls in the rejection region (z = 11.71 > 1.96), H_0 is rejected. There is sufficient evidence to indicate the difference in the population means is not equal to 5 at $\alpha = .05$.

f. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, Appendix A, $z_{.025}$ = 1.96. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (70 - 50) \pm 1.96\sqrt{\frac{100}{100} + \frac{64}{100}} \Rightarrow 20 \pm 2.51 \Rightarrow (17.49,\ 22.51)$$

We are 95% confident the difference in the two population means ($\mu_1 - \mu_2$) is between 17.49 and 22.51.

g. The confidence interval for $\mu_1 - \mu_2$ gives more information than the test of hypothesis. In the test, all we know is that $\mu_1 - \mu_2 \neq 5$, but not what $\mu_1 - \mu_2$ might be. With the confidence interval, we do have a range of values that we believe will contain $\mu_1 - \mu_2$.

7.12 a. Let μ_1 = mean time on the Trail Making Test for schizophrenics and μ_2 = mean time on the Trail Making Test for normal subjects. The parameter of interest is $\mu_1 - \mu_2$.

b. To determine if the mean time on the Trail Making Test for schizophrenics is larger than that for normal subjects, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

c. Since the p-value is less than α (p = .001 < .01), H_0 is rejected. There is sufficient evidence to indicate the mean time on the Trail Making Test for schizophrenics is larger than that for normal subjects at $\alpha = .01$.

d. For confidence coefficient .99, $\alpha = .01$ and $\alpha / 2 = .005$. From Table III, Appendix A, $z_{.005}$ = 2.58. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.005}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \Rightarrow (104.23 - 62.24) \pm 2.58\sqrt{\frac{45.45^2}{41} + \frac{16.34^2}{49}}$$

$$\Rightarrow 41.99 \pm 19.28 \Rightarrow (22.71,\ 61.27)$$

We are 99% confident that the true difference in mean time on the Trail Making Test between schizophrenics and normal subjects is between 22.71 and 61.27.

7.14. a. Let μ_1 = mean IBI value for the Muskingum River Basin and μ_2 = mean IBI value for the Hocking River Basin. For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table III, Appendix A, $z_{.05}$ = 1.645. The 90% confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.05}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \Rightarrow (.035 - .340) \pm 1.645\sqrt{\frac{1.046^2}{53} + \frac{.960^2}{51}}$$
$$\Rightarrow -.305 \pm .324 \Rightarrow (-.629, .019)$$

We are 90% confident that the difference in mean IBI values between the two river basins is between −.629 and 0.019.

b. To determine if the mean IBI values differ for the two river basins, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(.035 - .340) - 0}{\sqrt{\frac{1.046^2}{53} + \frac{.96^2}{51}}} = -1.55$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.55 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean IBI values differ for the two river basins at $\alpha = .10$.

Both the confidence interval and the test of hypothesis used the same level of confidence – 90%. Both used the same values for the sample means and the sample variances. Thus, the results must agree. If the hypothesized value of the difference in the means falls in the 90% confidence interval, it is a likely value for the difference and we would not reject H_0. On the other hand, if the hypothesized value of the difference in the means does not fall in the 90% confidence interval, it is an unusual value of the difference and we would reject it.

7.16 a. Let μ_1 = mean trap spacing for the BT fishing cooperative and μ_2 = mean trap spacing for the PA fishing cooperative. The parameter of interest is $\mu_1 - \mu_2$.

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: BT, PA

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
BT	7	89.86	11.63	70.00	82.00	93.00	99.00	105.00
PA	8	99.63	27.38	66.00	76.50	96.00	115.00	153.00

The point estimate of $\mu_1 - \mu_2$ is $\bar{x}_1 - \bar{x}_2 = 89.86 - 99.63 = -9.77$.

c. Since the sample sizes are so small ($n_1 = 7$ and $n_2 = 8$), the Central Limit Theorem does not apply. In order to use the z statistic, both σ_1^2 and σ_2^2 must be known.

d. $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(7-1)11.63^2 + (8-1)27.38^2}{7 + 8 - 2} = 466.0917$$

For confidence coefficient .90, $\alpha = .10$ and $\alpha / 2 = .05$. From Table IV, Appendix A, with $df = n_1 + n_2 - 2 = 7 + 8 - 2 = 13$, $t_{.05} = 1.771$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.05}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \Rightarrow (89.86 - 99.63) \pm 1.771\sqrt{466.0917\left(\frac{1}{7} + \frac{1}{8}\right)}$$
$$\Rightarrow -9.77 \pm 19.79 \Rightarrow (-29.56,\ 10.02)$$

We are 90% confident that the true difference in mean trap space between the BT fishing cooperative and the PA fishing cooperative is between -29.56 and 10.02.

e. Since 0 is contained in the confidence interval, there is no evidence to indicate a difference in mean trap spacing between the 2 cooperatives.

f. We must assume that the distributions of trap spacings for the two cooperatives are normal, the variances of the two populations are equal, and that the samples are random and independent.

7.18 $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(28-1)8.43^2 + (32-1)9.56^2}{28 + 32 - 2} = 81.9302$$

The test statistic is $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{82.64 - 84.81}{\sqrt{81.9302\left(\dfrac{1}{28} + \dfrac{1}{32}\right)}} = -.93$

This agrees with the researcher's test statistic.

Using MINITAB with df = $n_1 + n_2 - 2 = 28 + 32 - 2 = 58$, the p-value is $p = P(t \le -.93) + P(t \ge .93) = 2(.1781) = .3562$. This is close to the p-value of .358. This agrees with the researcher's conclusion.

7.20 a. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Text-line, Witness-line, Intersection

```
Variable   N    Mean  Median    StDev  Minimum   Maximum       Q1       Q3
Text-lin   3 0.3830  0.3740   0.0531   0.3350    0.4400   0.3350   0.4400
Witness-   6 0.3042  0.2955   0.1015   0.1880    0.4390   0.2045   0.4075
Intersec   5 0.3290  0.3190   0.0443   0.2850    0.3930   0.2900   0.3730
```

Let μ_1 = mean zinc measurement for the text-line, μ_2 = mean zinc measurement for the witness-line, and μ_3 = mean zinc measurement for the intersection.

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_3-1)s_3^2}{n_1+n_3-2} = \frac{(3-1).0531^2+(5-1).0443^2}{3+5-2} = .00225$$

For $\alpha = .05$, $\alpha/2 = .05/2 = .025$. Using Table IV, Appendix A, with df = $n_1 + n_3 - 2$ = 3 + 5 – 2 = 6, $t_{.025}$ = 2.447. The 95% confidence interval is:

$$(\bar{x}_1-\bar{x}_3) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_3}\right)} \Rightarrow (.3830-.3290) \pm 2.447\sqrt{.00225\left(\frac{1}{3}+\frac{1}{5}\right)}$$
$$\Rightarrow 0.0540 \pm .0848 \Rightarrow (-0.0308,\ 0.1388)$$

We are 95% confident that the difference in mean zinc level between text-line and intersection is between -0.0308 and 0.1388.

To determine if there is a difference in the mean zinc measurement between text-line and intersection, we test:

H_0: $\mu_1 - \mu_3 = 0$
H_a: $\mu_1 - \mu_3 \neq 0$

The test statistic is $t = \dfrac{(\bar{x}_1-\bar{x}_3)-D_o}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_3}\right)}} = \dfrac{(.3830-.3290)-0}{\sqrt{.00225\left(\frac{1}{3}+\frac{1}{5}\right)}} = 1.56$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t-distribution with df = $n_1 + n_3 - 2$ = 3 + 5 – 2 = 6. From Table IV, Appendix B, $t_{.025}$ = 2.447. The rejection region is $t < -2.447$ or $t > 2.447$.

Since the observed value of the test statistic does not fall in the rejection region (t = 1.56 $\not>$ 2.447), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean zinc measurement between text-line and intersection at $\alpha = .05$.

b. $$s_p^2 = \frac{(n_2-1)s_2^2+(n_3-1)s_3^2}{n_2+n_3-2} = \frac{(6-1).1015^2+(5-1).0443^2}{6+5-2} = .00660$$

For $\alpha = .05$, $\alpha/2 = .05/2 = .025$. Using Table IV, Appendix A, with df = $n_2 + n_3 - 2$ = 6 + 5 – 2 = 9, $t_{.025}$ = 2.262. The 95% confidence interval is:

$$(\bar{x}_2-\bar{x}_3) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_2}+\frac{1}{n_3}\right)} \Rightarrow (.3042-.3290) \pm 2.262\sqrt{.00660\left(\frac{1}{6}+\frac{1}{5}\right)}$$
$$\Rightarrow -0.0248 \pm .1113 \Rightarrow (-0.1361,\ 0.0865)$$

We are 95% confident that the difference in mean zinc level between witness-line and intersection is between -0.1361 and 0.0865.

To determine if there is a difference in the mean zinc measurement between witness - line and intersection, we test:

H_0: $\mu_2 - \mu_3 = 0$
H_a: $\mu_2 - \mu_3 \neq 0$

The test statistic is $t = \dfrac{(\bar{x}_2 - \bar{x}_3) - D_0}{\sqrt{s_p^2\left(\dfrac{1}{n_2} + \dfrac{1}{n_3}\right)}} = \dfrac{(.3042 - .3290) - 0}{\sqrt{.00660\left(\dfrac{1}{6} + \dfrac{1}{5}\right)}} = -0.50$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t-distribution with df $= n_2 + n_3 - 2 = 6 + 5 - 2 = 9$. From Table IV, Appendix B, $t_{.025} = 2.262$. The rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -0.50 \not< -2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean zinc measurement between witness-line and intersection at $\alpha = .05$.

c. From parts **a** and **b**, we know that there is no difference in the mean zinc measurement between text-line and intersection and that there is no difference in the mean zinc measurement between witness-line and intersection. However, we did not compare the mean zinc measurement between text-line and witness-line which have sample means that are the furthest apart. Thus, we can make no conclusion about the difference between these two means.

d. In order for the above inferences to be valid, we must assume:

1. The three samples are randomly selected in an independent manner from the three target populations.
2. All three sampled populations have distributions that are approximately normal.
3. All three population variances are equal (i.e. $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$)

Since there are so few observations per treatment, it is hard to check the assumptions.

7.22 Let μ_1 = mean number of high frequency vocal responses for piglets castrated using Method 1 and μ_2 = mean number of high frequency vocal responses for piglets castrated using Method 2.

Some preliminary calculations are:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(24-1).09^2 + (25-1).09^2}{24 + 25 - 2} = .0081$$

To determine if the mean number of high frequency vocal responses differ for piglets castrated by the 2 methods, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \dfrac{.74 - .70}{\sqrt{.0081\left(\dfrac{1}{24} + \dfrac{1}{25}\right)}} = 1.56$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution. From Table IV, Appendix A, with df = $n_1 + n_2 - 2 = 24 + 25 - 2 = 47$, $t_{.025} \approx 2.021$. The rejection region is $t < -2.021$ or $t > 2.021$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.56 \not> 2.021$), H_0 is not rejected. There is insufficient evidence to indicate that the mean number of high frequency vocal responses differ for piglets castrated by the 2 methods at $\alpha = .05$.

