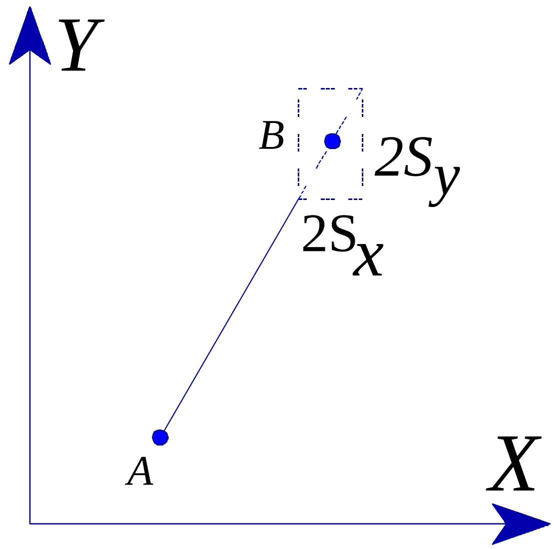
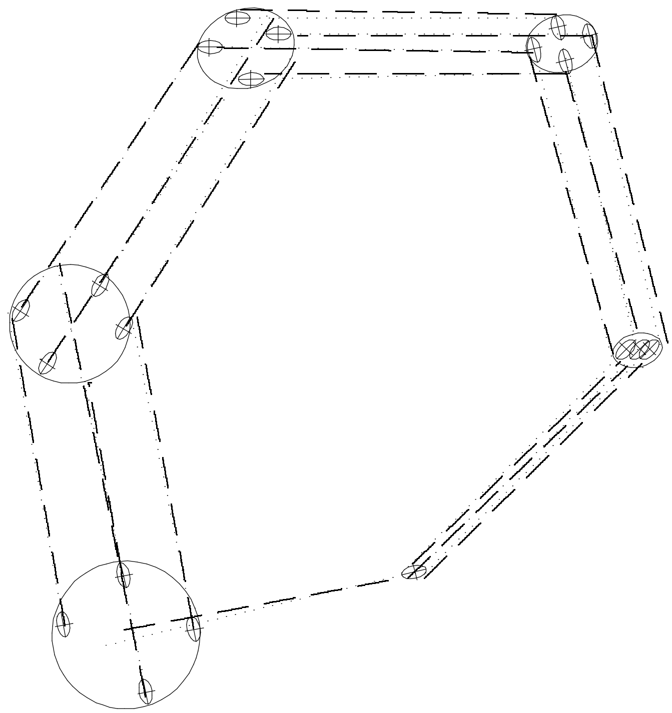
THE ERROR ELLIPSE



$2S_x$ and $2S_y$ define the dimension of the *standard error rectangle.*

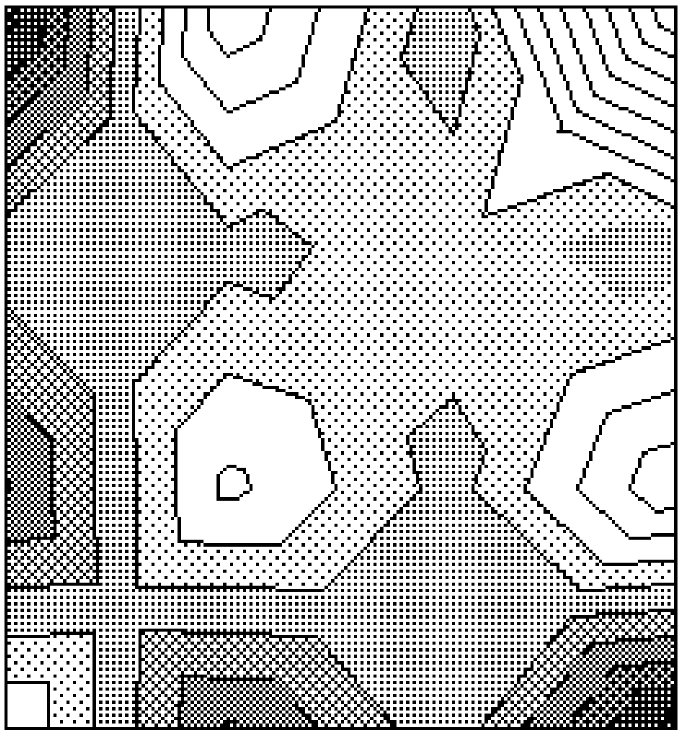
S_x and S_y show estimated error in station in direction of coordinate axes. However, position of largest error at a station not necessarily aligned with either axes.

