Estimating Dynamic Dimensionality in Roll Call Data: An Application to UNGA Voting

WU, TUNG-WEN

1 UNGA Voting Patterns and Their Significance in International Relations Research

Understanding the dimensional structure of ideological preferences is a core challenge in both political science and item response theory (IRT). The voting behavior of states in the United Nations General Assembly (UNGA) has long served as a framework for estimating these ideological dimensions, providing insights into global political alignments and shifts over time. Recent UNGA voting research has predominantly employed ideal point estimation models rooted in IRT, which assume that voting behavior reflects latent ideological positions within a low-dimensional space.

While previous models offer valuable insights into voting behavior, they often assume fixed dimensionality, overlooking the dynamic and evolving nature of ideological preferences. Additionally, these models typically focus on estimating ideal points for individual states, but they may fail to identify important clusters of countries that share similar ideological profiles. Such clusters, which reflect substantively significant groupings of countries with aligned voting patterns, can reveal deeper insights into the dynamics of global politics.

To address these limitations, this paper develops a flexible nonparametric Bayesian model that captures two key features of global voting behavior in the UNGA: the changing salience of ideological dimensions over time and the clustering of countries with similar ideological vectors. By allowing the relative importance of different issue dimensions to evolve over time, the model reflects how ideological conflict adapts to shifting global agendas. At the same time, the integration of a clustering mechanism facilitates the identification of blocs based on similarities in estimated ideological vectors—patterns that may or may not align with prevailing theoretical expectations. This capacity to uncover unexpected alignments opens new avenues for hypothesis generation and future research. Taken together, these contributions offer a more dynamic and interpretable account of global political alignments, particularly during periods of geopolitical transformation such as the end of the Cold War.

Moreover, this research provides valuable insights for policymakers and scholars of international relations. By analyzing the evolving dimensions of ideological preferences and the clusters, decision-makers can better anticipate shifts in global alliances and craft more informed diplomatic strategies. The methodology introduced in this paper also has broad applicability across various fields, including political science, economics, and computer science. For instance, the model could be adapted to analyze voting patterns in other legislative bodies, enhance discrete choice models for economic decision-making by automatically detecting unobserved changes in utility functions, or address complex challenges related to model complexity in machine learning and artificial intelligence. This includes applications in areas like recommendation systems and language models, which often assume a fixed and static number of dimensions but could benefit from the flexibility offered by the proposed approach.

2 Methodology and Broader Implications

Early studies, predating the widespread use of IRT, such as Russett (1966), utilized static factor analysis to analyze countries' voting patterns, with the aim of moving beyond the conventional Cold War dichotomy. Subsequent work, such as Voeten (2000), introduced dynamic models like `NOMINATE` to examine changes in ideological dimensions over time, estimating separate models for the periods 1946–1988 and 1991–1996. More recent research, such as Bailey, Anton Strezhnev, and Voeten (2017), has further refined this approach by estimating time-varying ideal points, providing a more nuanced understanding of the evolution of ideological preferences.

Despite these advancements, most existing approaches in both political science and IRT impose rigid parametric assumptions about dimensionality. Many studies assume a fixed number of ideological dimensions—often one (Bailey, Anton Strezhnev, and Voeten (2017))—or rely on cross-validation techniques Voeten (2000) to determine the appropriate number of dimensions. These approaches require multiple model estimations and raise concerns about overfitting. This method may lack the flexibility necessary to capture the complex and evolving nature of ideological structures, particularly when the salience of political dimensions shifts over time.

Additionally, while much of the literature has focused on domestic legislative voting behavior, researchers have also explored methods for identifying voting clusters among legislators. These clusters can reveal previously unnoticed or significant patterns of ideological alignment. Spirling and Quinn (2010) applied the Dirichlet process to cluster the ideological vectors of Members of the UK Parliament and visualize voting patterns within political parties. In a similar vein, Navarro et al. (2006) used the Dirichlet process in psychology to model the distribution of individual parameters, aiming to categorize behavior patterns across individuals.

However, in the formulations of Spirling and Quinn (2010) and Navarro et al. (2006), the parameters of the subjects being analyzed are directly sampled from the Dirichlet process, implying that all individuals share a small set of common parameters. This approach constrains the ability to identify meaningful similarity patterns, as everyone within a group shares the same parameters. When comparing politically significant cases—such as identifying "representatives similar to Representative A" or "countries similar to Country A"—this formulation limits the ability to draw meaningful comparisons between individuals or countries within the same group. While such grouping may suffice for some research objectives, visualizing the differences among representatives or countries offers a more nuanced analysis.

Moreover, as Ghosal and Van der Vaart (2017) point out, the discrete nature of the Dirichlet process makes it unsuitable for estimating probability densities. Estimating the probability density of ideologies or preferences is crucial when visualizing the spread of ideological vectors in the posterior distribution. Furthermore, when predicting the voting behavior of newly elected representatives, using probability densities enables more flexible and accurate predictions than relying on a finite set of fixed points.

In the context of UNGA voting research, IRT models often treat latent dimensions as underlying "voting groups" that explain voting behavior. However, it is essential to distinguish latent dimensions from the concept of voting groups. For instance, although Russett (1966) employed factor analysis, he directly equated the estimated factors with "groupings." A latent dimension refers to a continuous, unobserved ideological axis that captures variations in countries' preferences on political issues. In contrast, a voting group refers to a discrete set of countries that vote similarly on specific resolutions, often due to shared political or ideological interests. While both latent dimensions and voting groups can be used to interpret voting patterns, the former represents a continuous, unidimensional construct, whereas the latter reflects clusters of countries whose voting behavior may align along multiple dimensions, potentially without corresponding to a single, fixed axis. This distinction is critical for understanding the limitations of models that conflate these concepts, as it oversimplifies the complex, multidimensional nature of global political alignments.

To address these limitations, this paper proposes a nonparametric Bayesian model that allows for both time-invariant ideological vectors and time-varying dimensional salience. Unlike traditional IRT models, which assume a known number of dimensions or rely on ad hoc selection methods, our approach enables the data to determine the appropriate number of dimensions in a probabilistic framework. By fixing ideological positions while allowing the salience of different political dimensions to evolve, our model offers a novel approach to understanding global voting patterns, particularly during periods of rapid geopolitical transformation such as the Cold War and its aftermath.

At first glance, one might criticize the model's assumption of time-invariant ideological vectors as unrealistic—how can a country's ideology remain exactly the same over more than 70 years? However, as we demonstrate in this paper, the model's parameters allow us to compute *revealed ideological vectors* that evolve over time. This enables the measurement of dynamic ideological proximity between countries. For example, we can track how the revealed ideological difference between the United States and Japan has changed across decades.

3 A Flexible Nonparametric Bayesian Model for Analyzing UNGA Voting

To address the limitations outlined in Section 2, we introduce a nonparametric Bayesian model that provides both flexibility in the dimensionality of ideological preferences and the ability to identify clusters of countries with similar voting patterns. This model is designed to capture the temporal and multidimensional evolution of voting behavior in the UNGA, allowing for both the salience of political dimensions and the alignment of countries to shift over time.

The proposed approach builds on the strengths of IRT but expands on its static assumptions by incorporating dynamic element that adapt to the data. In particular, we allow the dimensionality, or salience, of ideological preferences to evolve dynamically, while also identifying latent clusters of countries with similar voting patterns. This flexibility enables the model to capture complex geopolitical shifts, such as the changing alignment of countries in response to global events or policy changes.

Unlike previous studies that use IRT models with dynamic structures, such as Martin and Quinn (2002) and Bailey, Anton Strezhnev, and Voeten (2017), which allow ideal points to evolve over time through an autoregressive process—with prior distributions based on the previous year's values—our model assumes that the ideological vector remains fixed over time.

While this static ideological vector might seem restrictive, if not unrealistic, the model accounts for this by weighting the country-specific ideological vectors and resolution-specific vectors with the year-specific salience vector when computing utility. This allows us to define a *revealed ideological vector*—an ideology as it manifests under the salience of the political agenda in a given year.

To illustrate this, consider a domestic legislative context: Legislator A is pro-choice and supports protectionism, while Legislator B is pro-life but also supports protectionism. In the year 2000, the political arena is dominated by debates over joining a regional trade organization. Both A and B vote against it due to their protectionist stance. In 2002, the central issue shifts to abortion legislation; A and B vote on opposite sides. Then, in 2004, a treaty to reduce non-tariff trade barriers is introduced—again prompting A and B to vote together.

Should we say that A and B's ideologies are constantly shifting because they sometimes align and sometimes diverge? Probably not. A more accurate interpretation is that their underlying ideological positions remain stable, while their *revealed* ideologies fluctuate based on which political dimension is salient at any given time. This dynamic—where the observed behavior is shaped by the agenda—is the foundational insight that motivates the model proposed in this paper.

When substantial historical events did happen that provoke significant ideological shifts, the model will incorporate this change by the introduction of new dimensions, preventing undue flexibility in the model. Thus, while the ideological vector as model parameter is static, this approach functions as a strong regularization mechanism.

Before going to the model description, it is important to stress the differences between "salience" and "importance" of issues. As defined in Bartle and Laycock (2012),

In the following subsections, we describe the key components of the model, starting with the underlying assumptions and the data structure, followed by the probabilistic formulation and the inference procedure.

The details regarding the well-definedness of the model are provided in the Appendix.

3.1 Utility Functions

The utility function for countries employed in this study is based on the ordered regression models widely utilized in the existing literature, such as in Bailey, Anton Strezhnev, and Voeten (2017). However, this model extends those formulations by incorporating multiple dimensions and salience vectors.

