STAT 510 Mathematical Statistics I, Spring 2020

Homework 2: Due at 5pm, Thu, Feb 20, 2020

Put your solution in the drop box STAT 510 in the Illini Hall

1. Let $\underline{X} = (X_1, X_2)^T$ be a two-dim random vector, and (R, Θ) be its polar coordinates, i.e. $X_1 = R\cos(\Theta)$ and $X_2 = R\sin(\Theta)$. Show that $\underline{X}$ is spherically symmetric if and only if R and Θ are independent, and $\Theta \sim$Uniform$(0, 2\pi)$. (In the class we proved one direction, in this problem you are asked to prove the converse).

2. Exercise 5.6.9

3. Exercise 5.6.16

4. Exercise 5.6.17

5. Suppose female customers come to a shop according to a Poisson process of rate λ per hour and male customers according to an independent Poisson process of rate μ per hour.

   a) What is the conditional distribution of the total number of female customers coming to the shop in the first hour given that the total number of customers coming to the shop in the first hour is n?

   b) What is the probability that the first customer coming to the shop is female?