數值方法 HW4 吳昱庭E14113144

1.

ansa =0.39615 ansb =0.38566 ansc =0.3647

2.

Calculate exact value	$\int_1^{1.5} x^2 \ln x \, dx$ $= \frac{1}{3}x^3 \cdot \ln x \Big\|_1^{1.5} - \int_1^{1.5} \frac{1}{3}x^2 \, dx$ $= \left(\frac{1}{3}x^3 \cdot \ln x - \frac{1}{9}x^3\right)\Big\|_1^{1.5}$
ans1(n = 3) =0.19238 ans2(n = 4) =0.19226 error(n = 3) =0.0001202 error(n = 4) =9.508e-07 n = 4 時更為準確	

3.

<table>
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<td>Calculate exact value</td>
<td>

$$\int_0^{\pi/4}\int_{\sin x}^{\cos x}\left(2y\sin x+\cos^2 x\right)dy\,dx$$

$$=\int_0^{\pi/4}\left(y^2\sin x+y\cos^2 x\right)\Big|_{\sin x}^{\cos x}dx$$

$$=\int_0^{\pi/4}\left(\cos^3 x-\sin^3 x\right)dx$$

$$\cos^3 x-\sin^3 x$$

$$=\cos x\left(1-\sin^2 x\right)-\sin x\left(1-\cos^2 x\right)$$

$$=\cos x-\sin x-\cos x\sin^2 x+\sin x\cos^2 x$$

$$=\left(\sin x+\cos\right)\Big|_0^{\pi/4}-\int_0^{\sqrt{2}/2}u^2du-\int_1^{\sqrt{2}/2}v^2dv$$

$$=\sqrt{2}-1-\frac{\sqrt{2}}{12}\times 2+\frac{1}{3}$$

$$=\frac{5}{6}\sqrt{2}-\frac{2}{3}$$

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ansa(simpson) =0.51182
ansb(gaussian quadrature) =0.5123
error(simpson) =2.1628e-05
error(gaussian quadrature) =0.00045178
simpson(n=4,m=4) 更為準確

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4.

transform	$\int_1^\infty x^{-4}\sin x\,dx$ $= -\int_1^\infty x^{-2}\sin x\,(-x^{-2})\,dx$ $t = x^{-1}$, $dt = -x^{-2}\,dx$ $\Rightarrow -\int_1^0 t^2\sin(\frac{1}{t})\,dt$
ansa =0.52767 ansb =-0.021986	