Notations

- $X = (X_1, X_2, ..., X_n)^T$ is a n-dim random vector of observed time series.
- $\mu = (\mu_1, \mu_2, ..., \mu_n)^T \in \Re^n$ is the unobserved truth.
- $e = (e_1, e_2, ..., e_n)^T$ is a n-dim random vector which represents the random error.

Observations

We assume the observed time series is a sample from:

$$X = \mu + e. \tag{1}$$

Errors

We assume the error e follows:

- $e_1, e_2, ..., e_n$ are i.i.d. distributed. For convenience, we denote the distribution as ε.
- $E\varepsilon = 0$
- $E\varepsilon^k$ are existed and bounded for $k = 2, 3, 4$

Moments

Suppose $E\varepsilon^k = M_k$, we have

- $Ee^T e = nM_2$
-

$$\begin{aligned} Eee^T e &= E \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix} \sum_{i=1}^{n} e_i^2 \\ &= M_3 1_n \end{aligned} \tag{2}$$

- $$\begin{aligned} Ee^Tee^Te &= E\left(\sum_{i=1}^n e_i^2\right)^2 \\ &= E\left(\sum_{i=1}^n e_i^4 + \sum i \neq j e_i^2 e_j^2\right) \\ &= nM_4 + n(n-1)M_2^2 \end{aligned} \tag{3}$$