Reinforcement Learning: FinalProject

Due on July 1st, 2020 at 11:59pm

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Problem 1

Solution
在此课程中，我学习到了许多与强化学习相关的算法，并加深了对基础数学知识的了解。总体来说，我认为此课程基本满足了我对强化学习课程的期望，但还有些部分还有改进的空间。比如，我希望了解更多与深度强化学习相关的内容，比如 DQN、PPO 等最近流行的算法。此外，我认为作业可以少一些重复性的复现类的内容，多一些和前沿内容接轨的内容，或数学推导类的内容。

Problem 2

Solution
The code implementation of the problem is in "problem2.ipynb"

a) MCMC method
First, we analysis the problem theoretically. The maximum cut problem is quite similar like the independent set problem, and we can solve it by similar ways. we have 2 sets A and B, the size of the cut (A, B) is the number of edges that crosses A and B. The basic idea is, use a binary sequence to represent the choice of . $s[i] = 0$ means choose vertex i in A, and $s[i] = 1$ means choose it in B. Then, construct a irreducible Markov Chain, each sate X_i represent a binary sequence, and in each iteration, we filp a bit randomly, and go to the new state.
Then, the stationary distribution π of this chain is an uniform distribution. Then, we can modify the chain to make it concentrate on states with larger size of the cut (i.e. larger number of crossing edges from A to B), by MH method. That is, filp a coin with $p_H = e^{\beta(V_{new} - V_{curr})}$, where $V_{s_i} = \#\ of\ 1s\ in\ s_i$. Then if the coin landing head, accept the proposal, otherwise reject the proposal.
That idea can be described in details as:

1) Label all vertices by a number $0, 1, \ldots, n$
2) Init the sequence to $s_i = (000\ldots0)_n$, 0 means the vertex is in the set A, and 1 means it's in set B.
3) Generate a random number $k = [0, n-1]$, and filp the bit at index k, to get the new state s_{new}.
4) Let $V_{s_i} = \text{cutSize}(A, B)$, filp a coin with probability of landing head $p = e^{\beta(V_{new} - V_{curr})}$
5) If the coin landing head, accept the proposal, otherwise reject the proposal.
6) Jump to step (3)

We do simulation 10^4 steps, and we can observe the maximum cut size is 24. we run it several times, and some of the simulation results are shown below:

```
========== MCMC method ==========
The maximum cut is:
S1 = {1,3,4,7,9,11,14,15,18,20},
S2 = {2,5,6,8,10,12,13,16,17,19},
with reward = 24
```

The result is the same as the optimal solution (size=24).

b) Cross-entropy method

The basic idea is quite simple. Same as the MCMC method, we still use a binary sequence to represent the selection of vertices. First, we generate N episodes. In each episode, for each vertex, we decide it in A or B with equal probability (i.e. for each sequence s, $\Pr[s[i] = 0] = \Pr[s[i] = 1] = 0.5$). Then, calculate the cut size of each episode, and select $k = N/2$ elite samples in these N samples. Then, update the probability of sampling each vertex by the CEM equation:

$$\hat{p}_{i,j} = \frac{\sum_{j=0}^{N} I\{S(X_i) \geq \hat{\gamma}_t\} I\{X_{i,j} = 1\}}{\sum_{j=0}^{N} I\{S(X_i) \geq \hat{\gamma}_t\}}$$

For example, if we choose 4 elite samples, 3 of them choosed vertex i in set A, and only 1 of them choosed vertex i in B, then we will update

$$\Pr[\text{choose vertex i in A}] = \frac{3}{4}$$

1) Label all vertices by a number $0, 1, \ldots, n$
2) Init the sequence to $s_i = (000\ldots0)_n$, 0 means the vertex is in the set A, and 1 means it's in set B.
3) Init the probability distribution $\mathbf{p} = [p_1, p_2, \ldots, p_n]$
4) Generate N samples from the probability distribution $\mathbf{p}$
5) Select k elite samples from N samples
6) Use these elite samples to update $\mathbf{p}$
7) Goto (4)

We do simulation 100 steps, which each step choose 500 samples, and we can observe the maximum cut size is 24. we run it several times, and some of the simulation results are shown below:

```
==========Cross Entropy method==========
The maximum cut is:
S1 = {1,3,4,7,10,11,14,17,18,20},
S2 = {2,5,6,8,9,12,13,15,16,19},
with reward = 24
```

The result is the same as the optimal solution (size=24).

c) MCTS method

Similarly, we still use a binary sequence to represent the the selection of vertices. In initial case (root of the tree), all nodes are in the same set A (the sequence is (000...00)). And each action is, "filp" a bit in the sequence (choose some vertex and put it in the other set), the value of a node is $V_i = \text{cutSize}(A, B)$. Then we follows the MCTS algorithm with UCB method (i.e. when choose a children, we follows the following equation):

$$\text{BestChildren of v} = argmax_{v' \in children(v)} \frac{Q(v')}{N(v')} + c\sqrt{\frac{2\ln N(v)}{N(v')}}$$

where $Q(v)$ is the value of the node, $N(v)$ is the visit count of v. We do simulation 5000 steps, and we can observe the maximum cut size is 24. we run it several times, and some of the simulation results are shown below:

```
========== MCTS method ==========
The maximum cut is:
S1 = {1,3,6,8,9,10,14,17,18,20},
S2 = {2,4,5,7,11,12,13,15,16,19},
with reward = 24
```

