STATE OF WISCONSIN INVESTMENT BOARD

July 2021

VIX Futures Strategy Research Summary

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Summary of Findings

1. We find futures tend to fall into settlement, on average. However, the outliers in the data were primarily instances when we saw significant increases in prices into settlement. We find both negative and positive outliers generally happen in high volatility environments and appear consistently with an uptrend in VIX and downtrend in SPX. This confirms the original research on settlement based on close prices only.
2. Futures sensitivity (beta) to spot VIX is increasing as it approaches maturity.
3. Given the upside skew (Table 1) into VIX futures settlement, closing VIX futures when the prices are low and have been declining is preferred within T-5 to T period. The reason is to avoid unexpected surprises vs. opportunity costs.

Table 1 Summary Statistics of Price Differences Distributions from T-5 to T-1

	T-5	T-4	T-3	T-2	T-1	*T
Max	27.81	24.51	18.86	15.76	13.16	4.51
Mean	-0.52	-0.56	-0.43	-0.3	-0.1	-0.06
Median	-0.93	-0.76	-0.53	-0.35	-0.11	0
Min	-9.4	-11.98	-11.03	-8.41	-9.29	-12.24
Skew	2.92	1.82	1.4	1.44	1.03	-3.89
Std Dev	3.16	2.72	2.39	2.01	1.57	1.1

*T only includes futures with the same dates for last trade and final settlement.

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DEFINITION

Price difference(t) = Final futures settlement price - Price(t)
(A positive difference means futures rise from day t into settlement.)

CHANGES IN FUTURES PRICES IN THEIR LAST 30 TRADING DAYS

From an overview of all futures (Fig. 1), we can see some large positive right tail prices in the distributions which indicates a short position could lose a significant amount of money if not closed before settlement. At the same time, the final settlement price is more often in the lower percentile range in the last 30 days (Fig. 2)

Figure 1. Price Difference Distributions of All Futures

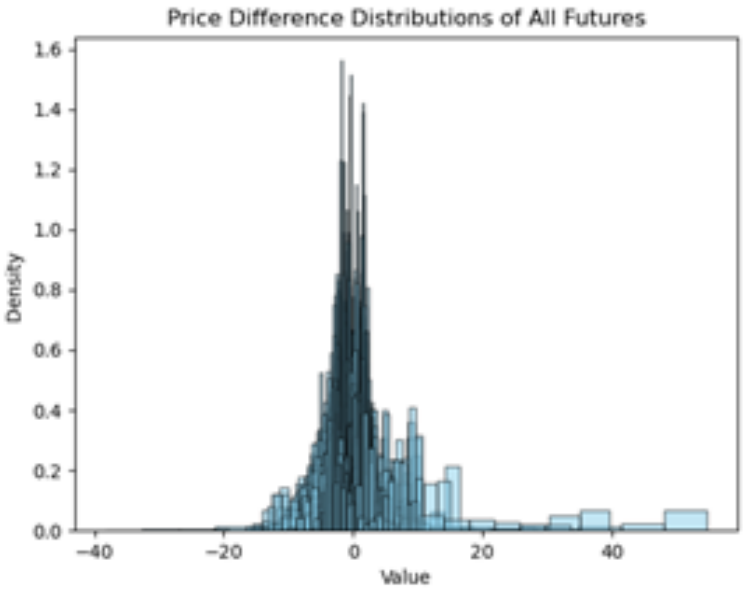
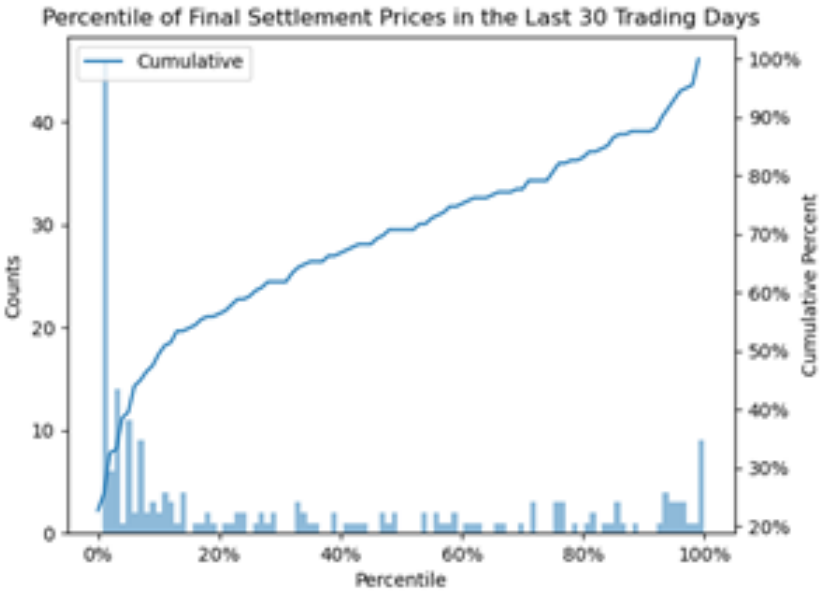


