

# The implementation of the renormalized complex MSSM in FeynArts and FormCalc



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## ABSTRACT

We describe the implementation of the renormalized complex MSSM (cMSSM) in the diagram generator FeynArts and the calculational tool FormCalc. This extension allows to perform UV-finite one-loop calculations of cMSSM processes almost fully automatically. The Feynman rules for the cMSSM with counterterms are available as a new model file for FeynArts. Also included are default definitions of the renormalization constants; this fixes the renormalization scheme. Beyond that all model parameters are generic, e.g. we do not impose any relations to restrict the number of input parameters. The model file has been tested extensively for several non-trivial decays and scattering reactions. Our renormalization scheme has been shown to give stable results over large parts of the cMSSM parameter space.

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## 1. Introduction

One of the main problems of Feynman-diagrammatic computations is the enormous growth of the number of Feynman diagrams, not only with the loop order, but also with the number of particles in a model. While many precision calculations in the Standard Model (SM) could still be performed by hand exactly, the same is very difficult in models like the Minimal Supersymmetric Standard Model (MSSM), even more so when parameters are allowed to take complex values (cMSSM). Yet it is highly desirable to perform unabridged calculations in the MSSM, too, since also this model allows to make precise predictions in terms of a set of input parameters.

With the availability of powerful software packages, the basic problem of bookkeeping and calculation of the diagrams has been solved for many common cases. Still, it is not entirely trivial to code a model of the complexity of the MSSM in such a system, since this has to be done in a reasonably general way (i.e. not only for special cases of the parameters) and many checks have to be performed to test all sectors of the model. Publicly available model files for the MSSM tree-level couplings are described in Refs. [1–3].

The present paper documents the implementation of the renormalized cMSSM with minimal flavor violation in the FeynArts [4] and FormCalc [5] packages. The counterterms and renormalization constants have been tested extensively and show stable results over large parts of the cMSSM parameter space.

Section 2 describes the new MSSM model file, with details of the renormalization given in Section 3. Section 4 contains usage information and Section 5 lists the calculations performed to test the model file.

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**Table 1**

The particle content of MSSMCT.mod.

leptons	$f = f^\dagger$	field	mass	sleptons	$f = f^\dagger$	field	mass
$\nu_g$		F[1, {g}]	0	$\tilde{\nu}_g$		S[11, {g}]	MSf
$\ell_g$		F[2, {g}]	MLE	$\tilde{\ell}_g^s$		S[12, {s, g}]	MSf
quarks				squarks			
$u_g$		F[3, {g, o}]	MQU	$\tilde{u}_g^s$		S[13, {s, g, o}]	MSf
$d_g$		F[4, {g, o}]	MQD	$\tilde{d}_g^s$		S[14, {s, g, o}]	MSf
gauge bosons				neutralinos, charginos			
$\gamma$	yes	V[1]	0	$\tilde{\chi}_n^0$	yes	F[11, {n}]	MNeu
$Z$	yes	V[2]	MZ	$\tilde{\chi}_c^\pm$		F[12, {c}]	MCha
$W^-$		V[3]	MW				
Higgs/Goldstone bosons				ghosts			
$h^0$	yes	S[1]	Mh0	$u_\gamma$		U[1]	0
$H^0$	yes	S[2]	MHH	$u_z$		U[2]	MZ
$A^0$	yes	S[3]	MAO	$u_+$		U[3]	MW
$G^0$	yes	S[4]	MZ	$u_-$		U[4]	MW
$H^-$		S[5]	MHp	$u_g$		U[5, {u}]	0
$G^-$		S[6]	MW				
gluon				gluino			
$g$	yes	V[5, {u}]	0	$\tilde{g}$	yes	F[15, {u}]	MG1

**Table 2**

Index labels and ranges used throughout this paper.

$g = \text{Index}[\text{Generation}] = 1 \dots 3,$ $o = \text{Index}[\text{Colour}] = 1 \dots 3,$ $u = \text{Index}[\text{Gluon}] = 1 \dots 8,$ $s = \text{Index}[\text{Sfermion}] = 1 \dots 2,$ $n = \text{Index}[\text{Neutralino}] = 1 \dots 4,$ $c = \text{Index}[\text{Chargino}] = 1 \dots 2.$	$(S)\text{fermions are indexed by } t = \begin{cases} 1 & (s)\text{neutrinos,} \\ 2 & \text{charged } (s)\text{leptons,} \\ 3 & \text{up-type } (s)\text{quarks,} \\ 4 & \text{down-type } (s)\text{quarks.} \end{cases}$
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**Table 3**

Symbols representing the MSSM parameters in MSSMCT.mod. On the Higgs masses see the discussion at the end of Section 3.2.

Mh0, MHH, MA0, MHP Mh0tree, MHHtree, MA0tree, MHPtree TB, CB, SB, C2B, S2B CA, SA, C2A, S2A CAB, SAB, CBA, SBA MUE	Higgs masses $m_h, m_H, M_A, M_{H^\pm}$ tree-level Higgs masses $\tan \beta, \cos \beta, \sin \beta, \cos 2\beta, \sin 2\beta$ $\cos \alpha, \sin \alpha, \cos 2\alpha, \sin 2\alpha$ (tree-level $\alpha$ ) $\cos(\alpha + \beta), \sin(\alpha + \beta), \cos(\beta - \alpha), \sin(\beta - \alpha)$ Higgs-doublet mixing parameter $\mu$
MG1 SqrTEG1 MNeu[n] ZNeu[n, n'] MCha[c] UCha[c, c'], VCha[c, c']	gluino mass $m_{\tilde{g}}$ root of the gluino phase, $e^{i\varphi_{\tilde{g}}/2}$ neutralino masses $m_{\tilde{\chi}_n^0}$ neutralino mixing matrix $\mathbf{N}_{nn'}$ chargino masses $m_{\tilde{\chi}_c^\pm}$ chargino mixing matrices $\mathbf{U}_{cc'}, \mathbf{V}_{cc'}$
MSf[s, t, g] USf[t, g][s, s'] Af[t, g, g']	sfermion masses $m_{f,sg}$ sfermion mixing matrix $U_{ss'}^{\tilde{f}g}$ soft-breaking trilinear A-parameters $(\mathbf{A}_f)_{gg'}$
MW, MZ CW, SW EL GS	gauge-boson masses $M_W, M_Z$ $c_w \equiv \cos \theta_w = M_W/M_Z, s_w \equiv \sin \theta_w$ electromagnetic coupling constant $e$ strong coupling constant $g_s$

## 2. The MSSMCT model file

The model file is the source of all physics information in FeynArts. It declares the properties of the fields, their propagators, and their couplings. In the model file the generic parameters of the Lagrangian are used, not a restricted set of input parameters.

There are two versions of the renormalized MSSM model file in FeynArts, both of which follow the conventions (for the MSSM at tree-level) of Refs. [6–8] and are based on the existing MSSM model file included in FeynArts [1]. The file MSSMCT.mod defines the complete (electroweak and strong) cMSSM including all counterterms. SQCDCT.mod contains only the SQCD part, i.e. the  $\alpha_{\text{em}} = 0$  limit, which is extracted from MSSMCT.mod at load time.

Table 1 gives the names of the fields defined in MSSMCT.mod and their masses, with index notation in Table 2. The symbols used for the MSSM parameters are specified in Table 3. Pre-defined filters to exclude certain groups of particles are listed in Table 4. In Table 5 we give an overview about the newly introduced symbols for the renormalization constants. The ones that appear already in the SM part are included for completeness.

**Table 4**

Particle-exclusion filters defined in MSSMCT.mod. Observe that indiscriminate use of these filters may destroy finiteness of the results.

NoGeneration1	exclude generation-1 (s)fermions
NoGeneration2	exclude generation-2 (s)fermions
NoGeneration3	exclude generation-3 (s)fermions
NoElectronHCCoupling	exclude all couplings of electrons and any Higgs/Goldstone particle
NoLightFHCoupling	exclude all couplings of light fermions (all fermions except the top) and any Higgs/Goldstone particle
NoSUSYParticles	exclude particles not present in the SM: sfermions, charginos, neutralinos, the Higgs particles $H, A, H^\pm$
THDMParticles	exclude particles not present in the two-Higgs-doublet model: sfermions, charginos, and neutralinos

**Table 5**

Renormalization constants (RCs) used in MSSMCT.mod, where  $a|b$  means ‘ $a$  or  $b$ ’ and  $dZ[\text{bar}]$  stands for both  $dZ$  and  $dZ_{\text{bar}}$ .

Higgs-boson Sector (Section 3.2)		Eqs.
$dZ[\text{bar}]Higgs1[h, h']$	Higgs field RCs	(5)
$dMHiggs1[h, h']$	Higgs mass RCs	(8)
$dTh01, dTHH1, dTA01$	Higgs tadpole RCs	(9)
$dZH1, dZH2, dTB1, dSB1, dCB1$	RCs related to $\beta$	(10)
Gauge-boson Sector (Section 3.3)		
$dMZsq1, dMWsq1$	gauge-boson mass RCs	(13)
$dZAA1, dZAZ1, dZZA1, dZZZ1, dZ[\text{bar}]W1$	gauge-boson field RCs	(14)
$dSW1, dZe1$	coupling-constant RCs	(15)
Chargino/Neutralino Sector (Section 3.4)		
$dMCha1[c, c']$	chargino mass RCs	(23)
$dMNeu1[n, n']$	neutralino mass RCs	(24)
$dMino11, dMino21, dMUE1$	RCs for $M_1, M_2, \mu$	(26)
$dZ[\text{bar}]f_R^L[12, c, c']$	chargino field RCs	(25)
$dZ[\text{bar}]f_R^L[11, n, n']$	neutralino field RCs	(25)
Fermion Sector (Section 3.5)		
$dMf1[t, g]$	fermion mass RCs	(28)
$dZ[\text{bar}]f_R^L[t, g, g']$	fermion field RCs	(30)
$dCKM1[g, g']$	CKM-matrix RCs	(31)
Squark Sector (Section 3.6.1)		
$dMSfsq1[s, s', 3 4, g]$	squark mass RCs	(36), (38)
$dAf1[3 4, g, g]$	trilinear squark coupling RCs	(39)
$dZ[\text{bar}]Sf_R^L[s, s', 3 4, g]$	squark field RCs	(40)
Slepton Sector (Section 3.6.2)		
$dMSfsq1[s, s', 1 2, g]$	slepton mass RCs	(42)
$dAf1[2, g, g]$	trilinear slepton coupling RCs	(44)
$dZ[\text{bar}]Sf_R^L[s, s', 1 2, g]$	slepton field RCs	(45)
Gluino Sector (Section 3.7)		
$dMG11$	gluino mass RC	(47)
$dZ[\text{bar}]G1_R^L$	gluino field RCs	(48)
Gluon Sector (Section 3.8)		
$dZgs1$	strong-coupling-constant RC	(49)
$dZGG1$	gluon field RCs	(50)

### 2.1. Renormalization and absorptive contributions

The counterterms have been derived via multiplicative renormalization applied to all two-, three- and four-point interactions in the Lagrangian. Special care has been taken to include counterterms that appear due to particle mixing for vertices that are zero at the tree level, such as the  $HZ\gamma$ -vertex (which obtains a counterterm contribution from the non-vanishing tree-level  $HZZ$ -vertex and one-loop  $Z-\gamma$  mixing). Feynman gauge is used throughout the model file.

Absorptive contributions arise from the product of imaginary parts of complex couplings in a diagram and imaginary parts of the loop functions in wave-function corrections (self-energy insertions on external legs), i.e. in processes with unstable external particles. These corrections are taken into account via wave-function correction factors  $\delta\tilde{Z}$ , not to be confused with the field renormalization constants  $\delta Z$  introduced by the multiplicative renormalization procedure. For the off-diagonal wave-function correction factors, this procedure has been checked against explicitly including the (renormalized) self-energy type corrections of the external legs, and full agreement was found. The corrections from the absorptive parts can be sizable [9–12]. Definitions  $dZx$  with external masses large enough to develop an

absorptive part are formulated in Section 3 and in the model file in terms of

$$\text{ReDiag} \equiv [\tilde{\text{Re}}] \quad \text{for diagonal } \delta Z_{xx}, \quad (1a)$$

$$\text{ReOffDiag} \equiv [\llbracket \tilde{\text{Re}} \rrbracket] \quad \text{for off-diagonal } \delta Z_{xy}. \quad (1b)$$

The default value for both is Identity, which means that absorptive parts are included ( $dZ_x = \delta Z_x + \delta \tilde{Z}_x$ ). Redefining  $\text{Re[Off]Diag} = \text{ReTilde}$  switches them off ( $dZ_x = \delta Z_x$ ).