The result is the same as the optimal solution (size=24).

d) Discussion & Comparison
In this problem, MCMC method without MH method will get a uniform distribution of all cuts, which may hard to find the optimal one. Suppose the graph is large, and the number of max cut is small (e.g. only one max cut), MCMC method need to run a large number of steps until find the max cut. If we use MH method in MCMC, then we have higher probability to visit the larger cuts, which may mitigate this problem. The advantage is, in each iteration, MCMC method is very fast. Unlike CEM or MCTS, in this problem, MCMC method only need to filp a bit and go to the next state in each iteration, which is faster than other methods.
CEM method is based on importance sampling. When we use CEM method, we need to consider the equilibrium between number of samples in each iteration (N) and the number of iterations. If N is small, we may need large number of iterations until converge, but if N is large, the calculation time in each iteration will be longer. In conclusion, CEM is a importance sampling based method, which is better than uniform sampling MCMC method. But we need a suitable N and proportion p of elite samples to make it converge faster.
MCTS is based on tree traversal. In each iteration, we go down the tree from root to the leaf. Each time we will choose the children with best value calculated by UCB algorithm. If we visit a node that we never visited, we will do a roll-out, otherwise we will expand it. This method is suitable for problems with large number of state spaces, and will get the optimal solution with high probability if we iterates enough time, which is much better than MCMC, but each iteration of MCTS is much slower than MCMC.

Problem 3

Solution

1) **Human-level control through deep reinforcement learning, by Mnih et al, Nature, 2015**
Main contributions: This article presents the Deep Q network, which is now widely used various subjets of deep reinforcement learning. This is a single algorithm that would be used in a wide range of challenging tasks. The main idea of Deep Q network is to combine the deep neural networks and the reinforcement learning algorithms. That is, use the deep neural networks to represent the Q value (state-action value) of some state $Q^*(s,a)$. We know that in traditional reinforcement learning algorithms, the target is to maximize the total reward, which is a quite difficult nonlinear problem:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots | s_t = s, a_t = a, \pi]$$

In this article, the author use deep convolutional networks to approximate $Q^*(s,a)$. That is, use $Q(s,a;\theta_i)$ to represent the Q value at (s,a), and θ_i is the parameters of the DQN at iteration i.

Then, use the observed samples to calculate the loss and update the network, the loss is shown as:

$$L_i(\theta_i) = \mathbb{E}_{(s,r,a,s')}[(r + \gamma \max_{a'} Q(s, a; \theta_i^-) - Q(s, a; \theta_i))^2]$$

Also, the author tests the performance of DQN on many classical games, and more than a half of them are at or above human level.

What is surprising is, the input is only the pixels which is the same as what's human seen, and the CNN can almost do the prediction as well as what's human did. That shows that CNNs can not only be used on fileds such as computer vision or image processing, but can also be used in more general problems.

What's difficult is, DQN is only suitable when the state space is discrete, and the performance of DQN is not stable (we can observe that the variance of avg. scores shown in Fig. 2(a) and 2(b) is quite high).

In my opinion, the experiments is convincing. Because the author tested the performance on various of games, most but not all of them are quite good. Also, the author offers the code, and I actually run part of them on my computer, the results are similar as which were shown in the paper.

Related works: The DQN is applied in multi-agent game StarCraft II: "Grandmaster level in StarCraft II using multi-agent reinforcement learning, Oriol Vinyals, Nature 2019".

2) **Mastering the game of Go with deep neural networks and tree search, by Silver et al, Nature 2016**

Main contribution: This article presents the AlphaGo, a new approach to computer Go that uses value networks to evaluate board positions and policy networks to select moves. The main difference between this algorithm and other algorithms is, AlphaGo not only learn from human expert games, but also learn from self-plays. The authors used MCTS method with value network and policy network to evaluate the optimal action. They first use supervised learning (SL) method to train policy 13-layered network p_π directly from human expert games, which uses stochastic gradient descent to maximize the likelihood of human move state-action pair (s, a). Then, they trained an reinfoecement learning(RL) policy network p_ρ, to improve the performance of SL policy network by optimizing the final outcomes of self-play. Finally, a value network v_θ is trained to predict the winner of the game. In the MCTS search process, each edge of the tree represents a state action pair (s, a), and for each time, choose action by:

$$a_t = arg\max_a(Q(s_t, a) + u(s_t, a))$$

where $u(s_t, a) \propto \frac{P(s,a)}{1+N(s,a)}$ that is proportional to the prior probability but decays with repeated visits to encourage exploration. Also, the author tested the performance of AlphaGo, it achieved 99.8% winning rate of other Go programs, and reached the expert level among human.

What is surprising is, they first trained a SL policy network by human experiments, then use RL methods to improve this network which can significantly reduce the training time and get better performance. This method can not only let the agent learn from the self-play experiments, but also bring human knowledges to the agent.