Each country is assumed to have an infinite-dimensional ideological vector $\theta_S = \{\theta_{S,d}\}_{d=1}^{\infty}$, where the index S denotes the country and d represents the dimension. Similarly, each resolution is represented by an infinite-dimensional resolution vector $\beta_R = \{\beta_{R,d}\}_{d=1}^{\infty}$, and each year is associated with an infinite-dimensional salience vector $\rho_Y = \{\rho_{Y,d}\}_{d=1}^{\infty}$.

The resolution vectors are assigned independent standard normal prior distributions, while the prior distributions for the ideological vectors and salience vectors will be detailed in the following section.

To define the ordered logistic regression model, we first need to establish the spatial preference, or the utility, that country S attaches to resolution R. Drawing from the literature on ideal point estimation and machine learning (see Gopalan et al. (2014)), the spatial preference is represented as:

$$\mu_{S,R} = \sum_{d=1}^{\infty} \theta_{S,d}\beta_{R,d}\rho_{year_R,d}$$

For each resolution, there exists a two-dimensional threshold vector $\gamma_R = (\gamma_{R,i})_{i \in 1,2}$, and country S's voting decision on resolution R is determined by the following rule:

$$\text{Result}_{S,R} = \text{Nay} \quad \text{if} \quad \mu_{S,R} < \gamma_{R,1}$$

$$\text{Result}_{S,R} = \text{Abstain} \quad \text{if} \quad \gamma_{R,1} < \mu_{S,R} < \gamma_{R,2}$$

$$\text{Result}_{S,R} = \text{Yea} \quad \text{if} \quad \mu_{S,R} > \gamma_{R,2}$$

Thus, the model can be viewed as an ordered logistic regression with an infinite number of latent covariates. Moreover, since μ's are constructed by summing an infinite number of elements, it is not immediately clear whether this summation will be finite or even summable. We will demonstrate the summability and finiteness of μ after introducing the prior distribution structures for the model parameters.

3.2 Ideological Vectors

The concept of an ideological regime refers to the prior distribution of ideological vectors for each country. This distribution is denoted as G.

While assuming a known distribution, such as the normal distribution, is a standard approach, it may not sufficiently capture the complexity of national ideologies. These ideologies are often multimodal due to latent ideological clusters, which are typically interpreted as indicators of ideological alignment—a phenomenon of central interest in international relations and political science. Consequently, in this study, we estimate G nonparametrically using a Dirichlet process mixture.

Following prior research such as Shiraito, Lo, and Olivella (2023), we construct the Dirichlet process using the stick-breaking process in the way proposed by Sethuraman (1994).

Specifically, we first sample the global hyperparameter α from a gamma distribution:

$$\alpha \sim Gamma(0.001, 0.001)$$

For each regime p within an infinite number of possible regimes, we sample the necessary variables as follows:

$$\pi_p \sim Beta(1, \alpha)$$

$$p_p = \pi_p \prod_{l=1}^{p-1} (1 - \pi_l)$$

The central ideological vector of regime $\bar{\theta}$ and its dispersion $\bar{\sigma}$ are sampled as follows:

$$\bar{\theta}_p \sim Normal(0, 1)$$

$$\bar{\sigma}_p \sim Gamma(0.001, 0.001)$$

When sampling the ideological vector for a given country S, we first draw an index from a categorical distribution parameterized by the stick-breaking probabilities:

$$\eta_S \sim Categorical(p)$$

Based on the sampled η_S, we then sample the ideological vector:

$$\theta_S \sim \text{Normal}(\bar{\theta}_{\eta_S}, \bar{\sigma}_{\eta_S})$$

As evident from this formulation, each θ_S is drawn from the normal distribution associated with the η_Sth regime, implying that ideological positions will be concentrated around a finite number of cluster centers, contingent on the number of regimes effectively utilized by the model. However, unlike parametric models that impose a normal prior, our approach permits the distribution of ideological vectors to deviate from predefined parametric forms, such as the normal distribution, when justified by the data. Moreover, in contrast to the models proposed by Spirling and Quinn (2010) and Navarro et al. (2006), the sampled ideological vectors in our framework do not perfectly coincide with those of other countries in the same regime, allowing for greater flexibility in capturing ideological variation.

This entire sampling process defines the distribution G, whose posterior distribution will be examined in the empirical analysis to illustrate the ideological alignments of countries since the inception of the UNGA, as inferred from their voting behavior within the assembly.

3.3 Salience Vectors

Efforts to estimate the number of dimensions have been extensively examined in prior research, both within political methodology and across other quantitative disciplines. One of the seminal works, Poole and Rosenthal (1991), highlighted the critical role of dimensionality estimation, offering visualizations that demonstrate how model classification performance varies with the number of dimensions. More recently, Kim, Londregan, and Ratkovic (2018) introduced a method utilizing a Bayesian Lasso prior, which shrinks specific dimensions to zero. Additionally, in the context of biological research on factor models for gene expression, formulations have been developed that drive the variance of normal distributions toward zero as the number of dimensions increases (Bhattacharya and Dunson (2011)).

In this study, we adopt an approach that utilizes a salience vector, or weight vector derived from the stick-breaking process, as proposed by Gopalan et al. (2014) in the context of recommender systems. This method facilitates the suppression of all but the first few dimensions, not through binary classification of dimensions as useful or not, but by assigning a continuous weight between zero and one to each dimension, which can be interpreted as its salience.

To account for temporal variation, we model the weights produced by the stick-breaking process in an autoregressive manner, using the framework of the hierarchical Dirichlet process (Yee Whye Teh and Blei (2006)).

Yee Whye Teh and Blei (2006) pointed out that if a separate Dirichlet process is applied to each group (e.g., each year), the dimensions across groups would inherently differ. This discrepancy arises because the Dirichlet process samples discrete values from a continuous probability distribution, making the probability of overlap between values drawn from different Dirichlet processes effectively zero.

To address this issue, Yee Whye Teh and Blei (2006) introduced the hierarchical Dirichlet process, which accommodates dependencies between groups while preserving flexibility. We adopt this regularization framework to allow the year-specific salience vectors to evolve over time, while simultaneously mitigating excessive deviations.

First, we sample the importance-weighted vector for the initial year using a stick-breaking process. Specifically, we first sample:

$$\tau \sim Gamma(0.001, 0.001)$$

Then, the importance-weighted vector for year 1, denoted as $rho_{1,d}$, is obtained through the stick-breaking process:

$$\delta_{1,d} \sim Beta(1, \tau)$$

$$\rho_{1,d} = \delta_{1,d} \prod_{l=1}^{d-1} (1 - \delta_{1,l})$$

This procedure is iterated for d = 1 to d = ∞. Next, we sample the parameter that governs the degree of temporal variation:

$$\zeta \sim Gamma(0.001, 0.001)$$

For year Y, the importance-weighted vector is sampled from a hierarchical stick-breaking process, using the vector from year Y-1 as the prior:

$$\delta_{Y,d} \sim Beta\left(\zeta \rho_{Y-1,d}, \zeta\left(1 - \sum_{l=1}^{d} \rho_{Y-1,l}\right)\right)$$

$$\rho_{Y,d} = \delta_{Y,d} \prod_{l=1}^{d-1} (1 - \delta_{Y,l})$$

This process is repeated for all d = 1 to d = ∞.

By incorporating hierarchical structure into the stick-breaking process, our approach allows for dynamic shifts in dimensional importance over time while maintaining consistency across periods. This framework provides a more flexible and theoretically grounded approach to modeling evolving ideological structures.

As explained earlier, the model can be interpreted as an ordered logistic regression with an infinite number of latent covariates. This raises important concerns about summability, as the sum of an infinite sequence of real numbers is not always guaranteed to exist or be well-defined.

For example, consider the infinite sequence whose i th element is $\left(\frac{1}{2}\right)^i$; this sequence is summable, and its sum converges to a finite value using a familiar geometric series formula. In contrast, if the i th element is $(-1)^i$, the partial sums alternate between -1 (for odd i) and 0 (for even i). Since infinity is neither odd nor even, the full sum of this sequence is not defined.

Therefore, to ensure that our model is well-posed, we must rigorously demonstrate that the infinite sum involved in our specification is indeed well-defined. Without this, we risk estimating a quantity that does not exist in any meaningful mathematical sense. The formal proof is provided in the Appendix.

4 Empirical Findings from UNGA Voting Data

In the following analysis, we draw upon data from Voeten, Strezhnev, and Bailey (2009). While a full Markov Chain Monte Carlo (MCMC) approach would be the most theoretically robust method for estimating the model, the large scale of the dataset—comprising over 700,000 country-resolution pair observations—and the complex hierarchical structure of the model necessitate an alternative approach. As a result, we implement variational inference, following Kucukelbir et al. (2017), using the Stan programming language (Stan Development Team (2024)). This method transforms the computationally intensive sampling process into a more tractable optimization problem.

Variational inference has gained traction among political scientists working with large datasets, as it enables the extraction of meaningful quantities while preserving the Bayesian framework (Grimmer (2011)). However, despite its advantages—such as scalability—variational inference has notable limitations, particularly its tendency to underestimate posterior variance (Blei, Kucukelbir, and McAuliffe (2017)).

Theoretically, the number of ideological regimes and salience dimensions in our model could be infinite. However, given computational constraints, handling infinite-dimensional arrays is infeasible. Instead, we approximate infinity with a sufficiently large finite value. For the empirical analysis, we set this value to 20 for both ideological regimes and salience dimensions.

Even with 20 dimensions, many ideological regimes and salience dimensions remain effectively unused, justifying this choice.

