Exercises for
Pattern Recognition
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Assignment 7, 2./5.12.2014

General Information:

Exercises (1 SWS):	Tue 12:15 – 13:45 (0.154-115) and Fri 08:15 – 09:45 (0.151-115)
Certificate:	Oral exam at the end of the semester
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Optimization

Exercise 1 The goal of this exercise is robust regression line fitting for N measurements (x_i, y_i). Thus, you should estimate parameters a, b for a line $ax_i + b$ that best explains your observations y_i. Here we employ the Huber norm to make the estimate more robust to outliers compared to simple least-square regression:

$$(a, b) = \arg\min_{a,b} D(a, b) = \sum_{i=1}^{N} \phi_{\text{Huber}}(y_i - ax_i - b) \quad (1)$$

The parameters (a, b) are determined using iterative numerical optimization. The Huber norm is defined as

$$\phi_{\text{Huber}}(x) = \begin{cases} x^2 & \text{if } |x| \leq M \\ M(2|x| - M) & \text{if } |x| > M \end{cases} \quad (2)$$

(a) Calculate the gradient of the cost function w.r.t. a and b. The gradient is necessary for many iterative numerical optimization techniques.
Hint: You need to calculate the derivative of the Huber norm.

(b) Show that the Huber norm is convex. Use the first-order convexity condition for differentiable functions $f(x)$

$$f(z) \geq f(x) + f'(x)(z - x)$$

Start by proving convexity for $g(x) = x^2$ and $h(x) = M(2|x| - M)$. Then, treat the special cases that occur due to the piece-wise definition of the Huber norm. For this exercise, focus only on positive values x, z, M.

(c) Download the provided measurements from the exercise homepage. Minimize the Huber norm using MATLAB. You do not need the Classification Toolbox. Use the MATLAB function *fminunc*.

(d) Compare the robust line fitting to a ordinary least-square approach. Find situations where the robust approach is superior. Show that due to convexity, the optimum is always found.

Exercise 2 Find the largest rectangle that can be inscribed in the ellipse $x^2 + 2 \cdot y^2 = 4$. In this example, it is sufficient to consider rectangles which are centered at the origin.

(a) What is the area of such rectangles? Formulate the constrained optimization problem.

(b) Solve using the Lagrange multiplier method.