STAT 571 — Section A1 — Fall 2018

Homework 6

1. [2 pts] For a both-sides model with (unweighted) least squares estimator $\widehat{\boldsymbol{\beta}}_{LS}$, consider the predicted values matrix $\widehat{\boldsymbol{Y}}_{LS} = \boldsymbol{x}\widehat{\boldsymbol{\beta}}_{LS}\boldsymbol{z}'$. Express it in terms of $\boldsymbol{Y}$ and two different projection matrices. What are the projection matrices?

2. [2 pts] MSOS, Exercise 6.6.3.

3. [9 pts] Read MSOS, Exercise 6.6.12. Do *not* answer the parts of that exercise, but instead answer the following:

   (a) Compute the matrix $\boldsymbol{C}_{\boldsymbol{x}}$. (b) Compute the least squares estimate of $\boldsymbol{\beta}$. (c) Compute the matrix $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{z}}$. (d) Compute the standard errors of the elements of your least squares estimate. (e) Compute the t-statistics for the elements of $\boldsymbol{\beta}$ (testing against zero). (f) Which elements of $\boldsymbol{\beta}$ are significantly different from zero? Use a threshold of 2 for $|t|$. (g) Based on your results, discuss whether there is any difference between the grade profiles of the women and the men.

4. [9 pts] Data file `coastalcities.csv` contains (in comma-separated values format) the latitudes (`Latitude`) and daily mean temperatures (°C) for each month of the year, in order (`Jan`, ..., `Dec`), for 31 different coastal cities around the world. Read the file into R using `read.csv("coastalcities.csv")`.

   Let response matrix $\boldsymbol{Y}$ contain the daily mean temperatures (with the months ordered `Jan` through `Dec`). Consider a both-sides model with

   - $\boldsymbol{x}$ matrix being the 31×3 $\boldsymbol{Q}$ matrix in a QR factorization of a *quadratic* polynomial matrix in latitude (see Homework 5, Problem 7).
   - $\boldsymbol{z}$ matrix being a 12×3 (first-order) trigonometric matrix, similar to (but not quite the same as) that described in MSOS expression (4.26).

   (a) Briefly describe what each of the parameters in $\boldsymbol{\beta}$ represents. (b) Compute $\widehat{\boldsymbol{\beta}}$.
   (c) Compute the standard errors of the elements of $\widehat{\boldsymbol{\beta}}$. (d) Which $\boldsymbol{\beta}$ elements have t-statistics that exceed 2 in absolute value? (e) Based on the results in part (d), draw conclusions.

5. [4 pts] Suppose $\boldsymbol{\beta} = (\beta_{ij})$ is a 3×5 matrix, and consider the null hypothesis

$$H_0: \quad \begin{pmatrix} \beta_{12} + \beta_{22}x_1 + \beta_{32}x_1^2 & \beta_{15} + \beta_{25}x_1 + \beta_{35}x_1^2 \\ \beta_{12} + \beta_{22}x_2 + \beta_{32}x_2^2 & \beta_{15} + \beta_{25}x_2 + \beta_{35}x_2^2 \end{pmatrix} = \boldsymbol{0}$$

   where x_1 and x_2 are known constants. Display matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ such that this null hypothesis is equivalent to

$$H_0: \quad \boldsymbol{C}\boldsymbol{\beta}\boldsymbol{D}' = \boldsymbol{0}$$

6. [4 pts] MSOS, Exercise 7.6.3. [Hint: What is the distribution of $\boldsymbol{U} = (\boldsymbol{G}')^{-1}\widehat{\boldsymbol{\beta}}^*$, where $\boldsymbol{G}$ is the upper-triangular Cholesky factor of $\boldsymbol{C}_{\boldsymbol{x}}^*$?]