$\text{ReTilde} \equiv \text{Re}$  takes the real part of loop integrals only and leaves complex couplings unaffected. The  $dZ_x$  including absorptive parts are in general valid for incoming  $x$  (outgoing  $\bar{x}$ ) only, for outgoing  $x$  (incoming  $\bar{x}$ )  $dZ_{\bar{x}}$  must be used.

### 3. Details of the renormalization

#### 3.1. Prolegomena

We use the following short-hands in this section:

- $c_x \equiv \cos x$ ,  $s_x \equiv \sin x$ ,  $t_x \equiv \tan x$ .
- $\Sigma_{ab}(p^2) \equiv \frac{b}{p} \rightarrow \text{a}$ .
- $\Sigma_f(p^2) \equiv \not{p}\omega_- \Sigma_f^L(p^2) + \not{p}\omega_+ \Sigma_f^R(p^2) + \omega_- \Sigma_f^{SL}(p^2) + \omega_+ \Sigma_f^{SR}(p^2)$  for fermions,  
where  $\omega_{\pm} = \frac{1}{2}(1 \pm \gamma_5)$ .
- $\Sigma_V(p^2) \equiv -\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \Sigma_V^T(p^2) - \frac{p_\mu p_\nu}{p^2} \Sigma_V^L(p^2)$  for vector bosons.
- $\Sigma'(m^2) \equiv \frac{\partial \Sigma(p^2)}{\partial p^2} \big|_{p^2=m^2}$ .
- $\tilde{\text{Re}}$  takes the real part of loop integrals but leaves complex couplings unaffected.
- $\hat{\Sigma}$  denotes a renormalized self energy.

#### 3.2. The Higgs-boson sector

The MSSM requires two Higgs doublets which results in five physical Higgs bosons, the light and heavy CP-even  $h$  and  $H$ , the CP-odd  $A$ , and the charged Higgs bosons  $H^\pm$ . As in the SM, the remaining degrees of freedom are taken up by the neutral and charged Goldstone bosons  $G$  and  $G^\pm$ . Taking higher-order corrections in the cMSSM into account, the three neutral Higgs bosons mix and give rise to three states,  $h_1, h_2, h_3$ .

The Higgs sector is fixed at lowest order by choosing a value for  $t_\beta = v_2/v_1$ , the ratio of the vacuum expectation values of the two Higgs doublets, and for the mass  $M_A$  of the CP-odd neutral Higgs boson  $A$  (for real input parameters) or the mass  $M_{H^\pm}$  of the charged Higgs boson  $H^\pm$  (for complex or real input parameters). The tree-level masses result from

$$\begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} = \mathcal{U}_\alpha \begin{pmatrix} c_\beta^2 M_A^2 + s_\beta^2 M_Z^2 & -s_\beta c_\beta (M_A^2 + M_Z^2) \\ -s_\beta c_\beta (M_A^2 + M_Z^2) & s_\beta^2 M_A^2 + c_\beta^2 M_Z^2 \end{pmatrix} \mathcal{U}_\alpha^T, \quad (2)$$

$$M_{H^\pm}^2 = M_A^2 + M_W^2. \quad (3)$$

Eq. (2) also establishes the  $\alpha$  used in the following as the angle that diagonalizes the tree-level CP-even Higgs-boson mass matrix through the rotation  $\mathcal{U}_\alpha = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$ .

Our multiplicative renormalization procedure shifts masses and couplings but not mixing matrices, with the subtle effect that, while the  $\beta$ 's in the mass matrices contribute to the counterterms as expected, the ones do not that enter through the  $\mathcal{U}_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}$  in

$$\begin{pmatrix} M_X^2 & 0 \\ 0 & 0 \end{pmatrix} = \mathcal{U}_\beta \begin{pmatrix} c_\beta^2 M_X^2 & -s_\beta c_\beta M_X^2 \\ -s_\beta c_\beta M_X^2 & s_\beta^2 M_X^2 \end{pmatrix} \mathcal{U}_\beta^T, \quad X = A, H^\pm. \quad (4)$$

More details on the renormalization of this sector can be found in Refs. [9,13].

The field renormalization constants are as follows. Note that those of the CP-violating self-energies are zero due to the Higgs potential being CP-conserving in lowest order.

$$dZ_{\text{Higgs}1}[1, 1] \equiv \delta Z_{hh} = s_\alpha^2 \delta Z_{\mathcal{H}1} + c_\alpha^2 \delta Z_{\mathcal{H}2}, \quad (5a)$$

$$dZ_{\text{Higgs}1}[2, 2] \equiv \delta Z_{HH} = c_\alpha^2 \delta Z_{\mathcal{H}1} + s_\alpha^2 \delta Z_{\mathcal{H}2}, \quad (5b)$$

$$dZ_{\text{Higgs}1}[3, 3] \equiv \delta Z_{AA} = s_\beta^2 \delta Z_{\mathcal{H}1} + c_\beta^2 \delta Z_{\mathcal{H}2}, \quad (5c)$$

$$dZ_{\text{Higgs}1}[4, 4] \equiv \delta Z_{GG} = c_\beta^2 \delta Z_{\mathcal{H}1} + s_\beta^2 \delta Z_{\mathcal{H}2}, \quad (5d)$$

$$dZ_{\text{Higgs}1}[1, 2] \equiv \delta Z_{hH} = s_\alpha c_\alpha (\delta Z_{\mathcal{H}2} - \delta Z_{\mathcal{H}1}), \quad (5e)$$

$$dZ_{\text{Higgs}1}[1, 3] \equiv \delta Z_{hA} = 0, \quad (5f)$$

$$dZ_{\text{Higgs}1}[2, 3] \equiv \delta Z_{HA} = 0, \quad (5g)$$

$$dZ_{\text{Higgs}1}[1, 4] \equiv \delta Z_{hG} = 0, \quad (5h)$$

$$\text{dZHiggs1}[2, 4] \equiv \delta Z_{HG} = 0, \quad (5i)$$

$$\text{dZHiggs1}[3, 4] \equiv \delta Z_{AG} = s_\beta c_\beta (\delta Z_{\mathcal{H}t_2} - \delta Z_{\mathcal{H}t_1}), \quad (5j)$$

$$\text{dZHiggs1}[5, 5] \equiv \delta Z_{H-H^+} + \delta \hat{Z}_{H-H^+} + \delta \tilde{Z}_{H-H^+} = -[\tilde{\text{Re}}] \Sigma'_{H^-}(M_{H^\pm}^2), \quad (5k)$$

$$\text{dZbarHiggs1}[5, 5] = \text{dZHiggs1}[5, 5], \quad (5l)$$

$$\text{dZHiggs1}[6, 6] \equiv \delta Z_{G^+G^-} = \delta Z_{GG}, \quad (5m)$$

$$\text{dZHiggs1}[5, 6] \equiv \delta Z_{H^+G^-} = \delta Z_{AG}, \quad (5n)$$

$$\text{dZHiggs1}[j, i] = \text{dZHiggs1}[i, j]. \quad (5o)$$

Of the three contributions in Eq. (5k),  $\delta Z_{H-H^+} (= \delta Z_{AA})$  contains only the UV-divergent part and is sufficient to yield UV-finite results, yet  $\delta \hat{Z}_{H-H^+}$  is needed to render the results IR-finite and ensure on-shell properties of an external charged Higgs boson. Corrections to the charged-Higgs-boson propagator give rise to the extra factor

$$\hat{Z}_{H-H^+} \equiv 1 + \delta \hat{Z}_{H-H^+} = \left(1 + \tilde{\text{Re}} \hat{\Sigma}'_{H^-}(M_{H^\pm}^2)\right)^{-1} \quad (6)$$

which, when expanded to one-loop order, leads to  $\delta \hat{Z}_{H-H^+} = -\tilde{\text{Re}} \Sigma'_{H^-}(M_{H^\pm}^2) - \delta Z_{H-H^+}$  and thus to Eq. (5k).

The variable `$MHPInput` (set *before* model initialization) chooses the input mass:

$$\text{\$MHPInput} = \text{False} \quad \text{input mass } M_A \text{ (case [A] below),} \quad (7a)$$

$$\text{\$MHPInput} = \text{True} \quad \text{input mass } M_{H^\pm} \text{ (default: case [H] below).} \quad (7b)$$

With  $C = e/(2M_Z s_W c_W)$  the mass counterterms read [13]<sup>5</sup>:

$$\begin{aligned} \text{dMHiggs1}[1, 1] \equiv \delta m_h^2 = & C(c_{\beta-\alpha} s_{\beta-\alpha}^2 \delta T_H - s_{\beta-\alpha} (1 + c_{\beta-\alpha}^2) \delta T_h) \\ & + 2c_\beta^2 (s_{\alpha+\beta} c_{\alpha+\beta} M_Z^2 - s_{\beta-\alpha} c_{\beta-\alpha} M_A^2) \delta t_\beta + c_{\beta-\alpha}^2 \delta M_A^2 + s_{\alpha+\beta}^2 \delta M_Z^2, \end{aligned} \quad (8a)$$

$$\begin{aligned} \text{dMHiggs1}[2, 2] \equiv \delta m_H^2 = & -C(c_{\beta-\alpha} (1 + s_{\beta-\alpha}^2) \delta T_H - s_{\beta-\alpha} c_{\beta-\alpha}^2 \delta T_h) \\ & - 2c_\beta^2 (s_{\alpha+\beta} c_{\alpha+\beta} M_Z^2 - s_{\beta-\alpha} c_{\beta-\alpha} M_A^2) \delta t_\beta + s_{\beta-\alpha}^2 \delta M_A^2 + c_{\alpha+\beta}^2 \delta M_Z^2, \end{aligned} \quad (8b)$$

$$\text{dMHiggs1}[3, 3] \equiv \delta M_A^2 = \begin{cases} \tilde{\text{Re}} \Sigma_A(M_A^2) & \text{[A]} \\ \delta M_{H^\pm}^2 - \delta M_W^2 & \text{[H]} \end{cases} \quad (8c)$$

$$\text{dMHiggs1}[4, 4] \equiv \delta m_G^2 = -C(c_{\beta-\alpha} \delta T_H + s_{\beta-\alpha} \delta T_h), \quad (8d)$$

$$\begin{aligned} \text{dMHiggs1}[1, 2] \equiv \delta m_{hH}^2 = & -C(s_{\beta-\alpha}^3 \delta T_H + c_{\beta-\alpha}^3 \delta T_h) - c_\beta^2 ((c_{\alpha+\beta}^2 - s_{\alpha+\beta}^2) M_Z^2 \\ & + (c_{\beta-\alpha}^2 - s_{\beta-\alpha}^2) M_A^2) \delta t_\beta - s_{\beta-\alpha} c_{\beta-\alpha} \delta M_A^2 - s_{\alpha+\beta} c_{\alpha+\beta} \delta M_Z^2, \end{aligned} \quad (8e)$$

$$\text{dMHiggs1}[1, 3] \equiv \delta m_{hA}^2 = -C s_{\beta-\alpha} \delta T_A, \quad (8f)$$

$$\text{dMHiggs1}[2, 3] \equiv \delta m_{HA}^2 = -C c_{\beta-\alpha} \delta T_A, \quad (8g)$$

$$\text{dMHiggs1}[1, 4] \equiv \delta m_{hG}^2 = -\delta m_{hA}^2, \quad (8h)$$

$$\text{dMHiggs1}[2, 4] \equiv \delta m_{HG}^2 = \delta m_{hA}^2, \quad (8i)$$

$$\text{dMHiggs1}[3, 4] \equiv \delta m_{AG}^2 = C(s_{\beta-\alpha} \delta T_H - c_{\beta-\alpha} \delta T_h) - c_\beta^2 \delta t_\beta \begin{cases} M_A^2 & \text{[A]} \\ (M_{H^\pm}^2 - M_W^2) & \text{[H]} \end{cases} \quad (8j)$$

$$\text{dMHiggs1}[j, i] = \text{dMHiggs1}[i, j] \quad i, j = 1 \dots 4, \quad (8k)$$

$$\text{dMHiggs1}[5, 5] \equiv \delta M_{H^\pm}^2 = \begin{cases} \delta M_A^2 + \delta M_W^2 & \text{[A]} \\ \tilde{\text{Re}} \Sigma_{H^-}(M_{H^\pm}^2) & \text{[H]} \end{cases} \quad (8l)$$

$$\text{dMHiggs1}[6, 6] \equiv \delta m_{G^\pm}^2 = \delta m_G^2, \quad (8m)$$

$$\text{dMHiggs1}[5, 6] \equiv \delta m_{H^+G^-}^2 = C(s_{\beta-\alpha} \delta T_H - c_{\beta-\alpha} \delta T_h + i \delta T_A) - c_\beta^2 M_{H^\pm}^2 \delta t_\beta, \quad (8n)$$

$$\text{dMHiggs1}[6, 5] = \text{dMHiggs1}[5, 6]^*. \quad (8o)$$

At tree-level the CP-even Higgs fields do not mix with the Z boson and hence there are no counterterm contributions to this mixing at one-loop level.

