UIUC IE510 Applied Nonlinear Programming

Lecture 11: Lagrangian Multipliers Part b: Inequality Constraints

Ruoyu Sun

If $\nabla h_i(x^*)$'s are indep., then $\nabla f(x^*) + \sum_i \lambda_i^* \nabla h_i(x^*) = 0$ ①

Else. $\nabla h_i(x^*)$'s are dep. $0 \cdot \nabla f(x) + \sum_i \lambda_i^* \nabla h_i(x^*) = 0$. ②

Review Question for Lecture 11a

$\begin{cases} \min f(x), & x \in \mathbb{R}^n. \\ \text{s.t.} & h_i(x) = 0, \forall i \end{cases}$

- **Question 1**: Judge: For (smooth) equality constrained problems, at a local-min the gradient of the objective and gradients of constraints are always linearly dependent.

~~FALSE~~. TRUE.

Fritz-John. condition

Trap 1: we learned "regular" local-min last time.

Trap 2: always $\exists \lambda_0, \lambda_i$'s, s.t. $\lambda_0 \nabla f(x^*) + \sum_i \lambda_i h_i(x^*) = 0$. ③

- **Question 2**: Judge: For (smooth) equality constrained problems, at a regular local-min the Hessian of the objective is positive semi-definite on the null space defined by the gradients of the constraints.

FALSE.

"same" linear combination of objective & constraints

$\nabla^2 f(x^*) + \sum_i \lambda_i^* \nabla^2 h_i(x^*)$.

- **Question 3**: Judge: At a regular local-min, the Lagrangian multipliers are unique.

TRUE.

Summary of Last Lecture

- Optimality condition for

$$\min_x f(x), \text{ s.t. } h_i(x) = 0, i = 1, \ldots, m,$$

where f and h_i's are cts-differentiable.

- Lagrangian function $L(x; \lambda) =$ $f(x) + \sum_i \lambda_i h_i(x)$. $\cancel{(h_i(x))^2}$

- **Lagrangian multiplier theorem:** at a "regular" local-min x^* (gradients of constraints are linearly independent), we have
  - The gradient of the Lagrangian function is zero
    - $\nabla f(x^*)$ lies in span$\{\nabla h_i(x^*)\}$, with coefficients λ_i^*'s
    - $h_i(x) = 0, \forall i.$
  - $\nabla^2_{xx} L$ is positive semidefinite in the nullspace defined by all gradients of constraints
- If $\nabla^2_{xx} L$ is PD in that nullspace, then sufficient condition.

Summary of Last Lecture

- Optimality condition for

$$\min_x f(x), \text{ s.t. } h_i(x) = 0, i = 1, \dots, m,$$

  where f and h_i's are cts-differentiable.

- Lagrangian function $L(x; \lambda) =$.

- **Lagrangian multiplier theorem:** at a "regular" local-min x^* (gradients of constraints are linearly independent), we have
  - The gradient of the Lagrangian function is zero
    - ◇ $\nabla f(x^*)$ lies in span$\{\nabla h_i(x^*)\}$, with coefficients λ_i^*'s ($\nabla_x L = 0$).
    - ◇ $h_i(x) = 0, \forall i$. feasibility. ($\nabla_\lambda L = 0$).
  - $\nabla^2_{xx} L$ is positive semidefinite in the nullspace defined by all gradients of constraints

- If $\nabla^2_{xx} L$ is PD in that nullspace, then sufficient condition.

This Lecture

- Today: optimization with both equality/inequality constraints
- After the lecture, you should be able to
  - Write down KKT condition for inequality/equality constrained problems
  - Describe what is "complementary slackness"
  - Compute/verify optimal solutions for simple inequality constrained problems

Outline

Inequality Constrained Problem

$$
\begin{array}{ll}
\text{minimize} & f(x) \\
\text{subject to} & h_i(x) = 0,\ i = 1, \ldots, m \\
 & g_j(x) \le 0,\ j = 1, \ldots, r.
\end{array}
$$

where $f : R^n \mapsto R, h_i : R^n \mapsto R, g_j : R^n \mapsto R$ are continuously differentiable functions.

