Computational science and engineering: Homework #1

Due on November 22, 2020 at 11:59pm

Professor Qifeng Liao

Tianyuan Wu
63305667

Problem 1

Show the Gibbs phenomenon

Solution

In this problem, I use `python` to show the fourier series of $step(x)$ in range $[0, 1]$. We know that

$$s(x) = \sum_{k=1}^{\infty} \frac{2}{\pi} \cdot h_k \cdot sin(kx)$$

where

$$h_k = \begin{cases} 0, & \text{k is even} \\ \frac{2}{k}, & \text{k is odd} \end{cases}$$

and for delta function ($\delta(x)$):

$$\delta(x) = \sum_{k=1}^{\infty} \frac{1}{\pi} \cdot sin(kx)$$

The following diagram shows the fourier sums when $k = 1$, $k = 10$, $k = 100$ and $k = 500$ of step function and delta function. We can observe that when $x = 0$ and $x = \pi$, the Fourier series has large oscillations near the jump (Gibbs phenomenon).

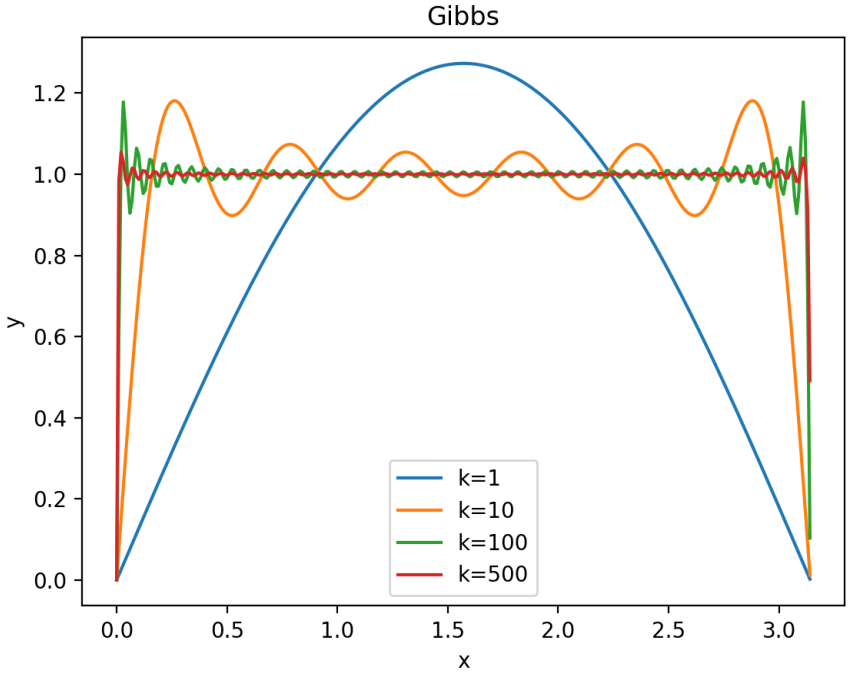

Figure 1: Gibbs phenomenon - step function

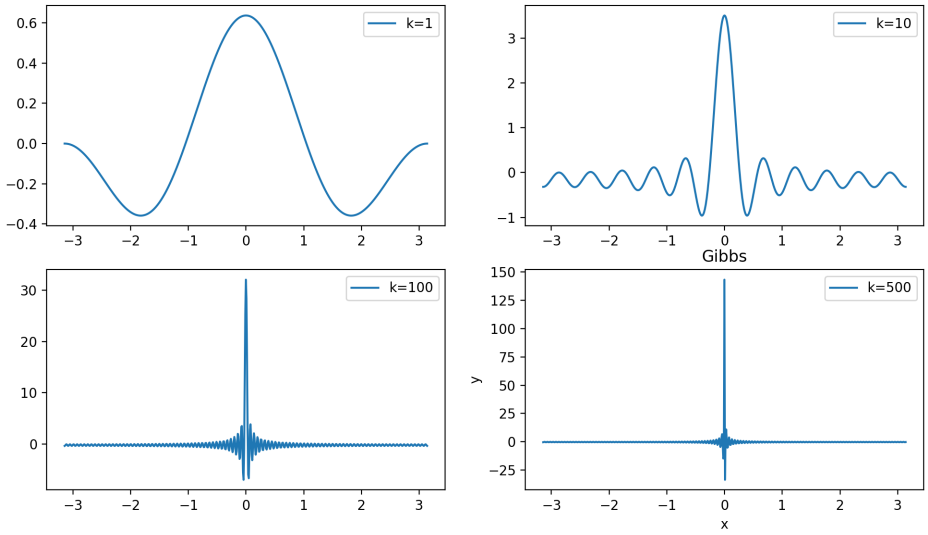

Figure 2: Gibbs phenomenon - delta function

Problem 2

Problem 4.1 on textbook

Solution

(a) $f(x) = sin^3(x)$

$$a_0 = \frac{2}{\pi}\int_{-\pi}^{\pi} 111$$

(b) $f(x) = |sin(x)|$

(c) $f(x) = x$

(d) $f(x) = e^x$, complex form