Qn.3 (34-5-8)

A certificate would be an assignment to input variables which causes exactly half the clauses to evaluate to 1, and the other half to evaluate to 0.
Since we can check this in polynomial time, half 3-CNF is in NP.
To prove that it's NP-hard, we have to show that 3-CNF-SAT ≤p HALF-3-CNF.
Proof:
Let φ be any 3-CNF formula with m clauses and input variables x1, x2, .. , xn. Let T be the formula $(y \vee y \vee \neg y)$, and let F be the formula $(y \vee y \vee y)$.
Let $\varphi' = \varphi \wedge T \wedge . . . \wedge T \wedge F \wedge . . . \wedge F$ where there are m copies of T and 2m copies of F. Then ϕ' has 4m clauses and can be constructed from ϕ in polynomial time.
Suppose that ϕ has a satisfying assignment. Then by setting y = 0 and the xi 's to the satisfying assignment, we satisfy the m clauses of ϕ and the m T clauses, but none of the F clauses. Thus, ϕ' has an assignment which satisfies exactly half of its clauses. On the other hand, suppose there is no satisfying assignment to ϕ. The m T clauses are always satisfied. If we set y = 0 then the total number of clauses satisfies in ϕ' is strictly less than 2m, since each of the 2m F clauses is false, and at least one of the ϕ clauses is false. If we set y = 1, then strictly more than half the clauses of ϕ' are satisfied, since the 3m T and F clauses are all satisfied.
Thus, ϕ has a satisfying assignment if and only if ϕ' has an assignment which satisfies exactly half of its clauses. We conclude that HALF-3-CNF is NP-hard, and hence NP-complete.