THE FUZZY TRAVERSE



Results from error propagation in coordinates caused by distance and angle errors.

CONTOUR PLOT OF ADJUSTMENT SURFACE

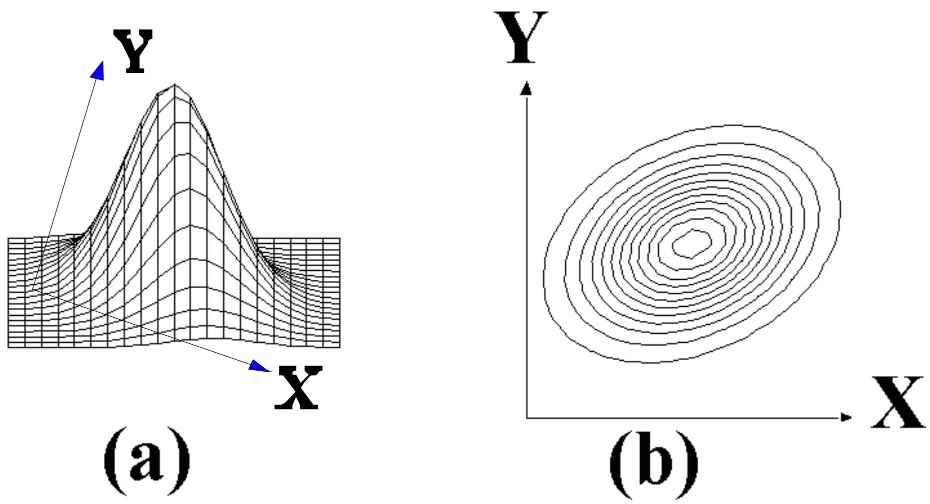


1. Each contour depicts estimated error in unknowns at specific probability level.

2. Can increase probability level by going to lower contour.

3. Can be used in network design

ERROR DISTRIBUTION AT A STATION



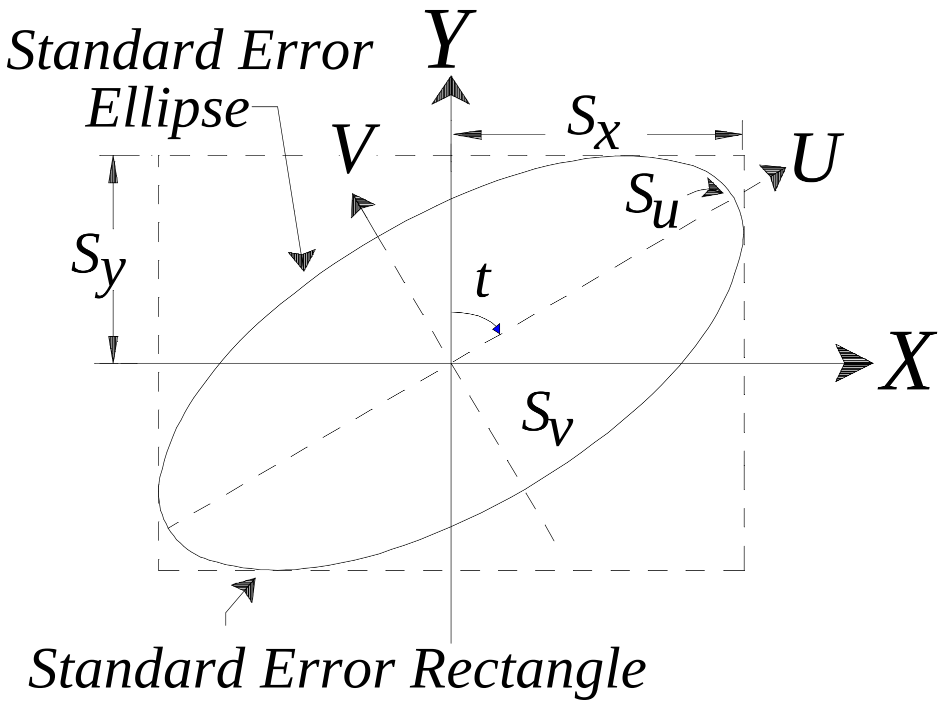
1. At each station, there is a bivariate distribution.

