DMIP - Exercise:

Defect Pixel Interpolation

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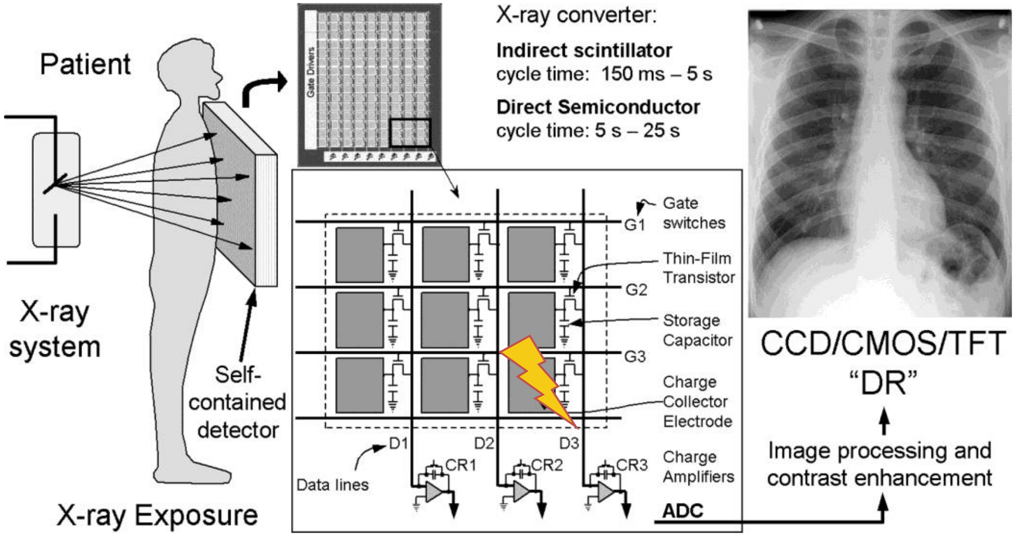
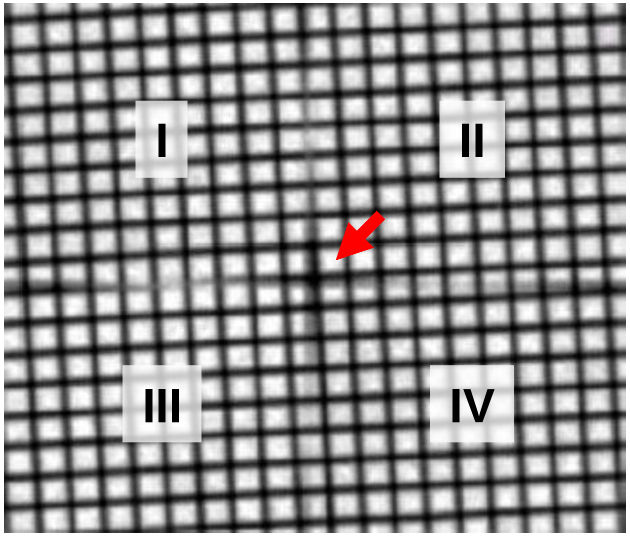
Flat Panel Detectors

Defect Pixels in Flat Panel Detectors

- Defect pixels are caused by defect detector cells.
- Small detectors are composed to generate a large one, which leads to butting cross effects.



Artifacts due to inactive pixels or rows



Butting Cross Artifact

Image taken from: Samei E et al. Radiographics 2004;24:313-334

Image taken from: Lecture DMIP (Maier, Hornegger)

