5.3 - NumPy Application: Random Walks

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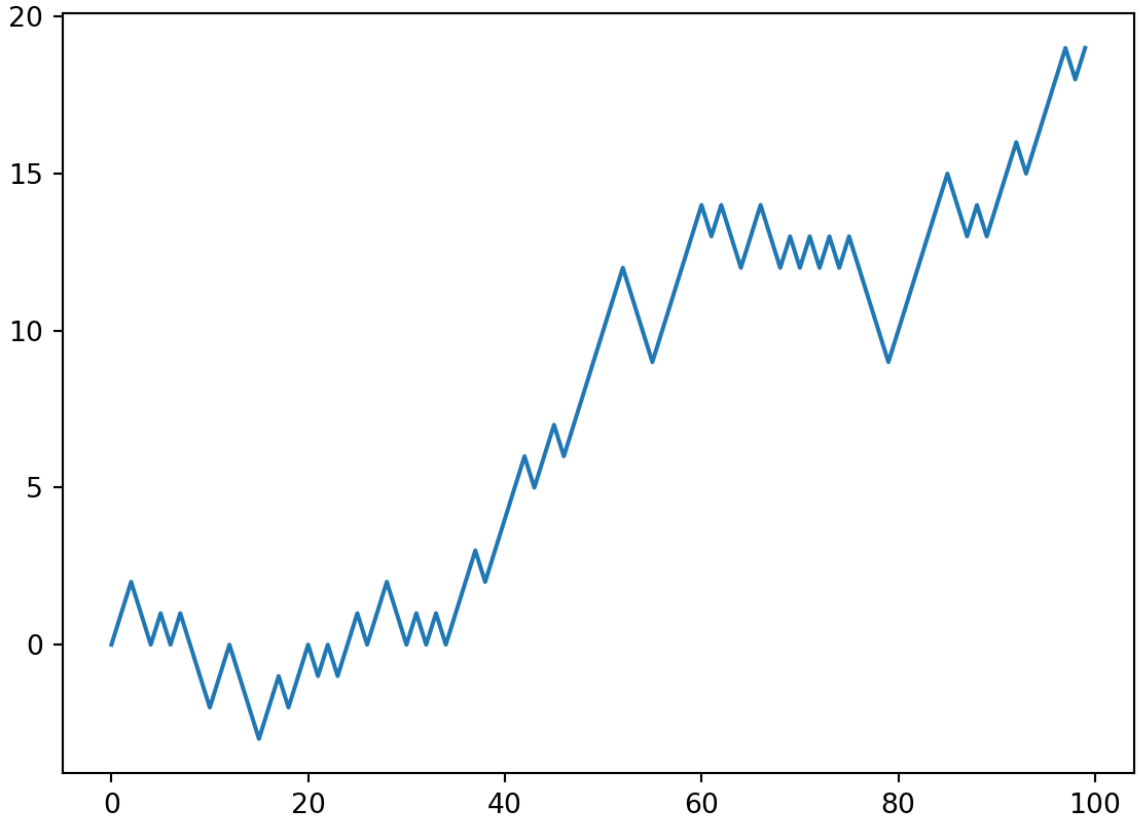
1. What are Random Walks?

- Here (https://en.wikipedia.org/wiki/Random_walk) is the Wikipedia page on Random Walk!
- In short, a random walk is a stochastic process.
- In this example, we will consider a simple random walk **starting at 0 with steps of 1 and -1 occuring with equal probability**.

```
import random

position = 0
walk = [position]
steps = 1000

for i in range(steps):
  step = 1 if random.randint(0, 1) else -1
  position += step
  walk.append(position)
```

- Now let's plot our walk!

```
import matplotlib.pyplot as plt

plt.plot(walk[:100])
```