7.24 Let μ_1 = mean performance level for students in the control group and μ_2 = mean performance level for students in the rudeness group. To determine if the mean performance level for the students in the rudeness group is lower than that for students in the control group, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

From the printout, the test statistic is $t = 2.683$ and the p-value is $p = .0043$. Since the p-value is so small, H_0 is rejected. There is sufficient evidence to indicate the mean performance level for the students in the rudeness group is lower than that for students in the control group for any value of α greater than .0043.

7.26 Let μ_1 = mean milk price in the "surrounding" market and μ_2 = mean milk price in the Tri-county market. If the dairies participated in collusive practices, then the mean price of milk in the Tri-county market will be greater than that in the "surrounding: market. To determine if there is support for the claim of collusive practices, we test:

H_o: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The test statistic is $z = -6.02$ (from the printout).

The p-value for the test is p-value = 0.000. Since the p-value is so small, H_0 will be rejected for any reasonable value of α. There is sufficient evidence to indicate that the mean milk price in the Tri-county market is larger than that in the surrounding market. There is evidence that the dairies participated in collusive practices.

7.28 By using a paired difference experiment, one can remove sources of variation that tend to inflate the variance, σ^2. By reducing the variance, it is easier to find differences in population means that really exist.

7.30 The conditions required for a valid large-sample inference about μ_d are:

1. A random sample of differences is selected from the target population differences.
2. The sample size n_d is large, i.e., $n_d \geq 30$. (Due to the Central Limit Theorem, this condition guarantees that the test statistic will be approximately normal regardless of the shape of the underlying probability distribution of the population.)

The conditions required for a valid small-sample inference about μ_d are:

1. A random sample of differences is selected from the target population differences.
2. The population of differences has a distribution that is approximately normal.

7.32 a. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution with df = $n_d - 1 = 16 - 1 = 15$. From Table IV, Appendix A, $t_{.10} = 1.341$. The rejection region is $t < -1.341$.

b. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{\dfrac{s_d}{\sqrt{n_d}}} = \dfrac{-7 - 0}{\dfrac{\sqrt{64}}{\sqrt{16}}} = -3.5$

The rejection region is $t < -1.341$. (Refer to part **a**).

Since the observed value of the test statistic falls in the rejection region ($t = -3.5 < -1.341$), H_0 is rejected. There is sufficient evidence to indicate $\mu_1 - \mu_2 < 0$ at $\alpha = .10$.

c. The necessary assumptions are:

1. The population of differences is normal.
2. The differences are randomly selected.

d. For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table IV, Appendix A, with df = 15, $t_{.05} = 1.753$. The confidence interval is:

$$\bar{x}_d \pm t_{.05} \frac{s_d}{\sqrt{n_d}} \Rightarrow -7 \pm 1.753 \frac{\sqrt{64}}{\sqrt{16}} \Rightarrow -7 \pm 3.506 \Rightarrow (-10.506, -3.494)$$

We are 90% confident the mean difference is between -10.506 and -3.494.

e. The confidence interval provides more information since it gives an interval of possible values for the difference between the population means.

7.34 a. Let μ_1 = mean of population 1 and μ_2 = mean of population 2.

To determine if the mean of the population 2 is larger than the mean for population 1, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d < 0$ where $\mu_d = \mu_1 - \mu_2$

b. Using MINITAB, the descriptive statistics are:

Descriptive Statistics: Pop1, Pop2, Diff

```
Variable   N    Mean  StDev  Minimum      Q1   Median      Q3  Maximum
Pop1      10   38.40  15.06    17.00   23.50    40.50   51.25    59.00
Pop2      10   42.10  15.81    20.00   26.25    43.50   55.00    66.00
Diff      10  -3.700  2.214   -7.000  -5.250   -4.000  -1.750     0.00
```

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-3.7 - 0}{2.214 / \sqrt{10}} = -5.285$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution with df = $n_d - 1$ = 10 – 1 = 9. From Table IV, Appendix A, $t_{.10} = 1.383$. The rejection region is $t < -1.383$.

Since the observed value of the test statistic falls in the rejection region ($t = -5.285 < -1.383$), H_0 is rejected. There is sufficient evidence to indicate the mean for population 1 is less than the mean for population 2 at $\alpha = .10$.

c. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, with df = $n_d - 1$ = 10 – 1 = 9, $t_{.05} = 1.833$. The 90% confidence interval is:

$$\bar{x}_d \pm t_{.05}\frac{s_d}{\sqrt{n_d}} \Rightarrow -3.7 \pm 1.833\frac{2.214}{\sqrt{10}} \Rightarrow -3.7 \pm 1.28 \Rightarrow (-4.98, -2.42)$$

We are 90% confident that the difference in the means of the 2 populations is between –4.98 and –2.42.

d. The assumptions necessary are that the distribution of the population of differences is normal and that the sample is randomly selected.

7.36 a. Let μ_1 = mean BMI at the start of camp and μ_2 = mean BMI at the end of camp. The parameter of interest is $\mu_d = \mu_1 - \mu_2$. To determine if the mean BMI at the end of camp is less than the mean BMI at the start of camp, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d > 0$

b. It should be analyzed as a paired-difference t-test. The BMI was measured on each adolescent at the beginning and at the end of camp. The samples are not independent.

c. The test statistics is $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{34.9 - 31.6}{\sqrt{\frac{6.9^2}{76} + \frac{6.2^2}{76}}} = 3.10$

d. The test statistic is $z = \frac{\bar{x}_d - 0}{\sqrt{\frac{s_d^2}{n_d}}} = \frac{3.3}{\sqrt{\frac{1.5^2}{76}}} = 19.18$

e. The test statistic in part c is much smaller than the test statistic in part d. The test statistic in part d provides more evidence in support of the alternative hypothesis.

f. Since the p-value is smaller than α ($p < .0001 < .01$), H_0 is rejected. There is sufficient evidence to indicate the mean BMI at the end of camp is less than the mean BMI at the beginning of camp at $\alpha = .01$.

g. No. Since the sample size is sufficiently large ($n_d = 76$), the Central Limit Theorem applies.

h. For confidence coefficient .99, $\alpha = .01$ and $\alpha / 2 = .005$. From Table III, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\bar{x}_d \pm z_{.005}\sqrt{\frac{s_d^2}{n_d}} \Rightarrow 3.3 \pm 2.58\sqrt{\frac{1.5^2}{76}} \Rightarrow 3.3 \pm .44 \Rightarrow (2.86,\ 3.74)$$

We are 99% confident that the true difference in mean BMI between the beginning and end of camp is between 2.86 and 3.74.

7.38 a. The data should be analyzed as a paired difference experiment because there were 2 measurements on each person or experimental unit, the number of laugh episodes as a speaker and the number of laugh episodes as an audience member. These 2 observations on each person are not independent of each other.

b. The study's target parameter is μ_d = difference in the mean number of laugh episodes between the speaker and the audience member.

c. No. With just the sample means of the speakers and audience, we do not have enough information to make a decision. We also need to know the variance of the differences.

d. With a p-value of $p < .01$, we would reject H_o for any value of $\alpha \geq .01$. There is sufficient evidence to indicate a difference in the mean number of laugh episodes between speakers and audience members at $\alpha \geq .01$

7.40 a. Let μ_1 = mean severity of the driver's chest injury and μ_2 = mean severity of the passenger's chest injury. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean severity of the driver's and passenger's chest injuries.

b. For each car, there are measures for the severity of the driver's chest injury and the severity of the passenger's chest injury. Since both measurements came from the same car, they are not independent.

c. Using MINITAB, the descriptive statistics for the difference data (driver chest – passenger chest) are:

Descriptive Statistics: Crash-Diff

```
Variable    N   Mean  StDev  Minimum     Q1 Median     Q3   Maximum
Crash-Diff 98 -0.561  5.517  -15.000 -4.000  0.000  3.000    13.000
```

For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table III, appendix A, $z_{.005} = 2.58$. The 99% confidence interval is:

$$\bar{x}_d \pm z_{.005}\frac{s_d}{\sqrt{n_d}} \Rightarrow -.561 \pm 2.58\frac{5.517}{\sqrt{98}} \Rightarrow -.561 \pm 1.438 \Rightarrow (-1.999,\ \ 0.877)$$

d. We are 99% confident that the difference in the mean severity of chest injury between drivers and passengers is between −1.999 and 0.877. Since 0 is contained in this interval, there is no evidence of a difference in the mean severity of chest injuries between drivers and passengers at $\alpha = .01$.

e. Since the sample size is so large (n = number of pairs = 98), the Central Limit Theorem applies. Thus, the only necessary condition is that the sample is random from the target population of differences.

7.42 a. The data should be analyzed using a paired-difference analysis because that is how the data were collected. Reaction times were collected twice from each subject, once under the random condition and once under the static condition. Since the two sets of data are not independent, they cannot be analyzed using independent samples analyses.

b. Let μ_1 = mean reaction time under the random condition and μ_2 = mean reaction time under the static condition. Let $\mu_d = \mu_1 - \mu_2$. To determine if there is a difference in mean reaction time between the two conditions, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

c. The test statistic is $t = 1.52$ with a p-value of .15. Since the p-value is not small, there is no evidence to reject H_0 for any reasonable value of α. There is insufficient evidence to indicate a difference in the mean reaction times between the two conditions. This supports the researchers' claim that visual search has no memory.

7.44 a. To determine if 2 population means are different, we not only need to know the difference in the sample means, but we also need to know the variance of the sample differences. If the variance is small, then we probably could conclude that the means are different. If the variance is large, then we might not be able to conclude the means are different.

b. Since the p-value is so small ($p < .001$), H_0 is rejected. There is sufficient evidence to indicate the mean score on the Quick-REST Survey at the end of the workshop is greater than the mean at the beginning of the workshop for any reasonable value of α. The program was effective.

c. We must assume that the sample is random. Since the number of pairs is large ($n = 238$), the Central Limit Theorem applies.

7.46 a. Let $\mu_1 =$ mean standardized growth of genes in the full-dark condition and $\mu_2 =$ mean standardized growth of genes in the transient light condition. Then $\mu_d = \mu_1 - \mu_2 =$ difference in mean standardized growth between genes in full-dark condition and genes in transient light condition.