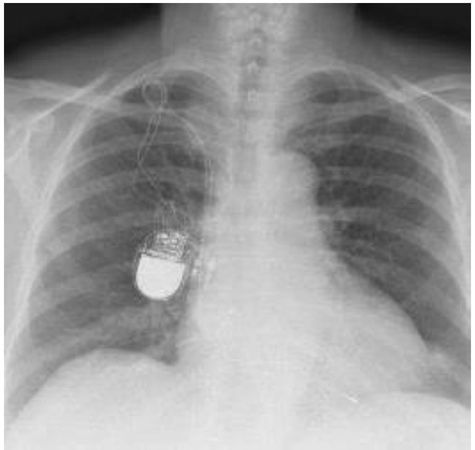
Flat Panel Detectors

Defect Pixels in Flat Panel Detectors

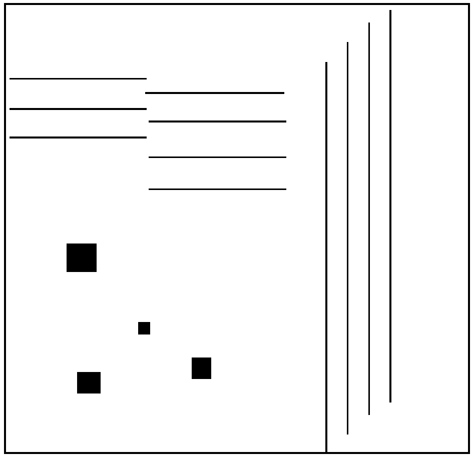
- Let $f_{i,j}$ denote the intensity value at grid point (i,j) of the **ideal image** f that has no defect pixels.
- Let $w_{i,j}$ denote the indicator value at (i,j) where w is **mask image** that indicates defect and uncorrupted pixels:

$$w_{i,j} = \begin{cases} 0 & \text{if pixel is defect} \\ 1 & \text{otherwise} \end{cases}$$

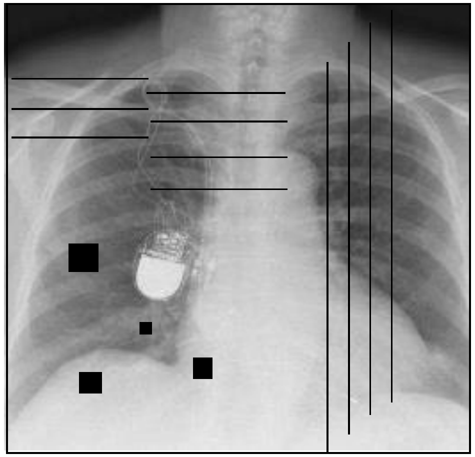
- Let $g_{i,j}$ denote the intensity value at grid point (i,j) of the **observed image** g that is acquired with the flat panel detector that has defect pixels.



Ideal image f



Mask image w



Observed image g

Flat Panel Detectors

Defect Pixels in Flat Panel Detectors

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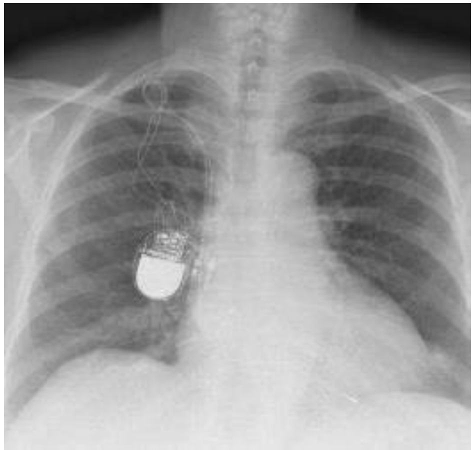

Ideal image f

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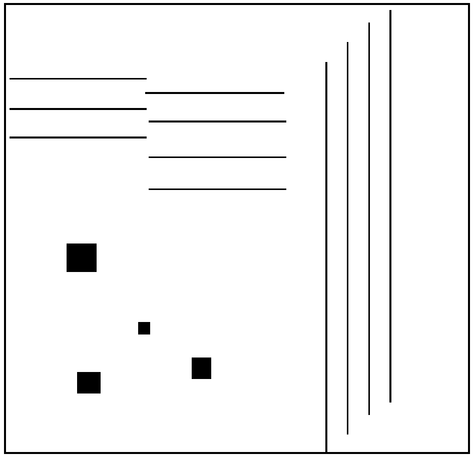

