

STAT 510 Mathematical Statistics I, Spring 2020

Homework 1: Due at 5pm, Thu, Feb 6, 2020

Put your solution in the drop box STAT 510 in the Illini Hall

1. Exercise 2.7.19
2. Exercise 2.7.20
3. The double exponential random variable X has space \mathbb{R} and pdf $f(x) = (1/2)e^{-|x|}$.
 - (a) Show that the moment generating function (mgf) of a double exponential is $M_X(t) = 1/(1 - t^2)$. [Break the integral into two parts according to the sign of x .] For which t is it finite?
 - (b) Suppose U and V are independent Exponential(1), and let $Y = U - V$. Find the mgf of Y . What is the distribution of Y ?
 - (c) The mean of a double exponential is 0 and the variance is 2. Suppose X_1, \dots, X_n are independent and identically distributed double exponentials, and let W_n be the standardized mean, $W_n = \frac{\bar{X}}{\sqrt{2/n}}$. Show that the mgf of W_n is $M_n(t) = \frac{1}{(1-t^2/(2n))^n}$.
 - (d) What is the limit of $M_n(t)$ as $n \rightarrow \infty$? What distribution corresponds to the limit of $M_n(t)$?
4. Suppose $Z \sim N(0, 1)$ and $U \sim \text{Uniform}(0, 1)$, and Z and U are independent. Let $\underline{Y} = (Y_1, Y_2) = g(Z, U)$ be given by

$$Y_1 = \frac{Z}{U} \text{ and } Y_2 = U.$$

Y_1 is said to have the “slash” distribution.

- (a) What is the space of \underline{Y} ?
- (b) Find $g^{-1}(\underline{y})$.
- (c) Find the pdf of \underline{Y} ?
- (d) Are Y_1 and Y_2 independent? Why or why not?
- (e) Show that the marginal pdf of Y_1 is

$$f_1(y_1) = c \frac{1 - e^{-y_1^2/2}}{y_1^2}$$

What is the constant c ?