Some preliminary calculations are:

Gene ID	Full-Dark	Transient Light	Difference
SLR2067	-0.00562	1.40989	-1.41551
SLR1986	-0.68372	1.83097	-2.51469
SLR3383	-0.25468	-0.79794	0.54326
SLR0928	-0.18712	-1.20901	1.02189
SLR0335	-0.20620	1.71404	-1.92024
SLR1459	-0.53477	2.14156	-2.67633
SLR1326	-0.06291	1.03623	-1.09914
SLR1329	-0.85178	-0.21490	-0.63688
SLR1327	0.63588	1.42608	-0.79020
SLR1325	-0.69866	1.93104	-2.62970

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{-12.11754}{10} = -1.212$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{29.67025188 - \frac{(-12.11754)^2}{10}}{10-1} = \frac{14.98677431}{9} = 1.665197146$$

$$s = \sqrt{1.665197146} = 1.290$$

To determine if there is a difference in mean standardized growth of genes in the full-dark condition and genes in the transient light condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-1.212 - 0}{1.290 / \sqrt{10}} = -2.97$.

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the t distribution with df = $n_d - 1 = 10 - 1 = 9$. From Table IV, Appendix A, $t_{.005} = 3.250$. The rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.97 \not< -3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the full-dark condition and genes in the transient light condition at $\alpha = .01$.

b. Using MINITAB, the mean difference in standardized growth of the 103 genes in the full-dark condition and the transient light condition is:

Descriptive Statistics: FD-TL

```
Variable    N    Mean  StDev  Minimum      Q1  Median     Q3  Maximum
FD-TL     103  -0.420  1.422   -2.765  -1.463  -0.296  0.803    2.543
```

The population mean difference is –.420. The test in part **a** did not detect this difference.

c. Let $\mu_3 =$ mean standardized growth of genes in the transient dark condition.
Then $\mu_d = \mu_1 - \mu_3 =$ difference in mean standardized growth between genes in full-dark condition and genes in transient dark condition.

Some preliminary calculations are:

Gene ID	Full-Dark	Transient Dark	Difference
SLR2067	–0.00562	–1.28569	1.28007
SLR1986	–0.68372	–0.68723	0.00351
SLR3383	–0.25468	–0.39719	0.14251
SLR0928	–0.18712	–1.18618	0.99906
SLR0335	–0.20620	–0.73029	0.52409
SLR1459	–0.53477	–0.33174	-0.20303
SLR1326	–0.06291	0.30392	-0.36683
SLR1329	–0.85178	0.44545	-1.29723
SLR1327	0.63588	–0.13664	0.77252
SLR1325	–0.69866	–0.24820	-0.45046

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{1.40421}{10} = 0.140$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{5.589984302 - \frac{(1.40421)^2}{10}}{10-1} = \frac{5.39280373}{9} = 0.599200414$$

$$s = \sqrt{0.599200414} = .774$$

To determine if there is a difference in mean standardized growth of genes in the full-dark condition and genes in the transient dark condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{.140 - 0}{.774 / \sqrt{10}} = 0.57$.

From part **a**, the rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 0.57 \not> 3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the full-dark condition and genes in the transient dark condition at $\alpha = .01$.

Using MINITAB, the mean difference in standardized growth of the 103 genes in the full dark condition and the transient dark condition is:

Descriptive Statistics: FD-TD

```
Variable   N    Mean   StDev  Minimum       Q1   Median       Q3  Maximum
FD-TD    103 -0.2274  0.9239  -2.8516  -0.8544  -0.1704   0.2644   2.3998
```

The population mean difference is –.2274. The test above did not detect this difference.

d. $\mu_d = \mu_2 - \mu_3$ = difference in mean standardized growth between genes in transient light condition and genes in transient dark condition.

Some preliminary calculations are:

Gene ID	Transient Light	Transient Dark	Difference
SLR2067	1.40989	–1.28569	2.69558
SLR1986	1.83097	–0.68723	2.51820
SLR3383	–0.79794	–0.39719	–0.40075
SLR0928	–1.20901	–1.18618	–0.02283
SLR0335	1.71404	–0.73029	2.44433
SLR1459	2.14156	–0.33174	2.47330
SLR1326	1.03623	0.30392	0.73231
SLR1329	–0.21490	0.44545	–0.66035
SLR1327	1.42608	–0.13664	1.56272
SLR1325	1.93104	–0.24820	2.17924

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{13.52175}{10} = 1.352$$

$$s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{34.02408743 - \frac{(13.52175)^2}{10}}{10-1} = \frac{15.74031512}{9} = 1.748923902$$

$$s = \sqrt{1.748923902} = 1.322$$

To determine if there is a difference in mean standardized growth of genes in the transient light condition and genes in the transient dark condition, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \frac{1.352 - 0}{1.322/\sqrt{10}} = 3.23$.

From part **a**, the rejection region is t < -3.250 or t > 3.250.

Since the observed value of the test statistic does not fall in the rejection region ($t = 3.23 \not> 3.250$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean standardized growth of genes in the transient light condition and genes in the transient dark condition at $\alpha = .01$.

Using MINITAB, the mean difference in standardized growth of the 103 genes in the full-dark condition and the transient dark condition is:

Descriptive Statistics: TL-TD

```
Variable   N  Mean  StDev  Minimum     Q1  Median     Q3  Maximum
TL-TD    103 0.192  1.499   -3.036 -1.166   0.149  1.164    2.799
```

The population mean difference is .192. The test above did not detect this difference.

7.48 Let μ_1 = mean density of the wine measured with the hydrometer and μ_2 = mean density of the wine measured with the hydrostatic balance. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance.

Using MINITAB, the descriptive statistics for the difference data are:

Descriptive Statistics: Diff

```
Var    N      Mean    StDev   Minimum        Q1    Median        Q3  Maximum
Diff  40 -0.000523 0.001291 -0.004480 -0.001078 -0.000165 0.000317 0.001580
```

We will use a 95% confidence interval to estimate the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, appendix A, $z_{.025}$ = 1.96. The 95% confidence interval is:

$$\bar{x}_d \pm z_{.025} \frac{s_d}{\sqrt{n_d}} \Rightarrow -.000523 \pm 1.96 \frac{.001292}{\sqrt{40}} \Rightarrow -.000523 \pm .000400 \Rightarrow (-.000923,\ -.000123)$$

We are 95% confident that the difference in the mean density of the wine measured with the hydrometer and the hydrostatic balance is between −.000923 and −.000123. Thus, we are 95% confident that the difference in the means ranges from .000123 and .000923 in absolute value. Since this entire confidence interval is less than .002, we can conclude that the difference in the mean scores does not exceed .002. Thus, we would recommend that the winery switch to the hydrostatic balance.

7.50 If the sample size calculation yields a value of n that is too large to be practical, we might decide to use a large sampling error (SE) in order to reduce the sample size, or we might decrease the confidence coefficient.

7.52 a. For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, Appendix A, $z_{.025}$ = 1.96.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.96)^2 (15^2 + 17^2)}{3.2^2} = 192.83 \approx 193$$

b. If the range of each population is 40, we would estimate σ by:

$\sigma \approx 60 / 4 = 15$

For confidence coefficient .99, $\alpha = 1 - .99 = .01$ and $\alpha / 2 = .01 / 2 = .005$. From Table III, Appendix A, $z_{.005}$ = 2.58.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(2.58)^2 (15^2 + 15^2)}{8^2} = 46.8 \approx 47$$

c. For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table III, Appendix A, $z_{.05}$ = 1.645. For a width of 1, the standard error is .5.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.645)^2 (5.8 + 7.5)}{.5^2} = 143.96 \approx 144$$

7.54 For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha / 2 = .10 / 2 = .05$. From Table III, Appendix A, $z_{.05}$ = 1.645. For width = 5, the standard error is SE = 5/2 = 2.5.

$$n_d = \frac{\left(z_{\alpha/2}\right)^2 \sigma_d^2}{(SE)^2} = \frac{1.645^2 (12)^2}{2.5^2} 62.35 \approx 63$$

In order to estimate μ_d using a 90% confidence interval of width 5, we would need to sample 63 paired observations. Since enough money was budgeted to sample 100 paired observations, sufficient funds have been allocated.

7.56 To determine the number of pairs to use, we can use the formula for a single sample. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table III, Appendix A, $z_{.05} = 1.645$.

$$n_d = \frac{\left(z_{\alpha/2}\right)^2 \sigma_d^2}{\left(SE\right)^2} = \frac{1.645^2(3)}{\left(.75\right)^2} = 14.4 \approx 15$$

7.58 For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table III, $z_{.025} = 1.96$.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.96)^2(4^2 + 3^2)}{2^2} = 24.01 \approx 25$$

Thus, 25 mice should be used in each group.

7.60 For confidence coefficient .90, $\alpha = 1 - .90 = .10$ and $\alpha/2 = .10/2 = .05$. From Table III, Appendix A, $z_{.05} = 1.645$.

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{1.645^2\left(5^2 + 5^2\right)}{1^2} = 135.3 \approx 136$$

7.62 The conditions that are required for a valid application of the Wilcoxon rank sum test are:

1. The two samples are random and independent.
2. The two probability distributions from which the samples are drawn are continuous.

7.64 a. The hypotheses are:

H_0: Two sampled populations have identical distributions
H_a: The probability distribution for population B is shifted to the right of that for A

b. First, we rank all the data:

A		B	
Observation	**Rank**	**Observation**	**Rank**
37	8	65	13
40	9	35	6.5
33	3.5	47	11
29	2	52	12
42	10		
33	3.5		
35	6.5		
28	1		
34	5		
	$T_1 = 48.5$		$T_2 = 42.5$

The test statistic is $T_2 = 42.5$ because $n_2 < n_1$.

The rejection region is $T_2 \geq 39$ from Table VI, Appendix A, with $n_1 = 9$, $n_2 = 4$ and $\alpha = .05$ for a one-tailed test.

Since the observed value of the test statistic falls in the rejection region ($T_2 = 42.5 \geq 39$), H_0 is rejected. There is sufficient evidence to indicate the distribution for population B is shifted to the right of the distribution for population A at $\alpha = .05$.

7.66 a. We first rank all the data:

Population 1		Population 2	
Observation	**Rank**	**Observation**	**Rank**
9.0	2	10.1	4
21.1	19	12.0	7
24.8	21	9.2	3
17.2	15	15.8	13
15.6	12	11.1	6
26.9	26	18.2	16
16.5	14	7.0	1
30.1	27	13.6	10
25.6	23	13.5	9
24.6	20	10.3	5
26.0	24	14.2	11
18.7	17	13.2	8
31.1	28		
20.0	18		
25.1	22		
26.1	25		
	$T_1 = 313$		$T_2 = 93$

To determine whether the probability distribution for population 2 is shifted to the left of that of population 1, we test:

H_0: The two sampled populations have identical probability distributions
H_a: The probability distribution for population 2 is shifted to the left of that of population 1

The test statistic is

$$z = \frac{T_2 - \frac{n_2(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} = \frac{93 - \frac{12(16+12+1)}{2}}{\sqrt{\frac{16(12)(16+12+1)}{12}}} = \frac{-81}{21.5407} = -3.76$$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = -3.76 < -1.645$), H_0 is rejected. There is sufficient evidence to indicate the probability distribution for population 2 is shifted to the left of that of population 1 at $\alpha = .05$.

b. The p-value is $P(z \le -3.76) \approx .5 - .5 = 0$ from Table III, Appendix A.

