

# CS270 Homework1

## Question 1

### Algorithm Description

In order to remove the haze in given image, I've tried several methods, and some combinations of these methods:

- Histogram equalization (HE)
- Contrast limited histogram equalization (CLHE)
- Contrast limited adaptive histogram equalization (CLAHE)
- Dark channel prior
- Gamma transformation

I implement each algorithm by opencv-python + numpy manually, the procedure are shown below.

- **Histogram equalization (HE)**
  - First, split the image into 3 channels (RGB).
  - For each channel, calculate the histogram:

$$Hist[k] = N(img[i, j] = k) = \sum_i \sum_j 1_{\{img[i][j]==k\}}$$

- Do the HE operation for each channel:

$$s = T(r) = \frac{L - 1}{M \cdot N} \sum_{j=0}^k n_j$$

- Merge 3 channels
- **Contrast limited histogram equalization (CLHE)**
  - First, split the image into 3 channels (RGB).
  - For each channel, calculate the histogram.
  - Clip the histogram by threshold.
    - If  $Hist[k] > \text{threshold}$ , assign  $Hist[k]$  to  $\text{threshold}$ .

- Otherwise, compensate the excess part to  $\text{Hist}[k]$ .
  - Do HE operation to the clipped histogram.
  - Merge 3 channels
- **Contrast limited adaptive histogram equalization (CLAHE)**
  - First, split the image into 3 channels (RGB).
  - For each channel:
    - Use a  $W \times W$  sliding window to traverse the image, do CLHE to this window and update  $M \times M$  central pixels.
    - Use bi-linear interpolation to reduce "grid effect"
  - Merge 3 channels
- **Gamma Transformation**
  - First, split the image into 3 channels (RGB).
  - Create a lookup table:

$$LUT(x) = c \cdot x^\gamma$$

- Apply the lookup table to the image for each channel:

$$Output[i, j, ch] = LUT[Input[i, j, ch]]$$

- Merge 3 channels
- **Dark Channel Prior**
  - This method is introduced in *Single Image Haze Removal Using Dark Channel Prior - CPVR 2009*
  - The dark channel is defined as:

$$Dark(x) = \min_{y \in \Omega(x)} (\min_{c \in [r, g, b]} Img(y))$$

- The basic idea is, the dark channel in non-foggy images is an almost all black image.
- The basic equation is:

$$I(x) = J(x)t(x) + A(1 - t(x))$$

where  $I(x)$  is the foggy image,  $J(x)$  is the output image,  $t(x)$  is the transmission rate,  $A$  is the global atmospheric light composition.

- A can be calculated as:
  - Find pixels with rank 0.1% highest intensity in Dark channel of Input image.
  - Find the highest intensity in input image at these positions, take the average to be the global atmospheric light composition (A).
- $t(x)$  can be calculated by:

$$t(x) = 1 - \omega_{Dark} \left( \frac{Img}{A} \right)$$

where  $\omega$  is a parameter ranges from 0 to 1.

- Hence, A and  $t(x)$  are known, we can obtain the output image:

$$Output(x) = \frac{Input(x) - A}{\max(t(x), t_0)} + A$$

where  $t_0$  is a parameter (I set it to 0.1)

In order to add haze to the input image, I used inverse Contrast-Stretching Transformation or Gamma Transformation first

- **Gamma Transformation**
  - Already introduced
- **Inverse Contrast-Stretching Transformation**
  - First, split the image into 3 channels (RGB).
  - For each channel:
    - Calculate the mean value  $m$
    - Create a lookup table:

$$LUT(x) = \frac{m}{(1-x)^E}$$

- Apply the lookup table to the image for each channel:

