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HW 3

Qn1,

Theorem Optimal substructure of A-S problem:

Let Sij be the set of activities that start after activity ai finishes and that finish before activity aj starts. Let A be the optimal solution set for the set Sij.

Let k be the one activity (anyone) in A.

We define that: $A_{ik} = A_{ij} \cap S_{ik}$, which mean A_{jk} contains the activities in A_{ij} that finish before k starts. And $A_{kj} = Ai_j \cap S_{kj}$, which means A_{kj} contains the activities in A_{ij} that start after k finishes.

1, A_{ik} is the optimal solution for sub-problem S_{ik}

2, A_{kj} is the optimal solution for sub-problem S_{kj}

Proof:

1,

if A_{jk} is not the optimal solution for sub-problem S_{ik}, then that means we could find a set A'_{ik} of mutually compatible activities in S_{ik} where $|A'_{ik}| > |A_{jk}|$, then we could use A'_{ik}, rather than A_{jk}, in the solution to the sub-problem for S_{ij}, we would have constructed a set of $|A'_{jk}| + |A_{kj}| + 1 > |A_{ik}| + |A_{kj}| + 1 = |A|$ mutually compatible activiries, which contradicts the assumption that A is an optimal solution.

2,

A symmetric argument applies to the activities in S_{kj}