

d. Horner's rule

1,  $y=0$

2, for  $i= n$  downto 0

3         $y=a_i +x*y$

**Initialization:**

When  $i=n$ , (the first loop iteration). The value of  $y$ , therefore, equals to  $a_n$ , which is in fact the last element of polynomial.

For example if  $n=0$ , from the algorithm, we will get that  $y = a_0$

And the polynomial gives us that  $y = a_0$ , therefore, they have the same result.

Showing that the loop invariant holds prior to the first iteration of the loop.

**Maintain:**

The for loop then makes  $y$  equals  $y = a_i + x*(a_{i+1} + x*y') =$

$$= a_i + x \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k = a_i + x \sum_{k=1}^{n-i} a_{k+i} x^{k-1} = \sum_{k=0}^{n-i} a_{k+i} x^k$$

which shows that the loop invariant holds prior to terminate.

**Termination:**

At termination,  $i=0$ , so is summing up to  $n-1$ , therefore, the last iteration give us the correct final output as shown before. We can see that the algorithm evaluated to the function. This is the value of the polynomial evaluated at  $x$ .