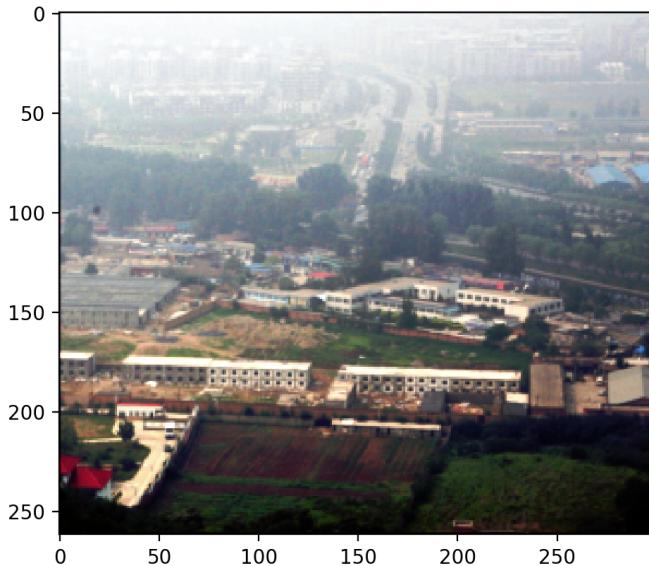
$$OutputCh[i, j] = LUT[InputCh[i, j]]$$

- Merge 3 channels  
Then, Add a bias to each pixel to make the image looks better:

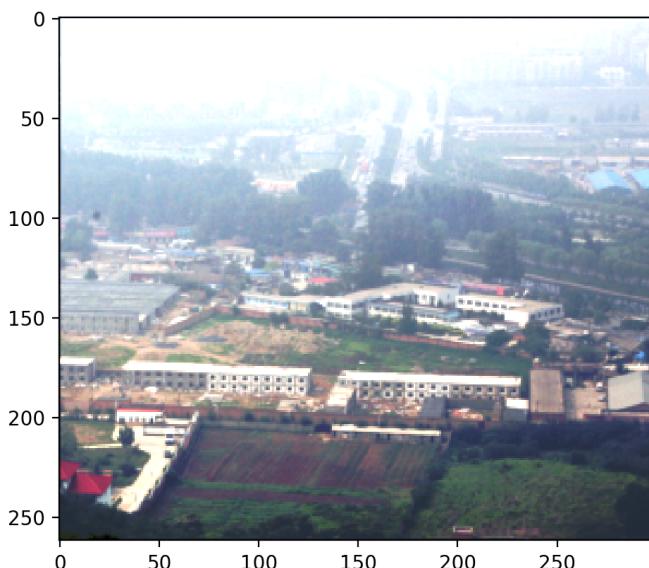
$$img[i, j, ch] \leftarrow img[i, j, ch] + Bias$$