- If all $g_j = 0$, becomes equality constrained problems.
- If all h_i are affine ("linear"), g_j are convex, becomes optimization over convex set.
  - If further, f is convex, becomes a **convex problem**

$\|x\|^2 = 1.$

$$\begin{cases} \min f(x) \\ \text{s.t. convex fn} \le 0 \\ \quad\ \ \text{affine fn} = 0. \end{cases} \neq \begin{cases} \min f(x) \\ \text{s.t. convex fn} \neq 0 \\ \quad\ \ \text{convex fn} \le 0 \end{cases}$$

One Constraint Case: Change of 1st Order Condition

For simplicity, consider one inequality constraint:

$$\min_x f(x), \text{ s.t. } g(x) \le 0.$$

$g(x^*)=0; \quad g(x^*)<0.$

- Suppose x^* is a local minimum. There are two possibilities of $g(x^*)$:
  - $g(x^*) < 0$: in the interior, the constraint does NOT matter
  - $g(x^*) = 0$: treated (almost) as equality constraint
- Assuming $\nabla g(x^*) \neq 0$, then

$$\begin{cases} \nabla f(x^*) = 0, & \text{if } g(x^*) < 0; \\ \nabla f(x^*) + \mu^* \nabla g(x^*) = 0, & \text{if } g(x^*) = 0, \end{cases}$$

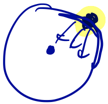
- Extra property: $\mu^* \ge 0$.

One Constraint Case: Change of 1st Order Condition

For simplicity, consider one inequality constraint:

$$\min_x f(x), \text{ s.t. } g(x) \le 0.$$

- Suppose x^* is a local minimum. There are two possibilities of $g(x^*)$:
  - ⋆ $g(x^*) < 0$: in the interior, the constraint does NOT matter
  - ⋆ $g(x^*) = 0$: treated (almost) as equality constraint



- Assuming $\nabla g(x^*) \neq 0$, then

$$\begin{cases} \nabla f(x^*) = 0, & \text{if } g(x^*) < 0; \\ \nabla f(x^*) + \mu^* \nabla g(x^*) = 0, & \text{if } g(x^*) = 0, \end{cases}$$

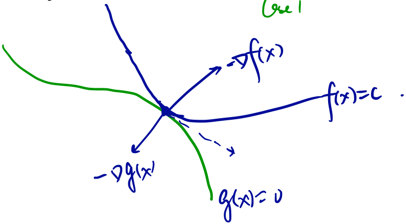
- Extra property: $\mu^* \ge 0$.

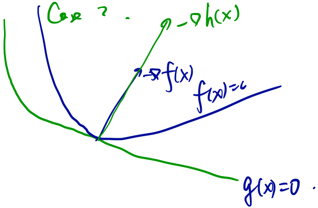
Nonnegativity of Lagrangian Multipliers: Intuition

$\cancel{\mu^2 - 1 \geq \mu,}$

We draw plots to illustrate why $\mu \geq 0$ is necessary.

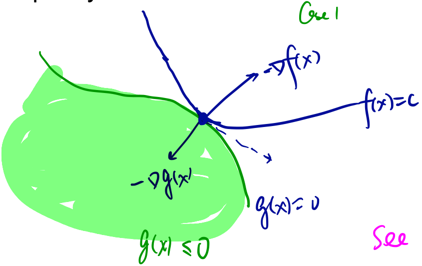
- Equality constraint case:

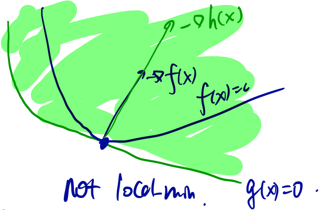




- Inequality constraint case:

Case 2.





See Example 1 for a concrete example.

One Constraint Case: Alternative Forms

Assuming $\nabla g(x^*) \neq 0$, then the local-min satisfies:

- **Form 0**:

$$\begin{cases} \nabla f(x^*) = 0, & \text{if } g(x^*) < 0; \\ \nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \text{for some } \mu^* \geq 0, & \text{if } g(x^*) = 0, \end{cases}$$

In-class exercise: form groups, discuss how to simplify it.