Mask image w

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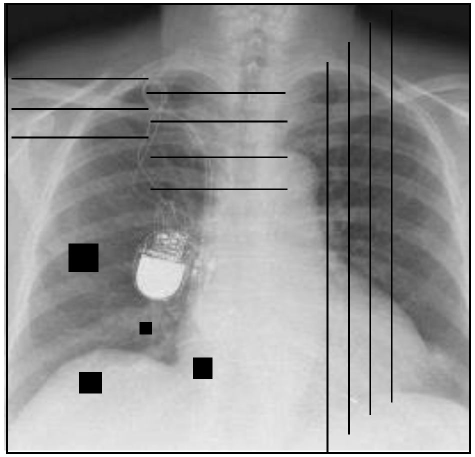
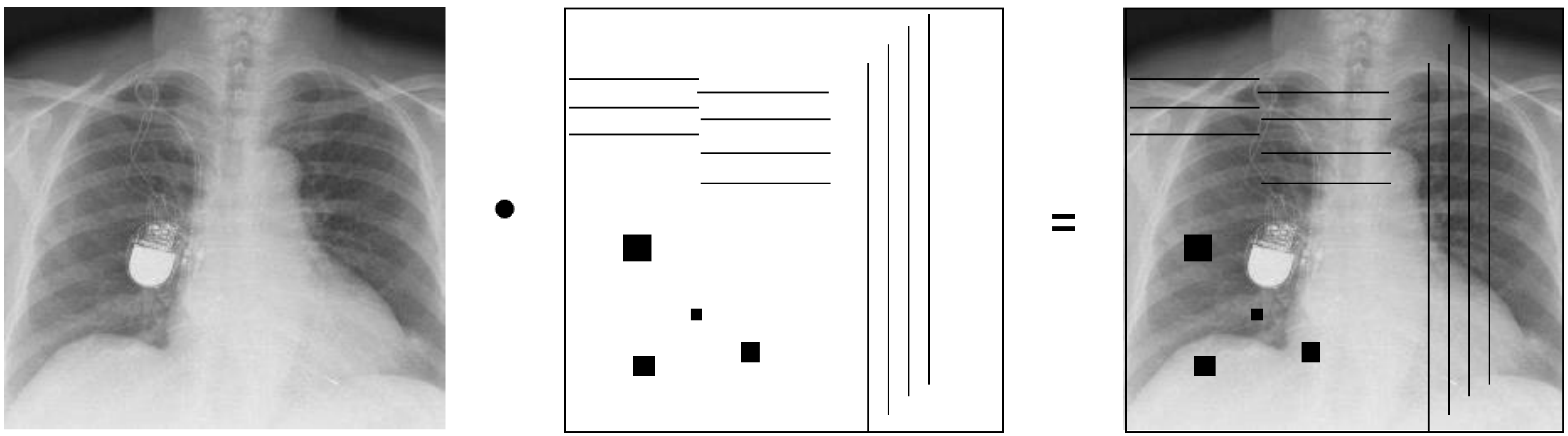


Observed image g

Flat Panel Detectors

Defect Pixels in Flat Panel Detectors



Ideal image f | Mask image w | Observed image g

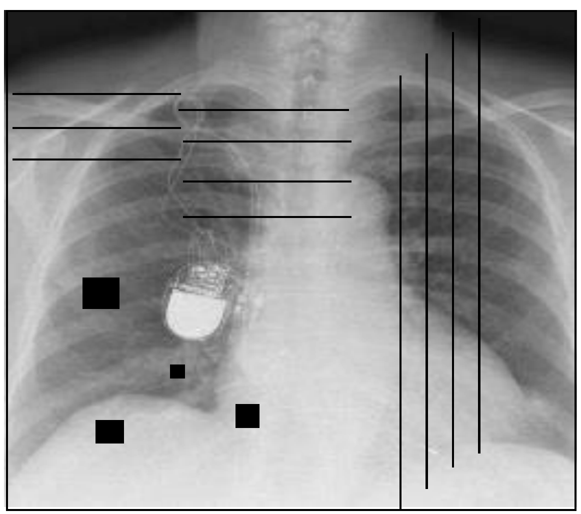
- Defect pixel problem: $f(n) \cdot w(n) = g(n)$
- Goal: Find $f(n)$, given the observed image $g(n)$ and the defect mask $w(n)$

