④ 用 Lagrange 对偶求解. 软间隔支持向量机.

$$L(\vec{w}, b, \vec{\xi}, \vec{\alpha}, \vec{\mu}) = \frac{1}{2}\|\vec{w}\|^2 + C\sum_{i=1}^{m}\xi_i + \sum_{i=1}^{m}\alpha_i\left(1-\xi_i-y_i(\vec{w}^T\vec{x}_i+b)\right) - \sum_{i=1}^{m}\mu_i\xi_i$$

primal: $\min\limits_{\vec{w}, b, \vec{\xi}} \max\limits_{\vec{\alpha}, \vec{\mu}} L(\vec{w}, b, \vec{\alpha}, \vec{\xi}, \vec{\mu}) =$ Dual: $\max\limits_{\vec{\alpha}, \vec{\mu}} \min\limits_{\vec{w}, b, \vec{\xi}} L(\vec{w}, b, \vec{\xi}, \vec{\alpha}, \vec{\mu})$

先求 $\min\limits_{\vec{w}, b, \vec{\xi}} L(\vec{w}, b, \vec{\xi}, \vec{\alpha}, \vec{\mu})$: 分别对 $\vec{w}$. b $\vec{\xi}$ 求偏导为 $\vec{0}$:

$$\begin{cases} \vec{w} - \sum_{i=1}^{m}\alpha_i y_i \vec{x}_i = \vec{0} \\ \sum_{i=1}^{m}\alpha_i y_i = 0 \end{cases}$$

$$\alpha_i + \mu_i = C \quad (i=1 \ldots m)$$

利用 $\sum_{i=1}^{m}\alpha_i y_i = 0$ 并把 $\vec{w} = \sum_{i=1}^{m}\alpha_i y_i \vec{x}_i$; $\alpha_i + \mu_i = C$ 代入 $L(\vec{w}, b, \vec{\xi}, \vec{\alpha}, \vec{\mu})$

得.

$$\min_{\vec{w}, b, \vec{\xi}} L(\vec{w}, b, \vec{\xi}, \vec{\alpha}, \vec{\mu})$$

$$= -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y_i y_j \langle \vec{x}_i, \vec{x}_j \rangle + \sum_{i=1}^{m}\alpha_i$$

利用 $\alpha_i + \mu_i = C$ $\alpha_i \geq 0$ $\mu_i \geq 0$. $\sum_{i=1}^{m}\alpha_i y_i = 0$ 对偶 Dual problem 可变为

(6)
$$\begin{cases} \min\limits_{\vec{\alpha}} \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_i\alpha_j y_i y_j \langle \vec{x}_i, \vec{x}_j \rangle - \sum_{i=1}^{m}\alpha_i \\ \text{s.t.} \sum_{i=1}^{m}\alpha_i y_i = 0 \\ 0 \leq \alpha_i \leq C \end{cases}$$

同线性可分支持向量机求解一样. (6) 作为带约束的凸优化问题, 可以用内点法求解.