To evaluate the model's predictive accuracy, we randomly held out 1,000 'yea', 1,000 'nay', and 1,000 'abstain' cases.

The variational inference algorithm, leveraging distributed evaluation of the log-likelihood function, converges in approximately 38 minutes on an M1 MacBook Air with 16 GB of memory and 8 cores.

4.1 Prediction Accuracy

To assess whether the model effectively captures meaningful patterns in UNGA voting data, we evaluate its predictive performance on a held-out set of 3,000 observations. Following the posterior predictive framework proposed by Meng (1994), we measure predictive accuracy using the F1 score—a widely adopted metric in classification tasks within the machine learning literature—as the test statistic.

Rather than computing a posterior predictive p-value against a fixed benchmark (e.g., 0.5), we visualize the entire posterior distribution of the F1 score. This approach provides a more comprehensive assessment of the model's predictive capability and its alignment with observed voting behavior.

More formally, we compute the posterior distribution of the F1 score for each class, as well as the macro F1 score—the arithmetic mean of the per-class F1 scores—conditioned on the observed data. Mathematically, this is expressed as:

$$p(\text{F1 score} \mid x) = \int p(\text{F1 score} \mid \theta) p(\theta \mid x) \, d\theta,$$

where x represents the set of training data, and θ denotes the parameters of the model. This formulation accounts for parameter uncertainty by integrating over the posterior distribution, ensuring a robust evaluation of the model's predictive performance.

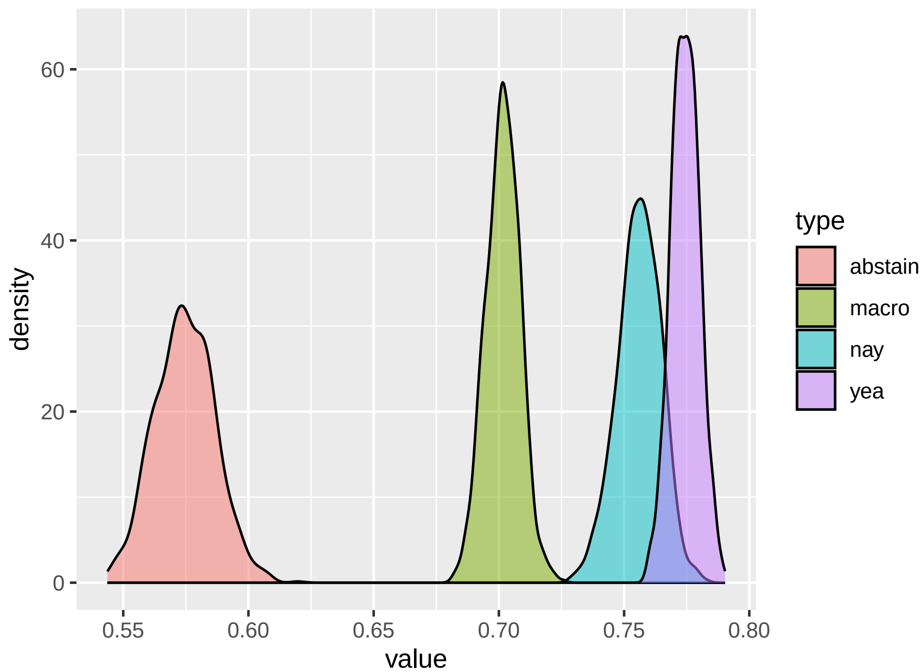

Figure 1: Posterior Distribution of the F1 Scores

Figure 1 presents the posterior distributions of the F1 scores for the three voting categories—‘yea’, ‘nay’, and ‘abstain’—as well as the macro F1 score, which provides an overall measure of predictive accuracy. The results indicate that the F1 scores for the ‘yea’ and ‘nay’ classes are significantly higher than 0.7, demonstrating strong predictive performance. In contrast, the F1 score for the ‘abstain’ class is lower, ranging from approximately 0.55 to 0.6.

The relatively lower predictive accuracy for the ‘abstain’ class is likely not a deficiency of the model itself but rather a reflection of the inherent nature of abstentions, which convey less clear information compared to affirmative or negative votes. Nevertheless, the F1 score for the ‘abstain’ class remains significantly above 0.5, indicating that the model performs substantially better than random chance.

The macro F1 score distribution is centered around 0.7, suggesting that the model achieves a high overall predictive accuracy across all voting categories. This result implies that the model effectively captures underlying patterns in UNGA voting behavior while maintaining balanced performance across different classes.

Overall, these findings suggest that the model successfully identifies meaningful patterns in UNGA voting behavior, particularly for decisive votes (‘yea’ and ‘nay’), while also demonstrating reasonable predictive capability in more ambiguous cases (‘abstain’).

4.2 Overview of the Estimated Salience Vectors

To gain a comprehensive understanding of the patterns identified by the model in the UNGA voting data, we begin by visualizing the estimated salience vectors. The model effectively utilizes six salience dimensions, while the remaining dimensions exhibit negligible values across all years, accompanied by near-zero credible intervals. Consequently, our analysis is concentrated on these six significant dimensions.

To capture the temporal shifts in salience, we visualize the estimated salience vectors, ρ_{year}, for each year. The salience vectors are plotted vertically, with connections drawn between the same dimension across different years, facilitating an examination of the evolution of salience over time. Specifically, we focus on six dimensions identified by the model, each representing a key global issue: human rights, north-south relations, nuclear weapons, the Cold War, the Yom Kippur War, and membership in the United Nations. The interpretation and naming of these dimensions will be addressed in the subsequent discussion.

Accompanying the line plot is a shaded ribbon, which represents the 5th and 95th percentiles of the salience vector estimates, visually indicating the uncertainty surrounding the mean values. A vertical dashed red line marks the year 1989, signaling the end of the Cold War, a pivotal geopolitical event.

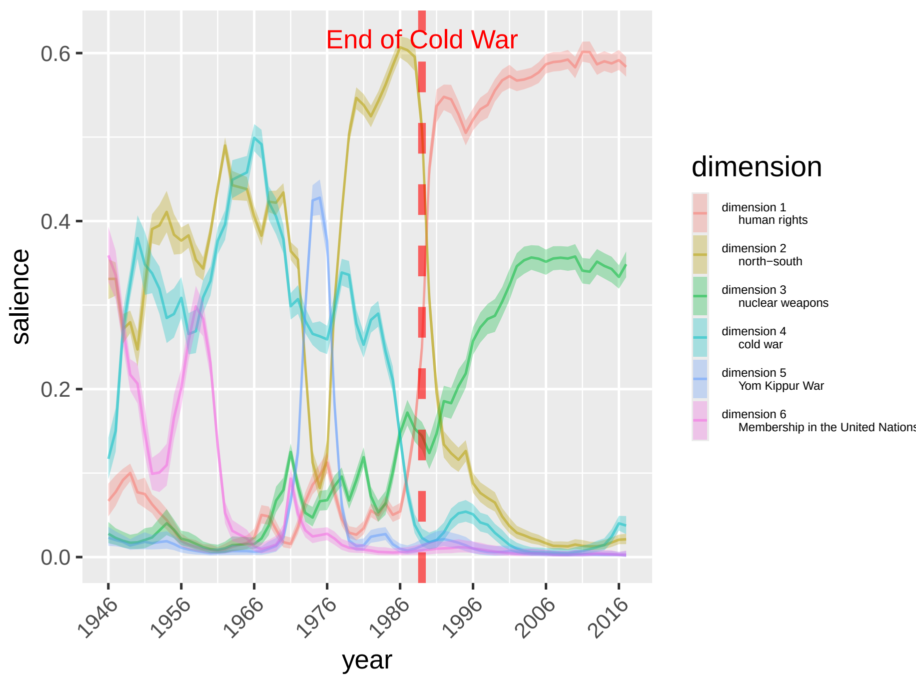

Figure 2: Estimated Salience Vectors

As illustrated in Figure 2, prior to the end of the Cold War, the movement of the salience vectors was pronounced, with many active dimensions reflecting significant shifts in global issues. However, following the end of the Cold War, the movement of the salience dimensions became more stable. By the year 2000, only two dimensions remained actively influencing the model, and the relative values of these two dimensions remained relatively constant thereafter.

To quantify the variability of salience vectors across years more effectively, we compute the entropy of the salience vectors for each year Y using the following formula:

$$entropy_Y = -\sum_{d=1}^{\infty} \rho_{Y,d} log(\rho_{Y,d})$$

Entropy is a key concept for quantifying uncertainty and disorder in a probability distribution. For instance, consider an unfair coin that always lands heads when tossed—its outcome is highly predictable, resulting in low entropy. In contrast, a fair coin that has an equal probability of landing heads or tails introduces more uncertainty, leading to a higher entropy. Although the salience vector in this context is not explicitly used as a probability distribution but rather as a weighting vector, which assigns higher weights to relevant dimensions while shrinking unnecessary ones to zero, its construction allows it to effectively represent a distribution. Consequently, we adopt the concept of entropy to visualize the uncertainty associated with the salience vectors.