Defect Pixel Correction

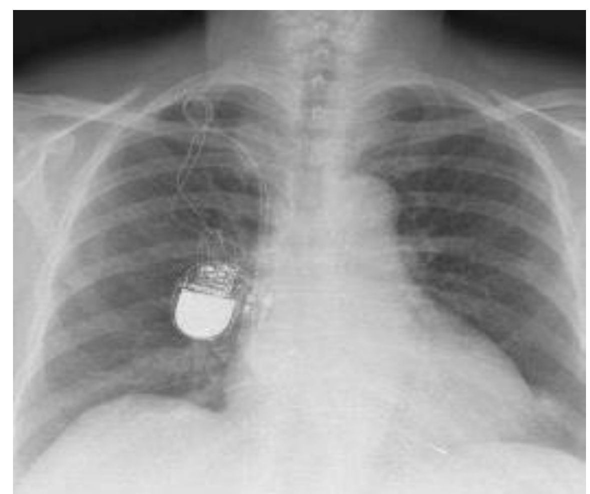
Defect Pixel Correction

Problem Statement

- Restore the **ideal image** based on the **observed image** and the known defect **pixel mask**.
- Defect pixel correction:
  - *Spatial Domain:* Interpolation
  - *Frequency Domain:* Band Limitation
  - *Frequency Domain:* Iterative Deconvolution



Observed image g



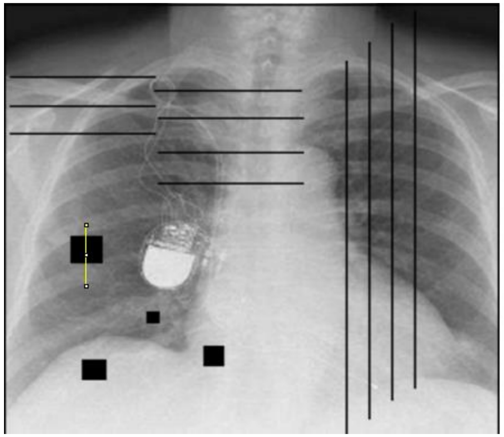
Ideal image f

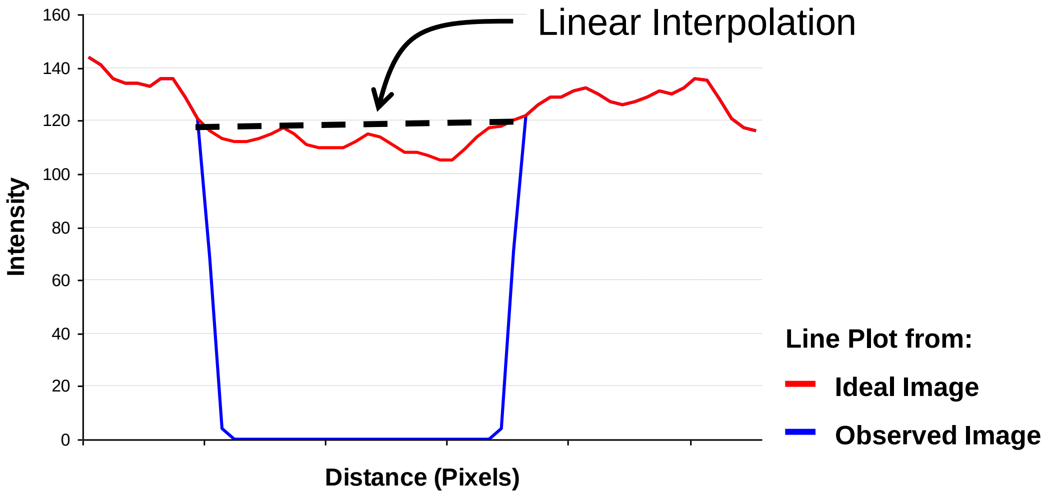
Defect Pixel Correction

Defect Pixel Correction by Spatial Interpolation

- Interpolate between active pixels to recover the inactive ones:
  - Bilinear interpolation
  - Median