## Results

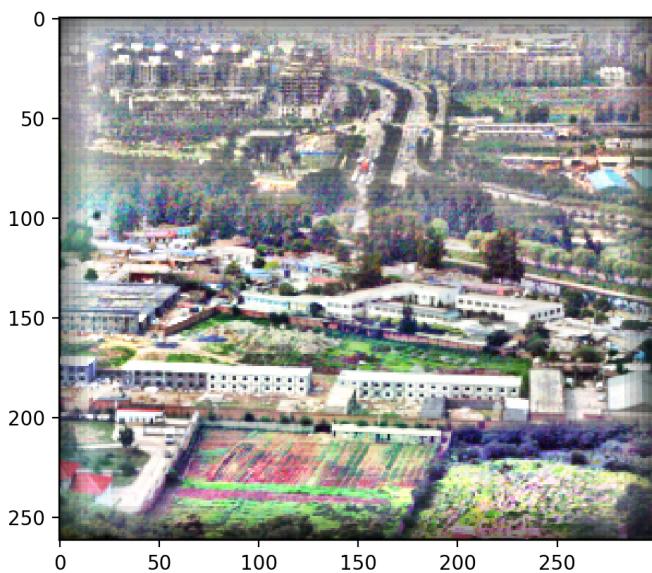
- Dehaze: HE



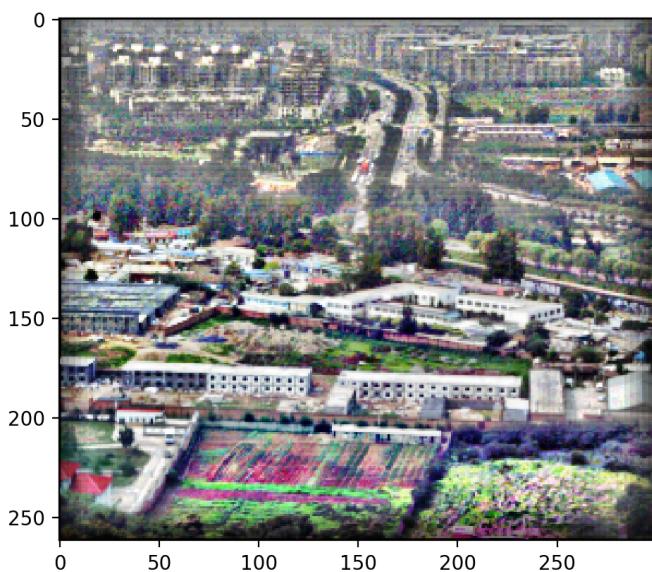
- Dehaze: CLHE (threshold = 0.013)



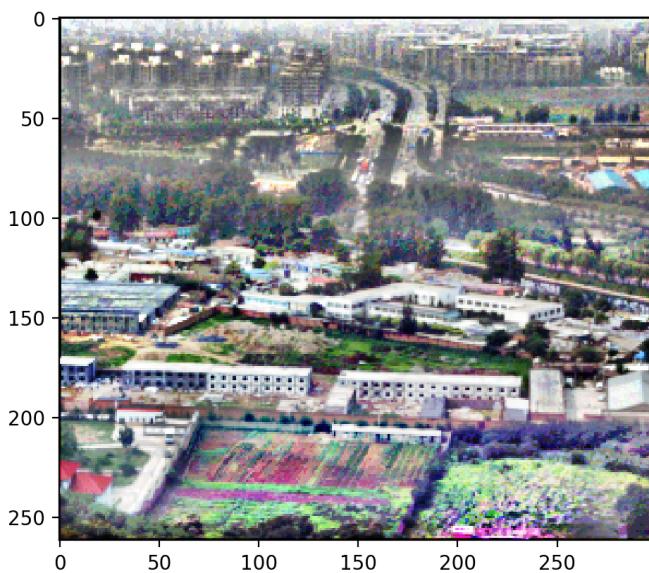
- Dehaze: CLAHE (threshold = 0.015)



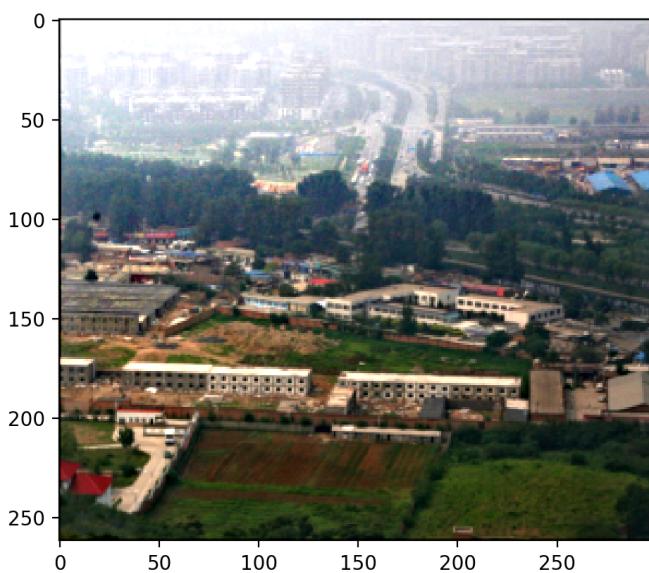
- Dehaze: CLAHE (threshold = 0.020)