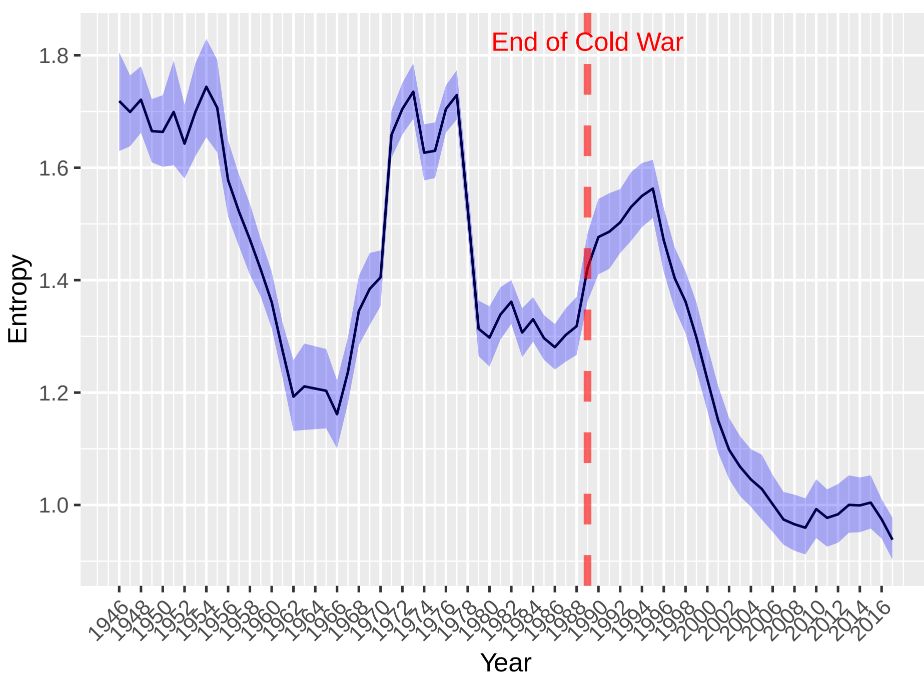

Figure 3: Entropy of Salience Vectors

Figure 3 highlights significant shifts following the conclusion of the Cold War. After the Cold War, entropy experienced a slight increase, peaking in 1995. However, it subsequently began a steady decline, ultimately reaching levels lower than the minimum entropy observed during the Cold War, specifically between 1962 and 1966. This ongoing downward trend indicates that, in terms of issue diversity measured by the entropy of salience vectors, the issue configuration within the UNGA resolutions is lower than at any point in its history.

This conclusion aligns closely with the findings of Voeten (2000), who suggests that, following the Cold War, voting patterns in the UNGA became increasingly driven by a single dimension. In contrast, our model reveals that post-Cold War voting is influenced by one dominant dimension, with a salience value of approximately 0.6, and a second, less significant dimension with a salience value around 0.35. In other words, our model indicates that approximately '1.5' dimensions govern post-Cold War voting behavior. Despite the differences in the precise number of dimensions, both studies reach the same overarching conclusion: UNGA voting during the Cold War was more complex compared to the post-Cold War period.

4.3 Interpretation of the Estimated Salience Dimensions and Their Substantive Implications

In this section, we interpret the substantive meaning of the estimated salience vector dimensions introduced in the previous subsection. Since the model relies solely on resolution IDs, country IDs, year IDs, and voting outcomes as inputs, it derives these dimensions purely from the underlying correlational structure in the data. Consequently, external information is required to meaningfully interpret the estimation results.

A promising direction for future research would be to incorporate textual data into the proposed model, allowing for a more direct association between estimated dimensions and substantive political themes. The estimation of ideal points, or ideological vectors as referred to in our model, has been explored in the machine learning literature. For example, Vafa, Naidu, and Blei (2020) propose a model that estimates ideal points using speech data, highlighting the potential for integrating text-based information into such analyses.

To interpret the estimated dimensions, we fit an additional model using the distributed multinomial regression approach proposed by Taddy (2015). This model incorporates resolution metadata along with the estimated resolution and salience vectors. Specifically, we first weight the resolution vectors by their corresponding salience vectors for the year of the vote. The resulting weighted resolution vectors serve as covariates in a distributed multinomial regression model, predicting the frequency of words in the resolution metadata provided by Voeten, Strezhnev, and Bailey (2009). This approach allows us to identify words that are strongly associated with each salience dimension.

To facilitate the interpretation of the salience dimensions, we include individual plots for each dimension over time, as well as a visualization of the ideological vector elements for each country. In the ideological vector plot, we standardize the direction for easier interpretation

Table 1: Distinctive Words for Each Dimension

dimension__1	dimension__2	dimension__3	dimension__4	dimension__5	dimension__6
rights__sudan	israel__south	res__alrev	development__organization	res__alrev	nepal__amendment
rights__cuba	res__alrev	nationals	appropriations__year	exercise__palestinian	council__admission
democratic__republic	rights__sudan	disarmament__general	moratorium	condemn__israels	palestine__question
moratorium	vote__operative	reducing__nuclear	budget__appropriations	moratorium	al__general
moratorium__use	possible__offer	nuclear__danger	appropriations__biennium	continuing__increasing	adopt__trusteeship
use__death	offer__necessary	legality	biennium__amount	increasing__collaboration	approved__th
penalty__resolution	resources__meet	legality__threat	appropriations	korea__continue	reject
rights__democratic	meet__needs	danger__resolution	nepal__amendment	israel__racist	consider__proposals
armaments__resolution	near__ea	arrangements__strengthening	expel	crime__humanity	section__ad
rights__iraq	us__motion	reducing	un__expel	rights__population	weapons__testing

by assigning blue to the direction aligned with the United States, and red to the opposite direction. This color scheme helps highlight ideological similarities and oppositions across countries in a visually intuitive manner. We also present the posterior estimates of ηs, which represent each country's degree of affiliation with the identified ideological regimes. These estimates serve as a historical summary of countries' voting patterns, offering insight into how consistently each country aligns with particular ideological regimes over time.

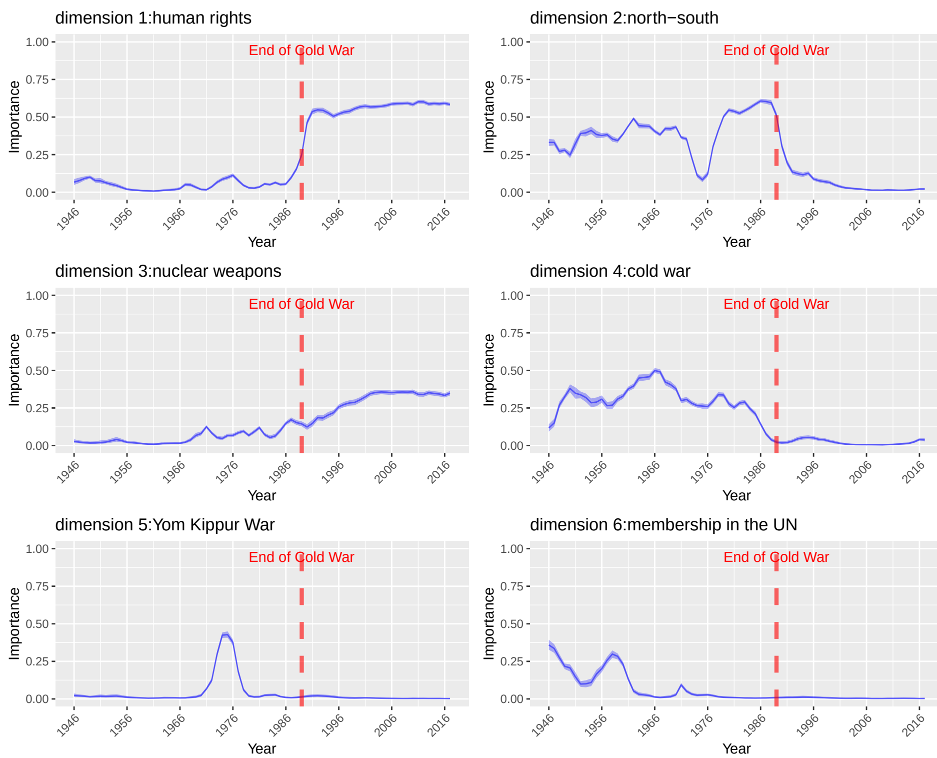

Figure 4: Individual Salience

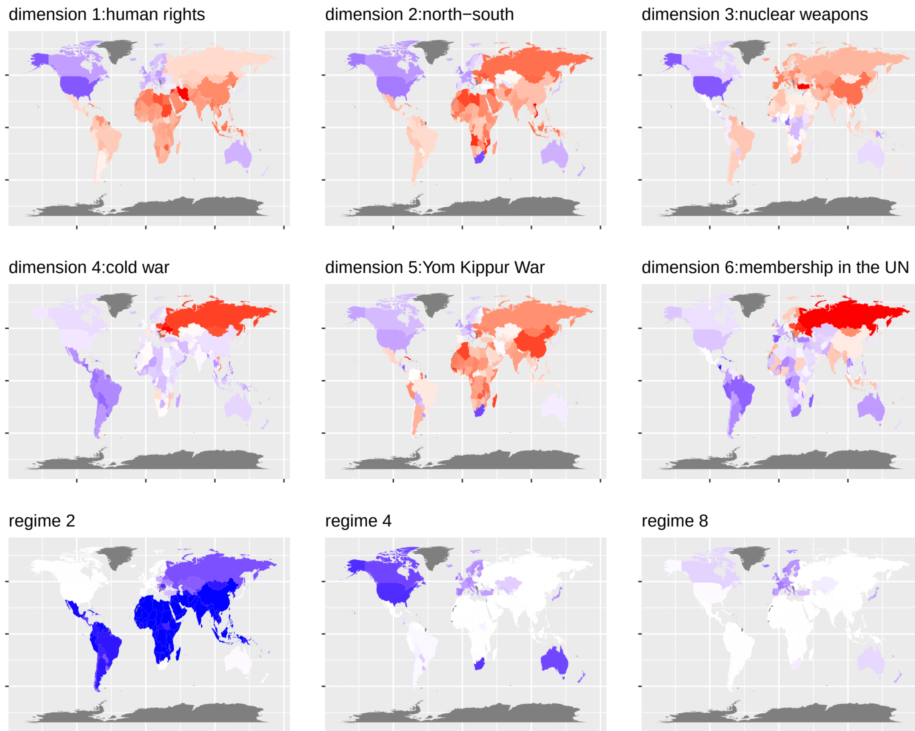

Figure 5: Values of Ideological Vectors and Ideological Regime

To better understand the substantive meaning of the dimensions uncovered by our model, we examine the most representative resolutions associated with each. The table lists the top ten resolutions or resolution components with the highest loadings per dimension, allowing us to infer their dominant themes.