The difficultity is, this method highly depends on the human experts' game knowledge, and the training and evaluation speed may need a large amount of computing resources.

I think the experiments is convincing, for the article provides enough information to prove the conclusion, and the performance of AlphaGo has already been proved by winning many Go competitions between AI and human experts.

Problem 3 continued on next page...

Related works: This is an article about computational chemistry, they uses chemists' knowledge and RL methods to predict and invent chemical reactions: "Modelling Chemical Reasoning to Predict and Invent Reactions, Marwin H. S. Segler and Mark P. Waller, Chem. Eur. J. 2017, 23, 6118 –6128"

3) **Mastering the game of Go without human knowledge, by Silver et al, Nature 2017**
Main contributions: This article presentes the AlphaGo Zero, a Go program without human knowledge. The neural networks of AlphaGo Zero is trained only from games of self-play by a reinforcement learning algorithm. For each board state s, the neural network f_θ is able to give a probability vector p, and a value v. $p_a = \Pr[a|s]$ is the probability of taking action a at state s. Same as AlphaGo, AlphaGo Zero still used MCTS method. The MCTS guided by the policy network $f_\theta(s)$ to get the probabilities of policy π (Use MCTS as a powerful policy improvement operator). Also, use self-play with search as a policy evaluation operator. That is, do the following procedure repeatedly,
(1). Update the parameters of network to make $(p, v) = f_\theta(s)$ more close to the winner's policy and the improved search probability
(2). Use the new parameters θ^- to make the search stronger.
The purpose of the neural network is to minimize the difference (error) between what it predicted (v) and the winner's history z, and to maximize the similarity between the move probabilities p given by the network and the search probabilities π. So, the loss is:

$$L = (z - v)^2 - \pi^T \log p + c||\theta||^2$$

Where $c||\theta||^2$ is the L2 weight regularization to prevent overfitting.
Then, the author tested the performance of AlphaGo Zero, and when the training time is enough, the performance finally be better than AlphaGo Lee and AlphaGo Master.
What's surprising is, the MCTS and neural networks can be combined together, and "help" each other to improve the performance. The evaluation results of AlphaGo Zero is better than AlphaGo is also a surprising result. That means the human experts' game knowledge may not be always correct, and AI may learning something new by self-play. What's difficult is, the training process without human knowledge will take longer time, and need more computing resources.
I think the experiments is convincing, for vaild evidents are given to prove the result of training process. Also, AlphaGo Zero is also proved by winning many worldwide Go competitions.
Related worls: This is also an article about computational chemistry, they use MCTS and similar training process, and it aims to help chemists with organic syntheses: "Planning chemical syntheses with deep neural networks and symbolic AI, Segler, Marwin H. S,Preuss, Mike,Waller, Mark P, Nature 2018"

4) **A general reinforcement learning algorithm that masters chess, shogi, and Go through self-play, by Silver et al, Science 2018**
Main contributions: This article presents AlphaZero, a general RL algorithm for chess, shogi, and Go. In earlier programs for chess-like games, a large amount of domain-specific human knowledge is needed in order to make the optimal decision. But AlphaZero used a self-play based RL algorithm, which means it can "learn" the game without human knowledge. Instead of using handcrafted evaluation functions and weights, AlphaZero uses a deep neural network $f_\theta(s)$ to give the values v and probability vector p indicates the probabilities of different moves.

$$(\mathbf{p}, v) = f_\theta, \text{where } \mathbf{p} = \Pr[a|s]$$

Same as AlphaGo Zero mentioned above, AlphaZero also uses a general MCTS method. In each iteration, it searches from root until a leaf node with high probability to move to high-value nodes (exploitation), and low probability to move to nodes with low visit count (exploration). Also the same as AlphaGo Zero, combine neural network and MCTS together to "help" each other to improve the performance. So the loss function of the neural network is also defined as $L = (z-v)^2 - \pi^T \log p + c||\theta||^2$. What different with AlphaGo Zero is, AlphaZero simply maintains a single neural network that is updated continually rather than waiting for an iteration to complete. AlphaZero is also tested on various chess-like games, and has superhuman performances on chess, shogi, and Go.

What's surprising is, almost all chess-like games can be solved by the MCTS + neural network moethod, I've also seen that such approachs used in chemistry, computer vision and many different fileds. That means the self-play based RL algorithm can work quite well when the state space is large and discrete. And the difficulties is also very clear, these methods (without human knowledge) needs more time to train and more computing resources to train and evaluate through it's more creative.

I also think that the results are convincing. For Go, chess and shogi are similar chess-like games, through the rules are different. They all have large and discrete state space, the agent's action are also similar. Due to AlphaGo Zero works quite well, this algorithm works also well on other similar games is convinced.