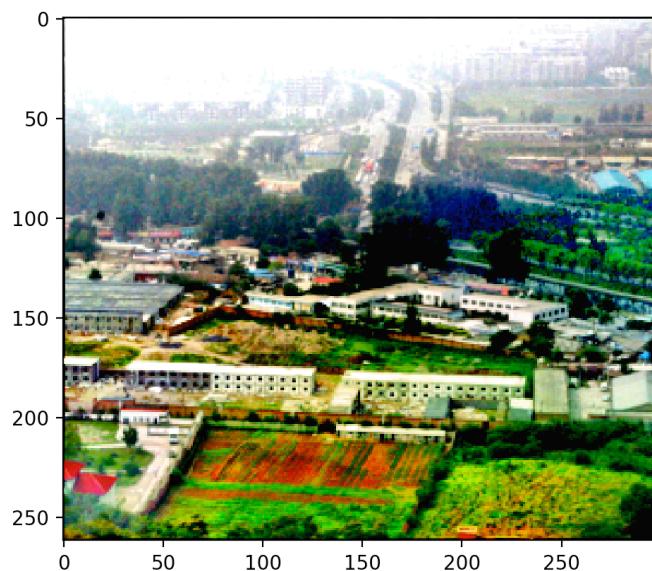
- Dehaze: Gamma Transformation + CLAHE (threshold = 0.02)



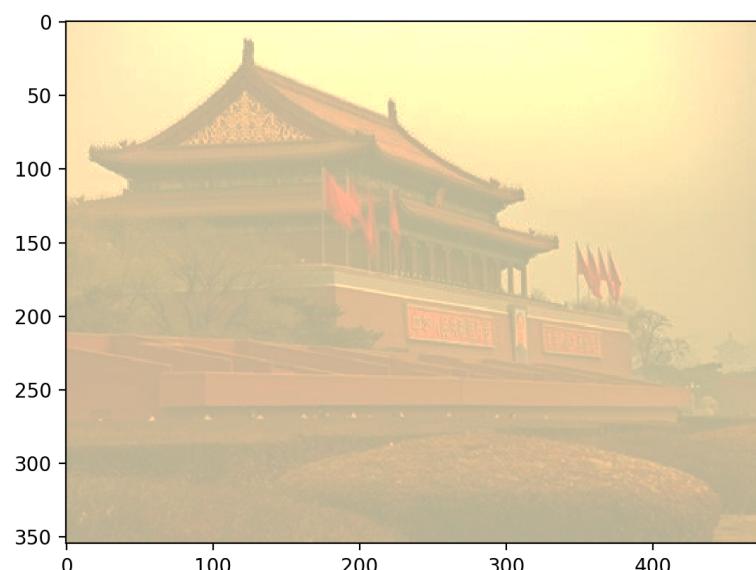
- Dehaze: Dark channel prior( $w=0.97$ )