<sup>5</sup> In Ref. [13] a slightly different renormalization prescription for  $t_\beta$  had been introduced,  $t_\beta \rightarrow t_\beta (1 + \delta t_\beta^{[13]})$ , such that  $\delta t_\beta = t_\beta \delta t_\beta^{[13]}$ .

The tadpole parameters are renormalized such that the complete one-loop tadpole contributions vanish, leading to

$$\text{dTh01} \equiv \delta T_h = -T_h^{(1)}, \quad (9a)$$

$$\text{dTHH1} \equiv \delta T_H = -T_H^{(1)}, \quad (9b)$$

$$\text{dTA01} \equiv \delta T_A = -T_A^{(1)} \quad (9c)$$

where  $T_\phi^{(1)}$  contains all irreducible one-loop tadpole diagrams of field  $\phi = h, H, A$ .

$\overline{\text{DR}}$  renormalization of  $\delta Z_{\mathcal{H}_{\ell_1}}$ ,  $\delta Z_{\mathcal{H}_{\ell_2}}$ , and  $\delta t_\beta$  avoids large (and unphysical) higher-order corrections in the Higgs-mass calculations [14]. It has been shown to yield stable numerical results [14–16] and is also gauge-independent at the one-loop level within the class of  $R_\xi$  gauges [15]. Furthermore, there is no obvious relation of  $t_\beta$  to a specific physical observable that would favor a particular on-shell definition and the divergent part of  $\delta t_\beta$  can be expressed by the UV-divergent parts of the field renormalization constants [17].

$$\text{dZH1} \equiv \delta Z_{\mathcal{H}_{\ell_1}} = -\text{Re} \Sigma'_H(0)|_{\alpha=0, \text{div}}, \quad (10a)$$

$$\text{dZH2} \equiv \delta Z_{\mathcal{H}_{\ell_2}} = -\text{Re} \Sigma'_H(0)|_{\alpha=0, \text{div}}, \quad (10b)$$

$$\text{dTB1} \equiv \delta t_\beta = \frac{1}{2} t_\beta (\delta Z_{\mathcal{H}_{\ell_2}} - \delta Z_{\mathcal{H}_{\ell_1}}), \quad (10c)$$

$$\text{dSB1} \equiv \delta s_\beta = c_\beta^3 \delta t_\beta, \quad (10d)$$

$$\text{dCB1} \equiv \delta c_\beta = -s_\beta c_\beta^2 \delta t_\beta, \quad (10e)$$

i.e. the counterterms in Eq. (10) contribute only UV-divergent parts,<sup>6</sup> and the finite result depends on the renormalization scale  $\mu_R$  (in FormCalc: MUDIM).

The Higgs-boson field renormalization constants are necessary to render the one-loop calculations of partial decay widths with external Higgs bosons UV-finite. The  $\overline{\text{DR}}$  scheme for the field renormalization constants is used in the calculation of the Higgs masses within FeynHiggs [13,18–20] (see Section 4.3 on how to use FeynHiggs with FormCalc) in order to avoid the possible occurrence of unphysical threshold effects.

When composing a vertex  $\Gamma_{h_i}$  ( $i = 1, 2, 3$ ) from the corresponding tree-level amplitudes  $\Gamma_h$ ,  $\Gamma_H$ , and  $\Gamma_A$ , another set of finite  $Z$ -factors is needed to ensure correct on-shell properties of the external Higgs boson  $h_i$  [13],

$$\Gamma_{h_i} = \hat{Z}_{i1} \Gamma_h + \hat{Z}_{i2} \Gamma_H + \hat{Z}_{i3} \Gamma_A + \dots, \quad (11)$$

where the ellipsis represents contributions from the mixing with the Goldstone and  $Z$  boson, which have to be taken into account explicitly. The  $Z$ -factor matrix  $\hat{Z}_{ij} \equiv \text{ZHiggs}[i, j]$  is not in general unitary. Its lower  $3 \times 3$  part is computed by FeynHiggs and application at the amplitude level automatically takes any absorptive contribution into account. Technically this is most easily accomplished using the FeynArts add-on model file HMix.mod [21] which mixes  $h = \text{S}[1]$ ,  $H = \text{S}[2]$ , and  $A = \text{S}[3]$  into two variants of the loop-corrected states  $h_i$ ,

$$\text{S}[0, \{i\}] = \sum_{j=1}^3 \text{UHiggs}[i, j] \text{S}[j], \quad \text{with unitary UHiggs (no absorptive contrib.), for use on internal lines}, \quad (12a)$$

$$\text{S}[10, \{i\}] = \sum_{j=1}^3 \text{ZHiggs}[i, j] \text{S}[j], \quad \text{inserted only on external lines}. \quad (12b)$$

For an external charged Higgs boson a similar factor  $\sqrt{\hat{Z}_{H-H^+}} = 1 + \frac{1}{2} \delta \hat{Z}_{H-H^+}$  is necessary; this is already included in Eq. (5k). Mixing with the Goldstone and the  $W$  boson must be computed explicitly, however, as in the neutral case.

Finally, a note on the Higgs masses used in the model file. Higher-order corrections are phenomenologically very important in the Higgs sector, yet the use of loop-corrected masses, besides mixing orders in perturbation theory, entails a certain risk of upsetting the relations necessary for the proper cancellation of UV and IR divergences. For instance, the masses on the Higgs propagators should be consistent with the mixing angle  $\alpha$  parameterizing the vertices, but this is not easy to achieve in practice since at loop level there is mixing between all three states  $h, H, A$ , which is not expressible through a single angle  $\alpha$ .

We opted therefore to formulate the vertices with tree-level  $\alpha$  and insert tree-level masses  $\text{Mh0tree}$ ,  $\text{MHHtree}$ ,  $\text{MA0tree}$ ,  $\text{MHptree}$  on loop propagators, where the chance of violating supersymmetric relations (and double-counting higher-order contributions) is highest, but use loop-corrected masses  $\text{Mh0}$ ,  $\text{MHH}$ ,  $\text{MA0}$ ,  $\text{MHp}$  on all other propagators. At the level of the Feynman rules it is not possible to generally avoid incomplete cancellations due to a mismatch between tree-level and loop-corrected masses, though there are typically process-specific solutions (see e.g. Refs. [9,22]). Our recommendation is to test UV and IR finiteness with loop-corrected masses and revert to tree-level masses as far as necessary.

<sup>6</sup> The divergences in Eqs. (10a) and (10b) are momentum-independent.

### 3.3. The Gauge-boson sector

For the SM gauge bosons we impose the usual set of on-shell renormalization conditions and obtain [9,23]

$$\delta M_{Z^2}^2 \equiv \delta M_Z^2 = \tilde{\text{Re}} \Sigma_Z^T(M_Z^2), \quad (13a)$$

$$\delta M_{W^2}^2 \equiv \delta M_W^2 = \tilde{\text{Re}} \Sigma_W^T(M_W^2), \quad (13b)$$

$$\delta Z_{AA1} \equiv \delta Z_{\gamma\gamma} = -\tilde{\text{Re}} \Sigma_{\gamma}^T(0), \quad (14a)$$

$$\delta Z_{AZ1} \equiv \delta Z_{\gamma Z} = -\frac{2}{M_Z^2} \tilde{\text{Re}} \Sigma_{\gamma Z}^T(M_Z^2), \quad (14b)$$

$$\delta Z_{ZA1} \equiv \delta Z_{Z\gamma} = \frac{2}{M_Z^2} \tilde{\text{Re}} \Sigma_{\gamma Z}^T(0), \quad (14c)$$

$$\delta Z_{ZZ1} \equiv \delta Z_{ZZ} + \delta \tilde{Z}_{ZZ} = -[\tilde{\text{Re}}] \Sigma_Z^T(M_Z^2), \quad (14d)$$

$$\delta Z_{W1} \equiv \delta Z_{WW} + \delta \tilde{Z}_{WW} = -[\tilde{\text{Re}}] \Sigma_W^T(M_W^2), \quad (14e)$$

$$\delta Z_{\text{bar}W1} = \delta Z_{W1}. \quad (14f)$$

For convenience we define the (dependent) coupling-constant counterterms

$$\delta S_W \equiv \delta s_w = \frac{1}{2} \frac{c_w^2}{s_w} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right), \quad (15a)$$

$$\delta Z_e \equiv \delta Z_e = \frac{1}{2} \left( \frac{s_w}{c_w} \delta Z_{Z\gamma} - \delta Z_{\gamma\gamma} \right). \quad (15b)$$

Renormalization constants for the ghost fields are not defined as this is necessary only for two-loop calculations.

### 3.4. The Chargino/Neutralino sector

The chargino/neutralino sector contains two soft-SUSY-breaking gaugino mass parameters,  $M_1$  for the bino and  $M_2$  for the wino field, and the Higgs superfield mixing parameter  $\mu$ , all of which are in general complex.<sup>7</sup> Details on the renormalization can be found in Refs. [9,11,12].

The chargino masses  $m_{\tilde{\chi}_{1,2}^\pm}$  are obtained from the singular value decomposition

$$\mathbf{M}_{\tilde{\chi}^-} \equiv \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix} = \mathbf{V}^* \mathbf{X}^T \mathbf{U}^\dagger \quad \text{with } \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}. \quad (16)$$

The neutralino mass matrix

$$\mathbf{Y} = \begin{pmatrix} M_1 & 0 & -M_Z s_w c_\beta & M_Z s_w s_\beta \\ 0 & M_2 & M_Z c_w c_\beta & -M_Z c_w s_\beta \\ -M_Z s_w c_\beta & M_Z c_w c_\beta & 0 & -\mu \\ M_Z s_w s_\beta & -M_Z c_w s_\beta & -\mu & 0 \end{pmatrix} \quad (17)$$

is symmetric and the masses are determined from a Takagi factorization [24]

$$\mathbf{M}_{\tilde{\chi}^0} = \mathbf{N}^* \mathbf{Y} \mathbf{N}^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}). \quad (18)$$

Parameters and fields are renormalized multiplicatively, following the prescription of Ref. [25]. The parameter counterterms are complex and thus two renormalization conditions must be specified for each. The transformation matrices  $\mathbf{U}$  and  $\mathbf{V}$  do not obtain counterterms, only the mass matrices pick up shifts  $\mathbf{X} \rightarrow \mathbf{X} + \delta \mathbf{X}$  and  $\mathbf{Y} \rightarrow \mathbf{Y} + \delta \mathbf{Y}$ , given by

$$\delta \mathbf{X} = \begin{pmatrix} \delta M_2 & \sqrt{2} \delta(M_W s_\beta) \\ \sqrt{2} \delta(M_W c_\beta) & \delta \mu \end{pmatrix}, \quad (19)$$

$$\delta \mathbf{Y} = \begin{pmatrix} \delta M_1 & 0 & -\delta(M_Z s_w c_\beta) & \delta(M_Z s_w s_\beta) \\ 0 & \delta M_2 & \delta(M_Z c_w c_\beta) & -\delta(M_Z c_w s_\beta) \\ -\delta(M_Z s_w c_\beta) & \delta(M_Z c_w c_\beta) & 0 & -\delta \mu \\ \delta(M_Z s_w s_\beta) & -\delta(M_Z c_w s_\beta) & -\delta \mu & 0 \end{pmatrix}. \quad (20)$$

This leads to the mass shifts

$$\mathbf{M}_{\tilde{\chi}^-} \rightarrow \mathbf{M}_{\tilde{\chi}^-} + \delta \mathbf{M}_{\tilde{\chi}^-} = \mathbf{M}_{\tilde{\chi}^-} + \mathbf{V}^* \delta \mathbf{X}^T \mathbf{U}^\dagger, \quad (21)$$

$$\mathbf{M}_{\tilde{\chi}^0} \rightarrow \mathbf{M}_{\tilde{\chi}^0} + \delta \mathbf{M}_{\tilde{\chi}^0} = \mathbf{M}_{\tilde{\chi}^0} + \mathbf{N}^* \delta \mathbf{Y} \mathbf{N}^\dagger \quad (22)$$

<sup>7</sup> Our  $M_2$  is a complex parameter even though  $M_2$  can be chosen real without loss of generality. This is possible since not all phases of the MSSM Lagrangian are physical and there is a certain freedom of choice.