Related works: This is an article about solving the Rubik's cube game with similar approach. "Solving the Rubik's Cube Without Human Knowledge, McAleer, Stephen,Agostinelli, Forest,Shmakov, Alexander, 2018"

5) **DeepStack: Expert-level artificial intelligence in heads-up no-limit poker, by Moravcik et al, Science, 2017**

Main contributions: This article presents DeepStack, a new algorithm to solve problems with sequential imperfect information (i.e. sequential problems/games with asymmetric information) such as poker. It uses the recursive reasoning of counterfactual regret minimization (CFR) method to handle the asymmetric information, but it neither compute and store the complete strategy prior, nor has need for explicit abstraction. Instead, it considers each particular situation as it arises during play, but not in isolation. Also, the DeepStack solves an approximate Nash equilibrium in zero-sum games (it computes and plays a low-exploitability strategy). The basic workflow of DeepStack is: It has a deep counterfactual value network trained by randomly generated poker situations. When it plays the poker game, for every public state it needs to act, it resolves for its action by using a depth-limited lookahead where values of subtrees are computed by the network mentioned above. The author also tested the performance of DeepStack on HUNL games, it won 492mbb/g on average in as few as 3000 games, which reaches the level of human experts.

What's surprising is, the author use a depth limited search tree with and DNN to compute the value of subtrees instead of simulate the whole game. It gets over the difficultity of the extremely large state space, and get a quite good approximation. What's confusing is, I'm a little doubt the training process which uses randomly generated samples. For the state space is large ($\sim 10^{160}$ in HUNL), maybe randomly generated samples cannot cover many special cases.

I think this article is convincing in HUNL, for valid evidents are shown, and the performance is reasonable. But on other poker games, I think it's just theoretically convinced, it's lack of evidence and experiments.

Related works: This is an article about the optimtzation of DeepStack on Ludec Hold'em: "Neural network optimtzation of algorithm DeepStack for playing in Leduc Hold'em, Yaroslav Yuriiovych

DorogyiVasyl Vasylovych Tsurkan,Vladyslav Yuriiovych Lisoviy, 2017 ”

6) **Safe and Nested Subgame Solving for Imperfect-Information Games, by Brown et al, NeurIPS, 2017**
Main contributions: This article presents a new subgame solving technique on imperfect-information games which has better performance than prior methods. In imperfect-information games, it is not possible to determine a subgame’ s optimal strategy using only knowledge of that subgame (i.e. cannot be solved in isolation), for the previous decisions will make influence on current strategy. Although that cannot be solved only by subgames, this article shows that it can be approximated by analyzing many disjoint subgames. Also, this article shows that by the game progress down the game tree, the subgame solving can be repeated, which leads to far lower exploitation. Then, based on this subgame solving theory, the authors designed the first AI that can win top humans by 147 mbb/hand in heads-up no-limit Texas hold’ em poker, Libratus.
What’s surprising is, although the imperfect-imformation games cannot be solved directly by using subgames, it can be well approximated by analyzing disjoint subgames. That is a creative and novel approach, especially for the large space state games, even games with continues state space (need to be discritized). What’s confusing is, for I know little about the game theory and relative algorithms, I cannot realize part of theory such as `reach margin`.
Personally speaking, the experimental results is convincing, for the AI, Libratus, actually won the top level humans (Jason Lee, Dong Kim, Daniel McAulay and Jimmy Chou) in January 2017, Pittsburgh with a fairly judgement, which can be seen in many reports.
Related works: this article is the previous work of this paper published on NIPS 2017. It introduces the game decomposition theory: “Solving Imperfect Information Games Using Decomposition, Neil Burch， Michael Johanson， Michael Bowling, AAAI’14”

Problem 4

Solution
The code implementation of this part is in the folder “MyAlphaGomoku”

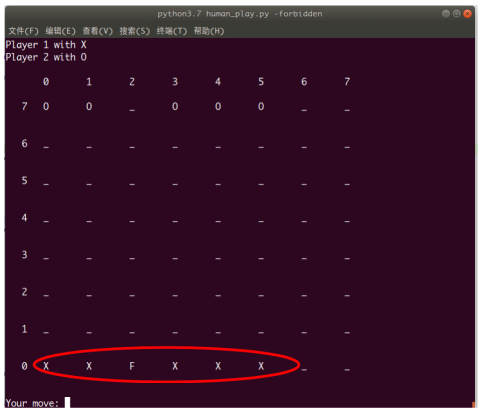
Usage of my implementation:

- This implementation needs dependencies `numpy`(for numeric calculations), `pygame`(for GUI), `theano==0.8.2` and `lasagne==0.1.0` for running the original implementation (with deep neural network).
- To start the game, use `python3 human_play.py -forbidden` to run with forbidden rule, `python3 human_play.py -forbidden -usegui` to run with GUI and forbidden rule. `python3 human_play.py --help` to show usage information.
- You will first need to input if you want to go first, input **y** to go first, **n** to go second.
- Then, you can choose your opponent, input **0** will play with another human, **1** will play with the original AlphaZero (with neural network), **2** to play with my RL agent which implemented without neural network.
- Player cannot put chess on forbidden locations, without GUI, forbidden locations are shown as **F**, with GUI, forbidden locations are red ‘X’.