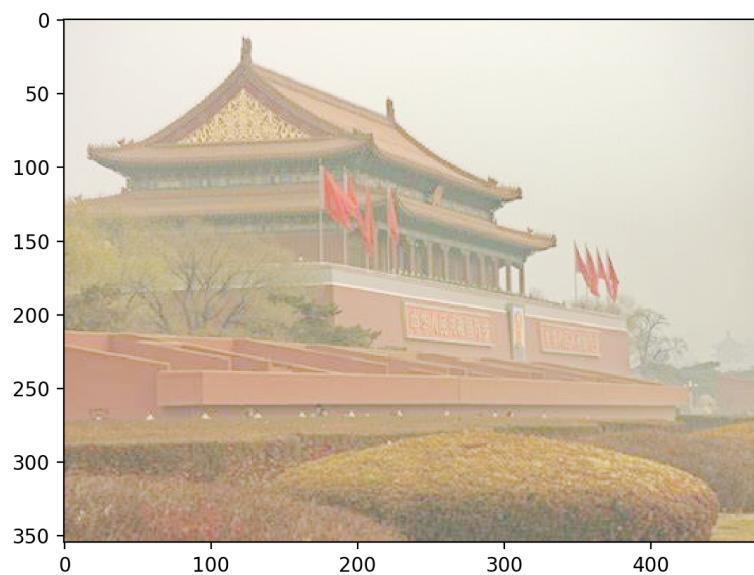
- Dehaze: Dark channel prior + CLAHE



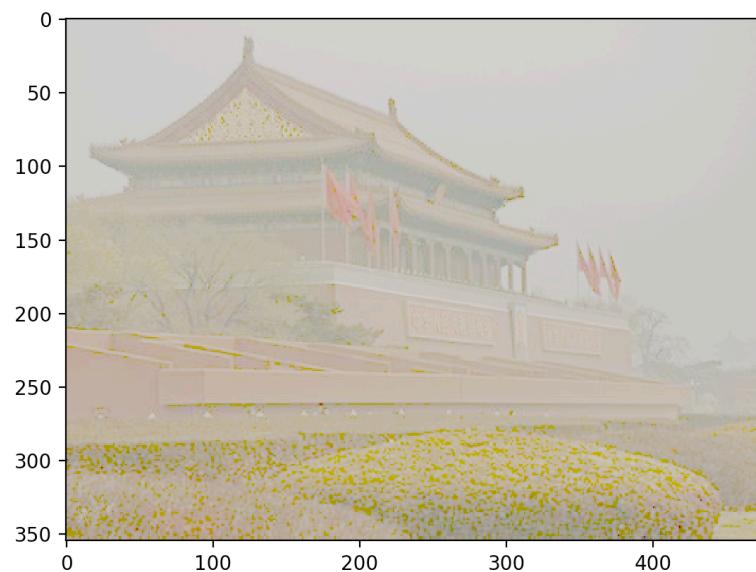
- Add Fog - Inverse Contrast-Stretching Transformation



- Add Fog - Gamma Transformation



- Add Fog - Gamma Transformation + Bias

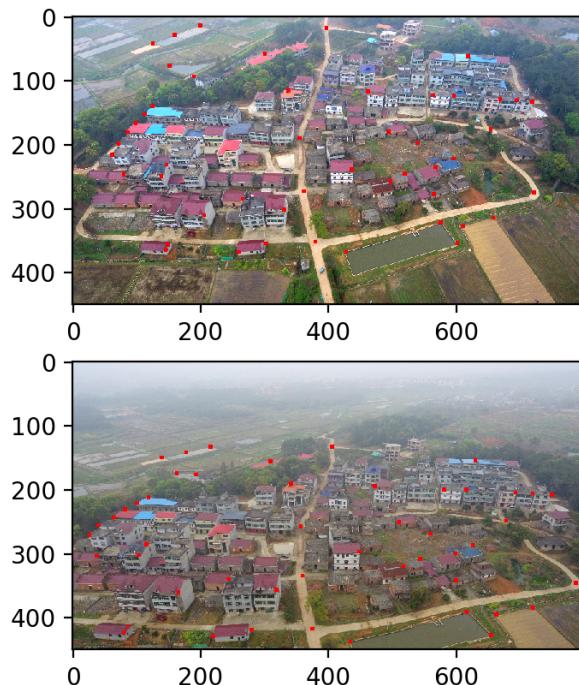


# Question2

## Algorithm Description

I use Thin plate spine (TPS) algorithm to fit two images.

- First, use `cpselect` in matlab, select 49 pairs of control points(Selected points are shown in following figure).