2. Contour plot of distribution produces concentric ellipses, called *error ellipses*.

3. Probability level given by volume under distribution.

4. Higher probability produces larger error ellipse.

COMPONENTS OF ERROR ELLIPSE



1. *t* is rotation angle from Y axis to axis of largest error.

2. S_u is the semi-major axis of ellipse. (Largest error)

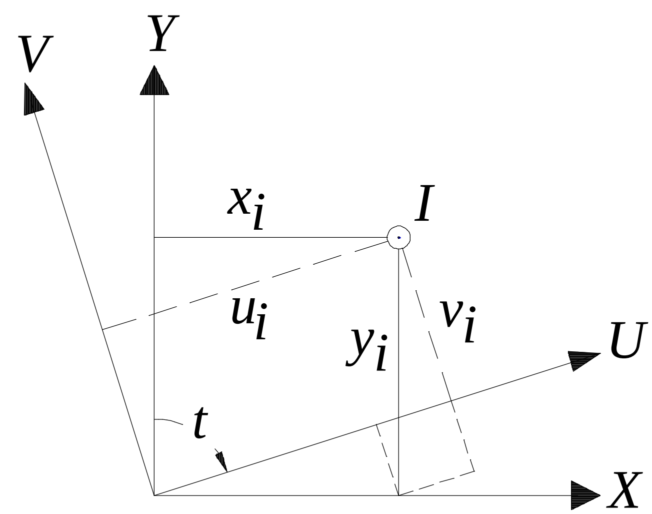
3. S_v is the semi-minor axis of ellipse. (Least error)

4. S_x is the standard deviation in X coordinate

5. S_y is the standard deviation in Y coordinate

COMPUTATION OF ELLIPSE AXIS

The method for calculating the *t* angle, that yields the maximum and minimum semi-axes involves a two-dimensional rotation.



For any point *I*

$$u_i = x_i Sin(t) + y_i Cos(t)$$

$$v_i = -x_i Cos(t) + y_i Sin(t)$$

or

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} Sin(t) & Cos(t) \\ -Cos(t) & Sin(t) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Simply $$Z = RX$$

where *R* is the rotation matrix.

COMPUTATION OF ELLIPSE AXIS

Problem is to develop a new covariance matrix from existing Q_{xx} matrix which removes correlation between unknown coordinates. From G.L.O.P.O.V.

$$Q_{zz} = R\, Q_{xx}\, R^T$$

where $$Q_{ZZ} = \begin{bmatrix} q_{uu} & q_{uv} \\ q_{uv} & q_{vv} \end{bmatrix}$$ and

$$Q_{xx} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix}$$

Expand Q_{zz} yields

$$Q_{ZZ} = \begin{bmatrix} \begin{pmatrix} Sin^2(t)\, q_{xx} + Cos(t)\, Sin(t)\, q_{xy} \\ +Sin(t)\, Cos(t)\, q_{xy} + Cos^2(t)\, q_{yy} \end{pmatrix} & \begin{pmatrix} -Sin(t)\, Cos(t)\, q_{xx} - Cos^2(t)\, q_{xy} \\ +Sin^2(t)\, q_{xy} + Cos(t)\, Sin(t)\, q_{yy} \end{pmatrix} \\ \begin{pmatrix} -Cos(t)\, Sin(t)\, q_{xx} + Sin^2(t)\, q_{xy} \\ -Cos^2(t)\, q_{xy} + Sin(t)\, Cos(t)\, q_{yy} \end{pmatrix} & \begin{pmatrix} Cos^2(t)\, q_{xx} - Sin(t)\, Cos(t)\, q_{xy} \\ -Cos(t)\, Sin(t)\, q_{xy} + Sin^2(t)\, q_{yy} \end{pmatrix} \end{bmatrix}$$

COMPUTATION OF ELLIPSE AXIS

If correlation between u and v is achieved then q_{uv} will equal zero.