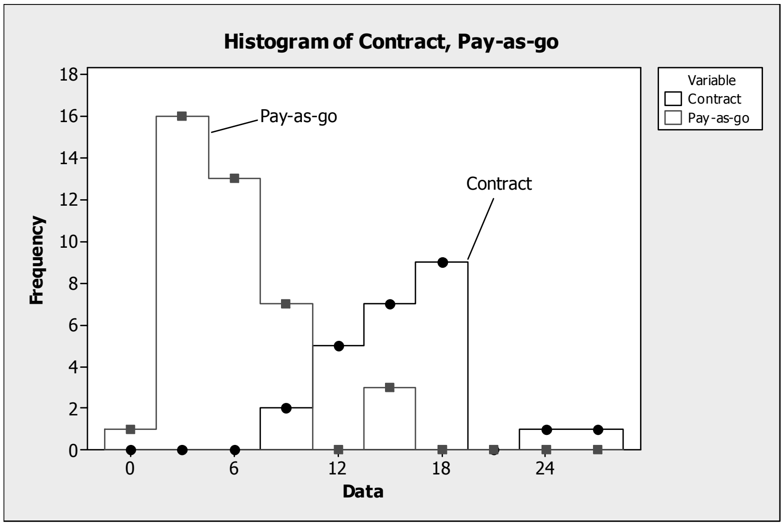
7.68 a. To determine if the distributions of the number of text messages sent and received during peak time differs for the two groups, we test:

H_0: The distributions of the number of text messages sent and received during peak time by the two groups are identical

H_a: The distribution of the number of text messages sent and received during peak time by those on an annual contract is shifted to the right or left of that for those with a pay-as-you-go option

b. The test statistic is $z = \dfrac{T_1 - \frac{n_2(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$

c. Answers will vary. Using MINITAB, a possible graph is:



The graph with the squares will represent the pay-as-you-go option, while the graph with the circles will represent the annual contract option.

7.70 a, b, & c. The ranks of the observations and sums are:

Old Design	Rank	New Design	Rank
210	9	216	16.5
212	13.5	217	18.5
211	11	162	4
211	11	137	1
190	7	219	20
213	15	216	16.5
212	13.5	179	6
211	11	153	3
164	5	152	2
209	8	217	18.5
	$T_1 = 104$		$T_2 = 106$

d. Since the sample sizes are the same, the test statistic is either T_1 or T_2.

e. To determine if the distributions of the bursting strengths for the two designs have different centers of location, we test:

H_0: The distributions of the bursting strengths of the two designs are identical
H_a: The distribution of the bursting strengths of the old design is shifted to the right or left of that for the new design

The test statistic is $T_1 = 104$.

The null hypothesis will be rejected if $T_1 \le T_L$ or $T_1 \ge T_U$ where $\alpha = .05$ (two-tailed), $n_1 = 10$ and $n_2 = 10$. From Table VI, Appendix A, $T_L = 79$ and $T_U = 131$. The rejection region is $T_1 \le 79$ or $T_1 \ge 131$.

Since the observed value of the test statistic does not fall in the rejection region ($T_1 = 104 \nleq 79$ and $T_1 = 104 \ngeq 131$), H_0 is not rejected. There is insufficient evidence to indicate the distributions of the bursting strengths for the two designs have different centers of location at $\alpha = .05$.

7.72 a. To determine if the recall of those receiving an audiovisual presentation is different from those receiving only a video presentation, we test:

H_0: The probability distributions of those receiving an audiovisual presentation and those receiving a video presentation are identical

H_a: The probability distribution of those receiving an audiovisual presentation is shifted to the right or left of the probability distribution of those receiving a video presentation

b. First, we rank the data:

A/V Group	Rank	Video Group	Rank
0	1.5	6	34.5
4	24	3	19
6	34.5	6	34.5
6	34.5	2	12
1	5	2	12
2	12	4	24
2	12	7	40
6	34.5	6	34.5
6	34.5	1	5
4	24	3	19
1	5	6	34.5
2	12	2	12
6	34.5	3	19
1	5	1	5
3	19	3	19
0	1.5	2	12
2	12	5	28
5	28	2	12
4	24	4	24
5	28	6	34.5
	$T_1 = 385.5$		$T_2 = 434.5$

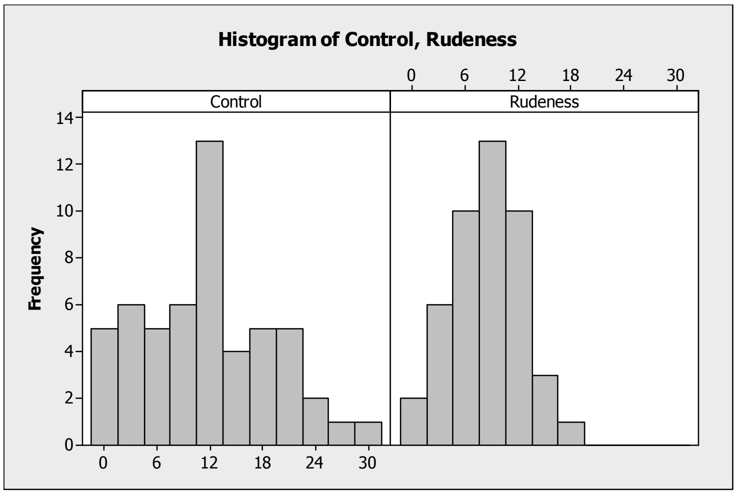
The test statistic is

$$z = \frac{T_1 - \dfrac{n_2(n_1+n_2+1)}{2}}{\sqrt{\dfrac{n_1 n_2(n_1+n_2+1)}{12}}} = \frac{385.5 - \dfrac{20(20+20+1)}{2}}{\sqrt{\dfrac{20(20)(20+20+1)}{12}}} = \frac{-24.5}{36.9685} = -.66$$

c. The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

d. Since the observed value of the test statistic does not fall in the rejection region ($z = -.66 \nless -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the recall of those receiving an audiovisual presentation is different from those receiving only a video presentation at $\alpha = .10$.

7.74 a. Using MINITAB, histograms of the two data sets are:



From the histogram for the control group, it appears that the data are skewed to the right. The histogram for the rudeness group looks to be approximately normal.

b. Some preliminary calculations are:

Control Group				Rudeness Group			
Control	Rank	Control	Rank	Rudeness	Rank	Rudeness	Rank
1	5.5	9	42	4	17	7	30.5
24	96	12	66.5	11	58.5	11	58.5
5	22	18	85.5	18	85.5	4	17
16	81.5	5	22	11	58.5	13	73
21	93.5	21	93.5	9	42	5	22
7	30.5	30	98	6	25.5	4	17
20	91	15	78	5	22	7	30.5
1	5.5	4	17	11	58.5	8	36
9	42	2	9	9	42	3	12.5
20	91	12	66.5	12	66.5	8	36
19	88	11	58.5	7	30.5	15	78
10	50	10	50	5	22	9	42
23	95	13	73	7	30.5	16	81.5
16	81.5	11	58.5	3	12.5	10	50
0	2	3	12.5	11	58.5	0	2
4	17	6	25.5	1	5.5	7	30.5
9	42	10	50	9	42	15	78
13	73	13	73	11	58.5	13	73
17	84	16	81.5	10	50	9	42
13	73	12	66.5	7	30.5	2	9
0	2	28	97	8	36	13	73
2	9	19	88	9	42	10	50
12	66.5	12	66.5	10	50		
11	58.5	20	91				
7	30.5	3	12.5				
1	5.5	11	58.5				
19	88						
			$T_1 = 2964.5$				$T_2 = 1886.5$

To determine if the true median performance level is lower for the rudeness group than for the control group, we test:

H_0: The distributions of performance levels for the two groups of students are identical

H_a: The distribution of the performance level for students in the rudeness condition is shifted to the left of that for students in the control group

The test statistic is

$$z = \frac{T_1 - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}} = \frac{2964.5 - \frac{53(53+45+1)}{2}}{\sqrt{\frac{53(45)(53+45+1)}{12}}} = \frac{341}{140.2721} = 2.43$$

The rejection region requires $\alpha = .01$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic falls in the rejection region ($z = 2.43 > 2.33$), H_0 is rejected. There is sufficient evidence to indicate that the true median performance level is lower for the rudeness group than for the control group at $\alpha = .01$.

c. Since both sample sizes are sufficiently large ($n_1 = 53$ and $n_2 = 45$), the sampling distribution of $\bar{x}$ will be approximately normal by the Central Limit Theory. Thus, the test conducted in Exercise 7.24 will be valid.

7.76 The ranks of the observations and the sums for the two groups are:

Parasitized	Rank	Non Parasitized	Rank
2.00	9	1.00	4
1.25	7.5	1.00	4
8.50	13	0	1
1.10	6	3.25	10
1.25	7.5	1.00	4
3.75	11	0.25	2
5.50	12		
	$T_1 = 66$		$T_2 = 25$

To determine if the vocalization rates of parasitized flycatchers are higher than those of nonparasitized flycatchers, we test:

H_0: The two sampled populations have identical probability distributions
H_a: The probability distribution of the parasitized flycatchers is shifted to the right of that for the nonparasitized flycatchers

The test statistic is $T_2 = 25$.

The null hypothesis will be rejected if $T_2 \le T_L$ where $\alpha = .05$ (one-tailed), $n_1 = 7$ and $n_2 = 6$. From Table VI, Appendix A, $T_L = 30$. The rejection region is $T_2 \le 30$.

Since the observed value of the test statistic falls in the rejection region ($T_2 = 25 \le 30$), H_0 is rejected. There is sufficient evidence to indicate the vocalization rates of parasitized flycatchers are higher than those of nonparasitized flycatchers at $\alpha = .05$.

7.78 The Wilcoxon signed rank test is a test of the location (center) of a distribution. The one-tailed test deals specifically with the center of one distribution being shifted in one direction (right or left) from the other distribution. The two-tailed test does not specify a particular direction of shift, only that there is a difference in the locations of the two distributions.

7.80 a. The test statistic for this two-tailed test is T, the smaller of the positive and negative rank sums, T_+ and T_-.

The null hypothesis of identical probability distributions will be rejected if $T \le T_0$ where T_0 is found in Table VII corresponding to $\alpha = .10$ (two-tailed) and $n = 20$:

Reject H_0 if $T \le 60$.

b. The test statistic is T_-, the negative rank sum.

Since it is necessary to reject the null hypothesis only if the distribution for A is shifted to the right of the distribution for B, small values of T_- would imply rejection of H_0. We will reject H_0 if $T_- \le T_0$ where T_0 is found in Table VII corresponding to $\alpha = .05$ (one-tailed) and $n = 39$:

Reject H_0 if $T_- \le 271$.