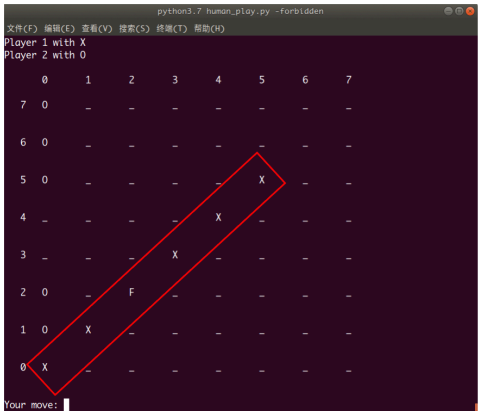
1) Gomoku with Forbidden Rule

In this problem, I implemented **almost all** forbidden rules.

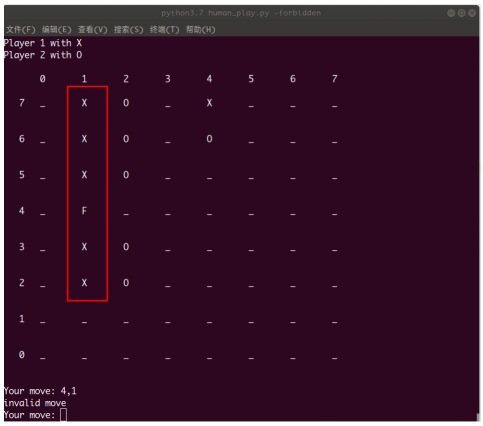
First, I implemented **Long forbiddens** (长连禁手 in Chinese). All four directions are supported:



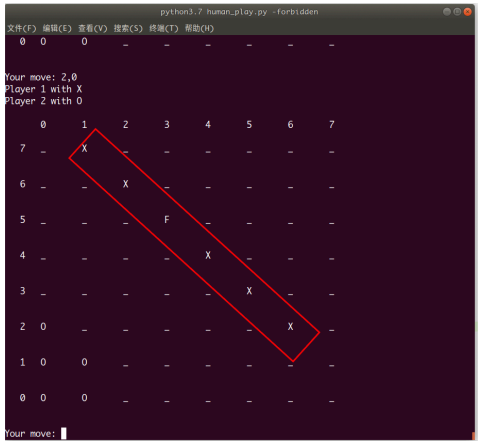
(a) long forbidden 1



(b) long forbidden 2



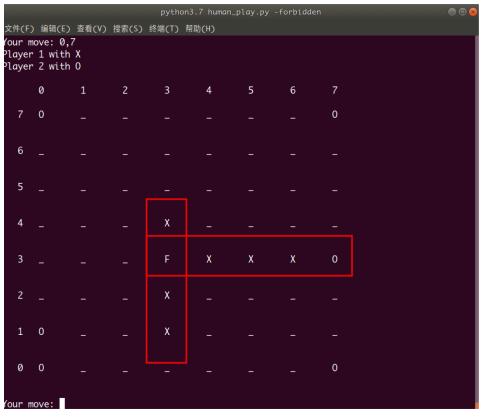
(c) long forbidden 3



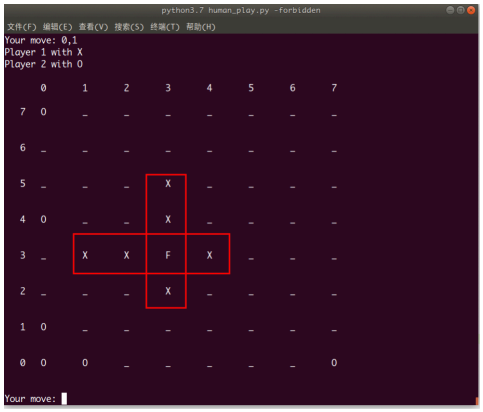
(d) long forbidden 4

Then four-four forbidden rules (四四禁手 in Chinese) are implemented.

First, consider the most simple cases:



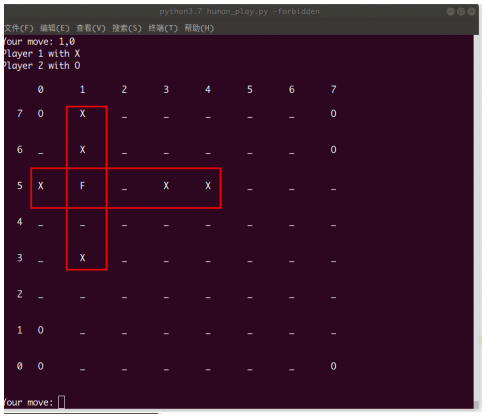
(e) 44 forbidden-simple



(f) 44 forbidden-simple

Let's check some special cases. The following 2 figures show that if the chess are not continues, but need to be banned. The program works quite well, it found the locations that need to be banned.

