CS498
HW1
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Question 1:

		Player B	
		L	R
Player A	U	$1, 2$	$3, 2$
	D	$2, 4$	$0, 2$

For player A, D is a strict best response to strategy L by player B and U is a strict best response to strategy R by player B.
For player B, he is indifferent between L and R to strategy U by player A and L is a strict best response to strategy D by player A.
Hence, **(D, L) and (U, R)** are pure strategy Nash equilibria since they are mutual best responses.

Question 2:

		Player B		
		L	M	R
Player A	t	$0, 3$	$6, 2$	$1, 1$
	m	$2, 3$	$0, 1$	$7, 0$
	b	$5, 3$	$4, 2$	$3, 1$

Figure 6.28: Payoff Matrix

(a)
Player B has a strictly dominant strategy: for player B, **L** is a strict best response to each strategy of player A. Hence, L is the dominant strategy for player B.
On the other hand, player A does not have a dominant strategy: t is the best strategy when player B plays M, m is the best strategy when player B plays R, b is the best strategy when player B plays L.

(b)
For player A, b is a strict best response to strategy L by player B, t is a strict best response to strategy M by player B and m is a strict best response to strategy R by player B.
Since player B has dominant strategy of playing L, **(b, L)** is the pure strategy Nash equilibria since they are mutual best responses.

Question 3:

(a)

		Player B L	Player B R
Player A	U	$2, 15$	$4, 20$
	D	$6, 6$	$10, 8$

Player A has a strictly dominant strategy: for player A, **D** is a strict best response to each strategy of player B.
Player B has a strictly dominant strategy: for player B, **R** is a strict best response to each strategy of player B.
Hence, **(D, R)** is the pure strategy Nash equilibria since they are mutual best responses.

(b)

		Player B L	Player B R
Player A	U	$3, 5$	$4, 3$
	D	$2, 1$	$1, 6$

Player A has a strictly dominant strategy: for player A, **U** is a strict best response to each strategy of player B.
For player B, L is a strict best response to strategy U by player A and R is a strict best response to strategy D by player A.
Hence, **(U, L)** is the pure strategy Nash equilibria since they are mutual best responses.

Question 3 (c):

		Player B	
		L	R
Player A	U	$1, 1$	$4, 2$
	D	$3, 3$	$2, 2$

Pure strategy equilibria:
For player A, D is a strict best response to strategy L by player B and U is a strict best response to strategy R by player B.
For player B, R is a strict best response to strategy U by player A and L is a strict best response to strategy D by player A.
Hence, **(D, L)** and **(U,R)** are the pure strategy Nash equilibria since they are mutual best responses.

Mixed strategy equilibrium:
First, suppose the player B chooses a probability of q for playing L.
Then the expected payoff to the player A playing U is:

$$(1)(q) + (4)(1 - q) = 4 - 3q$$

while the expected payoff to the player A playing D is:

$$(3)(q) + (2)(1 - q) = 2 + q$$

To make player A indifferent between its two strategies, we need to set $4 - 3q = 2 + q$, and hence q = 1/2.

Next, suppose the player A chooses a probability of q for playing U.
Then the expected payoff to the player B playing L is:

$$(1)(p) + (3)(1 - p) = 3 - 2p$$

while the expected payoff to the player B playing R is:

$$(2)(p) + (2)(1 - p) = 2$$

To make player B indifferent between its two strategies, we need to set $3 - 2p = 2$, and hence p = 1/2.

Thus, the mixed strategy equilibrium is when player A chooses strategy U with probability **1/2** and player B chooses strategy L with probability **1/2**.

Programming Question:

1.
For a search query on category Sports, the top-10 nodes in decreasing order of pagerank for network1 is:

```
top_10_sports
```