from which the mass counterterms can be read off as

$$\delta\mathbf{M}\text{Cha1}[c, c'] \equiv (\delta\mathbf{M}_{\tilde{\chi}^-})_{cc'} = (\mathbf{V}^* \delta\mathbf{X}^T \mathbf{U}^\dagger)_{cc'}, \quad (23)$$

$$\delta\mathbf{M}\text{Neu1}[n, n'] \equiv (\delta\mathbf{M}_{\tilde{\chi}^0})_{nn'} = (\mathbf{N}^* \delta\mathbf{Y} \mathbf{N}^\dagger)_{nn'}. \quad (24)$$

The field renormalization constants are [9,25] ( $c \neq c', n \neq n'$ ):

$$\text{dZfL1}[12, c, c] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^L)_{cc} = (\sigma_{cc}^L(\tilde{\chi}^-) + \tau_{cc}(\tilde{\chi}^-))_{cc}, \quad (25a)$$

$$\text{dZbarfL1}[12, c, c] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^L)_{cc} = (\sigma_{cc}^L(\tilde{\chi}^-) - \tau_{cc}(\tilde{\chi}^-))_{cc}, \quad (25b)$$

$$\text{dZfR1}[12, c, c] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^R)_{cc} = (\sigma_{cc}^R(\tilde{\chi}^-) - \tau_{cc}(\tilde{\chi}^-))_{cc}, \quad (25c)$$

$$\text{dZbarfR1}[12, c, c] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^R)_{cc} = (\sigma_{cc}^R(\tilde{\chi}^-) + \tau_{cc}(\tilde{\chi}^-))_{cc}, \quad (25d)$$

$$\text{dZfL1}[11, n, n] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^0}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^0}^L)_{nn} = (\sigma_{nn}^L(\tilde{\chi}^0) + \tau_{nn}(\tilde{\chi}^0))_{nn}, \quad (25e)$$

$$\text{dZbarfL1}[11, n, n] = \text{dZfR1}[11, n, n], \quad (25f)$$

$$\text{dZfR1}[11, n, n] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^0}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^0}^R)_{nn} = (\sigma_{nn}^R(\tilde{\chi}^0) - \tau_{nn}(\tilde{\chi}^0))_{nn}, \quad (25g)$$

$$\text{dZbarfR1}[11, n, n] = \text{dZfL1}[11, n, n], \quad (25h)$$

where

$$\sigma_{ii}^X(\tilde{\chi}) = -[\tilde{\text{Re}}] \left[ \Sigma_{\tilde{\chi}}^X(m_{\tilde{\chi}i}^2) + m_{\tilde{\chi}i}^2 \left( \Sigma_{\tilde{\chi}}^{\prime L}(m_{\tilde{\chi}i}^2) + \Sigma_{\tilde{\chi}}^{\prime R}(m_{\tilde{\chi}i}^2) \right) + m_{\tilde{\chi}i} \left( \Sigma_{\tilde{\chi}}^{\prime SL}(m_{\tilde{\chi}i}^2) + \Sigma_{\tilde{\chi}}^{\prime SR}(m_{\tilde{\chi}i}^2) \right) \right],$$

$$\tau_{ii}(\tilde{\chi}) = \frac{1}{2m_{\tilde{\chi}i}} \left( [\tilde{\text{Re}}] \left[ \Sigma_{\tilde{\chi}}^{SL}(m_{\tilde{\chi}i}^2) - \Sigma_{\tilde{\chi}}^{SR}(m_{\tilde{\chi}i}^2) \right] - \delta\mathbf{M}_{\tilde{\chi}} + \delta\mathbf{M}_{\tilde{\chi}}^\dagger \right).$$

$$\text{dZfL1}[12, c, c'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^L)_{cc'} = (\sigma_{cc'}^{L,R,SR,SL}(\tilde{\chi}^-) - \tau_{cc'}(\tilde{\chi}^-))_{cc'}, \quad (25i)$$

$$\text{dZbarfL1}[12, c, c'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^L)_{cc'} = (\sigma_{c'c}^{L,R,SL,SR}(\tilde{\chi}^-) - \tau_{c'c}^\dagger(\tilde{\chi}^-))_{cc'}, \quad (25j)$$

$$\text{dZfR1}[12, c, c'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^R)_{cc'} = (\sigma_{cc'}^{R,L,SL,SR}(\tilde{\chi}^-) - \tau_{cc'}^\dagger(\tilde{\chi}^-))_{cc'}, \quad (25k)$$

$$\text{dZbarfR1}[12, c, c'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^-}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^-}^R)_{cc'} = (\sigma_{c'c}^{R,L,SR,SL}(\tilde{\chi}^-) - \tau_{c'c}(\tilde{\chi}^-))_{cc'}, \quad (25l)$$

$$\text{dZfL1}[11, n, n'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^0}^L + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^0}^L)_{nn'} = (\sigma_{nn'}^{L,R,SR,SL}(\tilde{\chi}^0) - \tau_{nn'}(\tilde{\chi}^0))_{nn'}, \quad (25m)$$

$$\text{dZbarfL1}[11, n, n'] = \text{dZfR1}[11, n', n], \quad (25n)$$

$$\text{dZfR1}[11, n, n'] \equiv (\delta\mathbf{Z}_{\tilde{\chi}^0}^R + \delta\tilde{\mathbf{Z}}_{\tilde{\chi}^0}^R)_{nn'} = (\sigma_{nn'}^{R,L,SL,SR}(\tilde{\chi}^0) - \tau_{nn'}^\dagger(\tilde{\chi}^0))_{nn'}, \quad (25o)$$

$$\text{dZbarfR1}[11, n, n'] = \text{dZfL1}[11, n', n], \quad (25p)$$

where

$$\sigma_{ij}^{X,Y,SX,SY}(\tilde{\chi}) = \frac{2}{m_{\tilde{\chi}i}^2 - m_{\tilde{\chi}j}^2} [\tilde{\text{Re}}] \left[ m_{\tilde{\chi}j} (m_{\tilde{\chi}j} \Sigma_{\tilde{\chi}}^X(m_{\tilde{\chi}j}^2) + m_{\tilde{\chi}i} \Sigma_{\tilde{\chi}}^Y(m_{\tilde{\chi}j}^2)) + m_{\tilde{\chi}j} \Sigma_{\tilde{\chi}}^{SX}(m_{\tilde{\chi}j}^2) + m_{\tilde{\chi}i} \Sigma_{\tilde{\chi}}^{SY}(m_{\tilde{\chi}j}^2) \right],$$

$$\tau_{ij}(\tilde{\chi}) = \frac{2}{m_{\tilde{\chi}i}^2 - m_{\tilde{\chi}j}^2} [\tilde{\text{Re}}] \left[ m_{\tilde{\chi}i} \delta\mathbf{M}_{\tilde{\chi}} + m_{\tilde{\chi}j} \delta\mathbf{M}_{\tilde{\chi}}^\dagger \right].$$

Note again the inclusion of absorptive parts through the  $\tilde{\text{Re}}$ . More detailed information on this for the chargino/neutralino sector can be found in Refs. [9,11,12,26].

Rather than renormalizing the three complex input parameters  $M_1$ ,  $M_2$ , and  $\mu$  directly, we impose on-shell conditions for either two charginos and one neutralino (CCN) or one chargino and two neutralinos (CNN), and from them work out the counterterms  $\delta M_1$ ,  $\delta M_2$ , and  $\delta\mu$ .

### 3.4.1. CCN schemes

Inverting Eqs. (19)–(22) for neutralino  $\tilde{\chi}_n^0$  on-shell yields the following shifts of the breaking parameters [25,27]

$$\delta\mathbf{M}\text{ino11} \equiv \delta M_1 = \frac{1}{N_{n1}^{*2}} \left\{ \delta m_{\tilde{\chi}_n^0}^{\text{OS}} + \delta N_n - N_{n2}^{*2} \delta M_2 + 2N_{n3}^* N_{n4}^* \delta\mu \right\}, \quad (26a)$$



$$\text{dMino21} \equiv \delta M_2 = \frac{1}{u_o v_o - u_d v_d} \left\{ U_{12}^* V_{12}^* \delta m_{\tilde{\chi}_2^\pm}^{\text{OS}} - U_{22}^* V_{22}^* \delta m_{\tilde{\chi}_1^\pm}^{\text{OS}} + \sqrt{2} \left( c_\beta (u_d - u_o) V_{12}^* V_{22}^* + s_\beta (v_o - v_d) U_{12}^* U_{22}^* \right) M_W c_\beta^2 \delta t_\beta \right. \\ \left. + \left( s_\beta (u_d - u_o) V_{12}^* V_{22}^* - c_\beta (v_o - v_d) U_{12}^* U_{22}^* \right) \frac{\delta M_W^2}{\sqrt{2} M_W} \right\}, \quad (26b)$$

$$\text{dMUE1} \equiv \delta \mu = \frac{1}{u_o v_o - u_d v_d} \left\{ U_{21}^* V_{21}^* \delta m_{\tilde{\chi}_1^\pm}^{\text{OS}} - U_{11}^* V_{11}^* \delta m_{\tilde{\chi}_2^\pm}^{\text{OS}} - \sqrt{2} \left( s_\beta (u_d - u_o) V_{11}^* V_{21}^* + c_\beta (v_o - v_d) U_{11}^* U_{21}^* \right) M_W c_\beta^2 \delta t_\beta \right. \\ \left. + \left( c_\beta (u_d - u_o) V_{11}^* V_{21}^* - s_\beta (v_o - v_d) U_{11}^* U_{21}^* \right) \frac{\delta M_W^2}{\sqrt{2} M_W} \right\} \quad (26c)$$

where we use the short-hands

$$u_d = U_{11}^* U_{22}^*, \quad u_o = U_{12}^* U_{21}^*, \quad v_d = V_{11}^* V_{22}^*, \quad v_o = V_{12}^* V_{21}^*, \\ \delta N_n = 2c_\beta^2 \delta t_\beta \left( s_\beta N_{n3}^* + c_\beta N_{n4}^* \right) \left( M_W N_{n2}^* - M_Z s_W N_{n1}^* \right) + \left( c_\beta N_{n3}^* - s_\beta N_{n4}^* \right) \left( N_{n1}^* \left[ \frac{\delta M_Z^2}{M_Z} s_W + 2M_Z \delta s_W \right] - N_{n2}^* \frac{\delta M_W^2}{M_W} \right), \\ \delta m_{\tilde{\chi}_n^0}^{\text{OS}} = \tilde{\text{Re}} \left[ m_{\tilde{\chi}_n^0} \Sigma_{\tilde{\chi}_n^0}^L (m_{\tilde{\chi}_n^0}^2) + \Sigma_{\tilde{\chi}_n^0}^{SL} (m_{\tilde{\chi}_n^0}^2) \right]_{nn}, \\ \delta m_{\tilde{\chi}_c^\pm}^{\text{OS}} = \tilde{\text{Re}} \left[ \frac{m_{\tilde{\chi}_c^\pm}}{2} \left( \Sigma_{\tilde{\chi}_c^\pm}^L (m_{\tilde{\chi}_c^\pm}^2) + \Sigma_{\tilde{\chi}_c^\pm}^R (m_{\tilde{\chi}_c^\pm}^2) \right) + \Sigma_{\tilde{\chi}_c^\pm}^{SL} (m_{\tilde{\chi}_c^\pm}^2) \right]_{cc}.$$

### 3.4.2. CNN schemes

Renormalization schemes with one chargino  $\tilde{\chi}_c^\pm$  and two neutralinos  $\tilde{\chi}_n^0, \tilde{\chi}_{n'}^0$  taken as on-shell particles give better numerical stability in regions of  $|\mu| \approx |M_2|$ , as shown in Ref. [28].

