

$$Var(e) = Eee^T \quad (1)$$

$$= M_2 I_n \quad (2)$$

$$Cov(\mu^T e, e^T e) = \mu^T Eee^T e \quad (3)$$

$$= \mu^T M_3 1_n \quad (4)$$

$$Var(e^T e) = Ee^T ee^T e - (Ee^T e)^2 \quad (5)$$

$$= nM_4 - nM_2^2 \quad (6)$$

$$\begin{aligned} Var(X^T X) &= Var(2\mu^T e + e^T e) \\ &= 4\mu^T Var(e)\mu + 4Cov(\mu^T e, e^T e) + Var(e^T e) \\ &= 4M_2\mu^T \mu + 4M_3\mu^T 1_n + nM_4 - nM_2^2 \end{aligned} \quad (7)$$

$$Ee^T ee^T \Phi^T \Phi e = E\left(\sum_{k=0}^n e_k^2\right)\left(\sum_{i,j} \Lambda_{i,j} e_i e_j\right) \quad (8)$$

$$= E\left(\sum_{k=0}^n e_k^2\right)\left(\sum_{k=0}^n \Lambda_{k,k} e_k^2\right) \quad (9)$$

$$= E\left(\sum_{k=0}^n \Lambda_{k,k} e_k^2\right)\left(\sum_{i=0}^n e_i^2\right) \quad (10)$$

$$= (M_4 + (n-1)M_2^2) \sum_{k=0}^n \Lambda_{k,k} \quad (11)$$

$$= (M_4 + (n-1)M_2^2)(n-m) \sum_{k=0}^m \phi_k^2 \quad (12)$$

$$Cov(X^T X, e^T \Phi^T \Phi e) = Cov(2\mu^T e + e^T e, e^T \Phi^T \Phi e) \quad (13)$$

$$= 2\mu^T E e e^T \Phi^T \Phi e + E e^T e e^T \Phi^T \Phi e - (E e^T e)(E e^T \Phi^T \Phi e) \quad (14)$$

$$= 2M_3\mu^T diag(\Lambda) + (M_4 + (n-1)M_2^2)(n-m) \sum_{k=0}^m \phi_k^2 \quad (15)$$

$$-n(n-m)M_2^2 \sum_{k=0}^m \phi_k^2 \quad (16)$$

$$= 2M_3\mu^T diag(\Lambda) + (M_4 - M_2^2)(n-m) \sum_{k=0}^m \phi_k^2 \quad (17)$$

$$Var(e^T \Phi^T \Phi e) = E e^T \Phi^T \Phi e e^T \Phi^T \Phi e - (E e^T \Phi^T \Phi e)^2 \quad (18)$$

$$= M_4 \sum_{i=1}^n \Lambda_{i,i}^2 + M_2^2 \sum_{i \neq j} \Lambda_{i,i} \Lambda_{j,j} + \Lambda_{i,j}^2 \quad (19)$$

$$- \left(M_2(n-m) \sum_{k=0}^m \phi_k^2 \right)^2 \quad (20)$$

$$= (M_4 - 2M_2^2) \sum_{i=1}^n \Lambda_{i,i}^2 + M_2^2 \sum_{i,j} \Lambda_{i,i} \Lambda_{j,j} + \Lambda_{i,j}^2 \quad (21)$$

$$- \left(M_2 \sum_{i=1}^n \Lambda_{i,i} \right)^2 \quad (22)$$

$$= (M_4 - 2M_2^2) \sum_{i=1}^n \Lambda_{i,i}^2 + M_2^2 \sum_{i,j} \Lambda_{i,i} \Lambda_{j,j} \quad (23)$$

$$+ M_2^2 \sum_{i,j} \Lambda_{i,j}^2 - M_2^2 \sum_{i,j} \Lambda_{i,i} \Lambda_{j,j} \quad (24)$$

$$(25)$$