证明：如果$[k(\vec{x}_i\ \vec{x}_j)]_{m\times m}$矩阵半正定，则$k(\vec{x},\vec{y})$可用来表示
$\langle \phi(\vec{x}), \phi(\vec{y}) \rangle$

证、首先通过(1)(2)(3)(4)步构造希尔伯特再生核空间.

(1)定义映射 ϕ：$\vec{x} \to k(\cdot, \vec{x})$，其中$k(\cdot,\vec{x})$可以看作是
$(k(\vec{m}_1, \vec{x}),\ k(\vec{m}_2, \vec{x}) \ldots\ k(\vec{m}_n, \vec{x}) \ldots)$ 的无穷维向量.

(2)定义线性组合、构成向量空间S

$f(\cdot) = \sum_{i=1}^{m} \alpha_i k(\cdot, \vec{x}_i)$　$f(\cdot)$也是一个无穷维向量。$f(\cdot)$可以构成向量空间.

(3)在S向量空间上定义运算 *.

$f(\cdot) = \sum_{i=1}^{m} \alpha_i k(\cdot, \vec{x}_i)$　　$g(\cdot) = \sum_{j=1}^{l} \beta_j k(\cdot, \vec{y}_j)$

$f(\cdot) * g(\cdot) = \sum_{i=1}^{m} \sum_{j=1}^{l} \alpha_i \beta_j k(\vec{x}_i, \vec{y}_j)$

证明*满足以下1.2.3

1. $f(\cdot) * g(\cdot) = g(\cdot) * f(\cdot)$

2. $(af(\cdot)) * g(\cdot) = a(f(\cdot) * g(\cdot))$　　$(f(\cdot) + g(\cdot)) * h(\cdot) = f(\cdot) * h(\cdot) + g(\cdot) * h(\cdot)$

3. $f(\cdot) * f(\cdot) \geq 0$ 且 $f(\cdot) * f(\cdot) = 0 \Leftrightarrow f(\cdot) = \vec{0}$

1和2证明略.

以下证明3.

记$f(\cdot)$为f　$g(\cdot)$为g