- Save `MovingPoints` and `FixedPoints` into a txt file and load it into python.
- Calculate radio basis function for each pair of control points:

$$r_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$$

$$\varphi(r_{ij}) = r_{ij}^2 \log(r_{ij})$$

- Construct the following linear system:

$$\begin{bmatrix} \varphi(r_{11}) & \varphi(r_{12}) & \dots & \varphi(r_{1n}) & x_1 & y_1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \varphi(r_{11}) & \varphi(r_{12}) & \dots & \varphi(r_{1n}) & x_n & y_n & 1 \\ x_1 & \dots & \dots & x_n & 0 & 0 & 0 \\ y_1 & \dots & \dots & y_n & 0 & 0 & 0 \\ 1 & \dots & \dots & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{11} \\ \omega_{12} & \omega_{12} \\ \vdots & \vdots \\ \omega_{1n} & \omega_{2n} \\ a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} x'_1 y'_1 \\ x'_1 y'_2 \\ \vdots \\ x'_n y'_n \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Use `np.linalg.solve` to solve this linear system.

- For each pixel in src image, transform it by following equation:

$$x' = \sum_i \omega_{1i} \varphi(r_i) + a_{11}x + a_{12}y + b_1$$

$$y' = \sum_i \omega_{2i} \varphi(r_i) + a_{21}x + a_{22}y + b_2$$

- If  $x'$  and  $y'$  are in valid range, transform it to the dst image.
- I used nearest interpolation to reduce the meshgrid effect.

## Result



# Question3

## Algorithm Description

- GLCM calculating:
  - Construct a 8\*8 matrix GLCM
  - for each pixel in the image, update (0 degree for example):

```
GLCM[img[i,j], img[i, j+1]] += 1
```

- Features calculating:
  - Use a 7\*7 sliding window traverse the image, update the center pixel by calculating features in the window
  - For each angle, calculate by the given formula
- Energy (for each 7\*7 window):

$$Energy = \sum_i \sum_j P(i, j)^2$$

- Correlation

$$Corr = \sum_i \sum_j \frac{P(i, j)(i - \mu_i)(j - \mu_j)}{\sigma_i \sigma_j}$$

where

$$\begin{aligned}\mu_i &= \sum_i \sum_j i P(i, j) \\ \mu_j &= \sum_i \sum_j j P(i, j) \\ \sigma_i^2 &= \sum_i \sum_j (i - \mu_i)^2 P(i, j) \\ \sigma_j^2 &= \sum_i \sum_j (j - \mu_j)^2 P(i, j)\end{aligned}$$

- Homogeneity

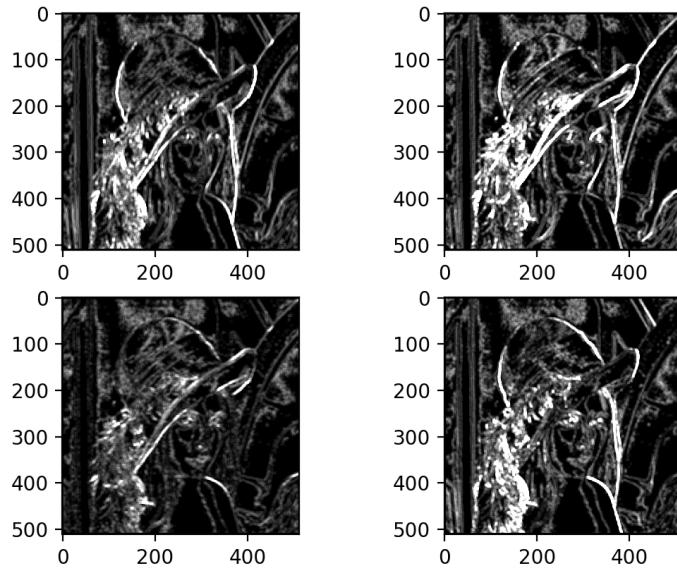
$$Homo = \sum_i \sum_j \frac{P(i, j)}{1 + \|i - j\|}$$