4.3.1 Dimension 1: Human Rights

The first dimension is clearly associated with human rights. Terms such as `rights_sudan`, `rights_cuba`, `rights_democratic`, and `rights_iraq` dominate this space, highlighting recurring debates over the protection of civil and political rights in specific national contexts. The inclusion of resolution topics like `use_death` and `penalty_resolution` suggests the presence of contested issues such as the death penalty, while entries like `armaments_resolution` and `moratorium_use` reflect broader humanitarian concerns about the regulation of weapons and militarization.

This dimension captures a persistent ideological divide regarding the universality and prioritization of human rights norms, often situated in tension with principles of national sovereignty and the geopolitical interests of states. During the Cold War, such debates were frequently polarized along East–West lines, with Western liberal democracies emphasizing individual rights, while many states in the Global South—often supported by the Soviet bloc—framed human rights critiques as instruments of Western political pressure.

An examination of the temporal salience of this dimension (see Figure 2) reveals that it is a relatively recent development, gaining prominence only in the final years of the Cold War. This pattern suggests that human rights emerged as a more structurally embedded concern in UN General Assembly politics beginning in the late 1980s. This shift likely reflects a confluence of global changes: the delegitimization of authoritarian regimes following the collapse of the Eastern bloc, the growing influence of international non-governmental organizations, and the global diffusion of liberal norms characteristic of the post-Cold War order.

This interpretation aligns with a broader body of scholarship identifying the post-Cold War period as a pivotal moment in the institutionalization of human rights discourse. For example, employing a dynamic item-response model that adjusts for evolving standards of accountability, Fariss (2014) finds that the global average of latent physical integrity rights—such as protection from political imprisonment and arbitrary arrest—began to rise in the late 1980s. This trend corresponds closely with our finding that the salience of the human rights dimension increased just prior to the end of the Cold War.

While some scholars, including Cingranelli and Richards (1999), argue that the improvement in respect for human rights may have been short-lived or regionally uneven, such qualifications do not contradict our model's estimation that human rights became a more prominent and structurally significant issue in the UN General Assembly in the post-Cold War era. Rather, our results highlight a shift in the global agenda-setting dynamics, even if substantive improvements in rights practices varied across contexts.

4.3.2 Dimension 2: North–South Relations

The second dimension appears to capture tensions between the Global North and South. Salient terms like `israel_south` and `meet_needs` point toward debates in which developing countries have historically voiced concerns over unequal treatment and biased framing of resolutions—particularly those related to the Middle East. Terms such as `resources_meet`, `offer_necessary`, and `possible_offer` further suggest themes of development assistance, economic justice, and negotiations over aid or compromise. Taken together, this dimension likely reflects patterns of coalition-building and rhetorical alignment rooted in post-colonial divides.

While one might be tempted to interpret this dimension as reflecting Cold War dynamics, evidence from Figure 5—as well as further analysis below—indicates that Dimension 4 more directly captures the ideological split between former Communist countries and the rest of the world. This is especially clear from the ideological vector plot, where only the USSR and its satellite states appear in red (i.e., opposite in direction to the United States). Thus, it is more appropriate to interpret Dimension 2 as representing North–South relations, rather than Cold War conflict.

4.3.3 Dimension 3: Nuclear Disarmament

The third dimension revolves around nuclear disarmament and related security concerns. Keywords such as `disarmament_general`, `reducing_nuclear`, `nuclear_danger`, and `danger_resolution` highlight the centrality of nuclear risk reduction. Additional terms like `legality` and `legality_threat` indicate legal debates surrounding the legitimacy of nuclear weapons use or possession. The presence of `arrangements_strengthening` and `reducing` suggests a technocratic strand within the disarmament discourse. Overall, this dimension captures a global normative cleavage on the role of nuclear weapons.

It is also substantively interesting that Dimension 3—the nuclear disarmament dimension—is the only one among the six salience dimensions where the United States is positioned in the opposite ideological direction from its European allies. As shown in Figure 5, in Dimensions 1, 2, 4, 5, and 6, both the United States and its European allies are shaded blue, indicating alignment. However, in Dimension 3, the United States remains blue while its European allies appear in red, signaling a divergence in voting behavior or ideological stance. This suggests that nuclear disarmament has been a distinct axis of disagreement even among otherwise closely aligned Western powers.

This divergence may also reflect deeper underlying trends that have contributed to the growing tensions between the United States and its European allies in recent years. Future research could benefit from incorporating more recent UNGA voting data to examine whether Dimension 3—the nuclear disarmament dimension—has gained or lost salience following the election of Donald Trump as U.S. president. Such analysis could shed light on whether this dimension persists as a site of transatlantic disagreement or whether it has been supplanted by emerging ideological divides, or new salience dimensions discovered by the model, in the evolving international order.

4.3.4 Dimension 4: Cold War Politics

As briefly discussed above, the visualization of the fourth element of the ideological vectors reveals a striking pattern: all countries—interestingly, including China—appear in blue, except for the USSR and its satellite states, which appear in red. This indicates a strongly isolating voting behavior on the part of the Soviet bloc, uniquely captured by this dimension.

From the distinguishing terms identified by the distributed multinomial regression model—such as `development_organization`, `budget_appropriations`, `appropriations_year`, and `biennium_amount`—we infer that Dimension 4 encodes the institutional and budgetary tensions emblematic of Cold War politics. These terms suggest bureaucratic and financial disputes within the UN system. However, the dimension also includes more overtly political terms like expel, likely referring to efforts to exclude controversial states (e.g., apartheid-era South Africa or Israel), and nepal_amendment, which frequently served as procedural tools in bloc-based maneuvering. This suggests that Dimension 4 captures both the administrative and ideological divides that characterized the Cold War's imprint on UNGA proceedings.

4.3.5 Dimension 5: Yom Kippur War and Middle East Conflict

As shown in Figure 4, the salience of Dimension 5 follows a distinctly temporal trajectory—it rises sharply in the early 1970s, peaks in the first half of the decade, and then declines just as rapidly by the end of the 1970s. Notably, this surge in Dimension 5 corresponds with a marked decline in the salience of Dimension 2, suggesting a substitution effect. It appears that Dimension 5 temporarily supplanted Dimension 2 during this period, only to recede later, allowing Dimension 2 to reemerge. This shift suggests a brief but intense ideological realignment—likely triggered by the Yom Kippur War and its broader geopolitical ramifications—that temporarily reshaped the landscape of conflict within the UN General Assembly.

Substantively, Dimension 5 is saturated with terms such as condemn_israels, exercise_palestinian, and israel_racist, all of which point to heated debates surrounding the Israeli–Palestinian conflict in the wake of the Yom Kippur War. The recurrent presence of the term moratorium further suggests calls to halt specific Israeli actions—most likely military operations or settlement expansion. Taken together, these patterns highlight how moments of heightened regional tension are encoded in the UNGA's voting structure through distinct, time-bound ideological dimensions.

4.3.6 Dimension 6: UN Membership and Institutional Questions

Finally, the sixth dimension appears to capture early debates surrounding membership, participation, and procedural legitimacy within the UN system. It is primarily active during the organization's formative years. Salient terms such as nepal_amendment, council_admission, palestine_question, and adopt_trusteeship point to discussions about state recognition and post-colonial administration, particularly in relation to Palestine and newly independent nations. The presence of terms like approved_th and reject underscores the procedural nature of these votes, often marked by contention over legitimacy and representation. While weapons_testing might seem thematically out of place, it may connect to nuclear issues tied to decolonization-era security concerns. Overall, this dimension reflects the early salience of institutional legitimacy and recognition in the post-war global order.