- **However, this is only suitable for small defect areas!**





Defect Pixel Correction

Fourier Transform Revisited

- Convolution theorem
- Symmetry property of Fourier transform of real signals

Defect Pixel Correction

Fourier Transform Revisited

- **Convolution theorem**

$$
\begin{aligned}
FT(f \star h)(\xi) &= \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} f(k)h(n-k)e^{\frac{-2\pi i n\xi}{N}} \\
&= \sum_{k=0}^{N-1} f(k) \sum_{n=0}^{N-1} h(n-k)e^{\frac{-2\pi i n\xi}{N}} \\
&= \sum_{k=0}^{N-1} f(k)e^{\frac{-2\pi i k\xi}{N}} H(\xi) = F(\xi)H(\xi) = G(\xi)
\end{aligned}
$$

$l = n - k$

$\Rightarrow n = l + k$

$\Rightarrow e^{\frac{-2\pi i (l+k)\xi}{N}} = e^{\frac{-2\pi i l\xi}{N}} \cdot e^{\frac{-2\pi i k\xi}{N}}$

$F(\xi) \underbrace{\sum_n h(l) e^{\frac{-2\pi i l\xi}{N}}}_{H(\xi)}$

The **convolution** of two signals in the **time domain**, corresponds to a **multiplication** in the **frequency domain**.

- **Symmetry property of Fourier transform of real signals**

Defect Pixel Correction

Fourier Transform Revisited

- **Convolution theorem**
- **Symmetry property of Fourier transform of real signals**
  If f(n) is a real valued discrete signal of length N, the Fourier transform F(ξ) fulfills the symmetry property:

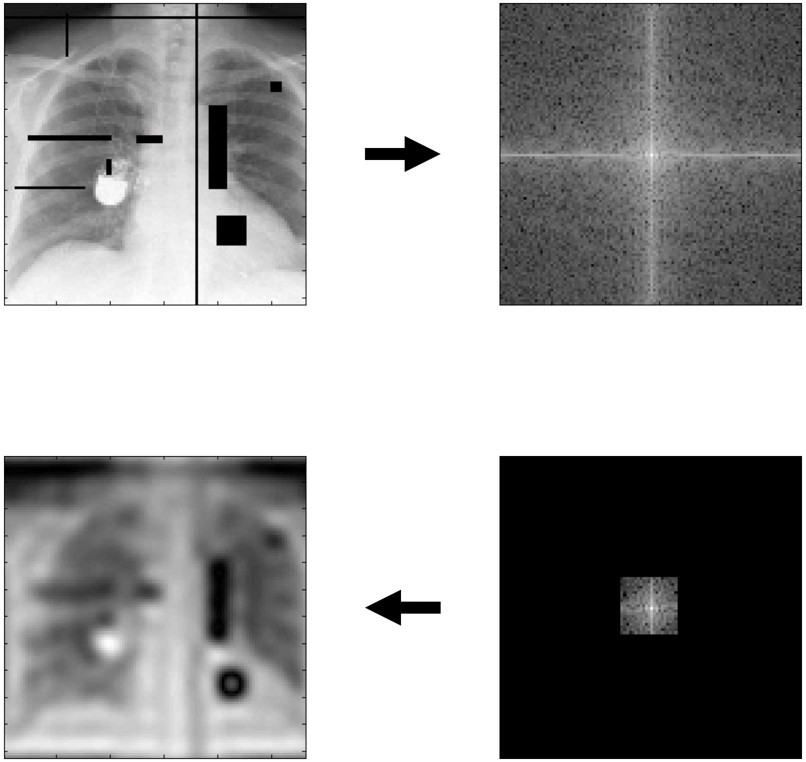
$$F(\xi) = F^*(N - \xi)$$

where ”* “ denotes the conjugate complex.

