Probability Theory

content

1 Basics

sample mean:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

sample variance:

$$S^2 = \frac{1}{n-1}$$

2 Distribution

2.0.1 Chi-Square Distribution

if sequence $X_1, \cdots, X_n$ are independent and $X_i \sim N(0,1)$

$$\chi^2 = \sum_{i=1}^{n} X_i$$

$$\chi^2 \sim \chi^2(n)$$

$$P\{\chi^2 > \chi^2_\alpha(n)\} = \int_{\chi^2_\alpha(n)}^{+\infty} f_{\chi^2}(x)dx = \alpha$$

$$E(\chi^2) = n \ \ D(\chi^2) = 2n$$

$$Y_1 \sim \chi^2(n_1)\ ,\ Y_2 \sim \chi^2(n_2)\ \Rightarrow\ Y_1 + Y_2 \sim \chi^2(n_1 + n_2)$$

$$\chi^2 \sim \chi^2(n)\ ,\ n > 45 \quad \chi^2 \sim N(n, 2n)$$

2.0.2 t-Distribution

if X and Y are indenpendent , $X \sim N(0,1)$, $Y \sim \chi^2(n)$

$$T = \frac{X}{\sqrt{\frac{Y}{n}}}\ \Rightarrow\ T \sim t(n)$$

$$0 < \alpha < 1\ ,\ P\{T > t_\alpha(n)\} = \int_{t_\alpha(n)}^{+\infty} f_T(x)dx = \alpha$$

$$\lim_{n\to\infty} f_T(x) = \phi(x)$$

$$t_\alpha(n) = 1 - t_{1-\alpha}(n)$$

2.0.3 F-Distribution

if X and Y are independent , $X \sim \chi^2(n_1)$, $Y \sim \chi^2(n_2)$

$$F = \frac{\frac{X}{n_1}}{\frac{Y}{n_2}} \Rightarrow F \sim F(n_1, n_2)$$

$$0 < \alpha < 1 \ , \ P\{F > F_\alpha(n_1, n_2)\} = \int_{F_\alpha(n_1,n_2)}^{+\infty} f_F(x)dx = \alpha$$

$$F \sim F(n_1, n_2) \ , \ \frac{1}{F} \sim F(n_2, n_1)$$

$$F_{1-\alpha}(n_1, n_2) = \frac{1}{F_\alpha(n_2, n_1)}$$

3 Sampling Distribution

if $X \sim N(\mu, \sigma^2)$ and $X_1, \cdots, X_n$ are samples

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$\frac{(n-1)}{\sigma^2} S^2 \sim \chi^2(n-1)$$

$$\frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1)$$

if $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, $X_1, \cdots, X_{n1}$ and $Y_1, \cdots, Y_{n2}$ are samples

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1)$$

$$\sigma_1^2 = \sigma_2^2 = \sigma^2 \;\Rightarrow\; T = \frac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{S_\omega \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

$$S_\omega = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

4 Parameter Estimation

4.1 Moment Estimation

$$\gamma_1 = E(X) = \mu$$

$$\gamma_2 = E(X^2) = D(X) + E^2(X) = \sigma^2 + \mu^2$$

4.2 Maximum Likelihood Estimation

4.3 Optima Criterion of Estimation

4.3.1 Unbaised

$$\hat{\theta} = \hat{\theta}(X_1, X_2, \cdots, X_n)\ ,\ E(\hat{\theta}) = \theta$$

4.3.2 effectiveness

$$\hat{\theta}_0 = \hat{\theta}_0(X_1, \cdots, X_n) \text{ is unbaised for } \hat{\theta} \quad D(\hat{\theta}_0) \leq D(\hat{\theta})$$

4.3.3 Consistence

$$\lim_{n \to \infty} P\{\mid \hat{\theta}_n - \theta \mid < \epsilon\} = 1$$

4.4 Interval Estimation

4.4.1 Confident Interval

$$\sigma^2 \text{ is kowned , } U = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$[\overline{X} - \frac{\sigma}{\sqrt{n}}\mu_{\frac{\alpha}{2}}, \overline{X} + \frac{\sigma}{\sqrt{n}}\mu_{\frac{\alpha}{2}}]$$

$$\sigma^2 \text{ is unkowned , } T = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim t(n-1)$$