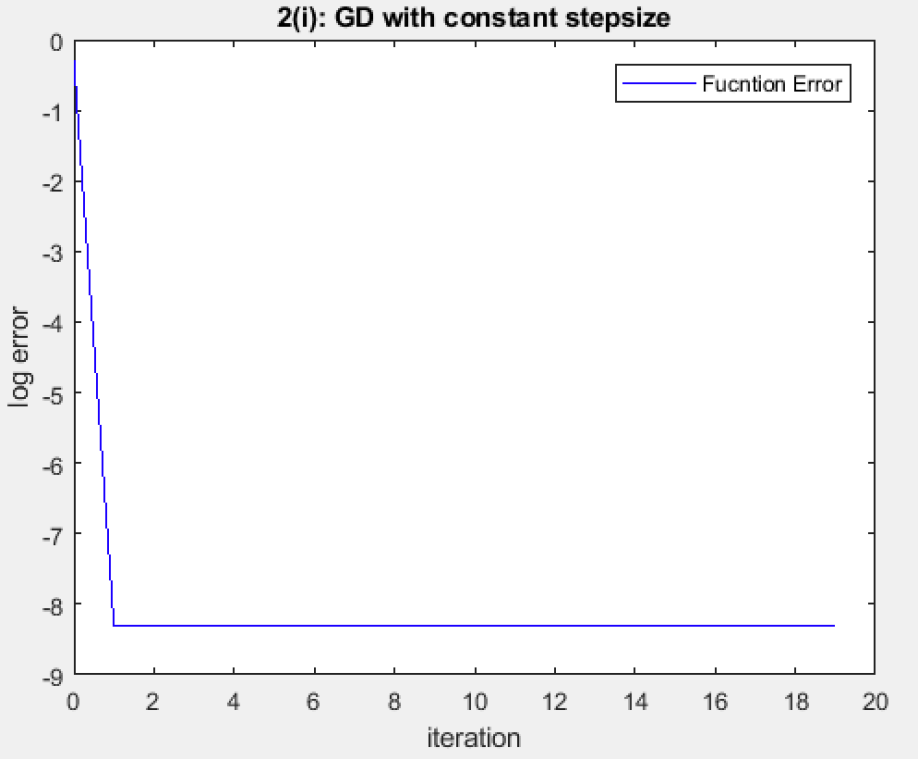
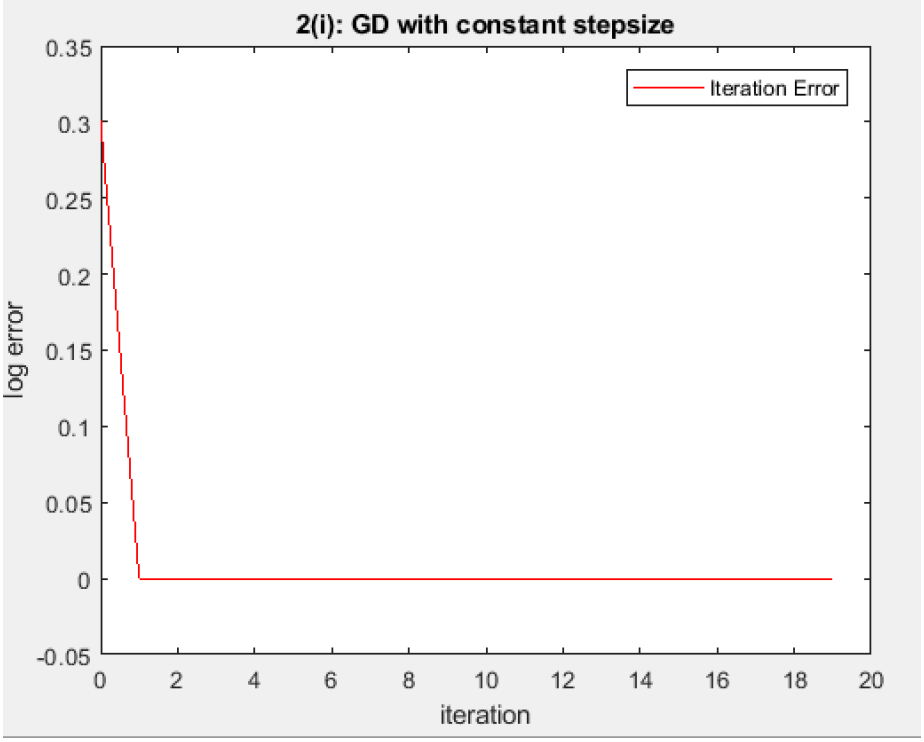
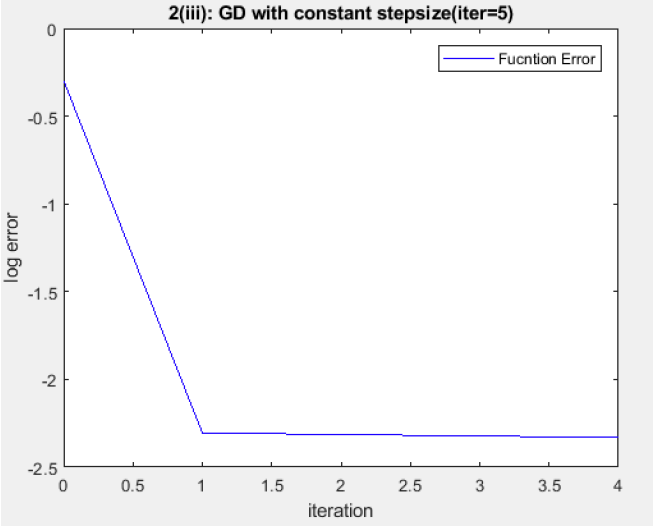
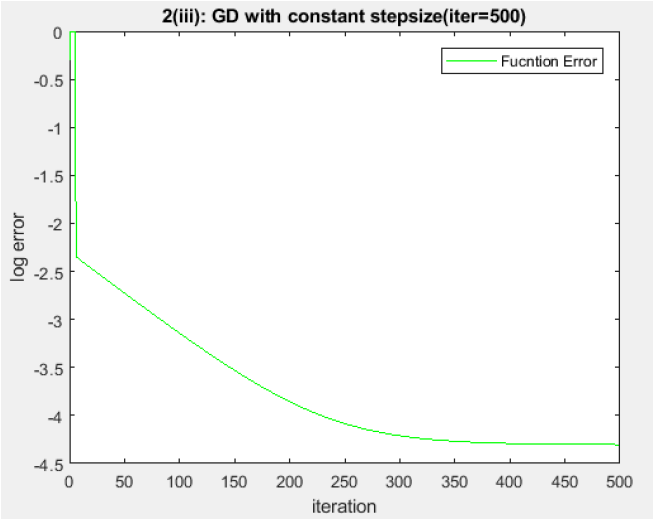
2. (i)