- **Form 1**:

$$f(x^*) + \mu^* \nabla g(x^*) \mathbb{I}(g(x^*) = 0) = 0, \text{ for some } \mu^* \geq 0.$$

Preferably in one line or two lines but simple.

generalizable to r constraints.

- **Form 2**:

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \quad \text{for some } \mu^* \geq 0,$$
$$\mu^* g(x^*) = 0. \qquad \text{[complementary slackness]}$$

- **Form 2b** (my choice) :

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0,$$
$$\mu^* \geq 0 \quad \perp \quad g(x^*) \leq 0.$$

One Constraint Case: Alternative Forms

Assuming $\nabla g(x^*) \neq 0$, then the local-min satisfies:

- **Form 0**:

$$\begin{cases} \nabla f(x^*) = 0, & \text{if } g(x^*) < 0; \\ \nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \text{for some } \mu^* \geq 0, & \text{if } g(x^*) = 0, \end{cases}$$

- **Form 1**:

$\text{Id}(\text{statement}) = \begin{cases} 1, & \text{true} \\ 0, & \text{false}. \end{cases}$

$$\nabla f(x^*) + \mu^* \nabla g(x^*) \text{Id}(g(x^*) = 0) = 0, \text{for some } \mu^* \geq 0.$$

- **Form 2**:

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \quad \text{for some } \mu^* \geq 0,$$

$$\mu^* g(x^*) = 0. \quad ① \quad \text{[complementary slackness]}$$

Assume ①. If $g(x^*) \neq 0$, then $\mu^* = 0$;
If $g(x^*) = 0$, then ① is meaningless.

- **Form 2b** (my choice) :

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0,$$

$$\mu^* \geq 0 \qquad g(x^*) \leq 0$$

require $\mu^* \geq 0$; if $\mu > 0$, there is "slack"
require $g(x^*) \leq 0$; if $g(x^*) < 0$, "slack".

Claim: Cannot have both "slacks".
Or: If one slack exists, the other disappear.
They are complementary.

One Constraint Case: Alternative Forms

Assuming $\nabla g(x^*) \neq 0$, then the local-min satisfies:

- **Form 0**:

$$\begin{cases} \nabla f(x^*) = 0, & \text{if } g(x^*) < 0; \\ \nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \text{for some } \mu^* \geq 0, & \text{if } g(x^*) = 0, \end{cases}$$

- **Form 1**:

$$f(x^*) + \mu^* \nabla g(x^*) \mathrm{Id}(g(x^*) = 0) = 0, \text{for some } \mu^* \geq 0.$$

- **Form 2**:

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0, \quad \text{for some } \mu^* \geq 0,$$
$$\mu^* g(x^*) = 0. \qquad \text{[complementary slackness]}$$

- **Form 2b** (my choice) :

$$\nabla f(x^*) + \mu^* \nabla g(x^*) = 0,$$
$$\mu^* \geq 0 \quad \perp \quad g(x^*) \leq 0.$$

Outline

First Order Condition in General

- **Form 1**: Assuming regularity of x^*, we have

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla h_i(x^*) + \sum_{j=1}^{r} \mu_j^* \nabla g_j(x^*) = 0,$$

$$u_j^* = 0, \quad \forall j \notin A(x^*).$$

$$\mu_j^* \geq 0; \quad \forall j.$$

where the set of active inequality constraints

$$A(x^*) = \{j \mid g_i(x^*) = 0\}.$$

"active" or "effective" or "important"

- **Form 2**: Assuming regularity of x^*, we have

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla h_i(x^*) + \sum_{j=1}^{r} \mu_j^* \nabla g_j(x^*) = 0,$$

$$\underbrace{\mu_j^* \geq 0}_{\text{extra property}} \quad \underbrace{\perp}_{\text{comp. slack.}} \quad \underbrace{g_j(x^*) \leq 0,}_{\text{feasibility}} \quad \forall\, j.$$