4.4 Interpretation of the Estimated Ideological Regimes

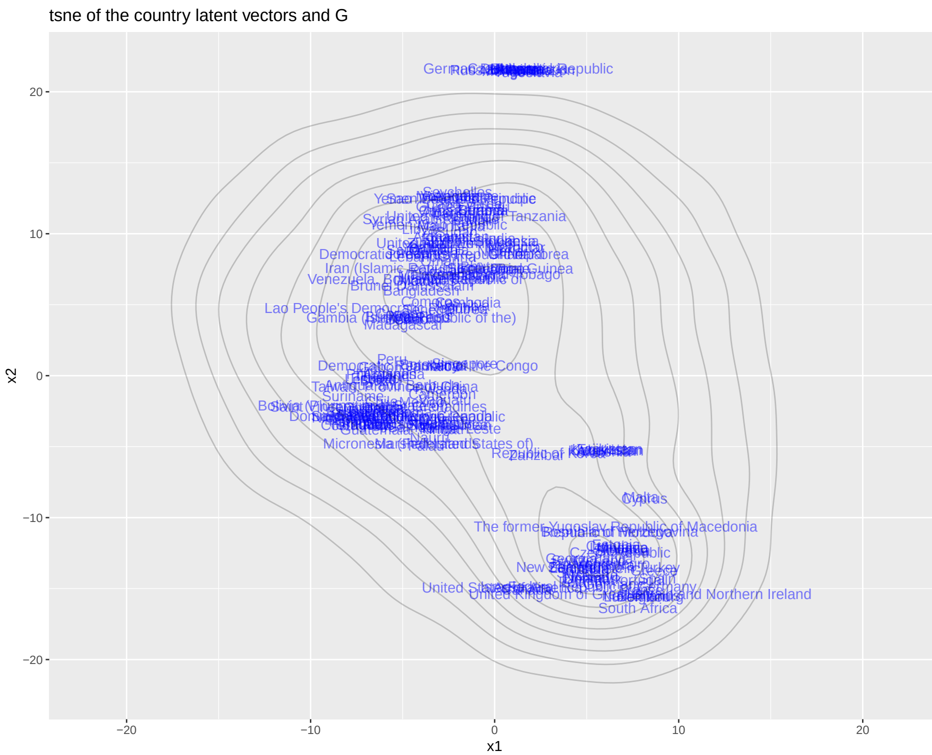

Figure 6: Tsne of Ideological Vectors and G

The model effectively identified three ideological regimes—Regimes 2, 4, and 8—from which countries' ideological vectors are sampled. It is important to note that the numerical labels of these regimes are arbitrary and arise purely from computational indexing; they do not carry inherent substantive meaning. Therefore, understanding each regime requires examining the actual country affiliations.

Regime 2 clearly represents the Global South, as it is characterized by strong affiliation among developing countries. In contrast, Regimes 4 and 8 are associated with developed countries. Among these, developed countries generally exhibit higher affiliation with Regime 4, while Regime 8 appears to function as a residual cluster. Notably, countries such as the United States, Canada, South Africa, and Australia show the highest affiliation with Regime 4.

The presence of Regime 8—composed of countries traditionally aligned with the U.S. but with a core led by European states—suggests an emerging divergence in voting behavior. This may reflect a growing ideological gap between the United States and its European allies, with Canada, South Africa, and Australia aligning more closely with the U.S. pattern, potentially indicating the early signs of a transatlantic split in foreign policy.

Also, it is important to note that the model does not assign a separate ideological regime for the USSR/Russia and its satellite countries. This suggests that, despite their distinct voting patterns—often in opposition to Western countries—their behavior is not sufficiently distinct across all dimensions to warrant a standalone regime. Instead, these countries may share enough ideological proximity with other blocs, such as the Global South or non-aligned states, to be grouped under broader regimes. Alternatively, their uniqueness may be captured more effectively through specific dimensions (such as Dimension 4, which isolates former Communist countries) rather than through regime affiliation. This highlights the model's flexibility in distinguishing between enduring ideological alignments (captured by regimes) and more transient or thematic oppositions (captured by dimensions).

To gain a more holistic understanding of the regime affiliations—or ideological clustering—of each country, we employed t-SNE, a dimensionality reduction technique proposed by van der Maaten and Hinton (2008), to project the ideological vectors and the distribution G (from which each country's ideological vector is drawn, as described in the "Ideological Vectors" subsection) onto a two-dimensional space. Specifically, we use the posterior mean of each country's ideological vector and draw 1,000 samples from G, applying t-SNE for visualization. In the resulting plot, country names indicate the location of each country's ideological position, while contour lines illustrate the density of samples from G.

As shown in Figure 6, the distribution G exhibits two prominent peaks, reinforcing the interpretation that Regimes 2 and 4 capture the dominant clustering structures, while Regime 8 acts more as a residual category. Developed countries tend to cluster in the southeastern region of the plot, while developing countries are concentrated in the northwest. Although many nuanced observations can be made from the t-SNE visualization—for example, the United States is slightly offset from the core group of developed countries—the most striking pattern is that the USSR/Russia and its satellite states are located in the far northern region of the plot, well outside the contour lines representing G. This spatial separation implies that their voting behavior differs in a *statistically significant* way from that of most other countries, even though we do not conduct a formal statistical test here.

4.5 Evolution of Revealed Ideological Proximity to the United States

As briefly discussed in the Methodology and Broader Implications section, the assumption of static, time-invariant ideological vectors may initially seem problematic, but the model allows us to define the *revealed ideological vector* of country S in year Y denoted as $\hat{\theta_{Y,S}} = \{\hat{\theta_{Y,S,d}}\}_{d=1}^{\infty}$, to be

$$\hat{\theta_{Y,S,d}} = \theta_{S,d} \cdot \rho_{Y,d},$$

which simply represents the elements of the country's ideological vector weighted by the corresponding year's salience vector. This formulation captures how a country's fixed ideological position interacts with the shifting importance of different political dimensions over time.

Here, we compute and visualize the cosine similarity between the *revealed ideological vector* of the United States and those of Australia, China, France, Japan, Russia/USSR, and the United Kingdom.

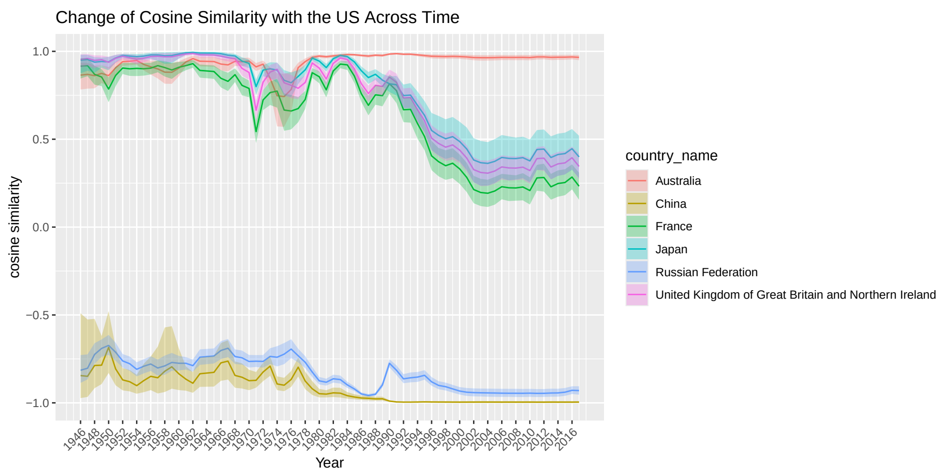

Figure 7: Cosine Similarity with the United States

Some might question how we are able to compute the complete time series of cosine similarity of the *revealed ideological vectors* between the United States and countries like the People's Republic of China and Japan—given that these countries were not members of the UNGA from the very beginning. However, this highlights the strength of the model's structure: the assumption of static ideological vectors combined with time-varying salience vectors.

The rationale is straightforward. Although China and Japan were not allowed to participate in the early sessions of the UNGA, the model still allows us to estimate their *revealed ideological vectors* for those periods. This is because most ideological dimensions—except for Dimension 5—persist over long stretches of time. For instance, if Country A is admitted in 1980, and a particular ideological dimension is present from 1940 to 2000, then data from 1980 to 2000 is informative in determining the value of that dimension for Country A. If the available information is insufficient, this will be reflected in the posterior credible interval, as the model is Bayesian in nature.

Further, the hierarchical structure of the model enables it to borrow statistical strength from countries that share the same ideological regime. As a result, even in the absence of direct observational data from early periods, the model can infer the ideological stance of these countries by leveraging their regime affiliations.

Therefore, once the salience vector for a given year is known, we can theoretically compute the *revealed ideological vector* for any country, regardless of whether it had formal UNGA membership at that time. The benefit of such a model structure is not only that it enables counterfactual simulations, but it also allows for a more comprehensive understanding of global political dynamics. This approach makes it possible to evaluate the ideological or preference positioning of actors even before they formally become members of a legislative body. It also holds strong potential for application in domestic legislative contexts, where legislators are more frequently removed and re-elected compared to the relatively stable membership of the UNGA.

From Figure 7, we observe that the cosine similarity of the *revealed ideological vectors* between the United States and the USSR/Russia was relatively high during the early stages of the Cold War. This similarity declined notably after the end of détente, particularly around the time of the Soviet intervention in Afghanistan, and began to rise again following the Cold War's conclusion.

While cross-model comparisons should be made cautiously—given differences in model structures and the specific quantities used to compute ideological similarity—our model successfully captures some of the major shifts in U.S.–USSR/Russian voting alignment also reported in previous work. For instance, while the model proposed by Bailey, Anton Strezhnev, and Voeten (2017) detects an earlier rise in similarity beginning in the 1980s, our model does not observe such an early upward trend. However, our model outperforms the S-score (also analyzed by Bailey, Anton Strezhnev, and Voeten (2017)), which only reflects increased similarity beginning in 1991.

Interestingly, both our model and the S-score indicate that similarity between the United States and the USSR/Russia was relatively high during the Cold War—contrasting with Bailey, Anton Strezhnev, and Voeten (2017)'s model, which shows lower alignment during this period. Moreover, while Bailey, Anton Strezhnev, and Voeten (2017)'s model presents a steady increase in similarity through the 1990s and beyond, our model reveals a different trend: an initial post–Cold War convergence followed by a decline in similarity from the late 1990s.

This divergence might be attributed to a fundamental difference in model structure: our model allows for multiple ideological dimensions, in contrast to the single-dimensional framework used by Bailey, Anton Strezhnev, and Voeten (2017).

