IE531
HW#4
Tianqi Wu

Question 1

Suppose we have vertex v and w.
In steady state, we have $\pi_v p_{vw} = \pi_w p_{wv}$
Since the graph is connected and undirected, the sum of the number of neighbors of each vertex would be 2 times of the total number of edges.
Hence, $\Sigma_v d_v = 2card(E)$
Then, $\Sigma_v \pi_v = \Sigma_v \frac{d_v}{2card(E)} = \frac{1}{2card(E)} * \Sigma_v d_v = \frac{1}{2card(E)} * 2card(E) = 1$

By lemma4.3 from the book, since $\pi_v p_{vw} = \pi_w p_{wv}$ and $\Sigma_v \pi_v = 1$,

$\pi_v = \frac{d_v}{2card(E)}$ is the unique stationary probability that satisfies $\pi P = \pi$

Question 2

(a) Mark the (8X8) chessboards with integers so that the first row is labeled as 1,2,3…8 and second row is labeled as 9,10…16 and so on.
Then, since the knight moves in "L-shape", we can count the legal moves that each spot have, which is also the number of neighbors that each spot has. For example, spot (1,1) has 2 neighbors and (1,2) has 3 neighbors.

For each spot i, it has total number of N neighbor j and we can formulate the probability matrix by setting the P_{ij} of the matrix being $\frac{1}{N}$. Otherwise, if the spot is not a neighbor of i, set the probability to be 0. Continuing this process, we will get 64*64 probability matrix .

(b)

Irreducible: Yes, since there is always a path that let knight go from one spot to any spot. It means that every state communicates with any other state.

Aperiodic: No. Taking (1,1) as example, the knight can move back to the spot with only even number of moves. Hence, it is not aperiodic.

(c)

Since knight moves in “L” shape, we can derive the following table illustrating the number of possible legal moves in each spot of the (8X8) chessboard. The Knight is essentially taking a random-walk based on the number of legal-moves from its current position. if x = (i , j), then π(x) is proportional to the number N(x) of the possible next states of the square x, meaning π(x) = CN(x).

2	**3**	**4**	**4**	**4**	**4**	**3**	**2**
3	**4**	**6**	**6**	**6**	**6**	**4**	**3**
4	**6**	**8**	**8**	**8**	**8**	**6**	**4**
4	**6**	**8**	**8**	**8**	**8**	**6**	**4**
4	**6**	**8**	**8**	**8**	**8**	**6**	**4**
4	**6**	**8**	**8**	**8**	**8**	**6**	**4**
3	**4**	**6**	**6**	**6**	**6**	**4**	**3**
2	**3**	**4**	**4**	**4**	**4**	**3**	**2**

$$2 \times 4 + 3 \times 8 + 4 \times 20 + 6 \times 16 + 8 \times 16 = 336$$

In total, there are 336 legal moves.
The probability matrix can be derived by dividing the number of possible legal moves by the total number of moves (336).
For example, π(1,1) is 2/336, π(1,2) is 3/336, π(2,2) is 4/336.

(d)

Looking at the stationary distribution, it is clear that the knight is most likely to be spotted in the center 16 spots (π = 8/336). It it least likely to be spotted in the corner 4 spots (π = 2/336).