Defect Pixel Correction

Iterative Band Limitation

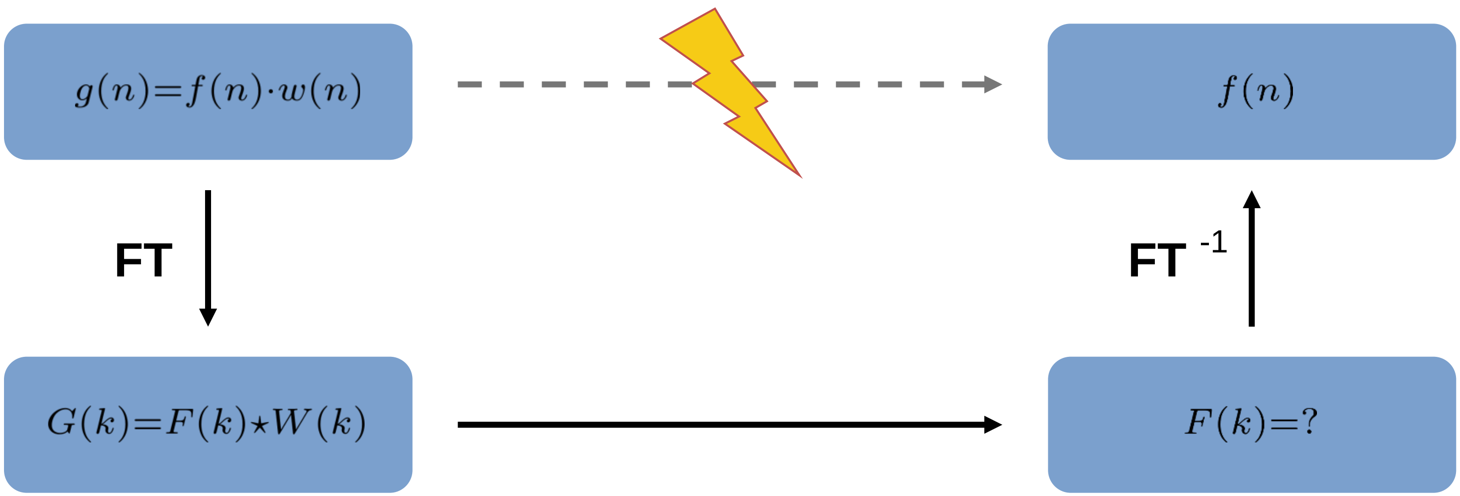
1. Fourier transform
2. Cut off high frequencies
3. Inverse Fourier transform
4. Replace only defect areas
5. Repeat from 1.



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **Application of Fourier convolution theorem:**



$$G(k) = \frac{1}{N} F(k) \star W(k) = \frac{1}{N} \sum_{l=0}^{N-1} F(l) \cdot W(k-l), \qquad 0 \leq n, k < N$$

Defect Pixel Correction

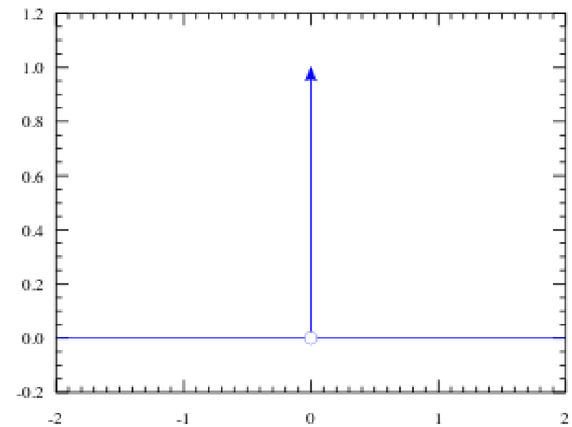
Defect Pixel Correction by Symmetry Properties