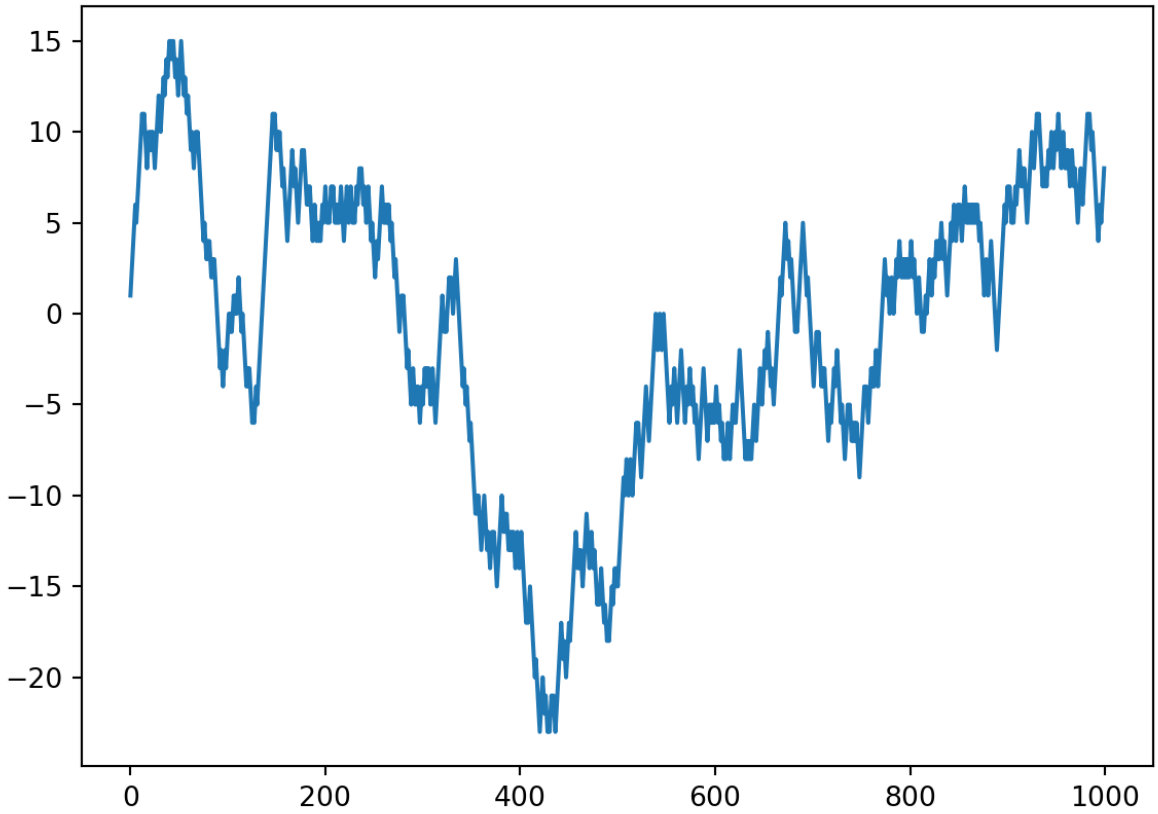
2. Using NumPy for Simulating Random Walks

- `walk` is simply the cumulative sum of the random steps and could be evaluated as an array expression.

```
import numpy as np
np.random.seed(430)

nsteps = 1000
draws = np.random.randint(0, 2, size=nsteps)
steps = np.where(draws > 0, 1, -1)
walk = steps.cumsum()
```

- **Note**: we have to use `np.random.randint()` to generate an array of random integers. `random.randint()` only returns 1 number at the time.
- Since we use 2 different random functions, even if we set the same seed, we won't get the same result (because the mechanisms inside the functions work differently).
- Here is the plot of new generated `walk` :

```
plt.plot(walk)
```



- From this we can begin to extract statistics like the minimum and maximum value along the walk's trajectory:

```
walk.min()
```

```
## -23
```

```
walk.max()
```

```
## 15
```

- *First crossing time* = the step at which the random walk reaches a particular value. This is a more advanced statistic.
- Let's say we want to know how long it takes the random walk to get at least 10 steps away from the starting point in either direction.

```
(np.abs(walk) >= 10).argmax()
```

```
## 11
```

- `np.abs(walk) >= 10` gives us a boolean array indicating where the walk has reached or exceeded 10.
- `argmax()` returns the *first* index of the maximum value of the array. In this case, it returns the index of the first `True` value.

3. Simulating Many Random Walks at Once

- Say our goal is to generate 5000 random walks at once! How do we do that?

```
nwalks = 5000
nsteps = 1000
draws = np.random.randint(0, 2, size=(nwalks, nsteps))
steps = np.where(draws > 0, 1, -1)
walks = steps.cumsum(1) # sum across the columns
walks
```

```
## array([[ -1,   0,   1, ..., -28, -27, -26],
##        [ -1,   0,  -1, ...,  54,  55,  56],
##        [ -1,  -2,  -3, ...,   2,   3,   2],
##        ...,
##        [ -1,   0,  -1, ..., -22, -21, -22],
##        [  1,   2,   1, ...,  48,  49,  48],
##        [  1,   0,   1, ..., -38, -39, -38]])
```

- Out of these walks, let's compute the minimum crossing time to 30 or -30!
- Now, note that not all of 5000 walks reach 30.

```
hits30 = (np.abs(walks) >= 30).any(1)
hits30
```

```
## array([ True,  True, False, ...,  True,  True,  True])
```

- `any(1)` returns `True` if at least 1 of the values of the row is `True`.

```
# number of walks that hit 30 or -30
hits30.sum()
```

```
## 3336
```

```
# estimate for probability a walk hitting 30 in either direction
hits30.sum()/nwalks
```

```
## 0.6672
```

- Use the boolean array `hist30` to select only the walks that actually hit 30 or -30! Then use `argmax()` across axis 1 (the column) to get the crossing times:

```
crossing_times = (np.abs(walks[hits30]) >= 30).argmax(1)
crossing_times.mean()
```

```
## 511.1636690647482
```

This lecture note is modified from Chapter 4 of Wes McKinney's Python for Data Analysis 2nd Ed (https://www.oreilly.com/library/view/python-for-data/9781491957653/).