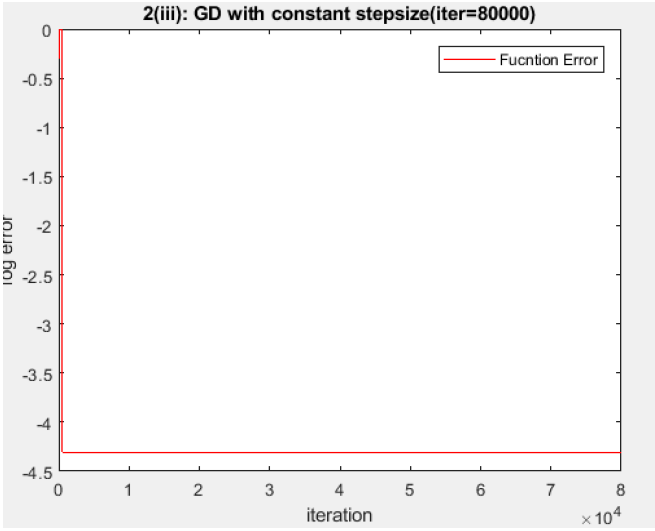

2(i): GD with constant stepsize
Fucntion Error
log error
iteration
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
0
2
4
6
8
10
12
14
16
18
20


2(i): GD with constant stepsize
Iteration Error
log error
iteration
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
2
4
6
8
10
12
14
16
18
20

2.(iii)


2(iii): GD with constant stepsize(iter=5)
Fucntion Error
log error
iteration
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
3
3.5
4


