Name: Shuo Wu wustl key: wushuo ID: 452666

CSE 541 HW4

Pro1.

From the theorem, we know that the theorem means that for integers m and n, each combination of subset of S (m-subset) should have equally chance of showing up, which should be ***Probability (each subset showing up) = 1/($\binom{n}{m}$).***

Induction:

1, Base case:

For m=0, the algorithm is page 130 will return empty set ϕ, the theorem is true as there is only one size m subset of [n].

2, Inductive Hypothesis: maintenance:

Assume m>0, and assume the invariant holds for m-1 which means "For any m' < m, n' < n, random-sample (m', n') returns a random m' -subset of {1, 2, . . . , n'} in which each m' -subset is equally likely".

When we pass m, the recursive call returns an m-1 sample with uniform probability, which is 1/($\binom{n-1}{m-1}$). And then there two choices – either the new m-subset includes n or not.

If n $\in$ S, with probability (m/n),

Since Prob(n $\in$ S) = prob(i $\notin$S &&i==n)+ prob(i$\in$S') =(m-1/n)+(n-m+1/n)*(1/n-m+1) =(m-1/n)+(1/n)= m/n

the probability for each combination including n is:

$$(m/n)*(1/(\binom{n-1}{m-1}))= (\binom{n}{m})$$

if n $\notin$S, with probability (n-m/n), which is 1-prob(n $\in$ S), so it includes one of (n-m) numbers not present. The chance for each is:

$$(n-m/n)*(1/(\binom{n-1}{m}))= (\binom{n}{m})$$