Figure 2. Percentile of Final Settlement Price



Consequently, we examine the price paths for 30 days till the final settlement to get a better sense of when these situations usually occur as well as their magnitudes and frequency. In Figure 3, we can see that the average price differences are decreasing from T-29 to T-1, with more and larger positive outliers than negative ones. The overall distribution appears to be positively skewed. This is also confirmed by Figure 4, which signifies that the mean is larger than the median for all 30 days.

Figure 3. Price Differences Path 30 Days till Settlement (T)

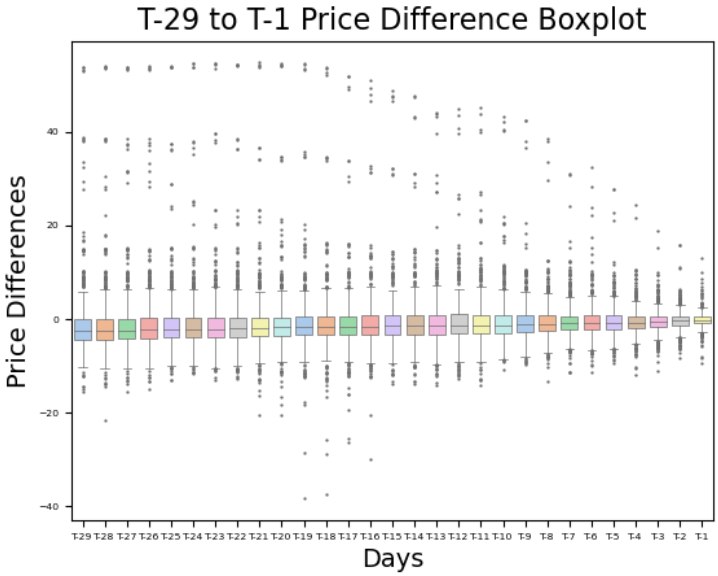
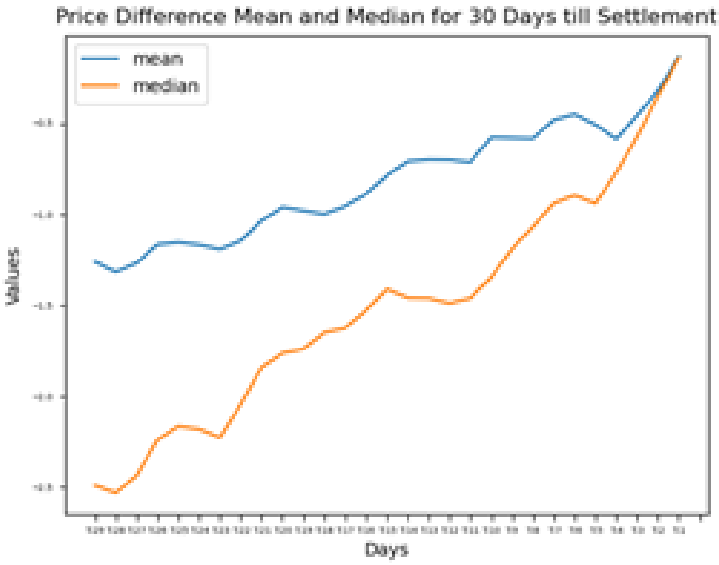


Figure 4. Mean and Median for 30 Day Price Differences



We also investigate which day has the most numbers of the lowest price for each future along the 30-day path, as well as the days with the highest likelihood of showing futures prices lower than their respective settlement prices. It appears that T-1 had most of the lowest prices out of all futures, and the likelihood of lower prices increase linearly toward the final settlement date from T-5.