2(iii): GD with constant stepsize(iter=500)
Fucntion Error
log error
iteration
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
0
50
100
150
200
250
300
350
400
450
500


2(iii): GD with constant stepsize(iter=80000)
Fucntion Error
log error
iteration
×10^4
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
0
1
2
3
4
5
6
7
8

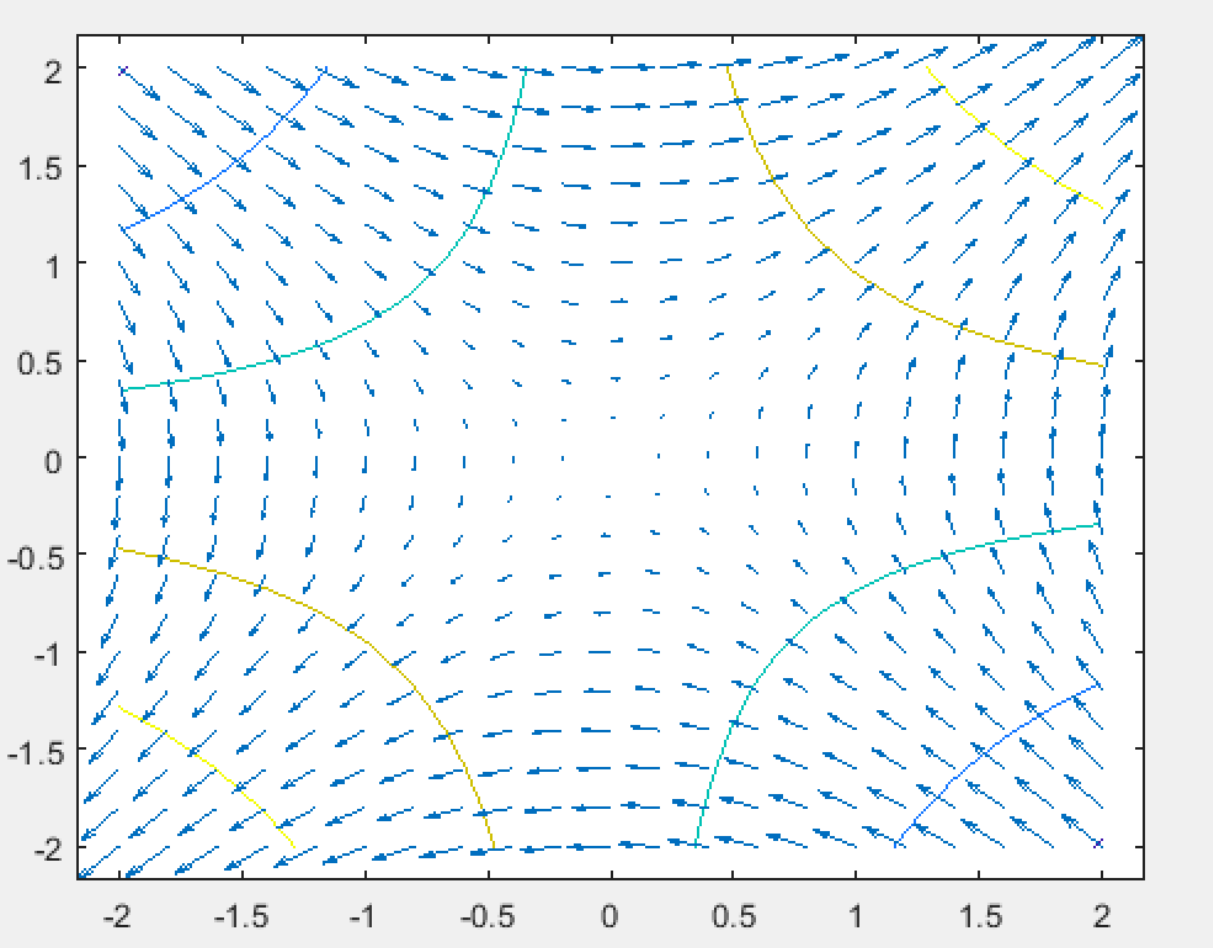
3.(v)

