Frechet Distribution: $F_u(\phi) = e^{-T\phi^{-\theta}}$

location i EK(2002)

labor

$y_u(\omega, i) = \phi(\omega, i) l$ upstream [perfect competitive]

$\omega_1\ \omega_2\ \omega_3 \cdots$ the unit of tasks: ν

labor $\tau(i,j)$ outsource location j firm z

$F(\phi) = e^{-T_0\phi^{-\theta}}$

$\omega_1\ \omega_2 \cdots$ in-house

downstream [monopolistically competitive]

$y = z l^{\alpha} \nu^{1-\alpha}$

final market

$u = \left[\int q_c(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}$

M total income

the price of ν =

$\mathbb{P}^{1-\rho} = \int_0^1 p_u(\omega)^{1-\rho} d\omega$

(CES)

perfect competitive:

$\pi = 0$

$\max_l \; p y - l w = p\phi l - l w$

$\Rightarrow p\phi = w \quad p = \frac{w}{\phi}$

due to $\tau(i,j)$

$\Rightarrow w(i)\,\tau(i,j) / \phi(\omega, i)$

* $w(j) = w(i) = W$

* both small and large firm face the same ν

monopolistically competitive:

Dixit and Stiglitz (1977) AER

not free entry $\pi \neq 0$, a single price p can not impact the price index $\mathbb{P}$

$\max_{q_c} \pi = p q_c - c^m q_c \quad c^m$ is constant $\quad c^f = 0$

$\min_{c} \int^1 p(z) q_c(z) dz$

$$\frac{d\pi}{dq_e} = p + \frac{dp}{dq_e}\cdot q_e - c = 0$$

$$p + \frac{dp}{dq_e}\frac{q_e}{p}\cdot p = c^m$$

$$p\left(1 + \frac{\frac{dp}{p}}{\frac{dq_e}{q_e}}\right) = c^m \iff \sigma = -\frac{\frac{dq_e}{q_e}}{\frac{dp}{p}}$$

$$p\left(1 - \frac{1}{\sigma}\right) = c^m$$

$$\Rightarrow p = \boxed{\frac{\sigma}{\sigma-1}} c^m$$

markup

$$s.t. \left[\int_0^1 q_e(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} = M$$

(CES).

$$\Rightarrow q_e(z) = \frac{p(z)^{-\sigma}}{\left[\int_0^n p(i)^{1-\sigma} di\right]^{\frac{\sigma}{\sigma-1}}} M$$

and $\mathbb{P} = \left[\int_0^n p(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$

Q

$$\Rightarrow q_e(z) = \frac{p(z)^{-\sigma}}{\mathbb{P}^{-\sigma}}\cdot M$$

household buying

$\ln q_e(z) =$

$-\sigma \ln p(z) + \cdots$

$$\Rightarrow \frac{\partial \ln q_e(z)}{\partial \ln p(z)} = -\sigma$$

in other papers:

$M\mathbb{P} = wL$

thus $q_e(z) = \frac{p(z)^{-\sigma}}{\mathbb{P}^{1-\sigma}} wL$

$$\pi = pq_e - c^m q_e$$

$$= pq_e - pq_e\frac{\sigma-1}{\sigma}$$

$$= pq_e\frac{1}{\sigma} = \boxed{\frac{\text{total sale}}{\sigma}} = c^f \text{ if free entry}$$

$$\pi = \frac{\text{sale}}{\sigma} = \frac{r}{\sigma}$$

$$r = pq_e = p\cdot\frac{p^{-\sigma}}{\mathbb{P}^{-\sigma}}M = Ap^{1-\sigma}$$

demand: $Ap^{-\sigma}$ A demand shifter.

in this paper $A = \frac{M}{\mathbb{P}^{-\sigma}} = w(1+\varphi)L\,\frac{1}{Q}\,\frac{1}{Q^{-\sigma}} = w(1+\varphi)LQ^{\sigma-1}$

pp. Appendix A

price index for final good.

$\frac{\partial TC}{\partial Y} = C^m$ with $\underline{W}$ P exo $\Rightarrow C^m$ is constant
(of V)

min $WL + PV$
$L\,V$

s.t. $Y = ZL^{\alpha}V^{1-\alpha}$ or $\mathcal{L} = \ldots\lambda(\quad)\ldots$

$$\frac{MPL}{MPV} = \frac{W}{P} = \frac{ZV^{1-\alpha}\alpha L^{\alpha-1}}{ZL^{\alpha}(1-\alpha)V^{-\alpha}} = \frac{\alpha}{1-\alpha}\cdot\frac{V}{L}$$

$\frac{W}{P}\frac{1-\alpha}{\alpha} = k = \frac{V}{L} \Rightarrow V = kL$ into production function