Karash-Kuhn-Tucker (KKT) Conditions

- x is regular iff $\nabla h_i(x)'s$ and $\nabla g_j(x)$ for active j's are linearly independent.
  $\{h_i(x)=0\}$. $\{g_j(x)=0.\}$.
- Lagrangian function $L(x;\lambda,\mu) = f(x) + \sum_i \lambda_i h_i(x) + \sum_j \mu_j g_j(x)$.
- Let x^* be a regular local minimum. Then there exist unique Lagrange multipliers $\lambda^* = (\lambda_1^*, ..., \lambda_m^*)$, $\mu^* = (\mu_1^*, ..., \mu_r^*)$, such that

$$\begin{aligned} \nabla_x L(x^*, \lambda^*, \mu^*) &= 0, \\ \mu_j^* &\geq 0, \ \ j = 1, ..., r; \\ \mu_j^* &= 0, \ \ \forall j \notin A(x^*). \end{aligned}$$

$\mu_j^* \geq 0 \perp g_j(x^*) \leq 0$.

- If $f,\ h_i's,$ and $g_j's$ are twice continuously differentiable,

$$y' \nabla_{xx}^2 L(x^*, \lambda^*, \mu^*) y \geq 0, \ \forall \ y \in V(x^*),$$

  where

$$V(x^*) = \{y \mid \nabla h_i(x^*)^T y = 0, \forall i;\ \nabla g_j(x^*)^T y = 0, \forall j \in A(x^*)\}.$$

- Similar sufficiency conditions, except that now it require strict complementarity [Prop 3.3.2]:

$$\mu_j^* > 0, \ \forall j \in A(x^*).$$

strict comp.

$(\mu_j, -g_j) = \cancel{(1,1)}$, $(1,0)$, $(0,1)$, $(0,0)$ | $(1,0)$, $(0,1)$, $\cancel{(0,0)}$

KKT Conditions in Human Language

- **Problem**: equality/inequality constrained (smooth) optimization
- **Assumption**: for a regular local-min x^*
  - ◇ Regular: gradients of all "active" constraints are linearly independent
- **1oC**: (a) Gradient of objective is spanned by the gradients of active constraints (equality and active inequality constraints); (b) Coefficients of active inequality constraints are non-negative. $\mu_j \geq 0$.
- **2oC**: The same linear combination of all Hessians is PSD in an orthogonal space (the space orthogonal to all gradients of active constraints).

KKT Conditions in Human Language (Langrangian version)

- Consider function

$$L(x;\lambda,\mu) = f(x) + \sum_i \lambda_i h_i(x) + \sum_j \mu_j g_j(x), \quad \mu_j \geq 0, \forall j.$$

- For a regular local-min of problem (P1), there exist unique Lagrangian multipliers such that
  - 1oC: $(x^*;\lambda^*,\mu^*)$ satisfies 1oC of L [exercise] $\mu_j \geq 0 + \nabla g_j(x) \leq 0.$
  - 2oC: Hessian of L is PSD in an orthogonal space (the space orthogonal to gradients of all active constraints)

KKT Condition for Linear Constraints

Consider the linearly constrained problem

$$\boxed{\min_{a_j' x \leq b_j,\ j=1,\ldots,r} f(x).}$$

- Remarkable property: No need for regularity.

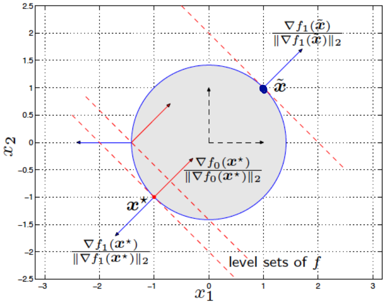
- Proposition: If x^* is a local-min, there exist $\mu^* = (\mu_1^*, ...\mu_r^*)$, such that

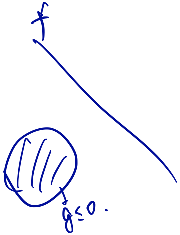
$$\nabla f(x^*) + \sum_{j=1}^{r} \mu_j^* a_j = 0,$$

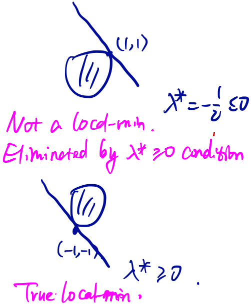
$$\mu_j^* \geq 0 \quad \perp \quad a_j' x^* \leq b_j,\ \forall j.$$