- Contrast

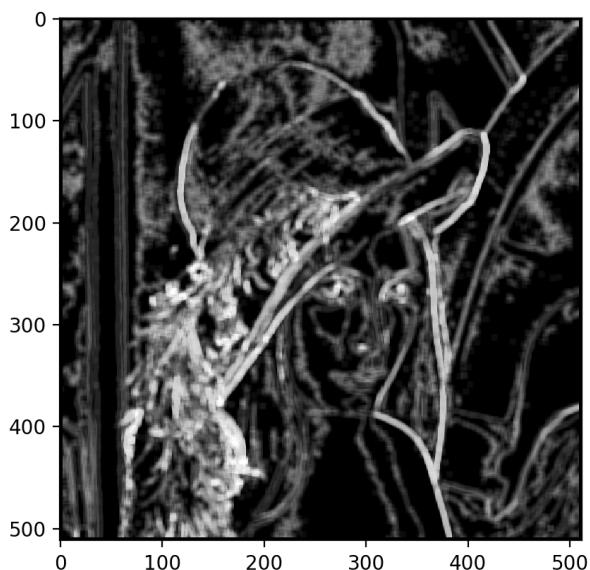
$$Contrast = \sum_i \sum_j P(i, j)(i - j)^2$$

## Results

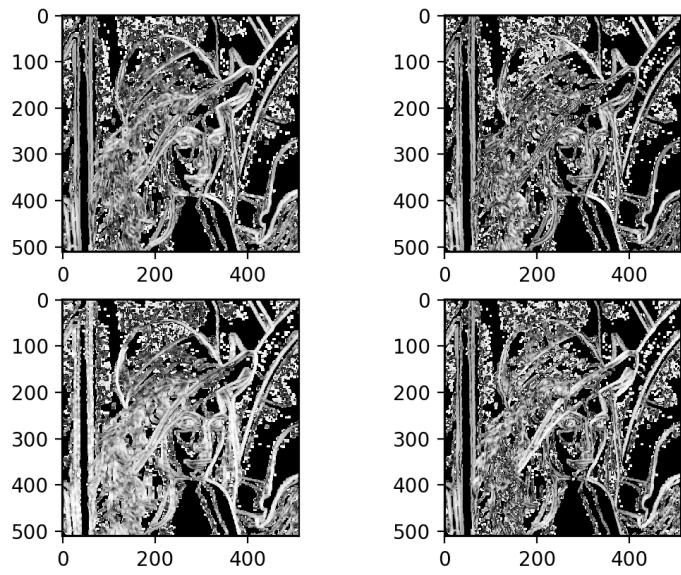
- Contrast – Different Angles



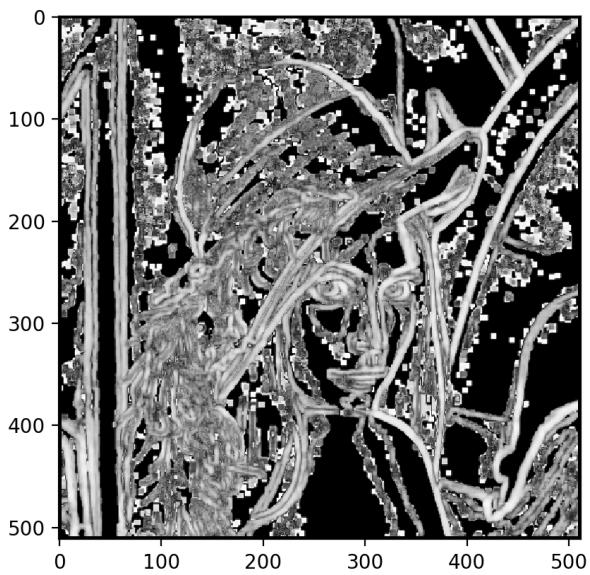
- Contrast – Average



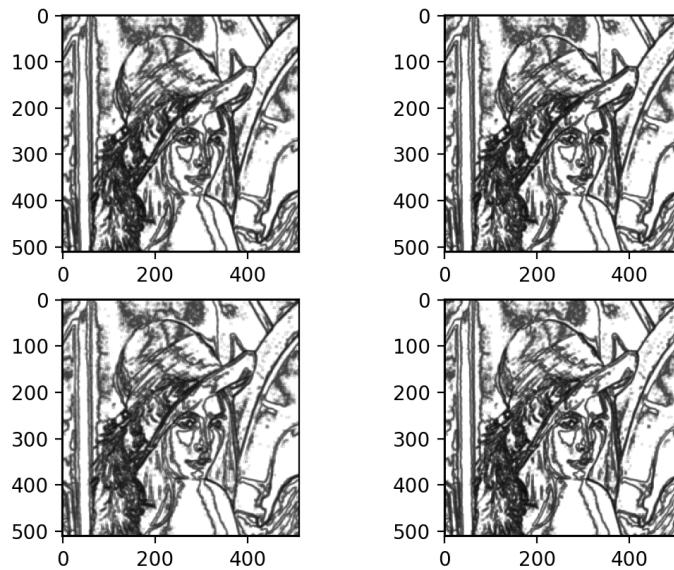