$\Rightarrow Y = ZL^{\alpha}(kL)^{1-\alpha} = Zk^{1-\alpha}L \Rightarrow L = \frac{Y}{Zk^{1-\alpha}}$

So $V = kL \Rightarrow V = \frac{Y}{Zk^{-\alpha}}$

$$\Rightarrow TC = \frac{WY}{Zk^{1-\alpha}} + \frac{PY}{Zk^{-\alpha}}$$

$$TC = \frac{WY}{Z\left(\frac{W}{P}\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} + \frac{PY}{Z\left(\frac{W}{P}\frac{1-\alpha}{\alpha}\right)^{-\alpha}}$$

$$TC = \frac{Y\,W^{\alpha}}{ZP^{\alpha-1}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} + \frac{Y\,W^{\alpha}}{ZP^{\alpha-1}\left(\frac{1-\alpha}{\alpha}\right)^{-\alpha}}$$

$$\frac{dTC}{dY} = c^m = w^\alpha p^{1-\alpha} \quad [A \text{ value}]$$

$$p(z,j) = \bar{m}\, w^\alpha p^{1-\alpha}$$ Appendix A.

Go Back

Labor $\bar{L}$ exo

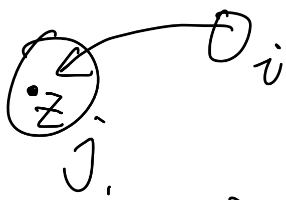
Entry

$m(i)$ exo household won firms.

$\Rightarrow w(1+\varphi)L$ income.

Outsourcing

source i



know freely : T ; $\tau(i,j)$

need effort : individual price of task ω

$f(i)$ same accross location

$\bar{\tau}(z,j)$ $\quad \tau(i,j)^{-\theta}$

Antras 2014 AER.
EK 2002

$$X(z,ij) = \frac{T\tau(i,j)^{-\theta}}{\Phi(z,j)} \quad (3).$$

(share from i) $\Phi(z,j)$

$$\Phi(z,j) = T_0 + \int_{1}^{\bar{\tau}(z,j)} T\tau^{-\theta} g(\tau,j)\,d\tau \quad (\text{※})$$

(~~mattbot~~ access)

$\tau = j$: $\frac{T^{-\theta}}{\Phi(z,j)}$ in-house

$$o(z,j) = 1 - \frac{T_0}{\Phi(z,j)}$$

$$P \text{ of } v = \lambda W \Phi(z,j)^{\frac{1}{\theta}} = P(z,j)$$

(input of v)

\# of searching places

$$\text{※} \quad \max_{\bar{\tau}} \left(\pi(z,j) - W f(j)\, n(z,j) \right)$$

$A = W(1+\varphi)\, \Sigma Q^{\sigma-1}$

exo

$$\pi(z,j) = \frac{\text{sale}}{\sigma} = \frac{r}{\sigma} = \frac{A\, p(z,j)^{1-\sigma}}{\sigma}$$

$$= \frac{A}{\sigma}\left[\bar{m}\, W^{\alpha} P(z,j)^{1-\alpha} / z\right]^{1-\sigma}$$

$$= \frac{A}{\sigma}\bar{m}^{1-\sigma} W^{\alpha(1-\sigma)} z^{\sigma-1} P(z,j)^{(1-\alpha)(1-\sigma)}$$

$(1-\alpha)(1-\sigma)$

$$= \cancel{\ldots} \left[\lambda W \Phi(z,j)^{\frac{1}{\theta}}\right]$$

$$= \frac{A}{\sigma} \bar{m}^{1-\sigma} Z^{\sigma-1} W^{\alpha(1-\sigma)} W^{(1-\alpha)(1-\sigma)} \lambda^{(1-\alpha)(1-\sigma)}$$

$$\Phi(z,j)^{\frac{1}{\theta}(\sigma-1)(1-\alpha)}$$

func of $\bar{\tau}$

$$\pi(z,j) = \frac{(\bar{m}\lambda^{1-\alpha})^{1-\sigma}}{\sigma} A W^{1-\sigma} \Phi(z,j)^{\frac{1}{\theta}(\sigma-1)(1-\alpha)} Z^{\sigma-1}$$

max $\bar{\tau}$

$$\bar{\tau} = K_1 \left[\frac{I}{W^{\sigma}} \frac{A}{f(j)}\right]^{1/\theta} \Phi(z,j)^{-k/\theta} Z^{(\sigma-1)/\theta}$$

$\sigma > 1$

$\bar{\tau}\uparrow \Rightarrow$ more search.

Appendix B:

$$\frac{\partial \bar{\tau}}{\partial z} > 0 \qquad \frac{\partial \bar{\tau}}{\partial f(j)} > 0 \qquad \frac{\partial \tau(z_0, j)}{\partial z_0} < 0$$

What drives firm to outsource? ✓

what is the benefit to outsource? ✗