(g) 44 forbidden-case2



(h) 44 forbidden-case2

And it also works when there are multiple locations need to be banned:

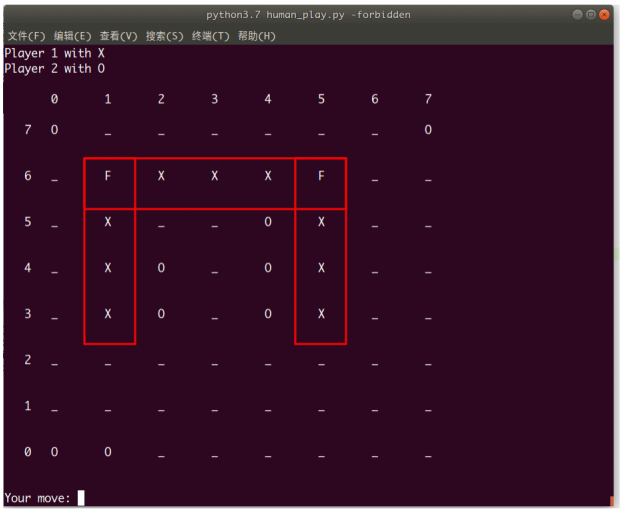


图 1: 44 forbidden-case3

And consider the following condition, the 4 chess in green rectangle is not "alive" (不为活四或冲四, in Chinese). So there is only one "alive" 4-chess chain here.

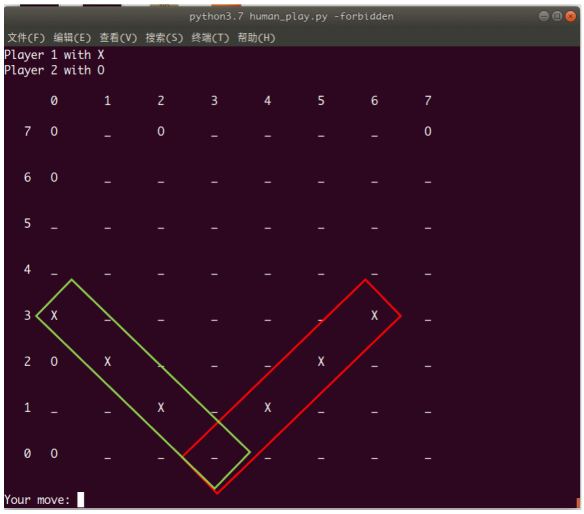


图 2: 44 forbidden special

Then, consider the most special case. First we know, the 4 chess chain in blue rectangle is not "alive", while that in yellow rectangle is alive. What's more, if you put chess at $(3, 3)$, you'll win the game directly. In Chinese terms, it not satisfies "有两个活四或冲四", for you will win directly. Hence, position $(3, 3)$ can't be banned.

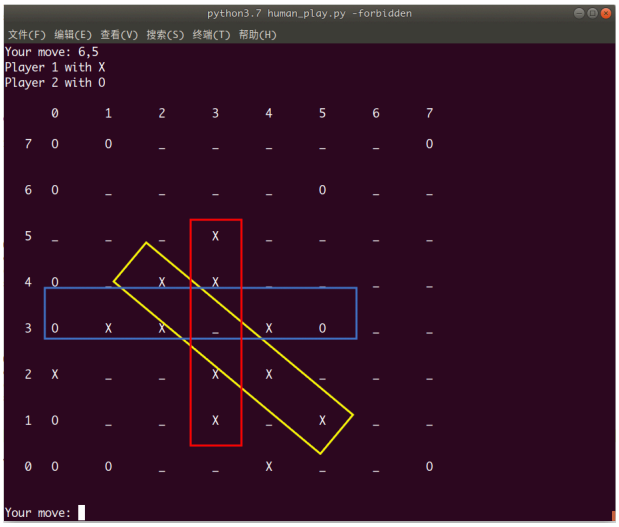
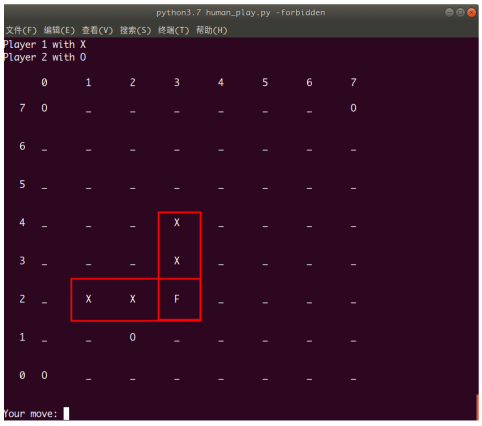
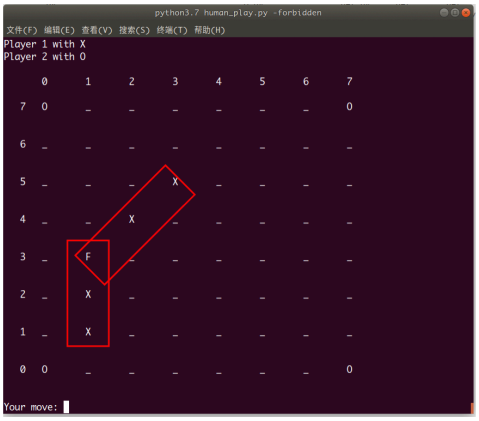


图 3: 44 forbidden special2

Finally, consider the 33 forbidden rules (三三禁手 in Chinese) We also start with the simplest conditions:



(a) 33 forbidden-simple



(b) 33 forbidden-simple

Then, some more complicated cases. First, the 33 chain which is not continues but need to be banned.