The on-shell conditions of the CNN schemes are analogous to those of the CCN schemes, see Refs. [9,11,12]. As above we impose conditions on the neutralino and chargino masses and solve for  $\delta M_1$ ,  $\delta M_2$ , and  $\delta \mu$ :

$$\text{dMino11} \equiv \delta M_1 = \frac{A_{2,nn'} + 2B_{2,nn'} \delta \mu}{C_{nn'}}, \quad (27a)$$

$$\text{dMino21} \equiv \delta M_2 = -\frac{A_{1,nn'} + 2B_{1,nn'} \delta \mu}{C_{nn'}}, \quad (27b)$$

$$\text{dMUE1} \equiv \delta \mu = \frac{1}{C_{nn'} U_{c2}^* V_{c2}^* - 2B_{1,nn'} V_{c1}^* U_{c1}^*} \left\{ U_{c1}^* V_{c1}^* A_{1,nn'} + C_{nn'} \left[ \delta m_{\tilde{\chi}_c^\pm}^{\text{OS}} - \sqrt{2} (c_\beta U_{c1}^* V_{c2}^* - s_\beta V_{c1}^* U_{c2}^*) c_\beta^2 \delta t_\beta M_W \right. \right. \\ \left. \left. - (s_\beta U_{c1}^* V_{c2}^* + c_\beta V_{c1}^* U_{c2}^*) \frac{\delta M_W^2}{\sqrt{2} M_W} \right] \right\}, \quad (27c)$$

with

$$A_{i,nn'} = N_{n'i}^{*2} (\delta m_{\tilde{\chi}_n^0}^{\text{OS}} + \delta N_n) - N_{ni}^{*2} (\delta m_{\tilde{\chi}_{n'}^0}^{\text{OS}} + \delta N_{n'}), \\ B_{i,nn'} = N_{n'i}^{*2} N_{n3}^* N_{n4}^* - N_{ni}^{*2} N_{n'3}^* N_{n'4}^*, \\ C_{nn'} = N_{n1}^{*2} N_{n'2}^{*2} - N_{n'1}^{*2} N_{n2}^{*2}.$$

### 3.4.3. Discussion of scheme selection

In a recent analysis [28] it was emphasized that for a CCN scheme to yield numerically stable results it must be the most bino-like neutralino that is chosen on-shell. Ref. [29] further discusses the problem of large unphysical contributions due to a non-binolike lightest neutralino.

Note that the Z-factors also ensure that the external (stable) particle does not mix with other fields, which is one of the on-shell properties. Which neutralino to take on-shell can be chosen (details below), currently the lightest one is the default. For more discussion, see the Appendix of Ref. [9].

A comparison of different renormalization schemes in the chargino/neutralino sector is given in Ref. [12]. The differences found with respect to another on-shell renormalization in the chargino/neutralino sector were small and of the expected size of two-loop contributions.

Special care has to be taken in the regions of the cMSSM parameter space where the gaugino–Higgsino mixing in the chargino sector is close to maximal, i.e. where  $|\mu| \approx |M_2|$ . Here  $\delta M_2$  and  $\delta \mu$  diverge as  $(U_{11}^* U_{22}^* V_{11}^* V_{22}^* - U_{12}^* U_{21}^* V_{12}^* V_{21}^*)^{-1}$  (cf. Eq. (26)) for the CCN schemes and the loop calculation does not yield a reliable result. A similar singularity for  $|\mu| \approx |M_2|$  arises also in the CNN[1, 1, 4] and CNN[2, 1, 2] schemes when  $|M_1| < |M_2|$ . These kind of divergences were also discussed in Refs. [26,28,30].

The choice of chargino/neutralino renormalization scheme is made through the variable \$InoScheme, which must be set before model initialization. Allowed selections are CCN[n] and CNN[c,n,n']; these may also be combined for run-time switching of the schemes, e.g.

- \$InoScheme = CCN[1] – fixed CCN scheme with on-shell  $\tilde{\chi}_1^0$ .  
This is currently the default but might change in the future based on experience with more calculations.
- \$InoScheme = CNN[2, 1, 3] – fixed CNN scheme with on-shell  $\tilde{\chi}_2^\pm, \tilde{\chi}_1^0, \tilde{\chi}_3^0$ .

- $\$InoScheme = IndexIf[cond, CNN[2, 1, 3], CCN[1]]$   
scheme chosen at run-time according to the logical Mathematica expression *cond* which may contain model parameters, e.g.  $Abs[Abs[MUE] - Abs[Mino2]] < 50$ . Note the use of *IndexIf* instead of *If*, necessary because *If* has delayed evaluation of its arguments.
- $\$InoScheme = CCN[nbino] - CCN$  scheme with the most bino-like neutralino on-shell, with *nbino* to be determined at run-time as the *n* which maximizes  $|N_{n1}|$ .

Observe that a run-time switch of the renormalization scheme requires a corresponding transition of the affected parameters from one scheme to the other (not yet implemented) for a fully consistent interpretation of the results.

Either scheme fixes three out of six chargino/neutralino masses to be on-shell. The other three masses then acquire a finite shift. It was shown in Ref. [9] that these shifts are small numerically. They are not implemented in the model file so far, which does not really count as an omission as propagators are parameterized with tree-level masses in canonical perturbation theory anyway.

### 3.5. The Fermion sector

We closely follow the renormalization of Ref. [9,31], enlarged to the full fermion sector and extended to include external bottom quarks, too, for which the “ $m_b, A_b \overline{DR}$ ” scheme proposed in Ref. [31] is inappropriate.

The fermion mass counterterms are defined as

$$dMf1[t, g] \equiv \delta m_{f_{tg}} = U_{tg} \left\{ \frac{1}{2} \tilde{\text{Re}} \left[ m_{f_{tg}} (\Sigma_{f_t}^L(m_{f_{tg}}^2) + \Sigma_{f_t}^R(m_{f_{tg}}^2))_{gg} + (\Sigma_{f_t}^{SL}(m_{f_{tg}}^2) + \Sigma_{f_t}^{SR}(m_{f_{tg}}^2))_{gg} \right] \right\}. \quad (28)$$

Note that for the (massless) neutrinos ( $t = 1$ ) this evaluates to  $\delta m_\nu = 0$ .

The function  $U_{tg} \equiv \text{UVMf1}[t, g]$  allows to choose between

$$\text{UVMf1}[4, 3] = \text{UVDivergentPart} \quad \overline{DR} \text{ renormalization for } m_b \text{ (default),} \quad (29a)$$

$$\text{UVMf1}[4, 3] = \text{Identity} \quad \text{on-shell (OS) renormalization for } m_b. \quad (29b)$$

Unless the bottom quark appears in external states, the  $\overline{DR}$  prescription is preferred for  $m_b$  [31]. The problems found in Ref. [31] with an on-shell renormalization condition for  $m_b$  (leading e.g. to unphysically large contributions to  $\delta A_b$ ) do not occur as long as there are no external sbottom quarks and the parameter  $A_b$  is only needed at leading order. An example for which both schemes would presumably fail is  $b\bar{b} \rightarrow \tilde{b}_i \tilde{b}_j$ . More details on the definition of  $m_b^{\text{OS}}$  can be found in Refs. [9,31].

The fermion field renormalization constants are given by

$$dZfL1[t, g, g] \equiv (\delta Z_{f_t}^L + \delta \check{Z}_{f_t}^L)_{gg} = (\sigma_{gg}^L(f_t) + \tau_{gg}(f_t))_{gg}, \quad (30a)$$

$$dZbarfL1[t, g, g] \equiv (\delta Z_{f_t}^L + \delta \check{Z}_{f_t}^L)_{gg} = (\sigma_{gg}^L(f_t) - \tau_{gg}(f_t))_{gg}, \quad (30b)$$

$$dZfR1[t, g, g] \equiv (\delta Z_{f_t}^R + \delta \check{Z}_{f_t}^R)_{gg} = (\sigma_{gg}^R(f_t) - \tau_{gg}(f_t))_{gg}, \quad (30c)$$

$$dZbarfR1[t, g, g] \equiv (\delta Z_{f_t}^R + \delta \check{Z}_{f_t}^R)_{gg} = (\sigma_{gg}^R(f_t) + \tau_{gg}(f_t))_{gg}, \quad (30d)$$

where

$$\sigma_{ii}^X(f) = -[\tilde{\text{Re}}] \left[ \Sigma_{f_i}^X(m_{f_i}^2) + m_{f_i}^2 (\Sigma_{f_i}^L(m_{f_i}^2) + \Sigma_{f_i}^R(m_{f_i}^2)) + m_{f_i} (\Sigma_{f_i}^{SL}(m_{f_i}^2) + \Sigma_{f_i}^{SR}(m_{f_i}^2)) \right],$$

$$\tau_{ii}(f) = \begin{cases} \frac{1}{2m_{f_i}} ([\tilde{\text{Re}}] [\Sigma_{f_i}^{SL}(m_{f_i}^2) - \Sigma_{f_i}^{SR}(m_{f_i}^2)]) & \text{if } m_{f_i} \neq 0, \\ 0 & \text{if } m_{f_i} = 0. \end{cases}$$

As we presently do not take neutrino masses into account, the following off-diagonal entries ( $g \neq g'$ ) appear only in the quark sector ( $t = 3, 4$ ) and only for non-trivial CKM matrix:

$$dZfL1[t, g, g'] \equiv (\delta Z_{f_t}^L + \delta \check{Z}_{f_t}^L)_{gg'} = (\sigma_{gg'}^{L,R,SR,SL}(f_t))_{gg'}, \quad (30e)$$

$$dZbarfL1[t, g, g'] \equiv (\delta Z_{f_t}^L + \delta \check{Z}_{f_t}^L)_{gg'} = (\sigma_{g'g}^{L,R,SL,SR}(f_t))_{gg'}, \quad (30f)$$

$$dZfR1[t, g, g'] \equiv (\delta Z_{f_t}^R + \delta \check{Z}_{f_t}^R)_{gg'} = (\sigma_{gg'}^{R,L,SL,SR}(f_t))_{gg'}, \quad (30g)$$

$$dZbarfR1[t, g, g'] \equiv (\delta Z_{f_t}^R + \delta \check{Z}_{f_t}^R)_{gg'} = (\sigma_{g'g}^{R,L,SR,SL}(f_t))_{gg'}, \quad (30h)$$

where

$$\sigma_{ij}^{X,Y,SX,SY}(f) = \frac{2}{m_{f_i}^2 - m_{f_j}^2} [\tilde{\text{Re}}] \left[ m_{f_j} (m_{f_j} \Sigma_{f_j}^X(m_{f_j}^2) + m_{f_i} \Sigma_{f_j}^Y(m_{f_j}^2)) + m_{f_j} \Sigma_{f_j}^{SX}(m_{f_j}^2) + m_{f_i} \Sigma_{f_j}^{SY}(m_{f_j}^2) \right].$$

The CKM matrix  $V_{ij}$  receives counterterms [23]

$$\delta\text{CKM1}[i, j] \equiv \delta V_{ij} = \frac{1}{4} \sum_{g=1}^3 \left( (\delta\mathbf{Z}_u^L - \delta\mathbf{Z}_u^{L\dagger})_{ig} V_{gj} - V_{ig} (\delta\mathbf{Z}_d^L - \delta\mathbf{Z}_d^{L\dagger})_{gi} \right). \quad (31)$$

A note on CKM mixing: The model file is presently limited to minimal flavor violation in the sfermion sector (see Section 3.6), which means that for non-trivial CKM matrix there is a slight imbalance between fermions and sfermions; for example, the  $b$ -quark has an admixture from  $d$  and  $s$  while the  $\tilde{b}$  does not. Because this violates delicate supersymmetric relations, processes involving squarks (in particular external ones) may not become finite and we have therefore chosen to make `$CKM = False` ( $V_{ij} = \delta_{ij}$ ,  $\delta V_{ij} = 0$ ) the default. CKM mixing can be turned on with `$CKM = True` (set *before* model initialization).

### 3.6. The Sfermion sector

In the absence of non-minimal flavor violation, the sfermion mass matrix is given by [6,7]

$$\mathbf{M}_{f_{tg}}^2 = \begin{pmatrix} (\mathbf{M}_{L,f_t}^2)_{gg} + m_{f_{tg}}^2 & m_{f_{tg}} (\mathbf{X}_{f_t})_{gg}^* \\ m_{f_{tg}} (\mathbf{X}_{f_t})_{gg} & (\mathbf{M}_{R,f_t}^2)_{gg} + m_{f_{tg}}^2 \end{pmatrix} \quad (32)$$

where

$$\begin{aligned} \mathbf{M}_{L,f_t}^2 &= M_Z^2 (I_3^f - Q_f^2 s_w^2) c_{2\beta} + \begin{cases} \mathbf{M}_L^2 & \text{for left-handed sleptons } (t = 1, 2), \\ \mathbf{M}_Q^2 & \text{for left-handed squarks } (t = 3, 4), \end{cases} \\ \mathbf{M}_{R,f_t}^2 &= M_Z^2 Q_f^2 s_w^2 c_{2\beta} + \begin{cases} \mathbf{M}_E^2 & \text{for right-handed sleptons } (t = 2), \\ \mathbf{M}_U^2 & \text{for right-handed } u\text{-type squarks } (t = 3), \\ \mathbf{M}_D^2 & \text{for right-handed } d\text{-type squarks } (t = 4), \end{cases} \\ \mathbf{X}_{f_t} &= \mathbf{A}_{f_t} - \mu^* \begin{cases} 1/t_\beta & \text{for isospin-up sfermions } (t = 3), \\ t_\beta & \text{for isospin-down sfermions } (t = 2, 4). \end{cases} \end{aligned}$$

The soft-SUSY-breaking parameters  $\mathbf{M}_{L,\tilde{Q},\tilde{E},\tilde{U},\tilde{D}}^2$  and  $\mathbf{A}_f$  are  $3 \times 3$  matrices in flavor space whose off-diagonal entries are zero in the minimally flavor-violating MSSM.

