Computer Science 673
Fall 2012
Homework 6:
Dynamic Programming
Due Friday, October 12th

For dynmamic programming algorithms, you should:

- Define the table – What does each element of the table hold?
- Give a formula for filling in table locations – this should include both a base case and a recursive case.
- Describe the order in which the table will be filled in (a picture is a good idea here)
- Give pseudocode for your algorithm

1. We want to multiply a chain of matrices together:

   $$M_1 M_2 M_3 M_4 \ldots M_n$$

   where M_k has dimensions $p_{k-1} \times p_k$ for $p_0 \ldots p_n$.

   We want to multiply these matrices in a way that minimizes the total number of scalar multiplications. Show that none of the following greedy algorithms produce optimal solutions in all cases:

   (a) First multiply the matrices M_i and M_{i+1} whose common dimension is smallest. Recursively find a solution for multiplying $M_1 * \ldots * M_{i-1} * (M_i * M_{i+1}) * \ldots * M_n$

   (b) First multiply the matrices M_i and M_{i+1} whose common dimension is largest. Recursively find a solution for multiplying $M_1 * \ldots * M_{i-1} * (M_i * M_{i+1}) * \ldots * M_n$

   (c) Split the problem of multiplying $M_i * \ldots * M_j$ into the subproblems of multiplying $M_i * \ldots * M_k$ and multiplying $M_{k+1} * \ldots * M_j$ so that $p_{i-1} p_k p_j$ is minimized. Recursively solve the subproblems of multiplying $M_i \ldots M_k$ and $M_{k+1} \ldots M_j$, then multiply the results of the subproblems.

2. Consider the alphabet $\Sigma = \{a, b, c\}$ the elements of Σ have the following multiplication table, which is neither communative nor associative:

	a	**b**	**c**
a	b	b	a
b	c	b	a
c	a	c	c

   so, ab = b, ba = c, bc = a, cb = b, and so on.

(a) Find a dynamic programming algorithm that examines a string $x = x_1x_2x_3 \ldots x_n$ and decides whether or not it is possible to parenthesize x such that the value of the resulting expression is a. For example, if $x = bbbba$, your algorithm should retun "yes", since $(b(bb))(ba) = a$ For the string $x = bac$, your algorithm should return "no" (since $(ba)c = c$ and $b(ac) = c$

(b) Modify your algoritm from part a so that instead of returning yes or no, it returns the number of was the expression can be parenthesized to ge the answer a.

3. Problem 15-8 *Image compression by seam carving*

4. Problem 15-9 *Breaking a string*