Using trigonometric identities it can be shown that

$$q_{uv} = \frac{q_{xx} - q_{yy}}{2} Sin(2t) + q_{xy} Cos(2t) = 0$$

Solving for t yields:

$$Tan(2t) = \frac{Sin(2t)}{Cos(2t)} = \frac{2q_{xy}}{q_{yy} - q_{xx}}$$

$$2t = Tan^{-1}\left(\frac{2q_{xy}}{q_{yy} - q_{xx}} \right)$$

and

$$q_{uu} = Sin^2(t)q_{xx} + 2Cos(t)Sin(t)q_{xy} + Cos^2(t)q_{yy}$$

$$q_{vv} = q_{xx}Cos^2(t) - 2q_{xy}Cos(t)\,Sin(t) + q_{yy}Sin^2(t)$$

COMPUTATION OF ELLIPSE AXIS

To compute S_u and S_v, do the following:

$$S_u = S_o \sqrt{q_{uu}}$$

$$S_v = S_o \sqrt{q_{vv}}$$

q_{uu} and q_{vv} are based on rotated covariance matrix.

EXAMPLE

Assume the following:

1) S_o = ± 0.136-ft.

2) The covariance Q_{xx} and X matrices were:

$$X = \begin{bmatrix} dX_{Wis.} \\ dY_{Wis.} \\ dX_{Campus} \\ dY_{Campus} \end{bmatrix} \qquad Q_{xx} = \begin{bmatrix} 1.198574 & -1.160249 & -0.099772 & -1.402250 \\ -1.160249 & 2.634937 & 0.193956 & 2.725964 \\ -0.099772 & 0.193956 & 0.583150 & 0.460480 \\ -1.402250 & 2.725964 & 0.460480 & 3.962823 \end{bmatrix}$$

Error Ellipse for Station Wisconsin.

$$Tan(2t) = \frac{2(-1.160249)}{2.634937 - 1.198574} = -1.6155$$

$$2t = -58° \ 14.5' + 360° = 301° \ 45.5'$$

Dividing by 2, $t = 150° \ 53'$

Compute S_u and S_v:

$$S_u = \pm 0.136\sqrt{1.198574\, Sin^2(t) + 2(-1.160249)\, Cos(t) Sin(t) + 2.634937\, Cos^2(t)}$$

$$= \pm 0.25$$

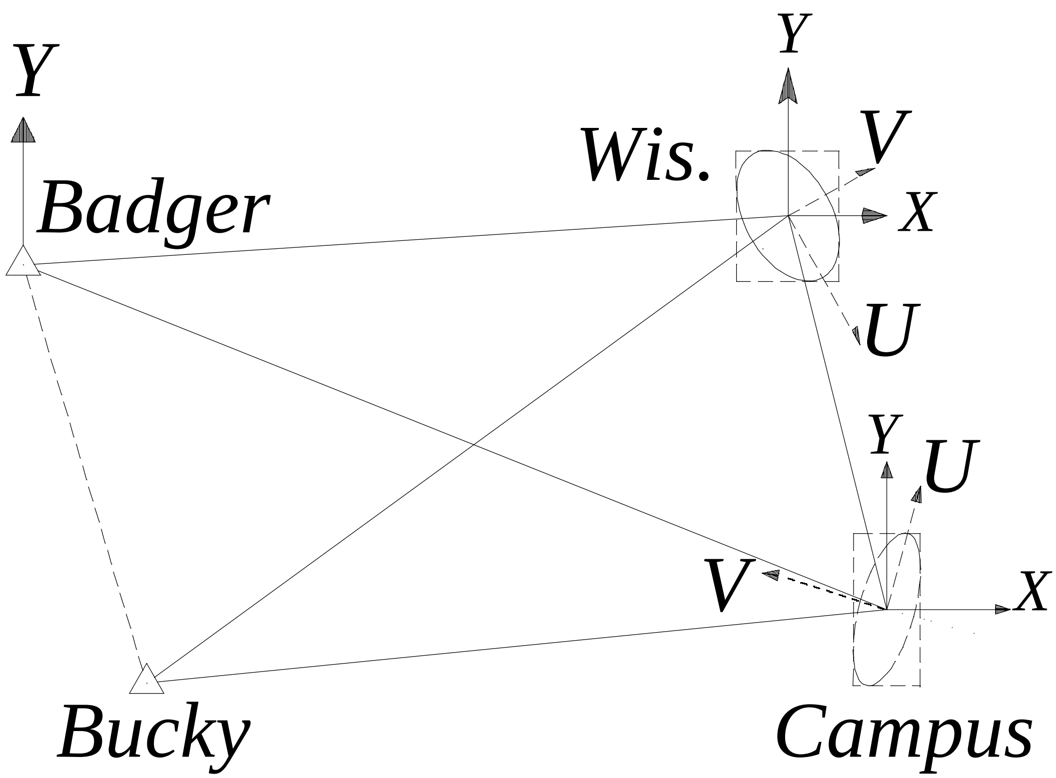
$$S_v = \pm 0.136\sqrt{1.198574\,Cos^2(t) - 2(-1.160249)\,Cos(t)\,Sin(t) + 2.634937\,Sin^2(t)}$$