X0	m	α
0.1	2	0.25
0.2	2	0.25
0.3	2	0.25
0.4	2	0.25
0.5	2	0.25
0.6	2	0.25
0.7	2	0.25
0.8	2	0.25
0.9	2	0.25
1	Infinite	Infinite
2	7	0.159090909090909
3	6	0.057692307692308
4	6	0.031914893617021
5	6	0.020270270270270
6	6	0.014018691588785
7	6	0.010273972602740
8	6	0.007853403141361
9	6	0.006198347107438
10	6	0.005016722408027

3.(vi)

X0	m	α
0.1	1	0.027777777777778
0.2	1	0.027777777777778
0.3	1	0.027777777777778
0.4	1	0.027777777777778
0.5	1	0.027777777777778
0.6	1	0.027777777777778
0.7	1	0.027777777777778
0.8	1	0.027777777777778
0.9	1	0.027777777777778
1	Infinite	Infinite
2	22	1.639246542680019e-04
3	22	2.824275624652743e-06
4	22	1.589633600438022e-07
5	22	1.706716316595877e-08
6	22	2.756388802067653e-09
7	22	5.900244753759777e-10
8	22	1.552206982480221e-10
9	22	4.779957406320654e-11
10	22	1.666667424242769e-11

4.(iv)



2.(i) Code from Matlab

```
%% Set up Problem
v = [1; 10^(-8)];
Q = diag(v);
x = [1; 1];
max_it = 20;

%% Set up iteration error
cur_ite_error = x'*x;
error_ite = zeros(max_it + 1, 1);
error_ite(1) = log10(cur_ite_error);

%% Set up function value error
cur_func_error = 0.5* x'* Q* x;
error_func = zeros(max_it +1, 1);
error_func(1) = log10(cur_func_error);

%% run the algorithm
alpha = 1;
ite = 1;
while (ite <= max_it)
   new_x = x - Q*x;
   x = new_x;
   error_ite(ite+1) = log10(x'*x);
   error_func(ite+1) = log10(0.5* x'* Q* x);
   ite = ite+1;
end

%% Plot the function
figure;
plot_length = max_it ;
plot_vec = 0:1:plot_length-1;
%plot(plot_vec, error_func(1:plot_length), 'b-');
%hold on;
%% Plot the iterates
plot(plot_vec, error_ite(1:plot_length), 'r-');
xlabel('iteration');
ylabel('log error');
legend('Iteration Error');
title('2(i): GD with constant stepsize');
```

2.(iii)

```
%% Set up Problem
v = [1; 10^(-2); 10^(-4); 10^(-6)];
Q = diag(v);
x = [1; 1; 1; 1];
max_it = 1000;
max_it1 = 500;
max_it2 = 80000;


%% Set up function value error
cur_func_error = 0.5* x'* Q* x;
error_func = zeros(max_it +1, 1);
error_func(1) = log10(cur_func_error);
error_func1 = zeros(max_it1 +1, 1);
error_func1(1) = log10(cur_func_error);
error_func2 = zeros(max_it2 +1, 1);
```

```
error_func2(1) = log10(cur_func_error);
%% run the algorithm
alpha = 1;
ite = 1;
while (ite <= max_it)
   new_x = x - [x(1); x(2)*10^(-2); x(2)*10^(-4); x(2)*10^(-6)];
   x = new_x;
   error_func(ite+1) = log10(0.5* x'* Q* x);
   ite = ite+1;
end
while (ite <= max_it1)
   new_x = x - [x(1); x(2)*10^(-2); x(2)*10^(-4); x(2)*10^(-6)];
   x = new_x;
   error_func1(ite+1) = log10(0.5* x'* Q* x);
   ite = ite+1;
end
while (ite <= max_it2)
   new_x = x - [x(1); x(2)*10^(-2); x(2)*10^(-4); x(2)*10^(-6)];
   x = new_x;
   error_func2(ite+1) = log10(0.5* x'* Q* x);
   ite = ite+1;
end
%% Plot the function
figure;
plot_length = max_it ;
plot_vec = 0:1:plot_length-1;
plot_length1 = max_it1 ;
plot_vec1 = 0:1:plot_length1-1;
plot_length2 = max_it2 ;
plot_vec2 = 0:1:plot_length2-1;

plot(plot_vec, error_func(1:plot_length), 'b-');
%plot(plot_vec1, error_func1(1:plot_length1), 'g-');
%plot(plot_vec2, error_func2(1:plot_length2), 'r-');
xlabel('iteration');
ylabel('log error');
legend('Fucntion Error');
title('2(iii): GD with constant stepsize(iter=80000)');
%% Plot the iterates
```

