STAT 510 Mathematical Statistics I, Spring 2020

Homework 3: Due at 5pm, Thu, March 5, 2020 Put your solution in the drop box STAT 510 in the Illini Hall

1. The matrix exponential is a function exp : $\mathbb{R}^{p \times p} \to \mathbb{R}^{p \times p}$ over all p-by-p square matrices defined as

$$\exp(A) = \sum_{k=0}^{\infty} \frac{A^k}{k!} = I_p + A + \frac{1}{2}A^2 + \frac{1}{6}A^3 + \cdots, \quad \text{for all } A \in \mathbb{R}^{p \times p}.$$

- a) Let the spectral decomposition of symmetric matrix $X \in \mathbb{R}^{p \times p}$ be $\Gamma \Lambda \Gamma^T$, where Λ is diagonal, and Γ is orthogonal. What is the spectral decomposition of $\exp(X)$?
- b) Suppose X is symmetric. Show that $\exp(-X)$ is the inverse of $\exp(X)$.
- 2. Exercise 7.8.10
- 3. Exercise 7.8.12
- 4. Exercise 6.8.17
- 5. Exercise 7.8.17