- **Application of Fourier symmetry properties**:
- Use Dirac's δ- function to select a line pair F(s) and F(N-s):

$$\hat{F}(k) = \hat{F}(s)\delta(k-s) + \hat{F}(N-s)\delta(k-N+s)$$

where $\hat{F}$ denotes an estimate of F, and δ - function is defined by:

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$



Dirac δ-function

Image taken from: Wikipedia

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **After frequency selection, we convolve with the mask**

$$\hat{G}(k) = \frac{1}{N}\hat{F}(k) \star W(k) = \frac{1}{N}\sum_{l=0}^{N-1}\hat{F}(l) \cdot W(k-l)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- $\hat{F}(l)$ is only non-zero if $l = s \vee l = N - s$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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$$\hat{G}(k) = \frac{1}{N}\left(\hat{F}(s)W(k-s) + \hat{F}(N-s)W\left(k-(N-s)\right)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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$$\hat{G}(k) = \frac{1}{N}\left(\hat{F}(s)W(k-s) + \underbrace{\hat{F}(N-s)}_{= \hat{F}^*(s)}W\left(k-(N-s)\right)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- We are only interested in $\hat{G}(k)$ at position $k = s$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- We are only interested in $\hat{G}(k)$ at position $k = s$

$$\hat{G}(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^{*}(s)W\left(2s-N\right)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **After frequency selection, we convolve with the mask**

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$$\hat{G}(k) = \frac{1}{N}\left(\hat{F}(s)W(k-s) + \hat{F}^*(s)W\left(k-(N-s)\right)\right)$$

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Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **After frequency selection, we convolve with the mask**

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$$\hat{G}(k) = \frac{1}{N}\left(\hat{F}(s)W(k-s) + \hat{F}^*(s)W\left(k-(N-s)\right)\right)$$

- We are only interested in $\hat{G}(k)$ at position $k = s$

$$\hat{G}(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W\left(2s\right)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **Application of Fourier symmetry properties**:
  Select a line pair G(s) and G(N-s) of the Fourier transform of the observed image. The observed image is described by a convolution of the ideal image and the known defect pixel mask:

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s)\right)$$

- And for the conjugate complex:

$$G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)$$

- Using these two equations, we can compute:

$$\hat{F}(s) = N\frac{G(s)W^*(0) - G^*(s)W(2s)}{|W(0)|^2 - |W(2s)|^2}$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- Special case without symmetry property (for s=0 and s = N/2)

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s)\right)$$

$$G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- Special case without symmetry property (for s=0 and s = N/2)

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) \cancel{+ \hat{F}^*(s)W(2s)}\right)$$

