Probability Theory

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1 BASICS 2

1 Basics

sample mean:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

sample variance:

$$S^2 = \frac{1}{n-1}$$

2 Distribution

2.0.1 Chi-Square Distribution

if sequence X_1, \cdots, X_n are independent and $X_i \sim N(0, 1)$

$$\chi^2 = \sum_{i=1}^n X_i$$

$$\chi^2 \sim \chi^2(n)$$

$$P\{\chi^2 > \chi^2_\alpha(n)\} = \int_{\chi^2_\alpha(n)}^{+\infty} f_{\chi^2}(x) dx = \alpha$$

$$E(\chi^2) = n \quad D(\chi^2) = 2n$$

$$Y_1 \sim \chi^2(n_1) , Y_2 \sim \chi^2(n_2) \Rightarrow Y_1 + Y_2 \sim \chi^2(n_1 + n_2)$$

$$\chi^2 \sim \chi^2(n)$$
, $n > 45$ $\chi^2 \sim N(n, 2n)$

2.0.2 t-Distribution

if X and Y are independent , $X \sim N(0,1)$, $\, Y \sim \chi^2(n)$

$$T = \frac{X}{\sqrt{\frac{Y}{n}}} \Rightarrow T \sim t(n)$$

$$0 < \alpha < 1$$
, $P\{T > t_{\alpha}(n)\} = \int_{t_{\alpha}(n)}^{+\infty} f_{T}(x) dx = \alpha$

2 DISTRIBUTION

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$$\lim_{n \to \infty} f_T(x) = \phi(x)$$

$$t_{\alpha}(n) = 1 - t_{1-\alpha}(n)$$

2.0.3 F-Distribution

if X and Y are independent , $X \sim \chi^2(n_1)$, $\, Y \sim \chi^2(n_2)$

$$F = \frac{\frac{X}{n_1}}{\frac{Y}{n_2}} \implies F \sim F(n_1, n_2)$$

$$0 < \alpha < 1$$
, $P\{F > F_{\alpha}(n_1, n_2)\} = \int_{F_{\alpha}(n_1, n_2)}^{+\infty} f_F(x) dx = \alpha$

$$F \sim F(n_1, n_2) \ , \ \frac{1}{F} \sim F(n_2, n_1)$$

$$F_{1-\alpha}(n_1, n_2) = \frac{1}{F_{\alpha}(n_2, n_1)}$$

3 Sampling Distribution

if $X \sim N(\mu, \sigma^2)$ and X_1, \dots, X_n are samples

$$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$\frac{(n-1)}{\sigma^2} S^2 \sim \chi^2(n-1)$$

$$\frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1)$$

if $X \sim N(\mu_1,\sigma_1^2)$, $Y \sim N(\mu_2,\sigma_2^2)$, X_1,\cdots,X_{n1} and Y_1,\cdots,Y_{n2} are samples

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1)$$

$$\sigma_1^2 = \sigma_2^2 = \sigma^2 \implies T = \frac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{S_\omega \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

$$S_\omega = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

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4 Parameter Estimation

4.1 Moment Estimation

$$\gamma_1 = E(X) = \mu$$

 $\gamma_2 = E(X^2) = D(X) + E^2(X) = \sigma^2 + \mu^2$

- 4.2 Maximum Likelihood Estimation
- 4.3 Optima Criterion of Estimation
- 4.3.1 Unbaised

$$\hat{\theta} = \hat{\theta}(X_1, X_2, \cdots, X_n) , E(\hat{\theta}) = \theta$$

4.3.2 effectiveness

$$\hat{\theta}_0 = \hat{\theta}_0(X_1, \cdots, X_n)$$
 is unbaised for $\hat{\theta}$ $D(\hat{\theta}_0) \leq D(\hat{\theta})$

4.3.3 Consistence

$$\lim_{n \to \infty} P\{ || \hat{\theta}_n - \theta| < \epsilon \} = 1$$

- 4.4 Interval Estimation
- 4.4.1 Confident Interval

$$\begin{split} \sigma^2 \text{ is kowned }, \, U &= \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \\ & [\overline{X} - \frac{\sigma}{\sqrt{n}} \mu_{\frac{\alpha}{2}}, \overline{X} + \frac{\sigma}{\sqrt{n}} \mu_{\frac{\alpha}{2}}] \\ \\ \sigma^2 \text{ is unkowned }, \, T &= \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim t(n-1) \end{split}$$