Lowest Price Occurrence = Numbers of Futures that had the Lowest Price Fell on This Day in the 30 Day Period

No. Lower Px = Numbers of OHLC Prices below Final Settlement Price on this Day

% Lower Px = Likelihood of a Lower Than Settlement Price = Numbers of OHLC Prices below Final Settlement Price / Numbers of Total Prices (203 futures x 4 (OHLC))

Days with Lower Px = Count of Days with at Least One of the OHLC Prices Lower than the Final Settlement Price

% Days with Lower Px = Likelihood of This Day Having a Lower Price = Days with Lower Px / Numbers of Total Futures (203)

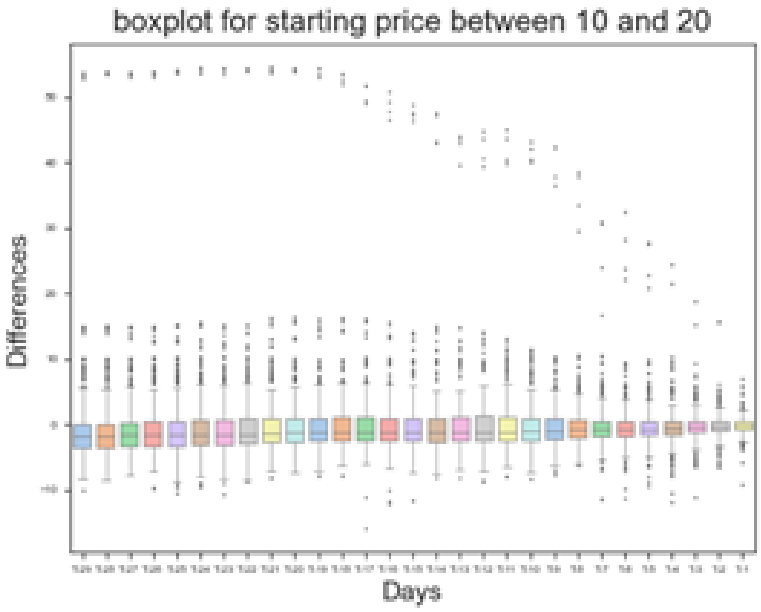
Table 2. Heatmap of Low Prices

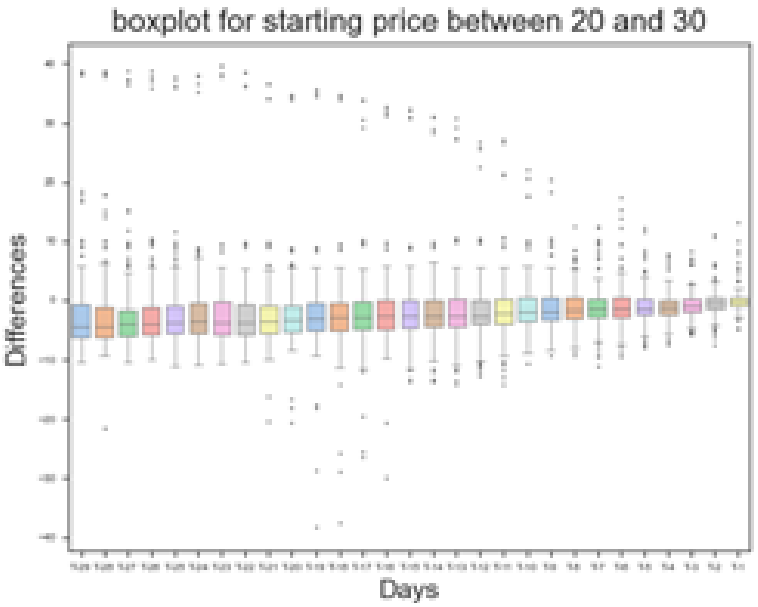
	T-29	T-28	T-27	T-26	T-25	T-24	T-23	T-22	T-21	T-20	T-19	T-18	T-17	T-16	T-15
Lowest Price Occurrence	11	6	0	2	2	0	3	1	0	2	5	3	2	1	1
No. Lower Px	201	198	205	216	218	222	235	227	237	233	236	245	248	256	259
% Lower Px	24.63%	24.26%	25.12%	26.47%	26.72%	27.21%	28.80%	27.82%	29.04%	28.55%	28.92%	30.02%	30.39%	31.37%	31.74%
Days with Lower Px	59	62	61	62	65	67	72	66	71	69	67	72	69	75	75
% Days with Lower Px	28.64%	30.10%	29.61%	30.10%	31.55%	32.52%	34.95%	32.04%	34.47%	33.50%	32.52%	34.95%	33.50%	36.41%	36.41%