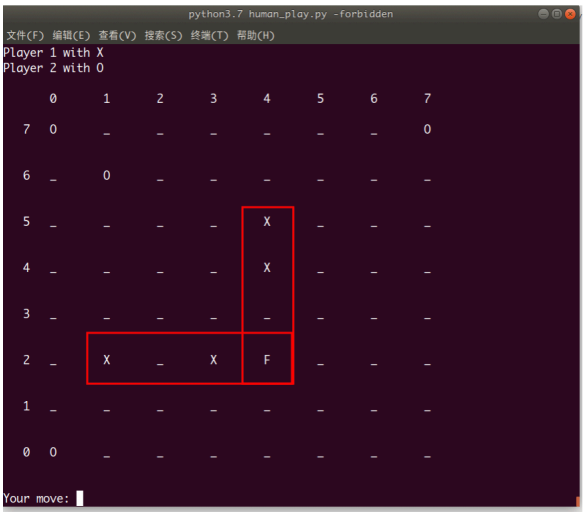


图 4: 33 forbidden-case2

It also works under multiple locations need to be banned:

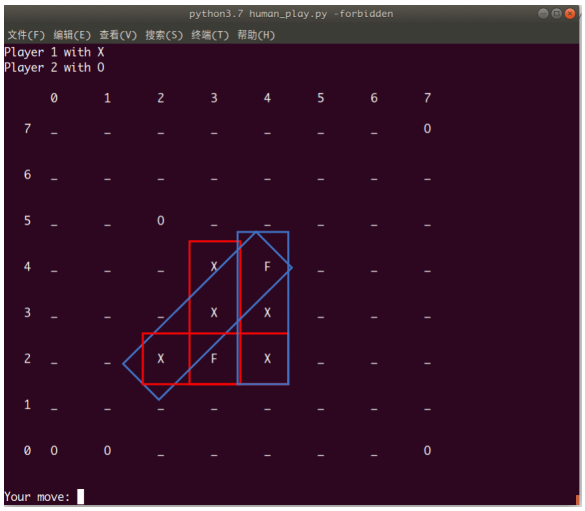


图 5: 33 forbidden multi-locations

Then, consider the following case. The chess in green rectangle is named "眠三" in Chinese terms, which is not a "alive" 3-chess chain. So, postion $(2, 4)$ can't be banned.

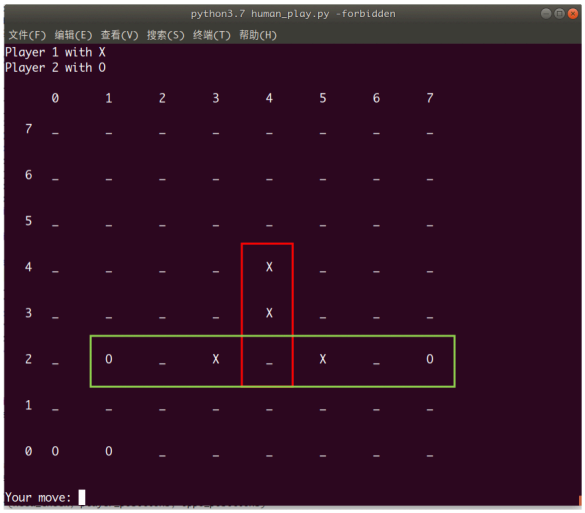


图 6: 33 forbidden-special

2) Gomoku RL agent with Forbidden Rule
In this problem, I use the approximate Q-learning algorithm.
The basic equation of this algorithm is:
We use a linear expression to present a Q value $Q(s, a)$, assume there are n parameters are used:

$$Q(s, a) = \sum_{i=1}^{n} w_i f_i(s, a)$$

where w_i are parameters that need to be learned, $f_i(s, a)$ are different feature functions to evaluate the state-action pair (s, a).
Then, instead of updating Q values in Bellman equation, we update the parameters w_i instead.
By the original Bellman equation:

$$Q(s, a) \leftarrow Q(s, a) + \alpha(R_s^a + \gamma \max_{a'} Q(s', a') - Q(s, a))$$

the Bellman equation in approximate Q learning is:

$$w_i(s, a) \leftarrow w_i(s, a) + \alpha(R_s^a + \gamma \max_{a'} Q(s', a') - Q(s, a)) f_i(s, a)$$

I consider following 4 features:

1) If there are agent's alive 3-chess chain after (s, a). If there exist such 3 chess chain, $f_1(s, a) = 1$, otherwise $f_1(s, a) = 0$
2) If there are agent's alive 4-chess chain after (s, a). If there exist such 4 chess chain, $f_2(s, a) = 1$, otherwise $f_2(s, a) = 0$
3) If there are opponent's alive 3-chess chain after (s, a). If there exist such 3 chess chain, $f_3(s, a) = 1$, otherwise $f_3(s, a) = 0$
4) If there are opponent's alive 4-chess chain after (s, a). If there exist such 4 chess chain, $f_4(s, a) = 1$, otherwise $f_4(s, a) = 0$

Then, the Q value can be represent as:

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + w_3 f_3(s,a) + w_4 f_4(s,a)$$

We know that, after taking action a, if there are agent's new 3 or 4 chess chain, it's a good thing for the agent, it need to get some reward, so the weights of f_1 and f_2 should be positive. But if there are opponent's chess chain, it's a bad thing for the agent, we need to punish it with negative weights.