For the People's Republic of China, as discussed earlier, it was not admitted to the UNGA from the outset. Nevertheless, we are still able to estimate its ideological similarity with the United States using the *revealed ideological vector*. Prior to the People's Republic of China's official admission—when it replaced the Republic of China as the recognized representative of China—the model's estimates are accompanied by wider credible intervals, reflecting increased uncertainty due to the absence of direct voting data. However, the available information is still sufficient to infer that its ideological similarity with the United States was negative even before formal participation began. The upper bound of the credible interval never exceeds -0.4.

From the 1990s onward, the cosine similarity between the United States and China becomes sharply negative, with values approaching minus one. This indicates that their *revealed ideological vectors* are nearly diametrically opposed in the latent ideological space, signaling a profound divergence in their voting behavior and underlying preferences.

The shift in cosine similarity between the United States and its close allies—Australia, France, Japan, and the United Kingdom—reveals an intriguing pattern. After the end of the Cold War, ideological alignment—as captured by the cosine similarity of their *revealed ideological vectors*—began to decline for most of these countries. Australia, however, stands out as an exception, consistently maintaining a similarity score close to one, indicating a remarkably high degree of ideological congruence with the United States. This suggests that while the post–Cold War era brought greater ideological diversification among traditional Western allies—as also observed in the context of the Nuclear Disarmament dimension—Australia continued to closely align with U.S. positions across the most salient ideological dimensions.

5 Conclusion

This paper introduces a novel nonparametric Bayesian model for uncovering the dynamic ideological structure underlying voting behavior in the United Nations General Assembly. By separating time-invariant ideological positions from time-varying salience of issue dimensions, the model offers a new lens through which to understand both long-run ideological alignment and short-term shifts in global political conflict. Crucially, the model allows for infinite-dimensional ideological spaces, accommodates missing data naturally, and enables estimation of *revealed ideological vectors* even for countries not yet admitted to the UNGA.

The empirical findings contribute to several long-standing debates in international relations. First, we show that Cold War ideological conflict was not the only salient global cleavage during the mid-20th century. North–South relations—rooted in postcolonial contestation and institutional marginalization—were equally, if not more, salient during key historical periods. Second, we find that human rights emerged as a durable ideological dimension only toward the end of the Cold War, aligning with existing accounts of normative diffusion and the rise of liberal internationalism in the post–Cold War order. Third, the model captures a subtle but important fragmentation within Western alliance structures: while the United States remained closely aligned with countries like Australia across most dimensions, its ideological distance from France, the United Kingdom, Japan, and other traditional Western allies grew in the post–Cold War era—particularly along the nuclear disarmament dimension. This divergence highlights that transatlantic and intra-allied ideological cohesion cannot be taken for granted, and may be contingent on shifting issue salience and normative priorities.

Methodologically, the proposed model offers a flexible and interpretable framework for analyzing high-dimensional preference spaces in roll-call voting contexts. Unlike standard ideal point models, it accounts for the reality that the importance of different ideological dimensions shifts over time in response to changes in the international agenda. Although the model assumes static ideological vectors—a feature that may initially appear unrealistic or restrictive—it compensates for this by introducing time-varying salience. This allows the model to generate *revealed ideological vectors* that reflect how a country's preferences manifest under the prevailing issue context of each period. As a result, it becomes possible to track ideological similarity between countries even during periods of non-membership, abstention, or partial participation. Additionally, the use of t-SNE for visualization and Dirichlet process clustering enables the identification of latent geopolitical blocs without requiring prior assumptions about their number or composition.

Looking forward, this framework opens up several promising avenues for future research. The model can be extended to other international organizations, domestic legislatures, or issue-specific subsets (e.g., climate, security, trade). Moreover, incorporating textual data into the resolution embeddings may further enhance interpretability. Finally, the framework can support counterfactual simulations, such as how a country would have voted in a resolution it did not participate in, or how ideological alignment might evolve under shifting salience trajectories.

In sum, this paper contributes to political science by bridging the gap between methodological innovation and substantive insight. It offers a scalable framework for understanding the evolving structure of global ideological conflict—and challenges the notion that such conflict can be meaningfully reduced to a single left–right or pro–anti bloc. Instead, it reveals a more nuanced, multi-dimensional landscape shaped by changing agendas, historical ruptures, and shifting coalitions.

6 Appendix

6.1 Finiteness and Summability of μ

In the Appendix, we focus on the issue of whether the $\mu_{S,R}$, which is constructed as the infinite summation of the sequence $\{\theta_{S,d}\beta_{R,d}\rho_{year_R,d}\}_{d=1}^{\infty}$, if well-defined ans finite.

As already explained in Section 3, the sequence $\mu_{S,R}$ as follows:

$$\mu_{S,R} = \sum_{d=1}^{\infty} \theta_{S,d}\beta_{R,d}\rho_{year_R,d}$$

where $\theta_{S,d}$, $\beta_{R,d}$, and $\rho_{year_R,d}$ represent the elements of the ideological vector, the elements of the resolution vector, and the weights in the salience vector constructed by stick-breaking process, respectively.

The goal of this proof is to demonstrate that the infinite sum defining μ is well-defined and summable. The argument proceeds in three main steps:

1. **Resolution Vectors**: We begin by addressing the resolution vectors $\beta_{R,d}$, which are independently drawn from standard normal distributions. Since the standard normal distribution is known to have finite variance, this guarantees that the resolution vectors

exhibit controlled behavior in expectation. This step does not require further elaboration, as the properties of the normal distribution are well-established.

2. **Ideological Vectors**: The primary focus is on the ideological vectors $\theta_{S,d}$, which are derived from an infinite mixture of normal distributions. To ensure the well-definedness of the infinite sum, we must demonstrate that these vectors have finite variance. Specifically, we aim to show that the contributions of $\theta_{S,d}$ decay sufficiently quickly as d $\to \infty$, thus ensuring that the sum converges.

3. **Salience Vectors**: Finally, we address the salience vectors $\rho_{year_R,d}$, which follow a stick-breaking process in the first year and a hierarchical stick-breaking process in subsequent years. We demonstrate that the elements of this sequence—both for the initial year, constructed using a stick-breaking process, and for subsequent years, modeled via a hierarchical stick-breaking process—decay at a sufficiently fast rate to guarantee the convergence of the infinite sum. This step is essential in ensuring that the salience vectors do not introduce instability, thereby maintaining the summability of μ.

By addressing these three components—resolution vectors, ideological vectors, and salience vectors—we establish the sufficient conditions for the well-definedness and summability of the infinite sum μ.

Since $\theta_{S,d}$ and $\beta_{R,d}$ originate from an infinite mixture of normals and a normal distribution, respectively, and have finite variance, their contributions remain controlled in expectation. Therefore, to ensure the well-definedness of the infinite sum, it is sufficient to demonstrate that $\rho_{year_R,d}$ converges to zero at a sufficiently fast rate as d $\to \infty$, ensuring absolute summability.

First, since both the construction of ideological vectors and salience vectors are governed by stick-breaking processes, we demonstrate that the elements of these vectors, derived from the stick-breaking process, decay exponentially fast.

6.1.1 Proposition 1: Exponential Decay of Stick-Breaking Weights

As introduced in the prior distribution part, typical stick-breaking process generates a sequence ρ_d as follows:

$$\rho_1 = v_1, \quad \rho_d = v_d \prod_{l=1}^{d-1}(1 - v_l), \quad d \geq 2$$

where $v_d \sim Beta(1, \alpha)$ independently.

Taking expectations, we examine:

$$\mathbb{E}[\rho_d] = \mathbb{E}\left[v_d \prod_{l=1}^{d-1}(1 - v_l)\right].$$

Since the v_d are independently sampled, the expectation of the product can be decomposed into the product of individual expectations, allowing us to analyze each term separately.

$$\mathbb{E}\left[v_d \prod_{l=1}^{d-1}(1-v_l)\right] = \mathbb{E}\left[v_d\right] \prod_{l=1}^{d-1} \mathbb{E}\left[1-v_l\right]$$

Since $v_d \sim Beta(1, \alpha)$, its expectation is:

$$\mathbb{E}[v_d] = \frac{1}{1+\alpha}.$$

For d $\geq$ 2,

$$\mathbb{E}[1-v_l] = \frac{\alpha}{1+\alpha}.$$

So,

$$\prod_{l=1}^{d-1} \mathbb{E}[1-v_l] = \left(\frac{\alpha}{1+\alpha}\right)^{d-1}.$$

Thus,

$$\mathbb{E}[\rho_d] = \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^{d-1}.$$

This demonstrates that the sequence exhibits exponential decay at a rate of $\frac{\alpha}{1+\alpha}$, which lies strictly between zero and one for any positive and finite α.

□

6.1.2 Proposition 2: Finiteness of the Variance of Ideological Vectors

As already shown, $\theta_{S,d}$ are the ideological vector components, drawn from a Dirichlet process mixture of normal distributions, which can be written in the following way:

$$\theta_{S,d} \sim \sum_{k=1}^{\infty} p_k Normal(\bar{\theta}_k, \sigma_k^2)$$

As previously established, the components of the ideological vector, $\theta_{S,d}$, are drawn from a Dirichlet process mixture of normal distributions, which can be expressed as follows:

To establish the finiteness of $\mathrm{Var}(\theta_{S,d})$, we observe that, as shown above, the distribution can be represented as an infinite summation of normally distributed random variables. Since each mixture component has finite variance, denoted as $\bar{\sigma}_k^2$, and the expectation of the total variance under the Dirichlet process prior is given by:

$$\mathbb{E}[\mathrm{Var}(\theta_{S,d})] = \sum_{k=1}^{\infty} p_k^2 \bar{\sigma}_k^2.$$

Since the expectation of p_k^2 decays quadratically due to the stick-breaking construction as shown in as shown in Proposition 1, and σ_k^2 is finite due to its prior distribution of $Gamma(0.001, 0.001)$, this sum converges and is finite.