Outline

Normal Example 1







Consider the problem

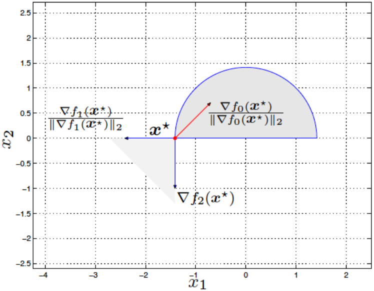
$$\underset{x \in \mathbb{R}^2}{\text{minimize}} \quad f_0(x) = x_1 + x_2$$
$$\text{subject to} \quad f_1(x) = x_1^2 + x_2^2 - 2 \leq 0.$$

This is a problem with a linear objective function $f(x)$ and one nonlinear inequality constraint $f_1(x) \leq 0$. At the solution $x^\star$, the gradient of the constraint $\nabla f_1(x^\star)$ is orthogonal to the level set of the function at x^*, and the following equality holds

$$\nabla f_0(x^\star) + \lambda^\star \nabla f_1(x^\star) = \mathbf{0},$$

for $\lambda^\star = \frac{1}{2} \geq 0$. Note that at the point $\tilde{x} = (1, 1)$, $\nabla f_0(\tilde{x}) + \lambda \nabla f_1(\tilde{x}) = \mathbf{0}$ holds as well, however $\lambda = -\frac{1}{2} \leq 0$.

Normal Example 2



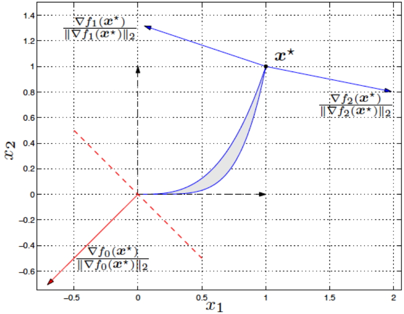
Consider the problem

$$\begin{aligned} \underset{\boldsymbol{x}\in\mathbb{R}^2}{\text{minimize}} \quad & f_0(\boldsymbol{x}) = x_1 + x_2 \\ \text{subject to} \quad & f_1(\boldsymbol{x}) = x_1^2 + x_2^2 - 2 \le 0, \\ & f_2(\boldsymbol{x}) = -x_2 \le 0. \end{aligned}$$

At the solution $\boldsymbol{x}^\star = (-\sqrt{2}, 0)$, $-\nabla f_0(\boldsymbol{x}^\star)$ belongs to the normal cone to the feasible set at point $\boldsymbol{x}^\star$, hence, there is $\boldsymbol{\lambda}^\star \ge \mathbf{0}$ that satisfies

$$\nabla f_0(\boldsymbol{x}^\star) + \lambda_1^\star \nabla f_1(\boldsymbol{x}^\star) + \lambda_2^\star \nabla f_2(\boldsymbol{x}^\star) = \mathbf{0}.$$

Normal Example 3

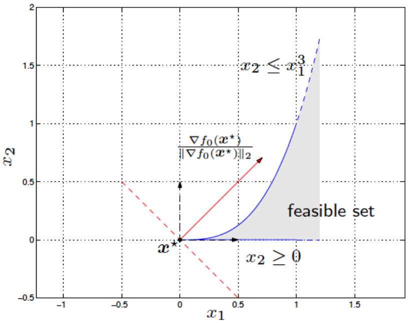


$$
\begin{aligned}
\underset{\boldsymbol{x}\in\mathbb{R}^2}{\text{maximize}} \quad & f_0(\boldsymbol{x}) = x_1 + x_2 \\
\text{subject to} \quad & f_1(\boldsymbol{x}) = x_2 - x_1^3 \le 0, \\
& f_2(\boldsymbol{x}) = x_1^5 - x_2 \le 0, \\
& f_3(\boldsymbol{x}) = -x_2 \le 0.
\end{aligned}
$$