$$\cancel{G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)}$$

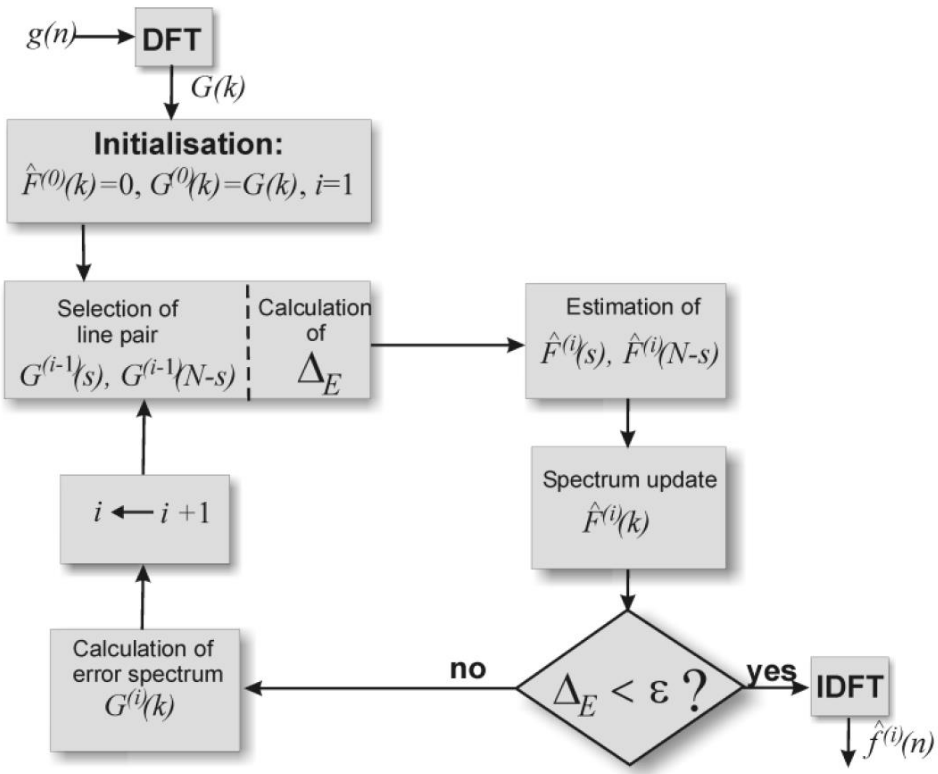
$$\Downarrow$$

$$\hat{F}(s) = N\left(\frac{G(s)}{W(0)}\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

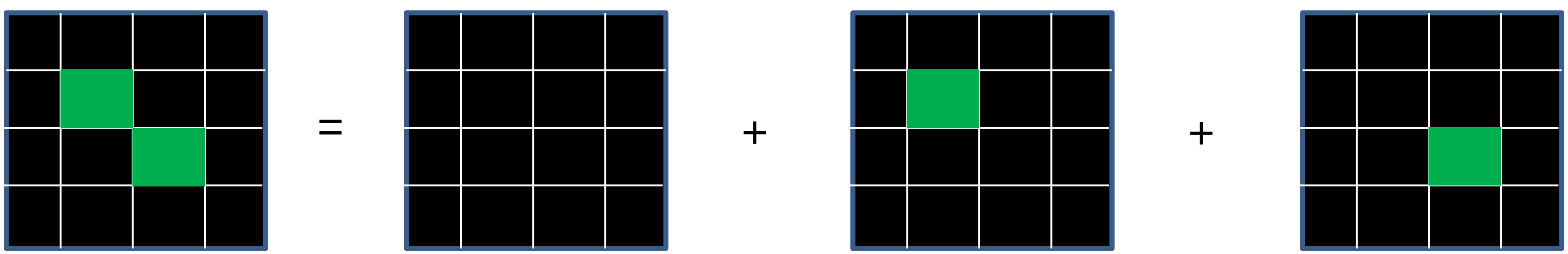
- So far only correction at line pair
  - ➔ Iterative correction of line pairs



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- So far we only found estimates for a single or a pair of selected lines
- We also need to update the global estimate of the spectrum after each linepair computation



$$\hat{F}^{(1)}(k) \qquad \hat{F}^{(0)}(k) = 0 \qquad \hat{F}(s^{(1)}) \qquad \hat{F}(N - s^{(1)}) = \hat{F}^{*}(s^{(1)})$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **How can we update the error spectrum G?**

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N}\left(\hat{F}(k) * W(k)\right)$$

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **How can we update the error spectrum G?**

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N}\left(\hat{F}(k) * W(k)\right)$$

- Just subtract the new estimate from the previous estimate
- Requires convolution!!! → Same complexity as FFT?
- F(k) changed only at two positions → Convolution much easier to compute

Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **How to select the line pairs each iteration?**

- Just select the maximum of the error spectrum

$$s^{(i)} = \underset{\hat{s}^{(i)}}{\operatorname{argmax}}\ G^{(i)}(\hat{s}^{(i)})$$

- Detailed derivation can be found in the paper.
- It is based on Parseval's theorem.
- Approach minimizes the mean square error (MSE) in valid areas:

$$\mathrm{MSE} = \sum_{n=0}^{N} (g(n) - \hat{f}(n)w(n))^2$$