□

6.1.3 Proposition 3: Exponential Decay of Hierarchical Stick-Breaking Weights

As shown in Section 3.3, for the second year, the salience vectors are constructed as

$$\delta_{2,d} \sim \mathrm{Beta}\left(\zeta\rho_{1,d}, \zeta\left(1 - \sum_{l=1}^{d} \rho_{1,l}\right)\right)$$

$$\rho_{2,d} = \delta_{2,d} \prod_{l=1}^{d-1} (1 - \delta_{2,l}).$$

As in Proposition 1, the expectation of $\rho_{2,d}$ can be expressed as

$$\mathbb{E}[\rho_{2,d}] = \mathbb{E}\left[\delta_{2,d} \prod_{l=1}^{d-1} (1 - \delta_{2,l})\right].$$

Since the variables $\delta_{2,d}$ are sampled independently, we can evaluate the expectation term by term:

$$\mathbb{E}\left[\delta_{2,d} \prod_{l=1}^{d-1} (1 - \delta_{2,l})\right] = \mathbb{E}\left[\delta_{2,d}\right] \prod_{l=1}^{d-1} \mathbb{E}\left[1 - \delta_{2,l}\right].$$

For a Beta-distributed variable $\delta_{2,d} \sim \mathrm{Beta}\left(\zeta\rho_{1,d}, \zeta\left(1 - \sum_{l=1}^{d} \rho_{1,l}\right)\right)$, its expectation is given by:

$$\mathbb{E}[\delta_{2,d}] = \frac{\zeta\rho_{1,d}}{\zeta\rho_{1,d} + \zeta\left(1 - \sum_{l=1}^{d} \rho_{1,l}\right)}.$$

Simplifying, we obtain

$$\mathbb{E}[\delta_{2,d}] = \frac{\rho_{1,d}}{1 - \sum_{l=1}^{d-1} \rho_{1,l}}.$$

Similarly, for the expectation of $1 - \delta_{2,l}$, we have:

$$\mathbb{E}[1 - \delta_{2,l}] = 1 - \mathbb{E}[\delta_{2,l}].$$

Since $\rho_{1,d}$ is constructed from a stick-breaking process, from Proposition 1, its expectation is:

$$\mathbb{E}[\rho_{1,d}] = \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^{d-1}.$$

Substituting this into our previous expression, we obtain:

$$\mathbb{E}[\delta_{2,d}] = \frac{\frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^{d-1}}{1 - \sum_{l=1}^{d-1} \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^{l-1}}.$$

Using the geometric series formula,

$$\sum_{l=1}^{d-1} \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^{l-1} = 1 - \left(\frac{\alpha}{1+\alpha}\right)^{d-1},$$

we obtain

$$\mathbb{E}[\delta_{2,d}] = \frac{1}{1+\alpha},$$

which is of the same form as in the non-hierarchical stick-breaking weights.

Thus, as in Proposition 1, for $d \geq 2$,

$$\mathbb{E}[1 - \delta_{2,d}] = \frac{\alpha}{1+\alpha}.$$

So,

$$\prod_{l=1}^{d-1} \mathbb{E}[1 - \delta_{2,d}] = \left(\frac{\alpha}{1+\alpha}\right)^{d-1}.$$

Thus,

$$\mathbb{E}[\rho_{2,d}] = \frac{1}{1+\alpha} \left(\frac{\alpha}{1+\alpha}\right)^{d-1}.$$

This demonstrates that the sequence exhibits exponential decay at a rate of $\frac{\alpha}{1+\alpha}$, which lies strictly between zero and one for any positive and finite α.

The same result trivially applies to the subsequent years.

□

6.1.4 Conclusion: Well-Definedness and Summability of μ

Having established the necessary conditions for the finiteness and summability of the infinite sum defining $\mu_{S,R}$, we can now summarize the key results:

1. **Exponential Decay of Stick-Breaking Weights**: We demonstrated that the expectation of the stick-breaking weights $\rho_{year_R,d}$ exhibits exponential decay at a rate of $\frac{\alpha}{1+\alpha}$, ensuring that their contributions to the sum diminish sufficiently fast as $d \to \infty$.
2. **Finiteness of the Variance of Ideological Vectors**: The ideological vector components $\theta_{S,d}$ were shown to have finite variance, derived from a Dirichlet process mixture of normal distributions, ensuring that their contributions remain controlled in expectation.
3. **Hierarchical Stick-Breaking Process Stability**: The hierarchical stick-breaking construction of salience vectors for subsequent years retains the exponential decay property, ensuring that $\rho_{year_R,d}$ continues to diminish at a sufficient rate.

With these properties established, we conclude that the infinite sum defining $\mu_{S,R}$ is absolutely summable and well-defined. This guarantees the theoretical soundness of our modeling approach, allowing for stable inference and meaningful interpretations of the latent ideological and salience structures over time.

Bibliography

Bailey, Michael A., Anton Strezhnev, and Erik Voeten. 2017. "Estimating Dynamic State Preferences from United Nations Voting Data." *Journal of Conflict Resolution* 61 (2): 430–56.

Bartle, John, and Samantha Laycock. 2012. "Telling More Than They Can Know? Does the Most Important Issue Really Reveal What Is Most Important to Voters?" *Electoral Studies* 31 (4): 679–88.

Bhattacharya, Anirban, and David B Dunson. 2011. "Sparse Bayesian Infinite Factor Models." *Biometrika* 98 (2): 291–306.

Blei, David M, Alp Kucukelbir, and Jon D McAuliffe. 2017. "Variational Inference: A Review for Statisticians." *Journal of the American Statistical Association* 112 (518): 859–77.

Cingranelli, David L, and David L Richards. 1999. "Respect for Human Rights After the End of the Cold War." *Journal of Peace Research* 36 (5): 511–34.

Fariss, Christopher J. 2014. "Respect for Human Rights Has Improved over Time: Modeling the Changing Standard of Accountability." *American Political Science Review* 108 (2): 297–318.

Ghosal, Subhashis, and Aad W Van der Vaart. 2017. *Fundamentals of Nonparametric Bayesian Inference*. Vol. 44. Cambridge University Press.

Gopalan, Prem, Francisco J Ruiz, Rajesh Ranganath, and David Blei. 2014. "Bayesian Nonparametric Poisson Factorization for Recommendation Systems." In *Artificial Intelligence and Statistics*, 275–83. PMLR.

Grimmer, Justin. 2011. "An Introduction to Bayesian Inference via Variational Approximations." *Political Analysis* 19 (1): 32–47.

Kim, In Song, John Londregan, and Marc Ratkovic. 2018. "Estimating Spatial Preferences from Votes and Text." *Political Analysis* 26 (2): 210–29.

Kucukelbir, Alp, Dustin Tran, Rajesh Ranganath, Andrew Gelman, and David M Blei. 2017. "Automatic Differentiation Variational Inference." *Journal of Machine Learning Research* 18 (14): 1–45.

Martin, Andrew D, and Kevin M Quinn. 2002. "Dynamic Ideal Point Estimation via Markov Chain Monte Carlo for the US Supreme Court, 1953–1999." *Political Analysis* 10 (2): 134–53.

Meng, Xiao-Li. 1994. "Posterior Predictive p-Values." *The Annals of Statistics* 22 (3): 1142–60.

Navarro, Daniel J, Thomas L Griffiths, Mark Steyvers, and Michael D Lee. 2006. "Modeling Individual Differences Using Dirichlet Processes." *Journal of Mathematical Psychology* 50 (2): 101–22.

Poole, Keith T, and Howard Rosenthal. 1991. "Patterns of Congressional Voting." *American Journal of Political Science*, 228–78.

Russett, Bruce M. 1966. "Discovering Voting Groups in the United Nations." *American Political Science Review* 60 (2): 327–39.

Sethuraman, Jayaram. 1994. "A Constructive Definition of Dirichlet Priors." *Statistica Sinica*, 639–50.

Shiraito, Yuki, James Lo, and Santiago Olivella. 2023. "A Nonparametric Bayesian Model for Detecting Differential Item Functioning: An Application to Political Representation in the US." *Political Analysis* 31 (3): 430–47.

Spirling, Arthur, and Kevin Quinn. 2010. "Identifying Intraparty Voting Blocs in the UK House of Commons." *Journal of the American Statistical Association* 105 (490): 447–57.

Stan Development Team. 2024. *Stan Reference Manual, Version 2.34.1*. https://mc-stan.org.

Taddy, Matt. 2015. "Distributed multinomial regression." *The Annals of Applied Statistics* 9 (3): 1394–414. https://doi.org/10.1214/15-AOAS831.

Vafa, Keyon, Suresh Naidu, and David Blei. 2020. "Text-Based Ideal Points." In *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, edited by Dan Jurafsky, Joyce Chai, Natalie Schluter, and Joel Tetreault, 5345–57. Online: Association for Computational Linguistics.

Voeten, Erik. 2000. "Clashes in the Assembly." *International Organization* 54 (2): 185–215.

Voeten, Erik, Anton Strezhnev, and Michael Bailey. 2009. "United Nations General Assembly Voting Data." Harvard Dataverse. https://doi.org/10.7910/DVN/LEJUQZ.

Yee Whye Teh, Matthew J Beal, Michael I Jordan, and David M Blei. 2006. "Hierarchical Dirichlet Processes." *Journal of the American Statistical Association* 101 (476): 1566–81.