The mass matrix is diagonalized by a unitary transformation  $\mathbf{U}_{\tilde{f}}$ ,

$$\mathbf{U}_{\tilde{f}} \mathbf{M}_{f_{\tilde{f}}}^2 \mathbf{U}_{\tilde{f}}^\dagger = \begin{pmatrix} m_{\tilde{f}_1}^2 & 0 \\ 0 & m_{\tilde{f}_2}^2 \end{pmatrix}, \quad \mathbf{U}_{\tilde{f}} = \begin{pmatrix} U_{11}^{\tilde{f}} & U_{12}^{\tilde{f}} \\ U_{21}^{\tilde{f}} & U_{22}^{\tilde{f}} \end{pmatrix}. \quad (33)$$

#### 3.6.1. The squark sector

We renormalize the up-type squarks ( $\tilde{u} = \{\tilde{u}, \tilde{c}, \tilde{t}\}$ ) on-shell (OS). For the down-type squarks ( $\tilde{d} = \{\tilde{d}, \tilde{s}, \tilde{b}\}$ ) we follow the discussion in Section 4 of Ref. [31].

Also the down-type squark masses could in principle both be renormalized on-shell (option 02 of Ref. [31]). They would then have to be computed from a mass matrix with shifted  $M_Q^2$ , however, which is not entirely straightforward to implement in practice. We therefore set only one down-type squark mass ( $m_{\tilde{d}_{1g}}$ ) on-shell and continue to work with a tree-level mass matrix. The one-loop-corrected on-shell value of the remaining mass  $m_{\tilde{d}_{2g}}$  then acquires a shift involving the dependent mass counterterm  $\delta m_{\tilde{d}_{2g}}^2$ ,

$$(m_{\tilde{d}_{2g}}^{\text{OS}})^2 = m_{\tilde{d}_{2g}}^2 + \delta m_{\tilde{d}_{2g}}^2 - \tilde{\text{Re}}(\Sigma_{\tilde{d}_g}(m_{\tilde{d}_{2g}}^2))_{jj}. \quad (34)$$

For the other parameters two scheme choices are available:

- By default we apply the “ $m_b, A_b$   $\overline{\text{DR}}$ ” scheme of Refs. [9,31]. (For completeness we note that a slight extension of this scheme was used there.) We shall refer to this as the **mixed scheme**—the original term “ $m_b, A_b$   $\overline{\text{DR}}$ ” is not quite accurate as we allow the bottom quark to be chosen on-shell, see Eq. (29).
- Alternately, an **on-shell scheme** can be used for the squark sector. Despite the name also here only one down-type squark mass is on-shell but the off-diagonal ‘mixing-angle’ counterterm  $\delta Y_{d_g}$  is fixed by an on-shell-type condition and the trilinear coupling’s counterterm  $\delta A_{d_g}$  becomes a dependent quantity.

The scheme affecting sfermions  $\tilde{d}_g$  is chosen with the variable `$SfScheme[4, g]`:

$$\text{\$SfScheme}[4, g] = \text{DR}[1] \quad \text{mixed scheme with } m_{\tilde{d}_{1g}} \text{ OS, } A_{d_g} \overline{\text{DR}}, \quad (35a)$$

$$\text{\$SfScheme}[4, g] = \text{DR}[2] \quad \text{mixed scheme with } m_{\tilde{d}_{2g}} \text{ OS, } A_{d_g} \overline{\text{DR}} \text{ (default)}, \quad (35b)$$

$$\text{\$SfScheme}[4, g] = \text{OS}[1] \quad \text{on-shell scheme with } m_{\tilde{d}_{1g}} \text{ OS, } Y_{d_g} \text{ OS}, \quad (35c)$$

$$\text{\$SfScheme}[4, g] = \text{OS}[2] \quad \text{on-shell scheme with } m_{\tilde{d}_{2g}} \text{ OS, } Y_{d_g} \text{ OS}. \quad (35d)$$

In the following,  $i$  denotes the index of DR or OS and  $j$  the other ( $j = 3 - i$ ). The index  $s$  runs over both values 1, 2. The two up-type and the chosen down-type sfermion are on-shell,

$$\text{dMSfsq1}[s, s, 3, g] \equiv \delta m_{\tilde{u}_{sg}}^2 = \tilde{\text{Re}}(\Sigma_{\tilde{u}_g}(m_{\tilde{u}_{sg}}^2))_{ss}, \quad (36a)$$

$$\text{dMSfsq1}[i, i, 4, g] \equiv \delta m_{\tilde{d}_{ig}}^2 = \tilde{\text{Re}}(\Sigma_{\tilde{d}_g}(m_{\tilde{d}_{ig}}^2))_{ii}. \quad (36b)$$

The up-type off-diagonal mass-matrix entries receive counterterms [31–33]

$$\text{dMSfsq1}[1, 2, 3, g] \equiv \delta Y_{\tilde{u}_g} = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{u}_g}(m_{\tilde{u}_{1g}}^2) + \Sigma_{\tilde{u}_g}(m_{\tilde{u}_{2g}}^2))_{12}, \quad (36c)$$

$$\text{dMSfsq1}[2, 1, 3, g] = \delta Y_{\tilde{u}_g}^* = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{u}_g}(m_{\tilde{u}_{1g}}^2) + \Sigma_{\tilde{u}_g}(m_{\tilde{u}_{2g}}^2))_{21}. \quad (36d)$$

For clarity of notation we furthermore define the auxiliary constants

$$\begin{aligned} \text{dMsq11Sf1}[4, g] \equiv \delta M_{\tilde{d}_g, 11}^2 &= |U_{11}^{\tilde{u}_g}|^2 \delta m_{\tilde{u}_{1g}}^2 + |U_{12}^{\tilde{u}_g}|^2 \delta m_{\tilde{u}_{2g}}^2 - 2\text{Re}[U_{22}^{\tilde{u}_g} U_{12}^{\tilde{u}_g*} \delta Y_{\tilde{u}_g}] \\ &\quad - 2m_{\tilde{u}_g} \delta m_{\tilde{u}_g} + 2m_{\tilde{d}_g} \delta m_{\tilde{d}_g} - c_{2\beta} \delta M_W^2 + 4M_W^2 c_\beta^3 s_\beta \delta t_\beta, \end{aligned} \quad (37a)$$

$$\text{dMsq12Sf1}[4, g] \equiv \delta M_{\tilde{d}_g, 12}^2 = m_{\tilde{d}_g} (\delta A_{\tilde{d}_g}^* - \mu \delta t_\beta - t_\beta \delta \mu) + (A_{\tilde{d}_g}^* - \mu t_\beta) \delta m_{\tilde{d}_g}. \quad (37b)$$

For the bottom quark two options are possible:  $\delta m_b = \delta m_b^{\overline{\text{DR}}}$  or  $\delta m_b = \delta m_b^{\text{OS}}$ , see Section 3.5.

In the **mixed scheme** the dependent mass counterterm is

$$\text{dMSfsq1}[j, j, 4, g] \equiv \delta m_{\tilde{d}_{jg}}^2 = \frac{1}{|U_{ij}^{\tilde{d}_g}|^2} \left\{ |U_{1i}^{\tilde{d}_g}|^2 \delta m_{\tilde{d}_{1g}}^2 + (i - j) (2\text{Re}[U_{11}^{\tilde{d}_g} U_{12}^{\tilde{d}_g*} \delta M_{\tilde{d}_g, 12}^2] + (|U_{11}^{\tilde{d}_g}|^2 - |U_{12}^{\tilde{d}_g}|^2) \delta M_{\tilde{d}_g, 11}^2) \right\}, \quad (38a)$$

and the down-type off-diagonal mass counterterms are related as

$$\text{dMSfsq1}[1, 2, 4, g] \equiv \delta Y_{\tilde{d}_g} = \frac{1}{|U_{11}^{\tilde{d}_g}|^2 - |U_{12}^{\tilde{d}_g}|^2} \left\{ U_{11}^{\tilde{d}_g} U_{21}^{\tilde{d}_g*} (\delta m_{\tilde{d}_{1g}}^2 - \delta m_{\tilde{d}_{2g}}^2) + U_{11}^{\tilde{d}_g} U_{22}^{\tilde{d}_g*} \delta M_{\tilde{d}_g, 12}^2 - U_{12}^{\tilde{d}_g} U_{21}^{\tilde{d}_g*} \delta M_{\tilde{d}_g, 12}^{2*} \right\}, \quad (38b)$$

$$\text{dMSfsq1}[2, 1, 4, g] = \delta Y_{\tilde{d}_g}^*. \quad (38c)$$

In the **on-shell scheme** we have instead

$$\text{dMSfsq1}[j, j, 4, g] \equiv \delta m_{\tilde{d}_{jg}}^2 = \frac{1}{|U_{ij}^{\tilde{d}_g}|^2} \left\{ -|U_{1i}^{\tilde{d}_g}|^2 \delta m_{\tilde{d}_{1g}}^2 + 2\text{Re}[U_{22}^{\tilde{d}_g} U_{12}^{\tilde{d}_g*} \delta Y_{\tilde{d}_g}] + \delta M_{\tilde{d}_g, 11}^2 \right\}, \quad (38d)$$

$$\text{dMSfsq1}[1, 2, 4, g] \equiv \delta Y_{\tilde{d}_g} = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{d}_g}(m_{\tilde{d}_{1g}}^2) + \Sigma_{\tilde{d}_g}(m_{\tilde{d}_{2g}}^2))_{12}, \quad (38e)$$

$$\text{dMSfsq1}[2, 1, 4, g] = \delta Y_{\tilde{d}_g}^* = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{d}_g}(m_{\tilde{d}_{1g}}^2) + \Sigma_{\tilde{d}_g}(m_{\tilde{d}_{2g}}^2))_{21}. \quad (38f)$$

In both schemes the trilinear couplings  $A_{q_{lg}} \equiv (\mathbf{A}_{q_l})_{gg}$  are renormalized by

$$\begin{aligned} \text{dAf1}[3, g, g] \equiv \delta A_{\tilde{u}_g} &= \frac{1}{m_{\tilde{u}_g}} \left[ U_{11}^{\tilde{u}_g} U_{12}^{\tilde{u}_g*} (\delta m_{\tilde{u}_{1g}}^2 - \delta m_{\tilde{u}_{2g}}^2) + U_{11}^{\tilde{u}_g} U_{22}^{\tilde{u}_g*} \delta Y_{\tilde{u}_g}^* + U_{12}^{\tilde{u}_g} U_{21}^{\tilde{u}_g*} \delta Y_{\tilde{u}_g} - (A_{\tilde{u}_g} - \mu^*/t_\beta) \delta m_{\tilde{u}_g} \right] \\ &\quad + \delta \mu^*/t_\beta - \mu^* \delta t_\beta / t_\beta^2, \end{aligned} \quad (39a)$$

$$\begin{aligned} \text{dAf1}[4, g, g] \equiv \delta A_{\tilde{d}_g} &= \left\{ \frac{1}{m_{\tilde{d}_g}} \left[ U_{11}^{\tilde{d}_g} U_{12}^{\tilde{d}_g*} (\delta m_{\tilde{d}_{1g}}^2 - \delta m_{\tilde{d}_{2g}}^2) + U_{11}^{\tilde{d}_g} U_{22}^{\tilde{d}_g*} \delta Y_{\tilde{d}_g}^* + U_{12}^{\tilde{d}_g} U_{21}^{\tilde{d}_g*} \delta Y_{\tilde{d}_g} - (A_{\tilde{d}_g} - \mu^* t_\beta) \delta m_{\tilde{d}_g} \right] \right. \\ &\quad \left. + t_\beta \delta \mu^* + \mu^* \delta t_\beta \right\}_{[\text{div}]}, \end{aligned} \quad (39b)$$

where the subscripted [div] means to take the divergent part in the mixed scheme only, to effect  $\overline{\text{DR}}$  renormalization of  $A_{\tilde{d}_g}$  [31].