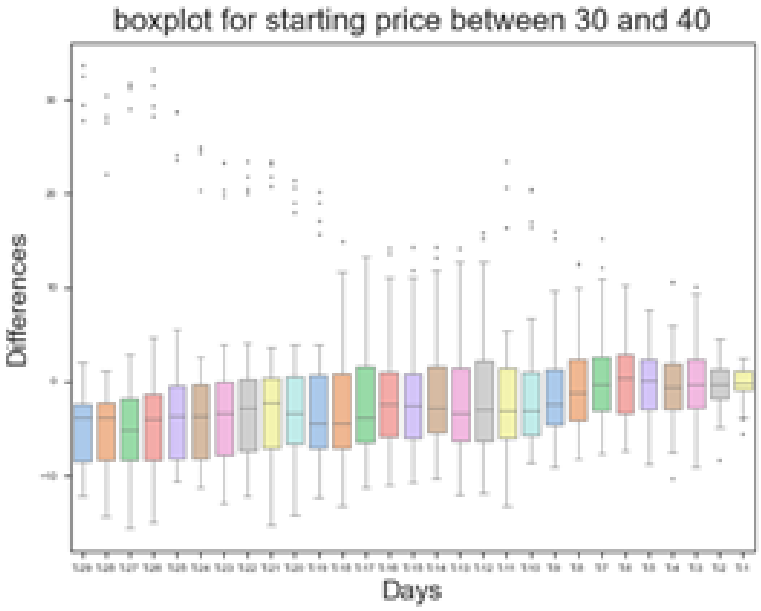
	T-14	T-13	T-12	T-11	T-10	T-9	T-8	T-7	T-6	T-5	T-4	T-3	T-2	T-1
Lowest Price Occurrence	3	3	3	7	3	4	3	12	10	7	8	13	15	73
No. Lower Px	262	269	272	272	280	279	270	265	264	254	255	283	305	357
% Lower Px	32.11%	32.97%	33.33%	33.33%	34.31%	34.19%	33.09%	32.48%	32.35%	31.13%	31.25%	34.68%	37.38%	43.75%
Days with Lower Px	74	75	76	78	82	85	81	83	78	77	88	98	112	138
% Days with Lower Px	35.92%	36.41%	36.89%	37.86%	39.81%	41.26%	39.32%	40.29%	37.86%	37.38%	42.72%	47.57%	54.37%	66.99%

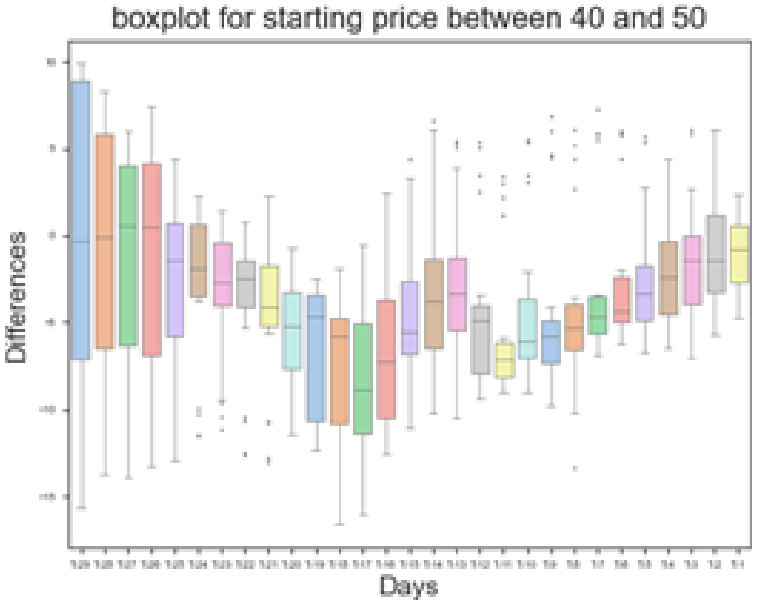
We then look into if starting prices would affect the path of prices in 30 days before settlement. Figure 5.1 to Figure 5.5 show that the paths appear to be more regular for starting futures prices between 10 and 30, it tends to fluctuate more when we enter the trade at futures prices above 30. Interestingly, it does not look optimum if we enter the trade when the price is above 50.

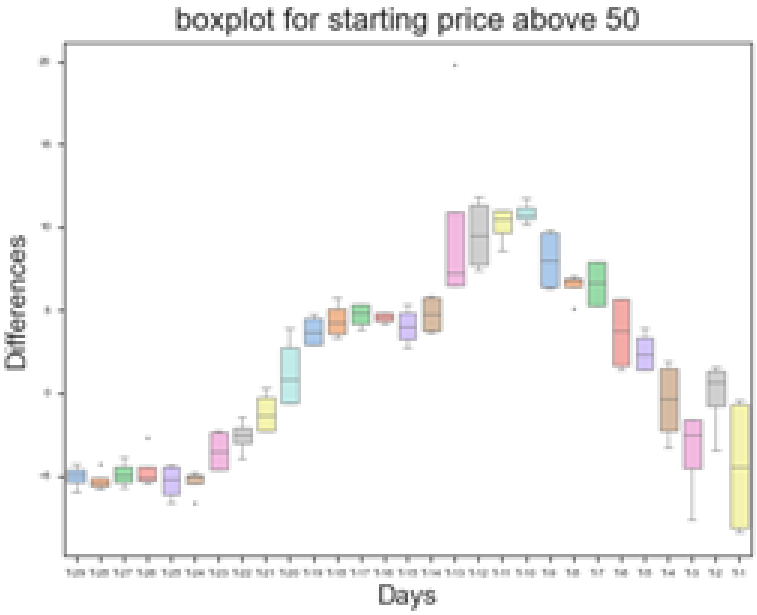
Figure 5.1-5.5. Starting Future Price and the Price Differences Distribution