- Correlation - Different Angles



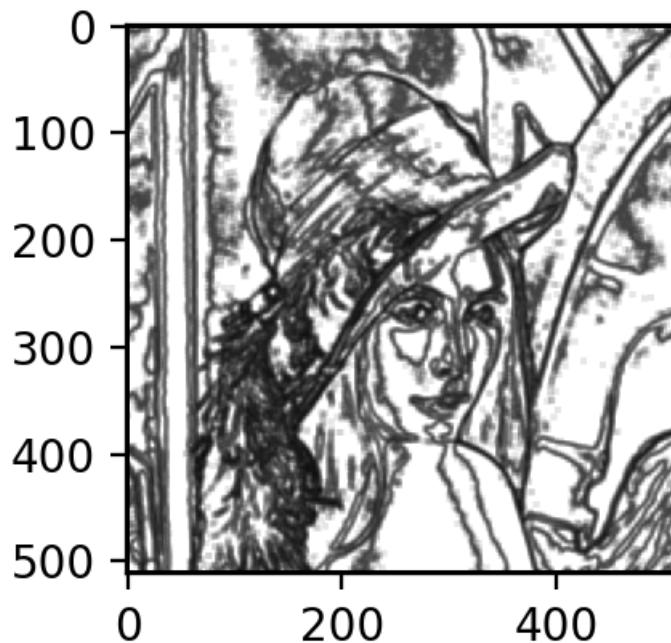
- Correlation - Average



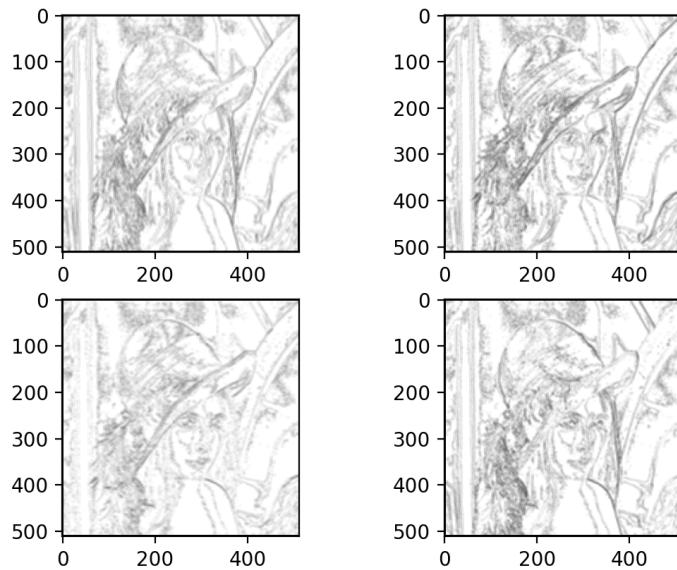
- Energy - Different Angles



- Energy - Average



- Homogeneity - Different Angles



- Homogeneity - Average