The sfermion  $Z$ -factors are derived in the OS scheme. The OS scheme has nothing to say on the imaginary parts of the diagonal  $Z$ -factors and since they contain no divergences, we implicitly set them to zero below.

$$\text{dZSf1}[s, s, 3, g] \equiv (\delta \mathbf{Z}_{\tilde{u}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{u}_g})_{ss} = -[\tilde{\text{Re}}](\Sigma'_{\tilde{u}_g}(m_{\tilde{u}_{sg}}^2))_{ss}, \quad (40a)$$

$$\text{dZbarSf1}[s, s, 3, g] = \text{dZSf1}[s, s, 3, g], \quad (40b)$$

$$\text{dZSf1}[s, s, 4, g] \equiv (\delta \mathbf{Z}_{\tilde{d}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{d}_g})_{ss} = -[\tilde{\text{Re}}](\Sigma'_{\tilde{d}_g}(m_{\tilde{d}_{sg}}^2))_{ss}, \quad (40c)$$

$$\text{dZbarSf1}[s, s, 4, g] = \text{dZSf1}[s, s, 4, g], \quad (40d)$$

$$\text{dZSf1}[1, 2, 3, g] \equiv (\delta \mathbf{Z}_{\tilde{u}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{u}_g})_{12} = +\sigma_2^{12}(\tilde{u}_g), \quad (40e)$$

$$\text{dZbarSf1}[1, 2, 3, g] \equiv (\delta \mathbf{Z}_{\tilde{u}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{u}_g})_{12} = -\sigma_2^{21}(\tilde{u}_g), \quad (40f)$$

$$\text{dZSf1}[2, 1, 3, g] \equiv (\delta \mathbf{Z}_{\tilde{u}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{u}_g})_{21} = +\sigma_1^{21}(\tilde{u}_g), \quad (40g)$$

$$\text{dZbarSf1}[2, 1, 3, g] \equiv (\delta \mathbf{Z}_{\tilde{u}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{u}_g})_{21} = -\sigma_1^{12}(\tilde{u}_g), \quad (40h)$$

$$\text{dZSf1}[1, 2, 4, g] \equiv (\delta \mathbf{Z}_{\tilde{d}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{d}_g})_{12} = +\sigma_2^{12}(\tilde{d}_g), \quad (40i)$$

$$\text{dZbarSf1}[1, 2, 4, g] \equiv (\delta \mathbf{Z}_{\tilde{d}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{d}_g})_{12} = -\sigma_2^{21}(\tilde{d}_g), \quad (40j)$$

$$\text{dZSf1}[2, 1, 4, g] \equiv (\delta \mathbf{Z}_{\tilde{d}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{d}_g})_{21} = +\sigma_1^{21}(\tilde{d}_g), \quad (40k)$$

$$\text{dZbarSf1}[2, 1, 4, g] \equiv (\delta \mathbf{Z}_{\tilde{d}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{d}_g})_{21} = -\sigma_1^{12}(\tilde{d}_g), \quad (40l)$$

where

$$\sigma_i^{12}(\tilde{f}) = \frac{2}{m_{f_1}^2 - m_{f_2}^2} (\llbracket \text{Re} \rrbracket (\Sigma_{\tilde{f}}(m_{f_i}^2))_{12} - \delta Y_f),$$

$$\sigma_i^{21}(\tilde{f}) = \frac{2}{m_{f_2}^2 - m_{f_1}^2} (\llbracket \text{Re} \rrbracket (\Sigma_{\tilde{f}}(m_{f_i}^2))_{21} - \delta Y_f^*).$$

CPT invariance requires that  $\tilde{\text{Re}}(\Sigma_{\tilde{f}}(p^2))_{ss'} = (\tilde{\text{Re}}(\Sigma_{\tilde{f}}(p^2))_{s's})^*$  and it is indeed true that, leaving out the absorptive parts,  $(\delta \mathbf{Z}_{\tilde{f}})_{ss'} = (\delta \mathbf{Z}_{\tilde{f}})_{s's}^*$ .

### 3.6.2. The Slepton sector

The discussion of the slepton renormalization can be fairly brief as we merely adapt the squark-sector results [9,11,12,31] to account for the absence of neutrino masses (and hence only a single sneutrino). We restrict ourselves to the on-shell scheme which we found to give numerically stable results up to relatively large values of  $t_\beta$ .

$$\text{\$SfScheme}[2, g] = \text{OS}[1] \quad \text{on-shell scheme with } m_{\tilde{e}_{1g}} \text{ OS, } Y_{e_g} \text{ OS,} \quad (41a)$$

$$\text{\$SfScheme}[2, g] = \text{OS}[2] \quad \text{on-shell scheme with } m_{\tilde{e}_{2g}} \text{ OS, } Y_{e_g} \text{ OS (default).} \quad (41b)$$

As before,  $i$  denotes the index of OS and  $j = 3 - i$  the other.

We fix two out of the three slepton masses on-shell ( $\tilde{e} = \{\tilde{e}, \tilde{\mu}, \tilde{\tau}\}$ ) [34,35],

$$\text{dMSfsq1}[1, 1, 1, g] \equiv \delta m_{\tilde{v}_{1g}}^2 = \tilde{\text{Re}}(\Sigma_{\tilde{v}_g}(m_{\tilde{v}_{1g}}^2))_{11}, \quad (42a)$$

$$\text{dMSfsq1}[i, i, 2, g] \equiv \delta m_{\tilde{e}_{ig}}^2 = \tilde{\text{Re}}(\Sigma_{\tilde{e}_g}(m_{\tilde{e}_{ig}}^2))_{ii}. \quad (42b)$$

The non-diagonal entries of the selectron-type mass matrix are determined by [31,35,36]

$$\text{dMSfsq1}[1, 2, 2, g] \equiv \delta Y_{e_g} = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{e}_g}(m_{\tilde{e}_{1g}}^2) + \Sigma_{\tilde{e}_g}(m_{\tilde{e}_{2g}}^2))_{12}, \quad (42c)$$

$$\text{dMSfsq1}[2, 1, 2, g] \equiv \delta Y_{e_g}^* = \frac{1}{2} \tilde{\text{Re}}(\Sigma_{\tilde{e}_g}(m_{\tilde{e}_{1g}}^2) + \Sigma_{\tilde{e}_g}(m_{\tilde{e}_{2g}}^2))_{21}, \quad (42d)$$

and the counterterm for the remaining dependent mass  $m_{\tilde{e}_{jg}}^2$  is

$$\text{dMSfsq1}[j, j, 2, g] \equiv \delta m_{\tilde{e}_{jg}}^2 = \frac{1}{|U_{1j}^{\tilde{e}_g}|^2} \left\{ -|U_{1i}^{\tilde{e}_g}|^2 \delta m_{\tilde{e}_{ig}}^2 + 2 \text{Re}[U_{12}^{\tilde{e}_g*} U_{22}^{\tilde{e}_g} \delta Y_{e_g}] + \delta M_{\tilde{e}_g, 11}^2 \right\} \quad (42e)$$

where we used the analogous auxiliary quantity

$$\text{dMsq11Sf1}[2, g] \equiv \delta M_{\tilde{e}_g, 11}^2 = \delta m_{\tilde{v}_{1g}}^2 + 2m_{e_g} \delta m_{e_g} - c_{2\beta} \delta M_W^2 + 4M_W^2 c_\beta^3 s_\beta \delta t_\beta. \quad (43a)$$

The trilinear couplings are renormalized by

$$\begin{aligned} \text{dAf1}[2, g, g] \equiv \delta A_{e_g} &= \frac{1}{m_{e_g}} [U_{11}^{\tilde{e}_g} U_{12}^{\tilde{e}_g*} (\delta m_{\tilde{e}_{1g}}^2 - \delta m_{\tilde{e}_{2g}}^2) + U_{11}^{\tilde{e}_g} U_{22}^{\tilde{e}_g*} \delta Y_{e_g}^* + U_{12}^{\tilde{e}_g*} U_{21}^{\tilde{e}_g} \delta Y_{e_g} - (A_{e_g} - \mu^* t_\beta) \delta m_{e_g}] \\ &\quad + \delta \mu^* t_\beta + \mu^* \delta t_\beta. \end{aligned} \quad (44)$$

The slepton Z-factors are:

$$\text{dZSf1}[1, 1, 1, g] \equiv (\delta \mathbf{Z}_{\tilde{v}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{v}_g})_{11} = -[\tilde{\text{Re}}](\Sigma'_{\tilde{v}_g}(m_{\tilde{v}_{1g}}^2))_{11}, \quad (45a)$$

$$\text{dZbarSf1}[1, 1, 1, g] = \text{dZSf1}[1, 1, 1, g], \quad (45b)$$

$$\text{dZSf1}[s, s, 2, g] \equiv (\delta \mathbf{Z}_{\tilde{e}_g} + \delta \tilde{\mathbf{Z}}_{\tilde{e}_g})_{ss} = -[\tilde{\text{Re}}](\Sigma'_{\tilde{e}_g}(m_{\tilde{e}_{sg}}^2))_{ss}, \quad (45c)$$

$$\text{dZbarSf1}[s, s, 2, g] = \text{dZSf1}[s, s, 2, g], \quad (45d)$$

$$dZSf1[1, 2, 2, g] \equiv (\delta Z_{\tilde{e}_g} + \delta \check{Z}_{\tilde{e}_g})_{12} = +\sigma_2^{12}(\tilde{e}_g), \quad (45e)$$

$$dZbarSf1[1, 2, 2, g] \equiv (\delta Z_{\tilde{e}_g} + \delta \check{Z}_{\tilde{e}_g})_{12} = -\sigma_2^{21}(\tilde{e}_g), \quad (45f)$$

$$dZSf1[2, 1, 2, g] \equiv (\delta Z_{\tilde{e}_g} + \delta \check{Z}_{\tilde{e}_g})_{21} = +\sigma_1^{21}(\tilde{e}_g), \quad (45g)$$

$$dZbarSf1[2, 1, 2, g] \equiv (\delta Z_{\tilde{e}_g} + \delta \check{Z}_{\tilde{e}_g})_{21} = -\sigma_1^{12}(\tilde{e}_g), \quad (45h)$$

where

$$\sigma_i^{12}(\tilde{f}) = \frac{2}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2} (\|\tilde{\text{Re}}\|(\Sigma_{\tilde{f}}(m_{\tilde{f}}^2))_{12} - \delta Y_f),$$

$$\sigma_i^{21}(\tilde{f}) = \frac{2}{m_{\tilde{f}_2}^2 - m_{\tilde{f}_1}^2} (\|\tilde{\text{Re}}\|(\Sigma_{\tilde{f}}(m_{\tilde{f}}^2))_{21} - \delta Y_f^*).$$

### 3.7. The Gluino sector

The gluino sector is determined by the soft-breaking gluino mass parameter  $M_3 = |M_3|e^{i\varphi_{\tilde{g}}}$ . A real gluino mass  $m_{\tilde{g}} = |M_3|$  is obtained through the transformation

$$\omega_{\pm}\tilde{g}^a \rightarrow e^{\mp i\varphi_{\tilde{g}}/2}\omega_{\pm}\tilde{g}^a \quad (46)$$

of the gluino field  $\tilde{g}^a$  which moves the gluino phase into the couplings, where it appears as  $\text{SqrtEGl} \equiv e^{i\varphi_{\tilde{g}}/2}$ .

The gluino is renormalized on-shell [9,37],

$$dMG11 \equiv \delta M_3 = \frac{1}{2}\tilde{\text{Re}}\left[m_{\tilde{g}}(\Sigma_{\tilde{g}}^L(m_{\tilde{g}}^2) + \Sigma_{\tilde{g}}^R(m_{\tilde{g}}^2)) + \Sigma_{\tilde{g}}^{SL}(m_{\tilde{g}}^2) + \Sigma_{\tilde{g}}^{SR}(m_{\tilde{g}}^2)\right]e^{i\varphi_{\tilde{g}}}, \quad (47)$$

$$dZG1L1 \equiv \delta Z_{\tilde{g}}^L + \delta \check{Z}_{\tilde{g}}^L = \sigma^L(\tilde{g}) + \tau(\tilde{g}) - i\delta\varphi_{\tilde{g}}, \quad (48a)$$

$$dZbarG1L1 = dZG1R1, \quad (48b)$$

$$dZG1R1 \equiv \delta Z_{\tilde{g}}^R + \delta \check{Z}_{\tilde{g}}^R = \sigma^R(\tilde{g}) - \tau(\tilde{g}) - i\delta\varphi_{\tilde{g}}, \quad (48c)$$

$$dZbarG1R1 = dZG1L1, \quad (48d)$$

where

$$\sigma^X(\tilde{g}) = -[\tilde{\text{Re}}]\left[\Sigma_{\tilde{g}}^X(m_{\tilde{g}}^2) + m_{\tilde{g}}^2\left(\Sigma_{\tilde{g}}^{'L}(m_{\tilde{g}}^2) + \Sigma_{\tilde{g}}^{'R}(m_{\tilde{g}}^2)\right) + m_{\tilde{g}}\left(\Sigma_{\tilde{g}}^{SL}(m_{\tilde{g}}^2) + \Sigma_{\tilde{g}}^{SR}(m_{\tilde{g}}^2)\right)\right],$$

$$\tau(\tilde{g}) = \frac{1}{2m_{\tilde{g}}}\left([\tilde{\text{Re}}]\left[\Sigma_{\tilde{g}}^{SL}(m_{\tilde{g}}^2) - \Sigma_{\tilde{g}}^{SR}(m_{\tilde{g}}^2)\right]\right).$$

We choose  $\delta\varphi_{\tilde{g}} = 0$  with a rationale as in the quark case with no flavor violation: There, the Yukawa coupling can be made real by a redefinition of the quark fields and a complex  $Z$ -factor keeps it that way also at one-loop order. For the gluinos the phase still appears in the Lagrangian after field redefinition but this phase factor can be considered a ‘transformation matrix’ and does not obtain a counterterm.