Note: Since $n \ge 25$, the large sample approximation could also be used.

c. The test statistic is T_+, the positive rank sum.

Since it is necessary to reject the null hypothesis of identical probability distributions only if the distribution of A is shifted to the left of the distribution of B, small values of T_+ would imply rejection of H_0. We will reject H_0 if $T_+ \le T_0$ where T_0 is found in Table VII corresponding to $\alpha = .005$ (one-tailed) and $n = 7$:

There is no critical value given in the table for $n = 7$. Therefore, we will never reject H_0.

7.82 a. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

b. The large sample test statistic is $z = \dfrac{T_+ - \dfrac{n(n+1)}{4}}{\sqrt{\dfrac{n(n+1)(2n+1)}{24}}} = \dfrac{273 - \dfrac{25(26)}{4}}{\sqrt{\dfrac{25(26)(51)}{24}}} = 2.97$

Since the observed value of the test statistic falls in the rejection region ($z = 2.97 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the responses for A tend to be larger than those for B at $\alpha = .05$.

c. p-value $= P(z \ge 2.97) = .5 - P(0 < z < 2.97) = .5 - .4985 = .0015$
(from Table III, Appendix A)

Thus, we can reject H_0 for any preselected α greater than .0015.

7.84 a. Since two measurements were obtained from each patient (before and after), the measurements are not independent of each other. Each "before" measurement is paired with an "after" measurement.

b. The distributions for both before and after treatment are skewed to the right and the spread of the distribution before treatment is much larger than the spread after treatment. The distributions are not normal.

c. Since the p-value is so small ($p < .0001$), H_0 would be rejected. There is sufficient evidence to indicate the ichthyotherapy was effective in treating psoriasis for any reasonable value of α.

7.86 To determine if the photo-red enforcement program is effective in reducing red-light-running crash incidents at intersections, we test:

H_0: The distribution of crash data before the camera has the same location as the distribution of crash data after the camera
H_a: The distribution of crash data before the camera is shifted to the right of that for crash data after the camera

From the printout, the test statistic is $T_+ = 79$ and the p-value is $p = .011$. No α value was given so we will use $\alpha = .05$. Since the p-value is less than α ($p = .011 < .05$), H_0 is rejected. There is sufficient evidence to indicate the photo-red enforcement program is effective in reducing red-light-running crash incidents at intersections at $\alpha = .05$.

7.88 a. To determine if the distributions of chest injury ratings for the driver and passenger differ, we test:

H_0: The distribution of driver chest injury ratings has the same location as the distribution of passenger chest injury ratings
H_a: The distribution of driver chest injury ratings is shifted to the right or left of that for passenger chest injury ratings

b. Using MIINITAB, the results are:

Wilcoxon Signed Rank Test: Diff

```
Test of median = 0.000000 versus median not = 0.000000

             N
               for   Wilcoxon         Estimated
        N  Test  Statistic       P     Median
Diff   18    16       23.0   0.021     -4.000
```

The test statistic is $T_- = 23$.

c. Reject H_0 if $T_- \le T_0$ where T_0 is based on $\alpha = .01$ and $n = 16$ pairs (two-tailed)

Reject H_0 if $T_- \le 19$ (From Table VII, Appendix A)

d. Since the observed value of the test statistic does not fall in the rejection region ($T_- = 23 \not\le 19$), H_0 is not rejected. There is insufficient evidence to indicate the distributions of chest injury ratings for the driver and passenger differ at $\alpha = .01$.

From the printout, the p-value is .021.

7.90 Some preliminary calculations:

Beach Zone	Before Nourishing	After Nourishing	Difference *Before-After*	Rank of Absolute Differences
401	0	.003595	-.003595	6
402	.001456	.007278	-.005822	7
403	0	.003297	-.003297	5
404	.002868	.003824	-.000956	3
405	0	.002198	-.002198	4
406	0	.000898	-.000898	2
407	.000626	0	.000626	1
408	0	0	0	(eliminated)
			Positive rank sum $T_+ = 1$	

To determine if sea turtle nesting densities differ before and after beach nourishing, we test:

H_0: The distribution of sea turtle nesting densities before beach nourishing has the same location as the distribution of sea turtle nesting densities after beach nourishing

H_a: The distribution of sea turtle nesting densities before beach nourishing has been shifted to the right or left of the distribution of sea turtle nesting densities after beach nourishing

The test statistic is $T_+ = 1$.

Reject H_0 if $T_+ \le T_0$ where T_0 is based on $\alpha = .05$ and $n = 7$ (two-tailed):

Reject H_0 if $T_+ \le 2$ (from Table VII, Appendix A)

Since the observed value of the test statistic falls in the rejection region ($T_+ = 1 \le 2$), H_0 is rejected. There is sufficient evidence to indicate that the sea turtle nesting densities differ before and after beach nourishing at $\alpha = .05$.

7.92

POW	379 Days After Release	157 Days After Release	Difference	Rank of Absolute Difference
1	3.73	2.46	1.27	6
2	5.46	4.11	1.35	7
3	7.04	3.93	3.11	10
4	4.73	4.51	.22	2
5	4.71	4.96	–0.25	3
6	6.19	4.42	1.77	9
7	1.42	1.02	.40	4
8	8.70	4.30	4.40	11
9	7.37	7.56	–.19	1
10	8.46	7.07	1.39	8
11	7.16	8.00	–.84	5
			Negative rank sum $T_- = 9$	
			Positive Rank sum $T_+ = 57$	

To determine if the VEP measurements of POWs 379 days after their release is greater than the VEP measurements of POWs 157 days after their release, we test:

H_0: The VEP measurement of POWs the two time periods are the same
H_a: The VEP measurments of POWs 379 days after their release are shifted to the right of the VEP measurements of POWs 157 days after their release.

The test statistic is $T\_ = 9$.

The rejection region is $T\_ \leq 14$ based on $n = 11$ and $\alpha = .05$ (one-tailed test from Table VII, Appendix A).

Since the observed value of the test statistic falls in the rejection region ($T\_ = 9 \leq 14$), H_0 is rejected. The VEP measurements of POWs 379 days after their release are shifted to the right of the VEP measurements of POWs 157 days after their release at $\alpha = .05$.

7.94 Some preliminary calculations are:

Bowler	After 4 Strikes	After 4 Non-Strikes	Difference	Rank of Absolute difference
1	.683	.432	.251	3
2	.684	.400	.284	4
3	.632	.421	.211	2
4	.610	.529	.081	1
			Negative rank sum $T\_= 0$	
			Positive rank sum $T_+ = 10$	

To determine if the data support the "hot hand" theory in bowling, we test:

H_0: The distribution of the proportion of strikes after rolling 4 strikes has the same location as the distribution of the proportion of strikes after rolling 4 non-strikes

H_a: The distribution of the proportion of strikes after rolling 4 strikes is shifted to the right of the distribution of the proportion of strikes after rolling 4 non-strikes

The test statistic is the smaller of $T\_$ and T_+ which is $T\_ = 0$.

There are no table values in Table VII when $n = 4$. Even if all the observations support rejecting H_o, the p-value is greater than .05. Thus, we are unable to reject H_0. There is insufficient evidence to support the "hot hand" theory at $\alpha = .05$.

b. When all 43 bowlers are included, the p-value is $p = 0$. Since this p-value is so small, we would reject H_0 for any reasonable value of α. There is sufficient evidence to support the "hot hand" theory at any reasonable value of α.

7.96 The conditions required for a valid ANOVA F-test are:

1. The samples are randomly selected in an independent manner from k treatment populations.

2. All k sampled populations have distributions that are approximately normal.

3. The k population variances are equal (i.e., $\sigma_1^2 = \sigma_2^2 = ... = \sigma_k^2$)

7.98 a. Using Table IX, $F_{.05} = 6.59$ with $\nu_1 = 3$, $\nu_2 = 4$.

b. Using Table XI, $F_{.01} = 16.69$ with $\nu_1 = 3$, $\nu_2 = 4$.

c. Using Table VIII, $F_{.10} = 1.61$ with $\nu_1 = 20$, $\nu_2 = 40$.

d. Using Table X, $F_{.025} = 3.87$ with $\nu_1 = 12$, $\nu_2 = 9$.

7.100 a. Some preliminary calculations are:

$$\text{CM} = \frac{\left(\sum y_i\right)^2}{n} = \frac{40.9^2}{13} = 128.6777$$

$$\text{SS(Total)}=\sum y_i^2 - \text{CM} = 159.43 - 128.6777 = 30.7523$$

$$\text{SST}=\sum \frac{T_i^2}{n_i} - \text{CM} = \frac{17.2^2}{5} + \frac{19.5^2}{5} + \frac{4.2^2}{5} - 128.6777$$
$$= 141.098 - 128.6777 = 12.4203$$

$$\text{SSE} = \text{SS(Total)} - \text{SST} = 30.7523 - 12.4203 = 18.332$$

$$\text{MST} = \frac{\text{SST}}{k-1} = \frac{12.4203}{3-1} = 6.21015 \qquad \text{MSE} = \frac{\text{SSE}}{n-k} = \frac{18.332}{13-3} = 1.8332$$

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{6.21015}{1.8332} = 3.39$$

Source	**df**	**SS**	**MS**	***F***
Treatments	2	12.4203	6.21015	3.39
Error	10	18.3320	1.8332	
Total	12	30.7523		

b. H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 3.39$.

The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with $\nu_1 = k - 1 = 3 - 1 = 2$ and $\nu_2 = n - k = 13 - 3 = 10$. From Table XI, Appendix A, $F_{.01} = 7.56$. The rejection region is $F > 7.56$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 3.39 \ngtr 7.56$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the treatment means at $\alpha = .01$.

7.102 a. Let μ_1 = mean body length of whales entangled in set nets, μ_2 = mean body length of whales entangled in pots, and μ_3 = mean body length of whales entangled in gill nets. To determine if the average body length of entangled whales differs for the three types of fishing gear, we test:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

b. Since the p-value is less than $\alpha = .05$ (p < .0001 < .05), H_0 is rejected. There is sufficient evidence to indicate the average body length of entangled whales differs for the three types of fishing gear at $\alpha = .05$.