In the next step, we perform an analysis on all the outliers across 30 days. We examine the environment these outliers appear in from several aspects:
VIX Index price, VIX Index short, medium, and long term moving average trends, SPX Index short, medium, and long term moving average trends (Appendix I), SPX realized volatility (Garman Klass Estimation, Appendix II), and SPX realized and implied volatility spread. From Table 3, we can tell that on average, these outliers appear when VIX Index is above 20 and trending up, while SPX Index is trending down and with high (>30) realized volatility, as well as a large positive spread between realized and implied volatilities.

Table 3. Summary Statistics of the Environment When the Outliers Occur

	VIX Index	VIX Short	VIX Medium	VIX Long	SPX Short	SPX Medium	SPX Long	SPX RV	SPX RV-IV
Mean	40.81	0.77	0.9	0.95	0.25	0.26	0.36	41.18	14.3
Median	37.07	1	1	1	0	0	0	35.52	9.69
Std Dev	18.43	0.42	0.3	0.22	0.43	0.44	0.48	25.12	17.82

SENSITIVITY (BETA) OF FUTURES RETURN TO SPX RETURN, VIX RETURN, AND FUTURES BASIS

In the second half of the analysis, we estimate the sensitivity of futures returns to SPX return, VIX return, and futures basis. We also look at how the relationship evolves as the futures approach their expiration. In general, futures returns are negatively correlated with SPX returns with its sensitivity to SPX returns increasing towards maturity. Similarly, futures returns are usually more sensitive to the spot VIX movement as the futures approach maturity. We investigate in detail and found out that this trend is most reliable in low volatility (VIX<20) environment. The futures basis normalized by days to maturity does not seem to be a key driver for the daily futures return in the last 30-day. See Figures 6.1-6.9 below.

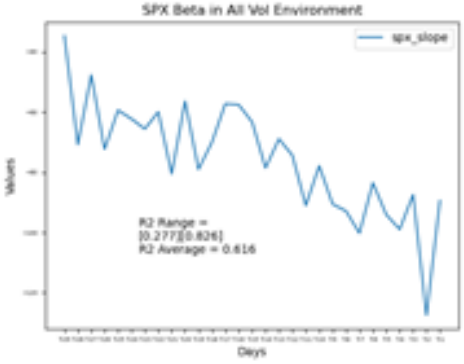
Normalized futures basist = (Futures Price(t-1) – VIX(t-1))/Days to Maturity(t-1)

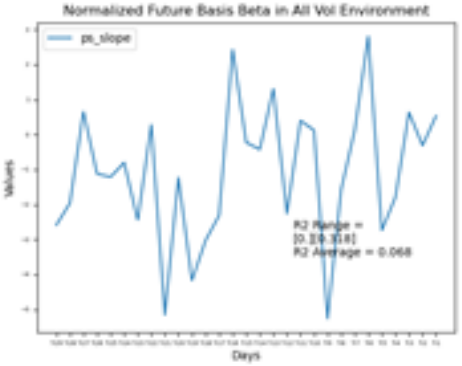
Futures returns = Futures Pricet – Futures Price(t-1)

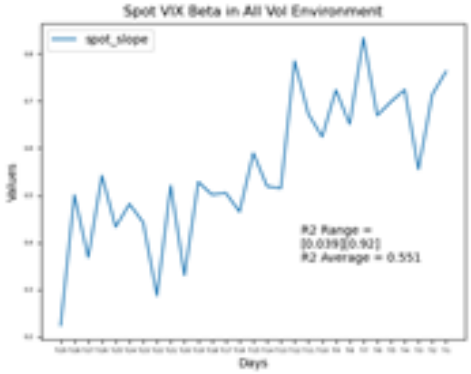
VIX returns = VIX Price(t) – VIX Price(t-1)

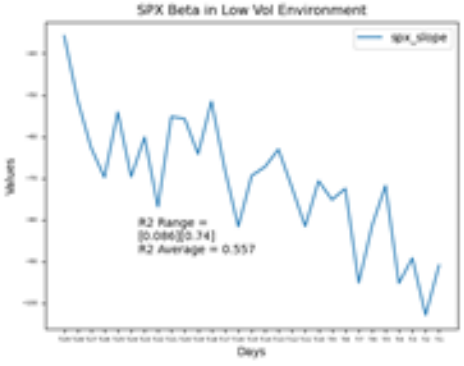
SPX returns = Log(SPX(t) / SPX(t-1))