### 3.8. The Gluon sector

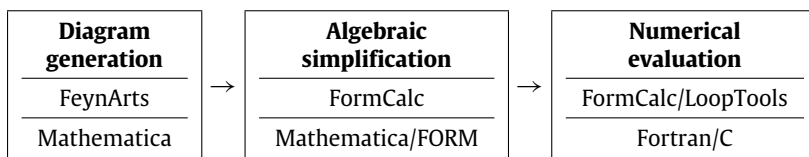
We use  $\overline{\text{DR}}$  renormalization for  $\alpha_s$  and the gluon field,

$$dZgs1 \equiv \delta Z_{\alpha_s} = \frac{1}{2}\delta Z_{gg}, \quad (49)$$

$$dZGG1 \equiv \delta Z_{gg} = -\tilde{\text{Re}}\Sigma_G^T(0)|_{\text{div}}. \quad (50)$$

## 4. Usage

The FeynArts–FormCalc system works in three stages, as sketched in the following diagram. A more detailed discussion of the interplay between FeynArts and FormCalc can be found in Ref. [38].



The model file can of course be used even if one does not wish to continue the calculation with FormCalc.

#### 4.1. Diagram generation

The `MSSMCT.mod` and `SQCDCT.mod` model files are used like any other FeynArts model files. At present, they *cannot* be used together with the FeynArts add-on model file `FV.mod` [39], which extends the minimal flavor mixing to full  $6 \times 6$  sfermion mixing. Extension of the sfermion renormalization of Section 3.6 to the non-minimal case is work in progress, however.

The model file can be influenced by redefining the following symbols:

- Wave-function correction factors  $\delta\tilde{Z}$  are omitted by setting `ReDiag` (for diagonal  $\delta\tilde{Z}_{ii}$ ) and/or `ReOffDiag` (for off-diagonal  $\delta\tilde{Z}_{ij}$ ) to `ReTilde`, see Eqs. (1).
- An on-shell bottom mass is chosen by setting `UVMf1[4, 3]` to `Identity`, see Eqs. (29).
- The variable `$MHPInput` (set *before* model initialization) decides whether  $M_A$  (`False`) or  $M_{H^\pm}$  (`True`, default) are taken as input, see Eqs. (7). Regarding the choice of tree-level vs. loop-corrected Higgs masses as discussed below Eq. (12), there is of course no such difference for the input mass.
- The variable `$InoScheme` (set *before* model initialization) determines the renormalization scheme for the chargino/neutralino sector. Choices are `CCN[n]` and `CNN[c, n, n']`; details in Section 3.4.3. The default is `CCN[1]`.
- The variable `$SfScheme[t, g]` determines the renormalization scheme for sfermion  $\tilde{f}_{tg}$ ,  $t = 4$  for squarks,  $t = 2$  for sleptons of generation  $g$ . Mathematica patterns may be used for  $t$  and  $g$  to combine definitions, e.g. `$SfScheme[2, _]`. Choices are `DR[i]` (for squarks) and `OS[i]` (for squarks and sleptons), where  $i = 1, 2$  identifies the on-shell sfermion; details in Eqs. (35) and (41). The default is `DR[2]` for squarks and `OS[2]` for sleptons.
- CKM mixing can be turned on with `$CKM = True` (set *before* model initialization). Be aware that in the present (minimally flavor-violating) version of the model file this may lead to incomplete cancellation of divergences if squarks, in particular external ones, are involved, see the discussion after Eq. (31).

The model file prints out the settings of all relevant flags during initialization.

#### 4.2. Algebraic simplification

Amplitudes generated with `MSSMCT.mod` can directly be simplified with `FormCalc`. Since the MSSM contains couplings that are relatively involved compared to QCD or the electroweak SM, it may become necessary to relax (some of) the fairly aggressive simplification functions to complete the algebraic simplification within reasonable run-time.

Potentially time-consuming transformations in the FORM part of the calculation can be suppressed with the `CalcFeynAmp` option `NoCostly  $\rightarrow$  True`.

Upon return from FORM, `FormCalc` wraps a zoo of simplification functions around various parts of the amplitude. All of these are ‘transparent’ in the sense that they can be replaced by `Identity` without affecting the numerical result. The important ones are listed below, a complete inventory is given in the `FormCalc` manual.

- `FormSub` is applied to subexpressions of an amplitude.
- `FormDot` is applied to combinations of dot products in an amplitude.
- `FormMat` is applied to the coefficients of matrix elements in an amplitude.
- `RCSUB` works like `FormSub` but is applied to subexpressions of a renormalization constant.
- `RCINT` works like `FormMat` but is applied to coefficients of loop integrals in renormalization constants.

#### 4.3. Numerical evaluation

In the `FormCalc` framework, the Mathematica expressions resulting from the algebraic simplification are translated to Fortran or C code for numerical evaluation. The generated code has to be provided with the proper numerical values for the parameters appearing in the model, i.e. the variables in Table 3, derived from a (reasonably small) set of input parameters. This is solved by a subroutine which is called at the beginning of the calculation to initialize all model parameters.

There are two ways to set up the MSSM parameters numerically:

- The `FormCalc` module `model_mssm.F` provides stand-alone initialization of the MSSM parameters, i.e. without linking to external libraries. The user can choose to link with `FeynHiggs` [13,18–20], however, to obtain the state-of-the-art Higgs-mass values (otherwise an approximation formula is used which includes the major two-loop shifts). Details on the usage and the parameter relations employed in this approach are given in Ref. [1].
- The module `model_fh.F` uses `FeynHiggs` as a frontend [13,40]. The code generated by `FormCalc` inherits thus the ability to read parameter files in either native `FeynHiggs` or SLHA format, and of course obtains all MSSM parameters and Higgs observables from `FeynHiggs`. There is no duplication of initialization code this way, and moreover the parameters are consistent between the Higgs-mass calculation and the evaluation of the `FormCalc`-generated matrix elements.

### 5. Tests

A model of the complexity of the MSSM needs exhaustive checks to be trustable in all sectors. For the tree-level couplings we could rely on the testing and maturity of the original `MSSM.mod` model file [1]. The new parts, i.e. the counterterm vertices and the renormalization constants, have been tested in two ways.



**Table 6**

Finite  $1 \rightarrow 2$  decays and  $2 \rightarrow 2$  scattering reactions computed with MSSMCT.mod. For the decays into photons, UV-finiteness was verified analytically.  $H^- \rightarrow GG^-$ ,  $H^- \rightarrow WZ$ , and  $H^- \rightarrow G^-Z$  are loop-induced processes. Processes with external Goldstone bosons are unphysical and not necessarily IR-finite.

$h \rightarrow \{AA, HH, GG, G^+G^-\}$
$h \rightarrow \{A\gamma, G\gamma, GZ, G^+W^-\}$
$h \rightarrow \tilde{c}c$
$H \rightarrow \{hh, AA, AG, GG, H^+H^-, G^+G^-\}$
$H \rightarrow \{A\gamma, G\gamma, AZ, GZ\}$
$H \rightarrow \{ZZ, WW\}$
$H \rightarrow \tilde{c}c$
$A \rightarrow \{hG, G^+G^-\}$
$A \rightarrow \{h\gamma, G\gamma, hZ, G^-W^+\}$
$A \rightarrow \{e^+e^-, \tilde{u}u\}$
$H^- \rightarrow \{hG^-, HG^-, AG^-, GG^-\}$
$H^- \rightarrow \{G^-Z, hW^-\}$
$H^- \rightarrow W^-Z$
$H^- \rightarrow \{\tilde{u}d, \tilde{\chi}_1^0 \tilde{\chi}_1^-\}$
$hH \rightarrow AG$
$hA \rightarrow HG$
$\{hA, hG, HA, HG\} \rightarrow H^+G^-$
$AA \rightarrow \{AA, GG, H^+H^-\}$
$AG \rightarrow \{H^+H^-, G^-H^+\}$
$GG \rightarrow \{GG, H^-G^+, G^-G^+, H^+H^-\}$
$\{H^-H^+, H^-G^+\} \rightarrow H^-G^+$
$hZ \rightarrow H^+W^-$
$HH^- \rightarrow \{\gamma W^-, ZW^-\}$
$AZ \rightarrow H^+W^-$
$AH^- \rightarrow ZW^-$
$\{AA, GG, \tilde{t}_2 \tilde{t}_2\} \rightarrow \tilde{t}_1 \tilde{t}_1$
$\{HH^-, AH^-\} \rightarrow \tilde{t}_1 \tilde{b}_1$
$\{AH^-, AG^-\} \rightarrow \tilde{\nu}_\tau \tilde{\tau}_1$
$e^+e^- \rightarrow \{Zh, W^+H^-\}$

- On a relatively technical level, we ascertained numerically (and sometimes analytically) for a significant number of processes, listed in Table 6, that the renormalization actually works, i.e. that the results are UV- and IR-finite.

A version of MSSMCT.mod with minor modifications (slightly different sbottom- and stau-sector renormalization, though with identical UV-finiteness properties, as well as finite mass shifts) had already been used to generate UV- and IR-finite heavy stop, sbottom, and stau decays [9,31,34], chargino and neutralino decays [11,12], and gluino decays [37], which are not listed in Table 6.

- The ‘gold standard’ are tuned comparisons of selected reactions, however. They are quite a bit more thorough but require a lot of work. For lack of literature results to compare with, these comparisons were mostly restricted to the MSSM with real parameters.

We calculated the decay  $\tilde{b}_{1,2} \rightarrow \tilde{t}_1 H^-$  and found good agreement with Ref. [41]. We checked that we are in good agreement with Ref. [42] using their input parameters, where a small difference remains due to the different renormalization schemes.  $H \rightarrow hh$  is in perfect agreement with Ref. [43], for both real and complex parameters, and  $e^+e^- \rightarrow t\bar{t}$  agrees to 11 digits with Ref. [44].

We performed a detailed comparison with Ref. [45] for the decay  $\tilde{\chi}_2^\pm \rightarrow \tilde{\chi}_1^0 W^\pm$ , where the chargino/neutralino sector is renormalized differently than in our prescription. After a correction of the charge renormalization in Ref. [45] we found good agreement at the level expected for different renormalization schemes in the chargino/neutralino sector.

We worked with the program SFOLD [46] to obtain numerical results for stau decays. SFOLD is restricted to the MSSM with real parameters and uses  $\overline{\text{DR}}$  renormalization throughout. Although OS masses can be substituted on internal and/or external lines in SFOLD, we preferred to use  $\overline{\text{DR}}$  masses in our calculation for the comparison. SFOLD also adopts a running electromagnetic coupling  $\alpha_{\text{em}}(Q^{\overline{\text{DR}}})$  with a numerical value significantly higher than the  $\alpha_{\text{em}}(0)$  used in our renormalization scheme (see Section 3.3) and thus our tree-level results differ substantially. At the one-loop level, the two results are in better agreement than expected, however. This agreement improves for lower values of  $Q$ , but differences at the level of 5% were found for  $Q \sim 2$  TeV.

A comparison of different renormalization schemes in the chargino/neutralino sector can be found in Ref. [12]. The differences found with respect to another on-shell renormalization in the chargino/neutralino sector were small and of the expected size of two-loop contributions.

## 6. Availability

Starting from Version 3.9, the MSSMCT.mod model file is included in the FeynArts distribution. The package can be downloaded from <http://feynarts.de>. The FormCalc features described in Sections 4.2 and 4.3 are available from Version 8.2 on, which can be obtained from <http://feynarts.de/formcalc>. FeynArts and FormCalc each include a comprehensive manual which explains installation and usage. Both are open-source programs and licensed under the LGPL.

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