7.104 a. The experimental units are the teeth. The treatments are the 3 different bonding times: 1 hour, 24 hours, and 48 hours. The response variable is the breaking strength (in Mpa).

b. To determine if there is a difference in the mean breaking strength among the 3 bonding times, we test:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

c. The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with $\nu_1 = k - 1 = 3 - 1 = 2$ and $\nu_2 = n - \text{k} = 300 - 3 = 297$. Using Table VI, Appendix A, $F_{.01} \approx 4.61$. The rejection region is $F > 4.61$.

d. Since the observed value of the test statistic falls in the rejection region ($F = 61.62 > 4.61$), H_0 is rejected. There is sufficient evidence to indicate a difference in mean breaking strength among the 3 bonding times at $\alpha = .01$.

e. The conditions required for a valid ANOVA F-test are:

1. The samples are randomly selected in an independent manner from k treatment populations.
2. All k sampled populations have distributions that are approximately normal.
3. The k population variances are equal (i.e., $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$)

7.106 a. To determine if there is a difference in the mean ages among the 3 groups of women, we test:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least 2 treatment means differ

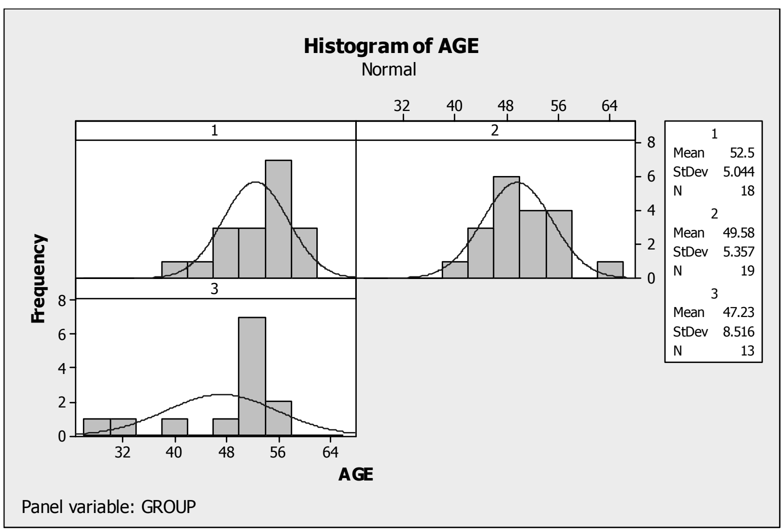
b. The sample means are variables. The next time the experiment is repeated, the sample means could be much different. We cannot tell if the population means are different unless we compute an appropriate test statistic which involves the standard deviation.

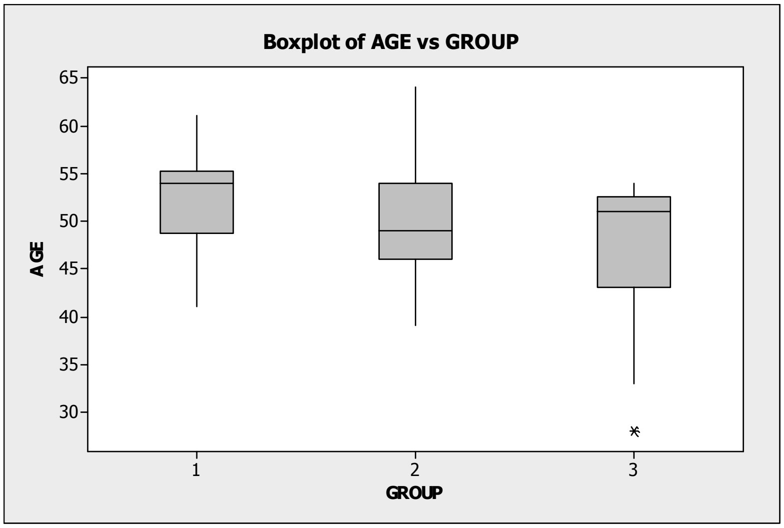
c. From the printout, the test statistic is $F = 2.784$ and the p-value is $p = .072$. Since the p-value is smaller than $\alpha = .10$ ($p = .072 < .10$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean ages among the 3 groups at $\alpha = .10$.

d. The necessary ANOVA assumptions are:

1. The samples are randomly selected in an independent manner from 3 treatment populations.
2. All 3 sampled populations have distributions that are approximately normal.
3. The 3 population variances are equal (i.e., $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$)

We cannot check whether the samples are randomly selected. To check for normality, we will simply look at histograms to see if the data are approximately normal. Using MINITAB, the histograms and the boxplots for the 3 groups are:





The data for all groups do not look particularly mound-shaped. With so few observations per group, it is hard to determine whether the data come from normal distributions. The sample standard deviations for the 3 groups are 5.044, 5.357, and 8.516. The standard deviations for groups 1 and 2 are pretty similar. The standard deviation for group 3 is somewhat larger than the first two. Thus, the assumption of equal variances may not be met.

7.108 a. This experiment is a completely randomized design.

b. Using MINITAB, the ANOVA is:

One-way ANOVA: AR, AC, A, P

```
Source  DF      SS      MS      F      P
Factor   3  0.9506  0.3169  10.29  0.000
Error   40  1.2317  0.0308
Total   43  2.1824

S = 0.1755   R-Sq = 43.56%   R-Sq(adj) = 39.33%


               Individual 95% CIs For Mean Based on
               Pooled StDev
Level   N    Mean   StDev  ---+---------+---------+---------+------
AR     11  0.4400  0.1705                    (------*------)
AC     11  0.2655  0.1526             (------*------)
A      11  0.0636  0.2180  (------*------)
P      11  0.4000  0.1525                 (------*------)
                           ---+---------+---------+---------+------
                            0.00      0.15      0.30      0.45

Pooled StDev = 0.1755
```

To determine if there are differences in the mean task scores among the 4 groups, we test:

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two treatment means differ

From the printout, the test statistic is $F = 10.29$ and the p-value is $p = 0.000$.

Since the p-value is less than $\alpha = .05$ ($p = 0.00 < .05$), H_0 is rejected. There is sufficient evidence to indicate a difference in mean task scores among the 4 groups at $\alpha = .05$.

c. The necessary assumptions are:

1. The samples are randomly selected in an independent manner from the 4 treatment groups.
2. All 4 sampled populations have distributions that are approximately normal.
3. The 4 population variances are equal (i.e. $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$)

7.110 Some preliminary calculations:

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{1{,}532}{50} = 30.64 \qquad s_1^2 = \frac{\sum x_1^2 - \frac{\left(\sum x_1\right)^2}{n_1}}{n_1 - 1} = \frac{66{,}610 - \frac{1{,}532^2}{50}}{50-1} = 401.4188$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{1{,}101}{42} = 26.2143 \qquad s_2^2 = \frac{\sum x_2^2 - \frac{\left(\sum x_2\right)^2}{n_2}}{n_2 - 1} = \frac{51{,}895 - \frac{1{,}101^2}{42}}{42-1} = 561.7822$$

$$\bar{x}_3 = \frac{\sum x_3}{n_3} = \frac{711}{47} = 15.12766 \qquad s_3^2 = \frac{\sum x_3^2 - \frac{\left(\sum x_3\right)^2}{n_3}}{n_3 - 1} = \frac{22{,}099 - \frac{711^2}{47}}{47-1} = 246.5920$$

$$\bar{x} = \frac{\sum x_1 + \sum x_2 + \sum x_3}{n_1 + n_2 + n_3} = \frac{1{,}532 + 1{,}101 + 711}{50 + 42 + 47} = \frac{3{,}344}{139} = 24.05755$$

$$\begin{aligned}\text{SST} &= \sum n_i (\bar{x}_i - \bar{x})^2 \\ &= 50(30.64 - 24.05755)^2 + 42(26.2143 - 24.05755)^2 + 47(15.12766 - 24.05755)^2 \\ &= 6{,}109.7163\end{aligned}$$

$$\begin{aligned}\text{SSE} &= (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2 \\ &= (50-1)401.4188 + (42-1)561.7822 + (47-1)246.5920 = 54{,}045.8234\end{aligned}$$

$$\text{MST} = \frac{\text{SST}}{p-1} = \frac{6{,}109.7163}{3-1} = 3{,}054.85815 \qquad \text{MSE} = \frac{\text{SSE}}{n-p} = \frac{5{,}404.8234}{139-3} = 397.3958$$

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{3{,}054.85815}{397.3958} = 7.687$$

To determine if the mean percentages of names recalled differ for the three name-retrieval methods, we test:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least two treatment means differ

The test statistic is $F = 7.687$.

The rejection region requires $\alpha = .05$ in the upper tail of the F distribution with $\nu_1 = k - 1 = 3 - 1 = 2$ and $\nu_2 = n - k = 139 - 3 = 136$. From Table IX, Appendix A, $F_{.05} \approx 3.00$. The rejection region is $F > 3.00$.

Since the observed value of the test statistic falls in the rejection region ($F = 7.687 > 3.00$), H_0 is rejected. There is sufficient evidence to indicate the mean percentages of names recalled differ for the three name retrieval methods at $\alpha = .05$.

7.112 a. Some preliminary calculations are:

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3} = \frac{26(10.5) + 25(3.9) + 25(1.4)}{26 + 25 + 25} = \frac{405.5}{76} = 5.34$$

$$SST = \sum n_i(\bar{x}_i - \bar{x})^2 = 26(10.5 - 5.34)^2 + 25(3.9 - 5.34)^2 + 25(1.4 - 5.34)^2 = 1{,}132.1956$$

b. $$SSE = (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2 = (26 - 1)7.6^2 + (25 - 1)7.5^2 + (25 - 1)7.5^2 = 4{,}144$$

c.

Source	**df**	**SS**	**MS**	**F**
Treatment	3 – 1 = 2	1,132.1956	566.0978	9.972
Error	76 – 3 = 73	4,144.0000	56.7671	
Total	76 – 1 = 75			

$$\text{MST} = \frac{\text{SST}}{k - 1} = \frac{1{,}132.1956}{3 - 1} = 566.0978 \qquad \text{MSE} = \frac{\text{SSE}}{n - k} = \frac{4{,}144}{26 + 25 + 25 - 3} = 56.7671$$

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{566.0978}{56.7671} = 9.972$$

d. To determine if there are differences in the mean drops among the three groups, we test:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: At least 2 means differ

The test statistic is $F = 9.972$.

The rejection region requires $\alpha = .01$ in the upper tail of the F distribution with numerator df $= \nu_1 = k - 1 = 3 - 1 = 2$ and denominator df $= \nu_2 = n - k$ $= 26 + 25 + 25 - 3 = 73$. From Table XI, Appendix A, $F_{.01} \approx 4.98$. The rejection region is $F > 4.98$.

Since the observed value of the test statistic falls in the rejection region ($F = 9.972 > 4.98$), H_0 is rejected. There is sufficient evidence to indicate a difference in mean drops in anxiety levels among the 3 groups at $\alpha = .01$.

e. 1. We must assume that the probability distributions for the 3 groups are normal. This assumption seems reasonable, but we do not have the actual data to check it.

2. We must assume that the variances for the three groups are equal. Since the standard deviations for the 3 groups are almost the same, this assumption seems to be valid.

3. We must assume that the observations are random and independent. This also seems to be a reasonable assumption because the problem states that the patients were randomly assigned to a group.

Since all assumptions seem valid, the results of the analysis should be valid.