Also, we need to balance the exploration and exploitation. I use ϵ-greedy algorithm, at first, $\epsilon = 0.1$, and by the game tree going down, we need to decrease the ϵ to make better decisions. Hence, I set

$$\epsilon = 0.1 * \frac{\text{number of available moves}}{\text{number of total moves}}$$

Some experiments are shown below (**WHITE** chess are agent's move):

First, it knows "How to win the game". It's shown that it "wants" to make a 5-chess chain:

图 7: It wins the game

Second, it knows to prevent its opponent from winning the game. It's shown that the agent puts chess on $(2,7)$ and $(2,2)$ to prevent me from winning the game.

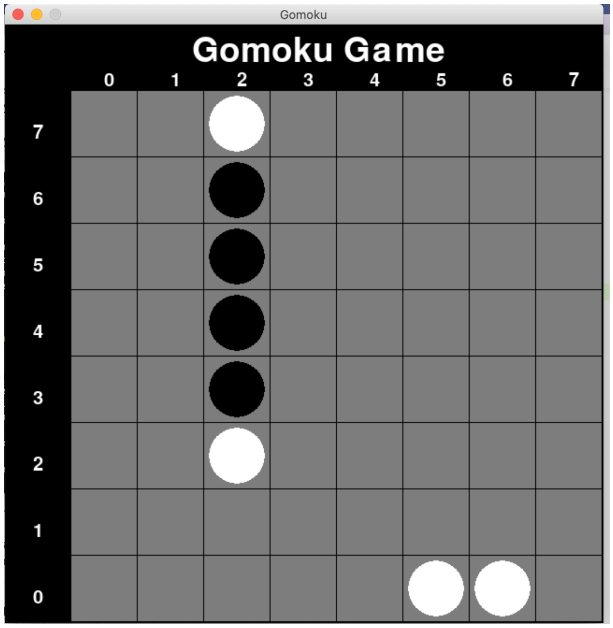


图 8: It prevents me from winning

In experiments, it works well, when it play with **Pure MCTS 2000**, it wins 3 of 5 games, and 1 tie. But it still cannot win the AlphaZero with deep neural network (9 of 10 loses).

3) Experiences

(a) When implementing forbidden rules, it's really hard to consider all the conditions. We need to traverse all empty positions, then assume there is a black chess. Then use search based algorithm to find if there are any positions need to be banned.

(b) Approximate Q learning works fine at this game. It's better and faster than pure MCTS algorithm, and has shown relative high win rate while playing with MCTS. For suitable feature functions are set, it can easily found some "good" locations.

(c) The disadvantage is, it needs human's prior knowledge. The feature functions are selected by me manully, which is not very elegant.

(d) It's important to balance the exploration and exploitation. At the beginning of the game, we need to explore more, and by the game tree going down, we need to make it choose to optimal policy with high probability.

4) Innovative parts

First, I make it much completed on the level of software engineering. The command line argument parsing, usage, and error handling are all much more completed, part of structure of the code is changed. (As shown in the Usage section)

Second, I use a search and hypothesis based approach to find forbidden locations. When implementing the forbidden rules, I use many testcases to vaildate my implementation, and all of cases are passed. That means, my implementation of forbidden rules almost included all possible conditions. Part of test results are shown in part(1).

Then, I provided a GUI (shown in next page) that improved the interaction between AI and human.

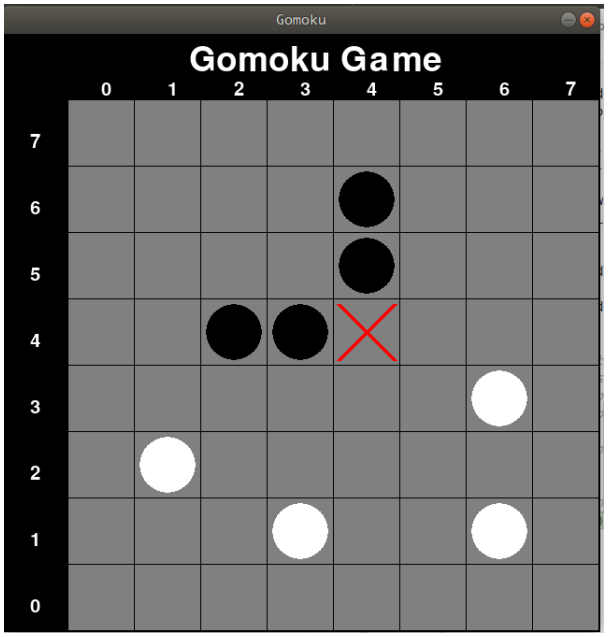


图 9: Pygame based GUI

You control the game using mouse instead of keyboard, which is much easier for human players.