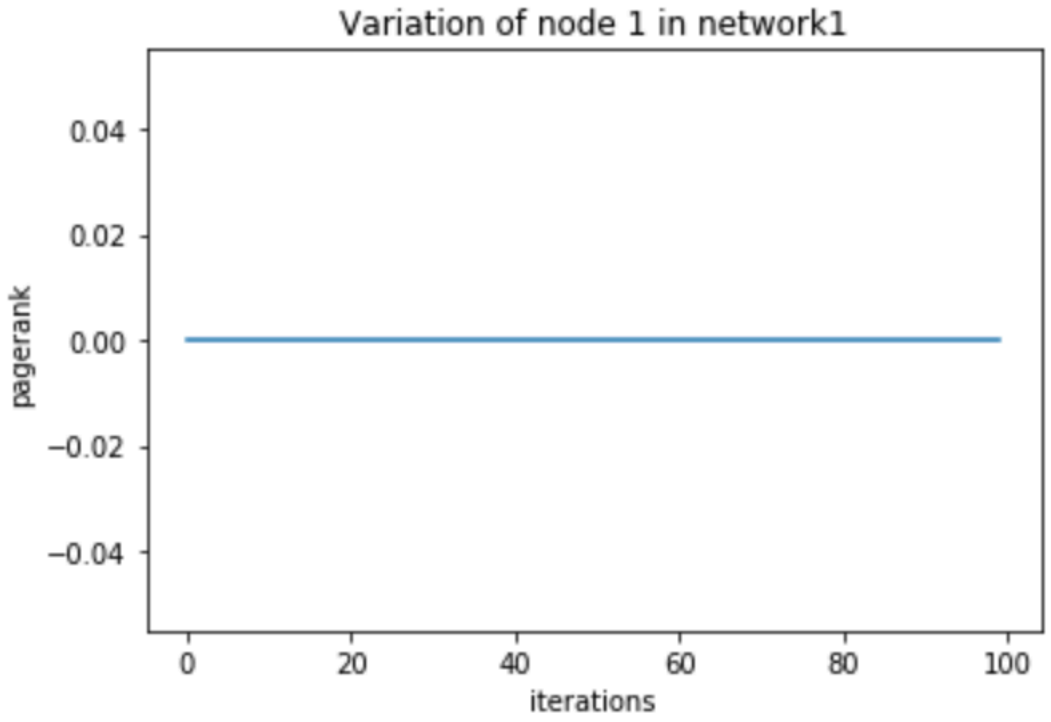
```
[22, 10, 3, 7, 2, 0, 17, 26, 16, 13]
```

2.
For a search query on category Politics, the top-10 nodes in decreasing order of pagerank for network1 is:

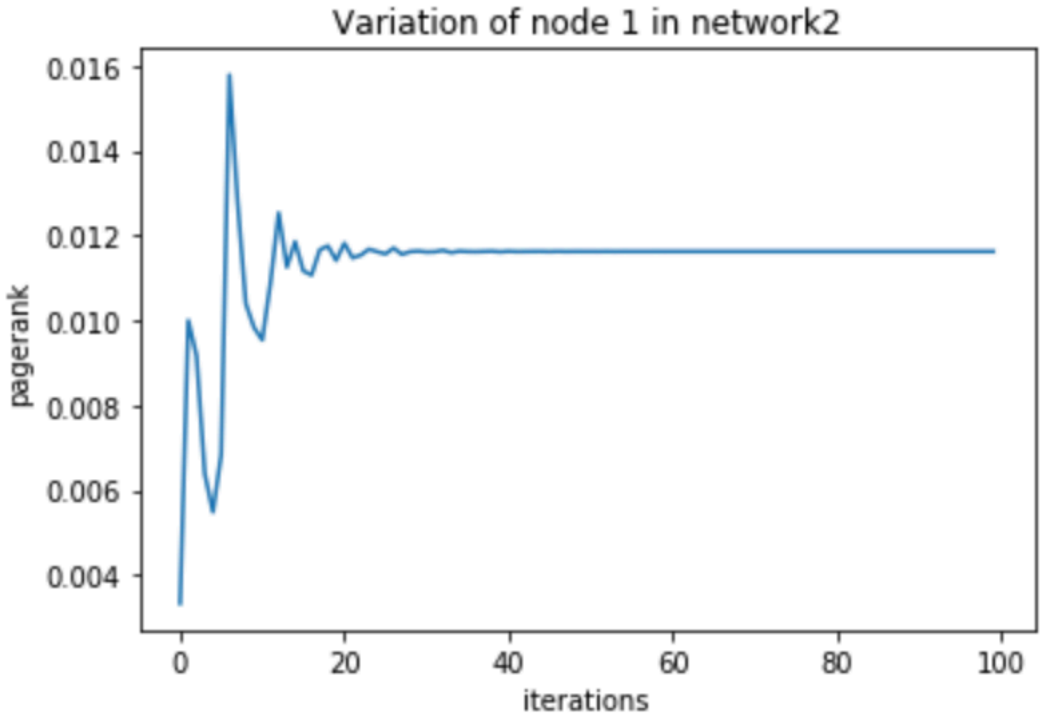
```
top_10_politics
```

```
[30, 59, 57, 56, 33, 49, 31, 39, 65, 60]
```

3.



4.



5.

The page rank value of node with id 1 for network1 is **0.0** after 100 iterations and the page rank value of node with id 1 for network2 is **0.0116** after 100 iterations, which is larger. It is because of one additional outgoing edge from node with id 2 to node with id 1 for network2. For network1, there is no edge pointing to node with id1.