3.(v)

```
%% Set up Problem
x = 0.2;
max_it = 100;

cur_func_error = (x^2-1)^2;
%cur_func_error = log10((x^2-1)^2);
error_func = zeros(max_it +1, 1);
error_func(1) = cur_func_error;
%deri_func = log10(4*x^3-4*x);
%error_deri = zeros(max_it +1, 1);
%error_deri(1) = deri_func;


%% run the algorithm
alpha = 1/(12*x^2-4);
%alpha = 1/8;
ite = 1;
```

```
while (ite <= max_it)
   new_x = x - alpha*(4*x^3-4*x);
   x = new_x;
   error_func(ite+1) = (x^2-1)^2;
   %error_func(ite+1) = log10((x^2-1)^2);
   %deri_func(ite+1) = log10(4*x^3-4*x);
   ite = ite+1;
end

%% Plot the function
figure;
plot_length = max_it ;
plot_vec = 0:1:plot_length-1;
plot(plot_vec, error_func(1:plot_length), 'b-');
%plot(plot_vec, deri_func(1:plot_length), 'b-');
xlabel('iteration');
ylabel('f(x)');
legend('Fucntion');
title('3(v): GD with constant stepsize');
%% Plot the iterates
```

3.(vi)

```
%% Set up Problem
x = 2;
max_it = 100;

cur_func_error = (x^6-1)^2;
%cur_func_error = log10((x^2-1)^2);
error_func = zeros(max_it +1, 1);
error_func(1) = cur_func_error;
%deri_func = log10(4*x^3-4*x);
%error_deri = zeros(max_it +1, 1);
%error_deri(1) = deri_func;


%% run the algorithm
alpha = 22/(12*(11*x^10-5*x^4));
%alpha = 1/36;
ite = 1;
while (ite <= max_it)
   new_x = x - alpha*(12*x^11-12*x^5);
   x = new_x;
   error_func(ite+1) = (x^6-1)^2;
   %error_func(ite+1) = log10((x^2-1)^2);
   %deri_func(ite+1) = log10(4*x^3-4*x);
   ite = ite+1;
end

%% Plot the function
figure;
plot_length = max_it ;
plot_vec = 0:1:plot_length-1;
plot(plot_vec, error_func(1:plot_length), 'b-');
%plot(plot_vec, deri_func(1:plot_length), 'b-');
xlabel('iteration');
ylabel('f(x)');
legend('Fucntion');
title('3(v): GD with constant stepsize');
```

4.(iii)

```
%% Set up Problem
x = normrnd(0,1);
y = normrnd(0,1);
z = [x;y];
max_it = 100;

cur_func_error = (x*y-1)^2;
%cur_func_error = log10((x^2-1)^2);
error_func = zeros(max_it +1, 1);
error_func(1) = cur_func_error;
%deri_func = log10(4*x^3-4*x);
%error_deri = zeros(max_it +1, 1);
%error_deri(1) = deri_func;

%% run the algorithm
%alpha = 1/abs(12*x*y-4);
%alpha = 1/8;
alpha = 2/(2*y^2+abs(4*x*y-2)*2+2*x^2);
ite = 1;
while (ite <= max_it)
   x0 = x;
   x = x - alpha*(2*x*y^2-2*y);
   y = y - alpha*(2*x0^2*y-2*x0);
   error_func(ite+1) = (x*y-1)^2;
   %error_func(ite+1) = log10((x^2-1)^2);
   %deri_func(ite+1) = log10(4*x^3-4*x);
   ite = ite+1;
end

%% Plot the function
figure;
plot_length = max_it ;
plot_vec = 0:1:plot_length-1;
plot(plot_vec, error_func(1:plot_length), 'b-');
%plot(plot_vec, deri_func(1:plot_length), 'b-');
xlabel('iteration');
ylabel('f(x)');
legend('Fucntion');
title('3(v): GD with constant stepsize');
%% Plot the iterates
```