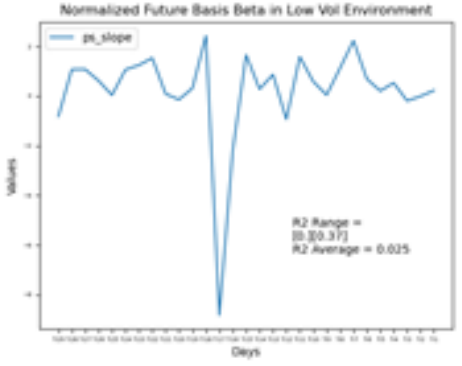
Figure 6.1-6.9 Betas of Spot VIX, SPX Returns, and Normalized Future Basis on Futures Return

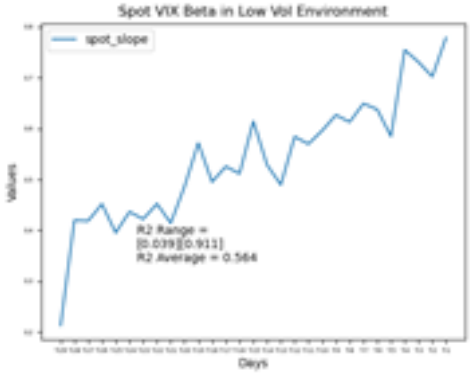


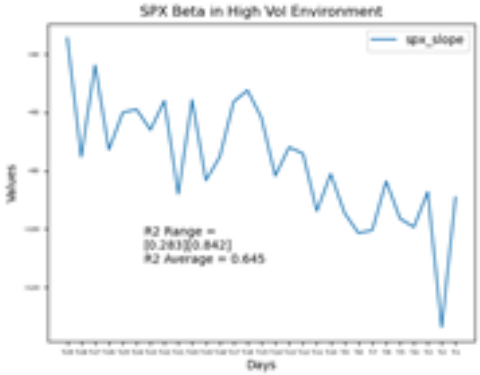


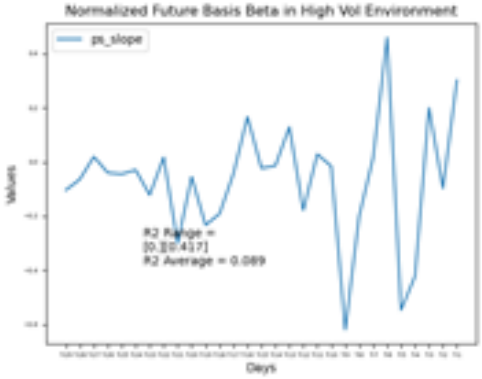


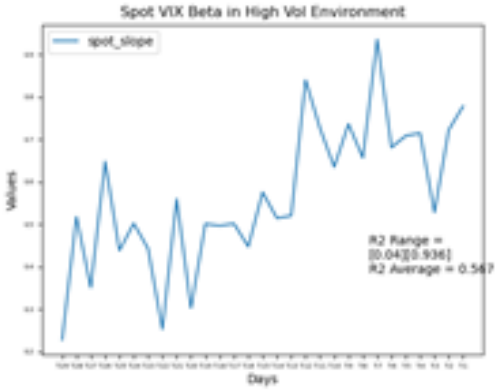






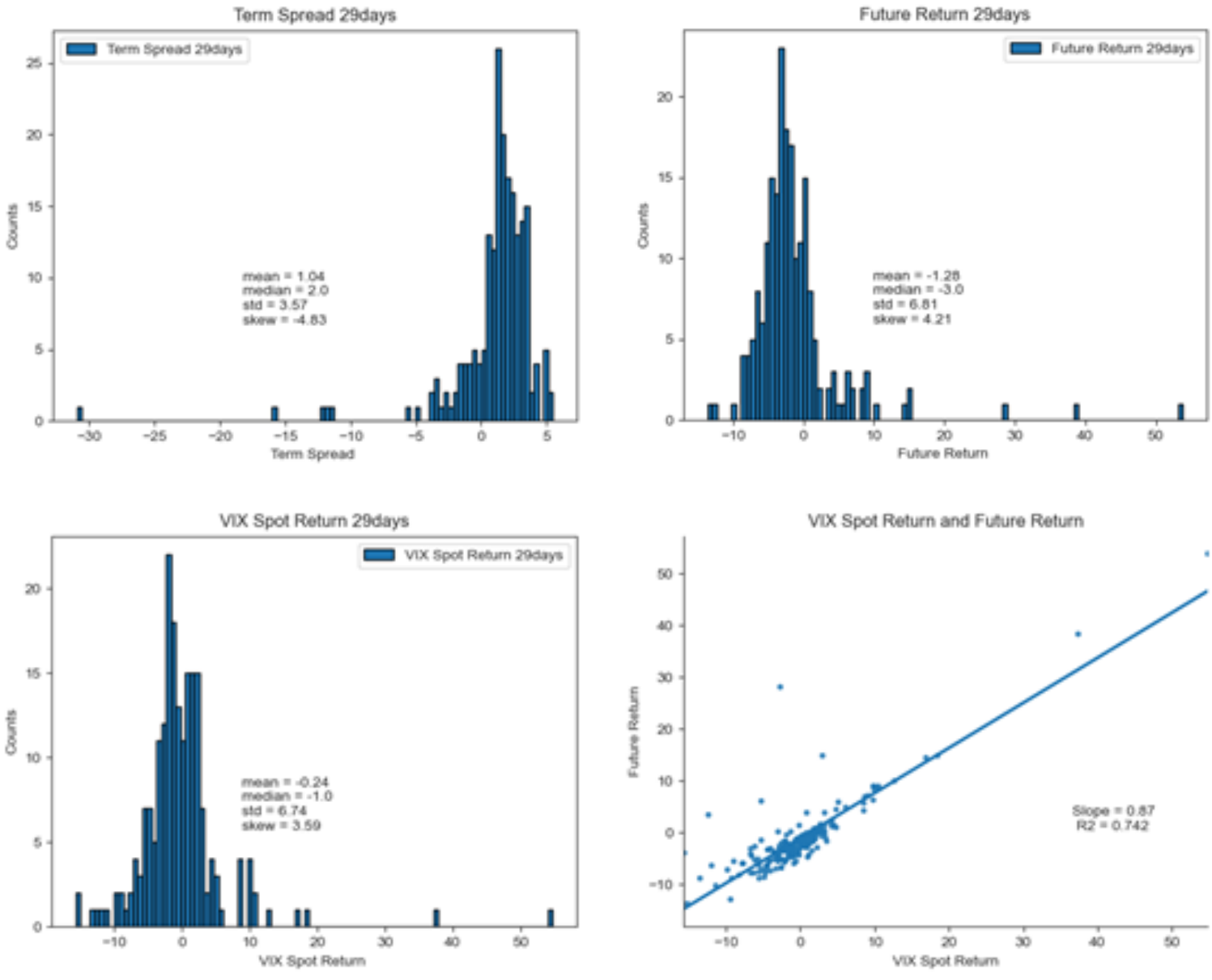






Although VIX does seem to drive the majority of the returns, the term spread contributes positively to the overall future returns on a longer time horizon, We decompose the future return on 30-day time horizons instead of on daily horizons, see Figures 7.1 - 7.3. We also find that future returns on 30-day horizons are positively correlated with VIX Index returns (Figure 7.4).