7.114 a. The 2 samples are randomly selected in an independent manner from the two populations. The sample sizes, n_1 and n_2, are large enough so that $\bar{x}_1$ and $\bar{x}_2$ each have approximately normal sampling distributions and so that s_1^2 and s_2^2 provide good approximations to σ_1^2 and σ_2^2. This will be true if $n_1 \geq 30$ and $n_2 \geq 30$.

b. 1. Both sampled populations have relative frequency distributions that are approximately normal.
2. The population variances are equal.
3. The samples are randomly and independently selected from the populations.

c. 1. The relative frequency distribution of the population of differences is normal.
2. The sample of differences is randomly selected from the population of differences.

d. 1. We must assume that the probability distributions for the 3 groups are normal.
2. We must assume that the variances for the three groups are equal.
3. We must assume that the observations are random and independent.

7.116 a. SSE = SSTot − SST = 62.55 − 36.95 = 25.60

df Treatment $= k - 1 = 4 - 1 = 3$ df Error $= n - k = 20 - 4 = 16$
df Total $= n - 1 = 20 - 1 = 19$

$$\text{MST} = \frac{\text{SST}}{k-1} = \frac{36.95}{3} = 12.32 \qquad \text{MSE} = \frac{\text{SSE}}{n-k} = \frac{25.60}{16} = 1.60$$

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{12.32}{1.60} = 7.70$$

The ANOVA table:

Source	**df**	**SS**	**MS**	***F***
Treatment	3	36.95	12.32	7.70
Error	16	25.60	1.60	
Total	19	62.55		

b. To determine if there is a difference in the treatment means, we test:

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: At least two of the means differ

where μ_i represents the mean for the ith treatment.

The test statistic is $F = 7.70$

The rejection region requires $\alpha = .10$ in the upper tail of the F distribution with $\nu_1 = (k - 1) = (4 - 1) = 3$ and $\nu_2 = (n - k) = (20 - 4) = 16$. From Table VIII, Appendix A, $F_{.10} = 2.46$. The rejection region is $F > 2.46$.

Since the observed value of the test statistic falls in the rejection region ($F = 7.70 > 2.46$), H_0 is rejected. There is sufficient evidence to conclude that at least two of the means differ at $\alpha = .10$.

7.118 a. This is a paired difference experiment.

Pair	**Difference (Pop. 1 – Pop. 2)**
1	6
2	4
3	4
4	3
5	2

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{19}{5} = 3.8 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{81 - \frac{19^2}{5}}{5 - 1} = 2.2$$

$$s_d = \sqrt{2.2} = 1.4832$$

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{3.8 - 0}{1.4832 / \sqrt{5}} = 5.73$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution with df $= n_d - 1 = 5 - 1 = 4$. From Table IV, Appendix A, $t_{.025} = 2.776$. The rejection region is $t < -2.776$ or $t > 2.776$.

Since the observed value of the test statistic falls in the rejection region ($t = 5.73 > 2.776$), H_0 is rejected. There is sufficient evidence to indicate that the population means are different at $\alpha = .05$.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha / 2 = .05 / 2 = .025$. Therefore, we would use the same t value as above, $t_{.025} = 2.776$. The confidence interval is:

$$\bar{x}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n_d}} \Rightarrow 3.8 \pm 2.776 \frac{1.4832}{\sqrt{5}} \Rightarrow 3.8 \pm 1.84 \Rightarrow (1.96,\ 5.64)$$

c. The sample of differences must be randomly selected from a population of differences which has a normal distribution.

7.120 The appropriate test for two independent samples is the Wilcoxon Rank Sum test. Some preliminary calculations are:

Sample 1	**Rank**	**Sample 2**	**Rank**
1.2	4	1.5	6
1.9	8.5	1.3	5
.7	1	2.9	12
2.5	10	1.9	8.5
1.0	2	2.7	11
1.8	7	3.5	13
1.1	3		
	$T_1 = 35.5$		$T_2 = 55.5$

To determine if there is a difference between the locations of the probability distributions, we test:

H_0: The two sampled populations have identical probability distributions
H_a: The probability distribution for population A is shifted to the left or right of that for B

The test statistic is $T_2 = 55.5$.

Reject H_0 if $T_2 \leq T_L$ or $T_2 \geq T_U$ where $\alpha = .05$ (two-tailed), $n_1 = 7$ and $n_2 = 6$:

Reject H_0 if $T_2 \leq 28$ or $T_2 \geq 56$ (from Table VI, Appendix A).

Since $T_2 = 55.5 \not\leq 28$ and $T_2 = 55.5 \not\geq 56$, H_0 is not rejected. There is insufficient evidence to indicate a difference between the locations of the probability distributions for the sampled populations at $\alpha = .05$.

7.122 a. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table III, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \Rightarrow (39.08 - 38.79) \pm 1.645\sqrt{\frac{6.73^2}{127} + \frac{6.94^2}{114}}$$
$$\Rightarrow 0.29 \pm 1.452 \Rightarrow (-1.162,\ 1.742)$$

We are 90% confident that the true difference in the mean service-rating scores for male and female guests at Jamaican 5-star hotels is between -1.162 and 1.742.

b. Since 0 is contained in the 90% confidence interval, there is insufficient evidence to indicate a difference the perception of service quality at 5-star hotels in Jamaica between the genders.

c. To determine if the variances of guest scores for males and females differ, we test:

H_0: $\sigma_1^2 = \sigma_2^2$
H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_2^2}{s_1^2} = \dfrac{6.94^2}{6.73^2} = 1.063$

The rejection region requires $\alpha/2 = .10/2 = .05$ in the upper tail of the F distribution with numerator df $\nu_1 = n_2 - 1 = 114 - 1 = 113$ and denominator df $\nu_2 = n_1 - 1 = 127 - 1 = 126$. From Table IX, Appendix A, $F_{.05} \approx 1.26$. The rejection region is $F > 1.26$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 1.063 \not> 1.26$), H_0 is not rejected. There is insufficient evidence to indicate that the variances of guest scores for males and females differ at $\alpha = .10$.

7.124 a, b, & c. The ranks of the observations and the sums for the two groups are:

Grade A	Rank	Grade B or C	Rank
53	14	40	11.5
42	13	28	6
40	11.5	22	4
39	10	21	3
34	8.5	20	2
34	8.5	16	1
30	7		
24	5		
	$T_1 = 77.5$		$T_2 = 27.5$

d. Since $n_1 = 8$ and $n_2 = 6$, the test statistic is $T_2 = 27.5$

e. To determine if the distributions of the number of books read by the two populations of students have different centers of location, we test:

H_0: The distributions of the number of books read by the two groups of students are identical
H_a: The distribution of the number of books read by the "A" group is shifted to the right or left of that for the "B & C" group

The test statistic is $T_2 = 27.5$.

The null hypothesis will be rejected if $T_2 \le T_L$ or $T_2 \ge T_U$ where $\alpha = .10$ (two-tailed), $n_1 = 8$ and $n_2 = 6$. From Table VI, Appendix A, $T_L = 32$ and $T_U = 58$. The rejection region is $T_2 \le 32$ or $T_2 \ge 58$.

Since the observed value of the test statistic falls in the rejection region ($T_2 = 27.5 \le 32$), H_0 is rejected. There is sufficient evidence to indicate the distributions of the number of books read by the two populations of students have different centers of location at $\alpha = .10$.

7.126 For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha / 2 = .05 / 2 = .025$. From Table III, Appendix A, $z_{.025} = 1.96$.

An estimate of σ_1 and σ_2 is obtained from:

$$\text{range} \approx 4s \quad \text{Thus, } s \approx \frac{\text{range}}{4} = \frac{4}{4} = 1$$

$$n_1 = n_2 = \frac{\left(z_{\alpha/2}\right)^2 \left(\sigma_1^2 + \sigma_2^2\right)}{(SE)^2} = \frac{(1.96)^2(1^2 + 1^2)}{.2^2} = 192.08 \approx 193$$

You need to take 193 measurements at each site.

7.128 a. Since the data were collected as paired data, they must be analyzed as paired data. Since all winners were matched with a non-winner, the data are not independent.

b. Let μ_1 = mean life expectancy of Academy Award winners and μ_2 = mean life expectancy of non-winners. Then $\mu_d = \mu_1 - \mu_2$.

To compare the mean life expectancies of Academy Award winners and non-winners, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \ne 0$

c. From the information given, $p = .003$. Since the p-value is so small, we reject H_0. There is sufficient evidence to indicate that the mean life expectancies of Academy Award winners and non-winners differ at $\alpha > .003$.

7.130 a. Let μ_1 = mean effective population size for outcrossing snails and μ_2 = mean effective population size for selfing snails.

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(17-1)1932^2+(5-1)1890^2}{17+5-2} = 3{,}700{,}519.2$$

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, with df = $n_1 + n_2 - 2 = 17 + 5 - 2 = 20$, $t_{.05} = 1.725$. The confidence interval is:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.05}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \Rightarrow (4{,}894 - 4{,}133) \pm 1.725\sqrt{3{,}700{,}519.2\left(\frac{1}{17}+\frac{1}{5}\right)}$$

$$\Rightarrow 761 \pm 1{,}688.19 \Rightarrow (-927.19,\ 2{,}449.19)$$

We are 90% confident that the difference in the mean effective population sizes for outcrossing snails and selfing snails is between −927.19 and 2,449.19.

b. Let σ_1^2 = variance of the effective population size of the outcrossing snails and σ_2^2 = variance of the effective population size of the selfing snails. To determine if the variances for the two groups differ, we test:

H_0: $\sigma_1^2 = \sigma_2^2$

H_a: $\sigma_1^2 \neq \sigma_2^2$

The test statistic is $F = \dfrac{\text{Larger sample variance}}{\text{Smaller sample variance}} = \dfrac{s_1^2}{s_2^2} = \dfrac{1{,}932^2}{1{,}890^2} = 1.045$

Since α is not given, we will use $\alpha = .05$. The rejection region requires $\alpha/2 = .05/2 = .025$ in the upper tail of the F distribution with $\nu_1 = n_1 - 1 = 17 - 1 = 16$ and $\nu_2 = n_2 - 1 = 5 - 1 = 4$. From Table X, Appendix A, $F_{.025} \approx 8.66$. The rejection region is $F > 8.66$.

Since the observed value of the test statistic does not fall in the rejection region ($F = 1.045 \ngtr 8.66$), H_0 is not rejected. There is insufficient evidence to indicate the variances for the two groups differ at $\alpha = .05$.

7.132 Some preliminary calculations:

Twin A	Twin B	Diff-A-B	Twin A	Twin B	Diff-A-B
113	109	4	100	88	12
94	100	−6	100	104	−4
99	86	13	93	84	9
77	80	−3	99	95	4
81	95	−14	109	98	11
91	106	−15	95	100	−5
111	117	−6	75	86	−11
104	107	−3	104	103	1
85	85	0	73	78	−5
66	84	−18	88	99	−11
111	125	−14	92	111	−19
51	66	−15	108	110	−2
109	108	1	88	83	5
122	121	1	90	82	8
97	98	−1	79	76	3
82	94	−12	97	98	−1

Since the data were collected as paired data, we must analyze it using a paired *t*-test.