Only the first two inequality constraints are active at the solution $\boldsymbol{x}^* = (1,1)$, which satisfies the KKT necessary conditions with $\lambda_1^\star = 3$, $\lambda_2^\star = 2$ and $\lambda_3^\star = 0$. This can be verified by solving the equation

$$
\begin{bmatrix} \nabla f_1(x_1) \\ \nabla f_2(x_1) \end{bmatrix}^T \begin{bmatrix} \lambda_1^\star \\ \lambda_2^\star \end{bmatrix} = -\nabla f(\boldsymbol{x}^\star) \Rightarrow \begin{bmatrix} -3 & 1 \\ 5 & -1 \end{bmatrix}^T \begin{bmatrix} \lambda_1^\star \\ \lambda_2^\star \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
$$

Degenerate Example 1



Consider the problem

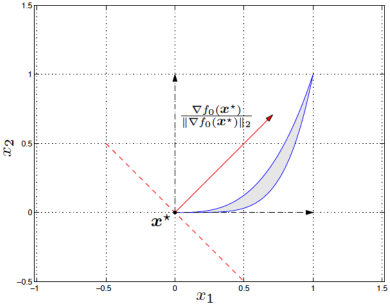
$$\begin{aligned} \underset{x \in \mathbb{R}^2}{\text{minimize}} \quad & f_0(x) = x_1 + x_2 \\ \text{subject to} \quad & f_1(x) = x_2 - x_1^3 \leq 0, \\ & f_2(x) = -x_2 \leq 0. \end{aligned}$$

This is a problem with a linear objective function $f_0(x)$, one nonlinear inequality constraint $f_1(x) \leq 0$ and one linear inequality constraint $f_2(x) \leq 0$. There are infinitely many feasible points, however, at $x^\star$ (where both inequality constraints are active), there is no $\lambda^\star$ satisfying

at locatni x^*.

$$\nabla f_0(x^\star) + \lambda_1^\star \nabla f_1(x^\star) + \lambda_2^\star \nabla f_2(x^\star) = \mathbf{0}.$$

Degenerate Example 2



Consider the problem

$$\begin{aligned} \underset{\boldsymbol{x}\in\mathbb{R}^2}{\text{minimize}} \quad & f_0(\boldsymbol{x}) = x_1 + x_2 \\ \text{subject to} \quad & f_1(\boldsymbol{x}) = x_2 - x_1^3 \leq 0, \\ & f_2(\boldsymbol{x}) = x_1^5 - x_2 \leq 0, \\ & f_3(\boldsymbol{x}) = -x_2 \leq 0. \end{aligned}$$

At the solution $\boldsymbol{x}^\star$, all inequality constraints are active and their gradients are co-linear. There is no $\boldsymbol{\lambda}^\star$ satisfying

$$\nabla f_0(\boldsymbol{x}^\star) + \lambda_1^\star \nabla f_1(\boldsymbol{x}^\star) + \lambda_2^\star \nabla f_2(\boldsymbol{x}^\star) + \lambda_3^\star \nabla f_3(\boldsymbol{x}^\star) = \mathbf{0}.$$

Outline

Proof Method 1: Penalty Approach

- Use equality-constraints result to obtain all the conditions except for $u_j^* \geq 0$ for $j \in A(x^*)$.
- Introduce the penalty functions $g_j^+(x) = \max\{0, g_j(x)\}, \ j = 1, ..., r$, and for $k = 1, 2, ...$, let x^k minimize

$$f(x) + \frac{k}{2}\|h(x)\|^2 + \frac{k}{2}\sum_{j=1}^{r}(g_j^+(x))^2 + \frac{1}{2}\|x - x^*\|^2$$

  over a neighborhood (closed sphere) of x^*.
- Using the same argument as for equality constraints,

$$\lambda_i^* = \lim_{k \Rightarrow \infty} k h_i(x^k), \quad i = 1, ..., m,$$

$$\mu_j^* = \lim_{k \Rightarrow \infty} k g_j^+(x^k), \quad j = 1, ..., r.$$

- Since $g_j^+(x^k) \geq 0$, we obtain $\mu_j^* \geq 0$ for all j.