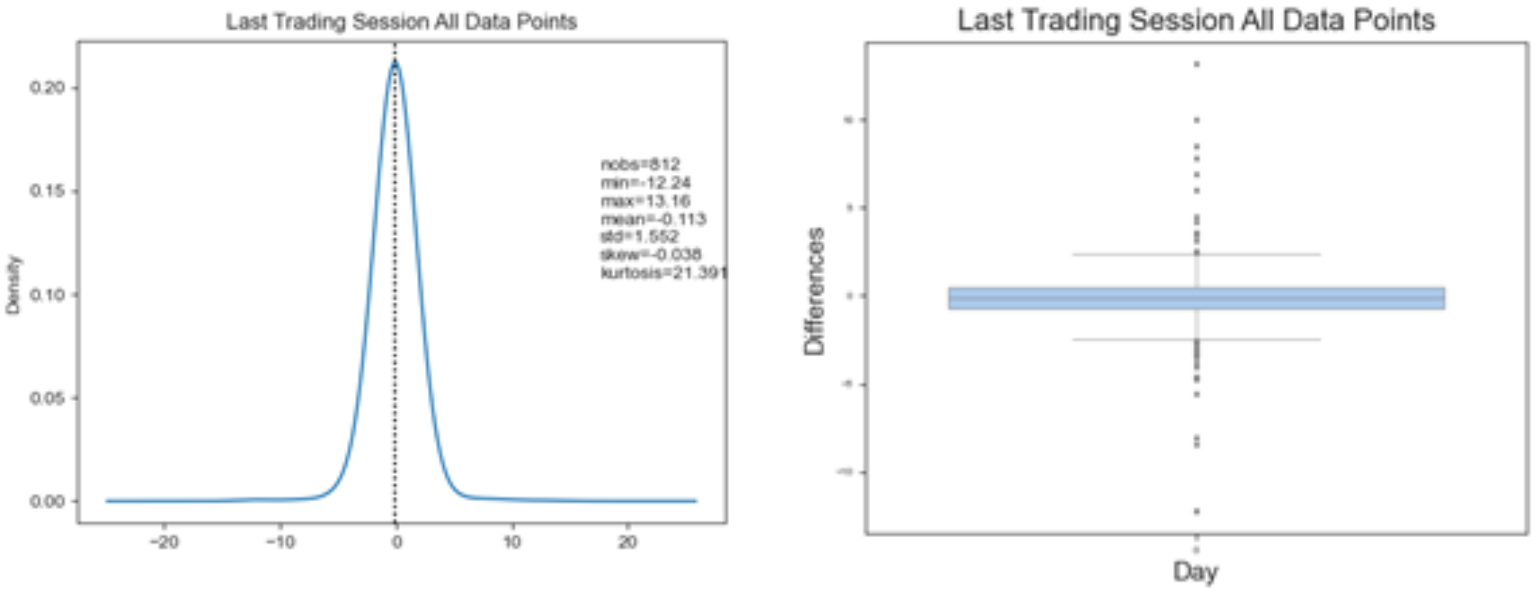
Figure 7.1 - 7.4. Decomposition of Future Returns on 30-Day Horizons



LAST TRADING SESSION ANALYSIS

In addition, we have performed an analysis on the last trading session actions right before the final settlement. This session pre-2014 was a full trading day with normal liquidity, post-2014, it became a half-day session and is generally illiquid with significantly lower volumes. The price differences in these last sessions appear to be slightly negatively skewed, although do not differentiate from the final settlement price much, excluding outliers (Figure 8.1 - 8.2).

Figure 8.1 - 8.2. Last Trading Session Distributions



Similar to the analysis on all prices for 30 days, we further perform an analysis on the last trading session outliers post 2014. They generally occur in high volatility environments. In detail, realized volatility on SPX is very high both in absolute terms and compared with implied volatility. For UXG18, it occurred during a low volatility environment, however, SPX realized volatility spiked ten -

- points in one day. The environment in which these outliers occur is consistent with the observations for all days in general. It is worth noting that on 4/15/2020, SPX Index short-term trend turned upward when the positive outlier happened (Table 4).

Table 4. Summary Statistics of the Environment When the Outliers Occur in the Last Session Post 2014

Date	Future	Price Diff.	VIX Index	VIX Short	VIX Medium	VIX Long	SPX Short	SPX Medium	SPX Long	SPX RV	SPX RV-IV
2/13/2020	N/A	N/A	14.15	0	1	1	1	1	1	12.01	-2.6976
2/14/2018	UXG18 Index	-4.63	19.26	1	1	1	0	0	1	22.09	5.6239
3/18/2020	UXH20 Index	-12.24	76.45	1	1	1	0	0	0	73.57	32.4266
4/15/2020	UXJ20 Index	4.21	40.84	0	0	1	1	0	0	86.16	57.9537

APPENDIX I

VIX and SPX moving average (MA) data from Bloomberg are categorized into short term, medium term, and long term.

For the short term category, MA1 = 5 days, MA2 = 15 days; For the medium term category, MA1 = 20 days, MA2 = 50 days; For the long term category, MA1 = 50 days, MA2 = 200 days.

The signals for these categories are coded into 0 (downtrend) and 1 (uptrend) using the following system:

if Price > MA1, it is 1, otherwise 0;
if Price > MA2, it is 1, otherwise 0;
If MA1 slope > 0, it is 1, otherwise 0;
If MA2 slope > 0, it is 1, otherwise 0;
If MA1 > MA2, it is 1, otherwise 0.

if the sum of five line items above >= 3, it is 1, otherwise 0.

APPENDIX II

The SPX historical realized volatility data utilizes Garman - Klass volatility model. It is calculated using the formula below. Bloomberg provides an option to directly pull the data estimated with this model, hence we do not need to perform this calculation.

$$GKHV = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}\left(ln\frac{h_i}{l_i}\right)^2 - \frac{1}{N}\sum_{i=1}^{N}(2ln2-1)\left(ln\frac{c_i}{o_i}\right)^2}$$

We utilize the data estimated with this particular model because it consists of using the returns of the open, high, low, and closing prices in its calculation, which is consistent with the magnitude of this research.
This model systematically underestimates volatility.