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{-93}{32} = -2.906 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{2723 - \frac{(-93)^2}{32}}{32-1} = 79.1200$$

$$s_d = \sqrt{s_d^2} = \sqrt{79.1200} = 8.895$$

To determine if there is a difference between the average IQ scores of identical twins, where one member of the pair is reared by the natural parents and the other member of the pair is not, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $z = \frac{\bar{x}_d - \mu_d}{s_d / \sqrt{n_d}} = \frac{-2.906 - 0}{8.895 / \sqrt{32}} = -1.848$

The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the z distribution. From Table III, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.848 \nless -1.96$), H_0 is not rejected. There is insufficient evidence to indicate there is a difference between the average IQ scores of identical twins, where one member of the pair is reared by the natural parents and the other member of the pair is not at $\alpha = .05$.

7.134 Let μ_1 = mean impulsive-sensation seeking score for cocaine abusers and μ_2 = mean impulsive-sensation seeking score for college students.

To determine if there is a difference in the mean impulsive-sensation scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(9.4 - 9.5) - 0}{\sqrt{\dfrac{4.4^2}{450} + \dfrac{4.4^2}{589}}} = -.36$

The rejection region requires $\alpha / 2 = .01/2 = .005$ in each tail of the z distribution. From Table III, Appendix A, $z_{.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.36 \not< -2.58$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean impulsive-sensation scores for the two groups at $\alpha = .01$.

Let μ_1 = mean sociability score for cocaine abusers and μ_2 = mean sociability score for college students.

To determine if there is a difference in the mean sociability scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(10.4 - 12.5) - 0}{\sqrt{\dfrac{4.3^2}{450} + \dfrac{4.0^2}{589}}} = -8.04$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = -8.04 < -2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean sociability scores for the two groups at $\alpha = .01$.

Let μ_1 = mean neuroticism-anxiety score for cocaine abusers and μ_2 = mean neuroticism-anxiety score for college students.

To determine if there is a difference in the mean neuroticism-anxiety scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(8.6 - 9.1) - 0}{\sqrt{\dfrac{5.1^2}{450} + \dfrac{4.6^2}{589}}} = -1.63$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.63 \nless -2.58$), H_0 is not rejected. There is insufficient evidence to indicate a difference in the mean neuroticism-anxiety scores for the two groups at $\alpha = .01$.

Let μ_1 = mean aggression-hostility score for cocaine abusers and μ_2 = mean aggression-hostility score for college students.

To determine if there is a difference in the mean aggression-hostility scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(8.6 - 7.3) - 0}{\sqrt{\dfrac{3.9^2}{450} + \dfrac{4.1^2}{589}}} = 5.21$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = 5.21 > 2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean aggression-hostility scores for the two groups at $\alpha = .01$.

Let μ_1 = mean activity score for cocaine abusers and μ_2 = mean activity score for college students.

To determine if there is a difference in the mean activity scores for the two groups, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 \neq 0$

The test statistic is $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(11.1 - 8.0) - 0}{\sqrt{\dfrac{3.4^2}{450} + \dfrac{4.1^2}{589}}} = 13.31$

The rejection region is $z < -2.58$ or $z > 2.58$ (from above).

Since the observed value of the test statistic falls in the rejection region ($z = 13.31 > 2.58$), H_0 is rejected. There is sufficient evidence to indicate a difference in the mean activity scores for the two groups at $\alpha = .01$.

7.136 a. Let μ_1 = mean number of swims by male rat pups and μ_2 = mean number of swims by female rat pups. Then $\mu_d = \mu_1 - \mu_2$.

Some preliminary calculations are:

Litter	Male	Female	Diff	Litter	Male	Female	Diff
1	8	5	3	11	6	5	1
2	8	4	4	12	6	3	3
3	6	7	-1	13	12	5	7
4	6	3	3	14	3	8	-5
5	6	5	1	15	3	4	-1
6	6	3	3	16	8	12	-4
7	3	8	-5	17	3	6	-3
8	5	10	-5	18	6	4	2
9	4	4	0	19	9	5	4
10	4	4	0				

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{7}{19} = .368 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{225 - \frac{7^2}{19}}{19-1} = \frac{222.4211}{18} = 12.3567$$

$$s_d = \sqrt{12.3567} = 3.5152$$

To determine if there is a difference in the mean number of swims required by male and female rat pups, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \frac{\bar{x}_d - 0}{\frac{s_d}{\sqrt{n_d}}} = \frac{.368 - 0}{\frac{3.5152}{\sqrt{19}}} = .456$

The rejection region requires $\alpha / 2 = .10 / 2 = .05$ in each tail of the t distribution with df = n_d -1 = 19 – 1 = 18. From Table IV, Appendix A, $t_{.05}$ = 1.734. The rejection region is t > 1.734 or t < -1.734.

Since the test statistic does not fall in the rejection region $(t = .456 \not> 1.734)$, H_0 is not rejected. There is insufficient evidence to indicate there is a difference in the mean number of swims required by male and female rat pups at $\alpha = .10$.

The differences are probably not from a normal distribution.

b. Some preliminary calculations are:

Litter	Male	Female	Diff	Rank Abs. Diff.	Litter	Male	Female	Diff	Rank Abs. Diff.
1	8	5	3	8	11	6	5	1	2.5
2	8	4	4	12	12	6	3	3	8
3	6	7	-1	2.5	13	12	5	7	17
4	6	3	3	8	14	3	8	-5	15
5	6	5	1	2.5	15	3	4	-1	2.5
6	6	3	3	8	16	8	12	-4	12
7	3	8	-5	15	17	3	6	-3	8
8	5	10	-5	15	18	6	4	2	5
9	4	4	0	Elim	19	9	5	4	12
10	4	4	0	Elim					
								$T_- = 70$	
								$T_+ = 83$	

H_0: The two sampled populations have identical probability distributions
H_a: The probability distribution for males is shifted to the right of that for females

The test statistic is $T_- = 70$

The rejection region is $T_- \leq 41$, from Table VII, Appendix A, with $n = 17$ and $\alpha = .10$.

Since the observed value of the test statistic does not fall in the rejection region ($T_- = 70 \not\leq 41$), H_0 is not rejected. There is insufficient evidence to indicate the mean number of swims for male pups differs from that of the female pups at $\alpha = .10$.

7.138 a. Let μ_1 = mean compression ratio for the standard method and μ_2 = mean compression ratio for the Huffman-coding method. The target parameter is $\mu_d = \mu_1 - \mu_2$, the difference in the mean compression ratios for the standard method and the Hoffman-coding method.

Some preliminary calculations are:

Circuit	Standard Method	Huffman-coding Method	Difference
1	.80	.78	.02
2	.80	.80	0
3	.83	.86	-.03
4	.53	.53	0
5	.50	.51	-.01
6	.96	.68	.28
7	.99	.82	.17
8	.98	.72	.26
9	.81	.45	.36
10	.95	.79	.16
11	.99	.77	.22

$$\bar{x}_d = \frac{\sum x_d}{n_d} = \frac{1.43}{11} = .13 \qquad s_d^2 = \frac{\sum x_d^2 - \frac{\left(\sum x_d\right)^2}{n_d}}{n_d - 1} = \frac{.3799 - \frac{1.43^2}{11}}{11-1} = \frac{.194}{10} = .0194$$

$$s_d = \sqrt{.0194} = .1393$$

For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, with df = n_d – 1 = 11 – 1 = 10, $t_{.025}$ = 2.228. The 95% confidence interval is:

$$\bar{x}_d \pm t_{.025}\frac{s_d}{\sqrt{n_d}} \Rightarrow 0.13 \pm 2.228\frac{.1393}{\sqrt{11}} \Rightarrow 0.13 \pm 0.094 \Rightarrow (0.036,\quad 0.224)$$

We are 95% confident that the difference in the mean compression ratios between the standard method and the Hoffman-coding method is between .036 and .224. Since all of the values in the confidence interval are positive, the Hoffman-coding method has the smaller compression ratio.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. The standard error is .03.

$$n_d = \frac{\left(z_{\alpha/2}\right)^2 \sigma_d^2}{(SE)^2} = \frac{1.96^2(.1393)^2}{.03^2} = 82.8 \approx 83$$

7.140 Let p_1 = proportion of patients receiving Zyban who were not smoking one year later and p_2 = proportion of patients not receiving Zyban who were not smoking one year later.

Some preliminary calculations are:

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{71}{309} = .230 \qquad \hat{p}_2 = \frac{x_2}{n_2} = \frac{37}{306} = .121$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{71 + 37}{309 + 306} = .176 \qquad \hat{q} = 1 = \hat{p} = 1 - .176 = .824$$

To determine if the antidepressant drug Zyban helped cigarette smokers kick their habit, we test:

H_0: $p_1 - p_2 = 0$
H_a: $p_1 - p_2 > 0$

The test statistic is $z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.230 - .121) - 0}{\sqrt{.176(.824)\left(\frac{1}{309} + \frac{1}{306}\right)}} = 3.55$

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table III, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = 3.55 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the antidepressant drub Zyban helped cigarette smokers kick their habit at $\alpha = .05$.

7.142 Using MINITAB, the descriptive statistics are:

Descriptive Statistics: AgeDiff, HtDiff, WtDiff

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
AgeDiff	10	0.800	3.88	-8.00	-0.500	1.00	3.25	6.00
HtDiff	10	1.38	3.23	-3.60	-1.25	0.950	4.05	6.20
WtDiff	10	-2.83	5.67	-16.10	-5.17	-2.35	1.47	4.50

(Note: The differences were computed by taking the measurement for those with MS – the measurement for those without MS.)

To determine if there is a difference in mean age between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \frac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \frac{0.8 - 0}{3.88 / \sqrt{10}} = 0.65$

We will use $\alpha = .05$ for all tests. The rejection region requires $\alpha / 2 = .05 / 2 = .025$ in each tail of the t distribution with df = $n_d - 1 = 10 - 1 = 9$. From Table IV, Appendix A, $t_{.025} = 2.262$. The rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 0.65 \ngtr 2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean age between those with MS and those without at $\alpha = .05$.

To determine if there is a difference in mean height between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{1.38 - 0}{3.23 / \sqrt{10}} = 1.35$

From above, the rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.35 \ngtr 2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean height between those with MS and those without at $\alpha = .05$.

To determine if there is a difference in mean weight between those with MS and those without, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

The test statistic is $t = \dfrac{\bar{x}_d - 0}{s_d / \sqrt{n_d}} = \dfrac{-2.83 - 0}{5.67 / \sqrt{10}} = -1.58$

From above, the rejection region is $t < -2.262$ or $t > 2.262$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.58 \nless -2.262$), H_0 is not rejected. There is insufficient evidence to indicate a difference in mean weight between those with MS and those without at $\alpha = .05$.

Thus, it appears that the researchers have successfully matched the MS and non-MS subjects.