

Question 4,

We know that, cut a list with n elements into n/k sub-lists of length k

a, the insertion sort can sort a list of length k in $\Theta(k^2)$, so for n/k sub-lists, the total time will be $\Theta(k^2 * (n/k)) = \Theta(nk)$

b, since the depth of recursion tree for normal merge sort is $\log(n)$,

so the depth of recursion tree for this problem is $\log(n) - \log(k)$, because the recursion tree stops at the sub-list at size k . therefore, the depth of recursion tree is $\log(n/k)$

and each level time consuming is cn , so the total time is $\log(n/k) * cn$ which is equals to $\Theta(n \log(n/k))$.

c, if $n \log(n/k) + nk = n \log(n)$ in terms of Θ notation

then we need to make sure

$n \log(n/k) = \Theta(n \log n)$ (if $n \log(n/k) > nk$) ~ 1

or

$nk = \Theta(n \log n)$ if $n \log(n/k) < nk$ ~ 2

so for 1,

k is a constant, not a function of n

for 2,

$k = \Theta(\log n)$, they have the same asymptotic.

So $k = \Theta(\log n)$, or we can say $k \in O(\log n)$

d, in practice, we can choose k depends on value c_1 and c_2 which are the coefficients of nk and $n \log(n/k)$ hidden by the asymptotic notation. We can build a mathematic model to analysis it by doing calculus 1 of k , and find the peak value which makes $(nk + n \log(n/k))' = 0$, then we can choose the k value, again the values of coefficients are important for choosing k value.