$$= \pm 0.10$$

EXAMPLE

When drawing ellipses, scale dimensions so that relative orientation and size of ellipses can be visualized.

Ellipse Scale factor = 4800



PROBABILITY LEVEL OF AN ERROR ELLIPSE

1. Confidence level of standard error ellipse only 39%.

2. To get higher level multiply by constant based on statistical table.

   a) Formula: $c = \sqrt{2\,F_{(\alpha,\,2,\,degrees\ of\ freedom)}}$

   b) Table of F values

NETWORK DESIGN

BASIC PRINCIPLES:

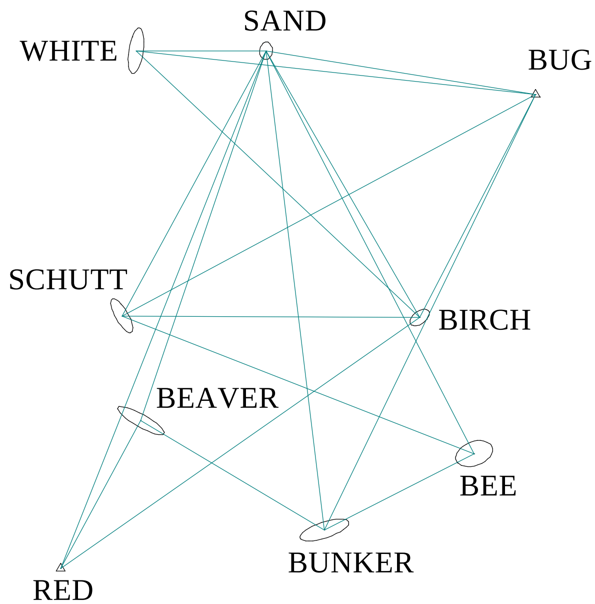
1. Distance measurements strengthen the positions of stations in the directions collinear with the measured lines.

2. Angle and direction observations strengthen the positions of stations perpendicular to the lines of sight.

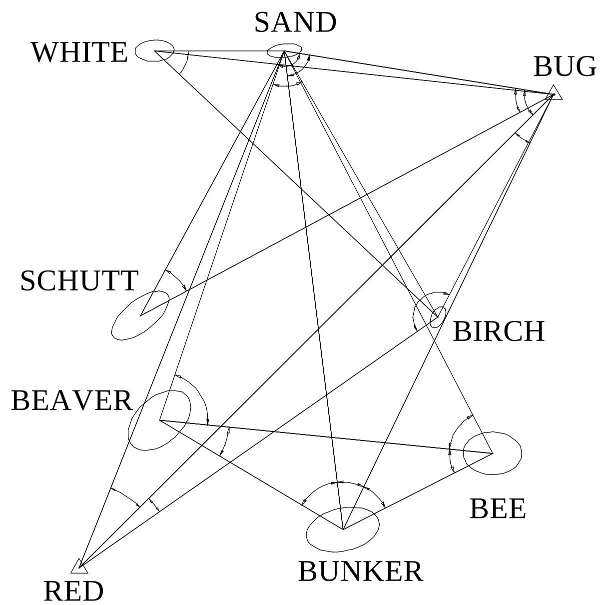
3. Largest errors occur farthest from control.

4. Adjustment can be simulated with measurements computed from approximate coordinates for stations.

EFFECTS OF DISTANCE AND ANGLE OBSERVATIONS



(a)
Trilateration
19 Distances



(b)
Triangulation
19 Angles

Note and explain shape of ellipse at:

1. Station White,
2. Station Schutt,
3. Station Birch

in figures (a) and (b).