

CSE541 HW2

Q1,

HW2

$T(n) = T(n-1) + T(n/2) + n$

Recursion tree:

depth = n

depth = $\log n$

The recursion tree ~~looks~~ on the left there is a long branch which length is "n", on the right the length of the branch is $\log n$.

So for upper bound, I guess should be $T(n) = 2T(n-1) + n$ using ~~master method~~ So $T(n) = 2[T(n-1)] + n$

$$= 2[2T(n-2) + n] + n$$

$$= 2^n \cdot \text{constant} + n^2 = 2^n$$

$\therefore T(n) = O(2^n)$

* Substitution method.

$$T(n) = T(n-1) + T(n/2) + n$$

$$\leq 2^{n-1} + 2^{\frac{n}{2}} + n \quad (\text{when } n \gg \text{constant})$$

$$\leq 2^{n-1} + 2^{n-1} = 2^n$$

$\therefore T(n) \leq 2^n$ where $n \gg 1$,

also $O(2^n)$ is a tight bound, Since if we have any polynomial upper bound like $T(n) \leq cn^k$ $\therefore T(n) = T(n-1) + T(n/2) + n$

$$\leq c(n-1)^k + c\frac{n^k}{2^k} + n$$

$$\leq cn^k(1 + \frac{1}{2^k}) > cn^k$$

\therefore this is $O(2^n)$ a pretty tight bound.