Proof Method 2: Transform to Equality Case

- The original problem

$$
\begin{aligned}
(P1) \quad \min_x \quad & f(x) \\
\text{subject to} \quad & h_i(x) = 0,\ i = 1, \ldots, m \qquad (1) \\
& g_j(x) \le 0,\ j = 1, \ldots, r.
\end{aligned}
$$

- Consider an equivalent problem

$$
\begin{aligned}
(P2) \quad \min_{x,z} \quad & f(x) \\
\text{subject to} \quad & h_i(x) = 0,\ i = 1, \ldots, m \qquad (2) \\
& g_j(x) + z_j^2 = 0,\ j = 1, \ldots, r.
\end{aligned}
$$

- From 1oC for (P2), get 1oC for (P1) plus $\mu_j^* = 0, \forall j \notin \mathcal{A}(x^*)$.
- From 2oC for (P2), get 2oC for (P1) and

$$\mu_j \ge 0, \forall j.$$

- See details in Sec. 3.3.2.
Requires twice differentiable, so slightly less general than Penalty approach.

Tip: Transformations Between Problems

- Transform inequality constraints to equality constraints?

$g(x) \le 0 \iff g(x) + z^2 = 0$, for some z. (introduce additional variable)

- Transform equality constraints to inequality constraints?

$h(x) = 0 \iff \begin{cases} h(x) \ge 0 \\ h(x) \le 0. \end{cases}$ (increase # of constraints)

- Transform constraints to objective?

$\min_x \; 0 \;\; \text{s.t. } h(x) = 0 \iff \min_x \|h(x)\|^2$;

$\min_x f(x) \;\; \text{s.t. } h(x) = 0 \xleftrightarrow{\text{"almost"}} \min_x f(x) + \lambda \|h(x)\|^2$ for huge λ.

(not rigorous; but practically often fine).

- Transform objective to constraints?

$(P1) \begin{cases} \min_x f(x) \\ \text{s.t. } g(x) \le 0 \end{cases} \iff (P2) \; \min_{x,t} t \;\; \text{s.t. } f(x) \le t, \; g(x) \le 0,$ (simple objective)

- **Lesson**: constrained-opt "equivalent" to solving equations.

(P2) can be solved by bisection on t^*

$+ \; \min_x t^*, \;\; \text{s.t. } f(x) \le t^*, \; g(x) \le 0.$ (no objective)

Tip: Transformations Between Problems

- Transform inequality constraints to equality constraints?
- Transform equality constraints to inequality constraints?
- Transform constraints to objective?
- Transform objective to constraints?
- **Lesson**: constrained-opt “equivalent” to solving equations.

Side: Move Objective to Constraints

Consider a problem $\min_{x \in X} f(x)$.

How to move f to the constraints?

$$(P1) \begin{cases} \min_{x} & f(x) \\ \text{s.t.} & x \in X \end{cases} \iff (P2) \begin{array}{ll} \min_{x,t} & t \\ \text{s.t.} & f(x) \le t \\ & x \in X. \end{array} \quad \text{(simple objective)}$$

We show (P2) can be solved by bisection on $\hat{t}$ + $(P3)$ $\begin{array}{ll} \min_{x} & 0 \\ \text{s.t.} & f(x) \le \hat{t} \\ & x \in X. \end{array}$

Assume the optimal value of (P1) is $t^* \in \mathbb{R}$.

Claim If for some fixed $\hat{t}$, problem (P3) is feasible, then $t^* \le \hat{t}$; otherwise, $t^* > \hat{t}$.

Based on this claim, we can solve (P2) by solving a series of (P3) with changing $\hat{t}$, where $\hat{t}$ is updated by bisection.

Summary

In this lecture, we learned the following (think yourself before reading):

- KKT conditions (1st and 2nd order) for inequality/equally constrained problems

- Crucial in KKT: complementary slackness
  - $\mu_j \geq 0 \perp g_j(x) \leq 0.$

- Two proofs: penalty, transform to equality-constrained case
  - Side: equality $\leftrightarrow$ inequality constraints; constraints $\leftrightarrow$ objectives

Summary

In this lecture, we learned the following (think yourself before reading):

- KKT conditions (1st and 2nd order) for inequality/equally constrained problems
- Crucial in KKT: complementary slackness
  - $\mu_j \geq 0 \perp g_j(x) \leq 0.$
- Two proofs: penalty, transform to equality-constrained case
  - Side: equality $\leftrightarrow$ inequality constraints; constraints $\leftrightarrow$ objectives