扩展卡尔曼滤波（EKF）仿真演示

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一、　问题描述

如图 1 所示，从空中水平抛射出的物体，初始水平速度 $v_x(0)$，初始位置坐标（$x(0), y(0)$）；受重力 g 和阻尼力影响，阻尼力与速度平方成正比，水平和垂直阻尼系数分别为 k_x, k_y；还存在不确定的零均值白噪声干扰力 δa_x 和 δa_y。在坐标原点处有一观测设备（不妨想象成雷达），可测得距离 r（零均值白噪声误差 δr）、角度 α（零均值白噪声误差 $\delta\alpha$）。

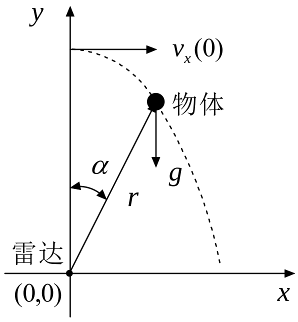

图 1　雷达观测示意图

二、　建模

系统方程： $\boldsymbol{f}$:

$$\begin{cases} \dot{x} = v_x \\ \dot{v}_x = -k_x v_x^2 + \delta a_x \\ \dot{y} = v_y \\ \dot{v}_y = k_y v_y^2 - g + \delta a_y \end{cases}$$

量测方程： $\boldsymbol{h}$:

$$\begin{cases} r = \sqrt{x^2 + y^2} + \delta r \\ \alpha = \mathrm{a}\tan(x/y) + \delta\alpha \end{cases}$$

选状态向量 $\boldsymbol{x} = \begin{bmatrix} x & v_x & y & v_y \end{bmatrix}^{\mathrm{T}}$, 量测向量 $\boldsymbol{z} = \begin{bmatrix} r & \alpha \end{bmatrix}^{\mathrm{T}}$

系统 Jacobian 矩阵 $\dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2k_x v_x & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2k_y v_y \end{bmatrix}$

?

量测 Jacobian 矩阵 $\dfrac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}} = \begin{bmatrix} \dfrac{1}{\sqrt{x^2+y^2}} & 0 & \dfrac{1}{\sqrt{x^2+y^2}} & 0 \\ \dfrac{1/y}{1+(x/y)^2} & 0 & \dfrac{-x/y^2}{1+(x/y)^2} & 0 \end{bmatrix}$

三、　Matlab 仿真

```
function test_ekf
    kx = .01; ky = .05; % 阻尼系数
    g = 9.8; % 重力
    t = 10; % 仿真时间
    Ts = 0.1; % 采样周期
    len = fix(t/Ts); % 仿真步数
```

```
    % 真实轨迹模拟
    dax = 1.5; day = 1.5;   % 系统噪声
    X = zeros(len,4); X(1,:) = [0, 50, 500, 0]; % 状态模拟的初值
    for k=2:len
        x = X(k-1,1); vx = X(k-1,2); y = X(k-1,3); vy = X(k-1,4);
        x = x + vx*Ts;
        vx = vx + (-kx*vx^2+dax*randn(1,1))*Ts;
        y = y + vy*Ts;
        vy = vy + (ky*vy^2-g+day*randn(1))*Ts;
        X(k,:) = [x, vx, y, vy];
    end
    figure(1), hold off, plot(X(:,1),X(:,3),'-b'), grid on
%       figure(2), plot(X(:,2:2:4))
    % 构造量测量
    mrad = 0.001;
    dr = 10; dafa = 10*mrad; % 量测噪声
    for k=1:len
        r = sqrt(X(k,1)^2+X(k,3)^2) + dr*randn(1,1);
        a = atan(X(k,1)/X(k,3)) + dafa*randn(1,1);
        Z(k,:) = [r, a];
    end
    figure(1), hold on, plot(Z(:,1).*sin(Z(:,2)), Z(:,1).*cos(Z(:,2)),'*')
    % ekf 滤波
    Qk = diag([0; dax; 0; day])^2;
    Rk = diag([dr; dafa])^2;
    Xk = zeros(4,1);
    Pk = 100*eye(4);
    X_est = X;
    for k=1:len
        Ft = JacobianF(X(k,:), kx, ky, g);
        Hk = JacobianH(X(k,:));
        fX = fff(X(k,:), kx, ky, g, Ts);
        hfX = hhh(fX, Ts);
        [Xk, Pk, Kk] = ekf(eye(4)+Ft*Ts, Qk, fX, Pk, Hk, Rk, Z(k,:)'-hfX);
        X_est(k,:) = Xk';
    end
    figure(1), plot(X_est(:,1),X_est(:,3), '+r')
    xlabel('X'); ylabel('Y'); title('ekf simulation');
    legend('real', 'measurement', 'ekf estimated');

%%%%%%%%%%%%%%%%%%%%%子程序%%%%%%%%%%%%%%%%%%%%%
function F = JacobianF(X, kx, ky, g) % 系统状态雅可比函数
    vx = X(2); vy = X(4);
    F = zeros(4,4);
    F(1,2) = 1;
    F(2,2) = -2*kx*vx;
    F(3,4) = 1;
    F(4,4) = 2*ky*vy;

function H = JacobianH(X) % 量测雅可比函数
    x = X(1); y = X(3);
    H = zeros(2,4);
    r = sqrt(x^2+y^2);
    H(1,1) = 1/r; H(1,3) = 1/r;
    xy2 = 1+(x/y)^2;
    H(2,1) = 1/xy2*1/y; H(2,3) = 1/xy2*x*(-1/y^2);
```

```
function fX = fff(X, kx, ky, g, Ts) % 系统状态非线性函数
    x = X(1); vx = X(2); y = X(3); vy = X(4);
    x1 = x + vx*Ts;
    vx1 = vx + (-kx*vx^2)*Ts;
    y1 = y + vy*Ts;
    vy1 = vy + (ky*vy^2-g)*Ts;
    fX = [x1; vx1; y1; vy1];

function hfX = hhh(fX, Ts) % 量测非线性函数
    x = fX(1); y = fX(3);
    r = sqrt(x^2+y^2);
    a = atan(x/y);
    hfX = [r; a];

function [Xk, Pk, Kk] = ekf(Phikk_1, Qk, fXk_1, Pk_1, Hk, Rk, Zk_hfX) % ekf 滤波函数
    Pkk_1 = Phikk_1*Pk_1*Phikk_1' + Qk;
    Pxz = Pkk_1*Hk';      Pzz = Hk*Pxz + Rk;      Kk = Pxz*Pzz^-1;
    Xk = fXk_1 + Kk*Zk_hfX;
    Pk = Pkk_1 - Kk*Pzz*Kk';
```

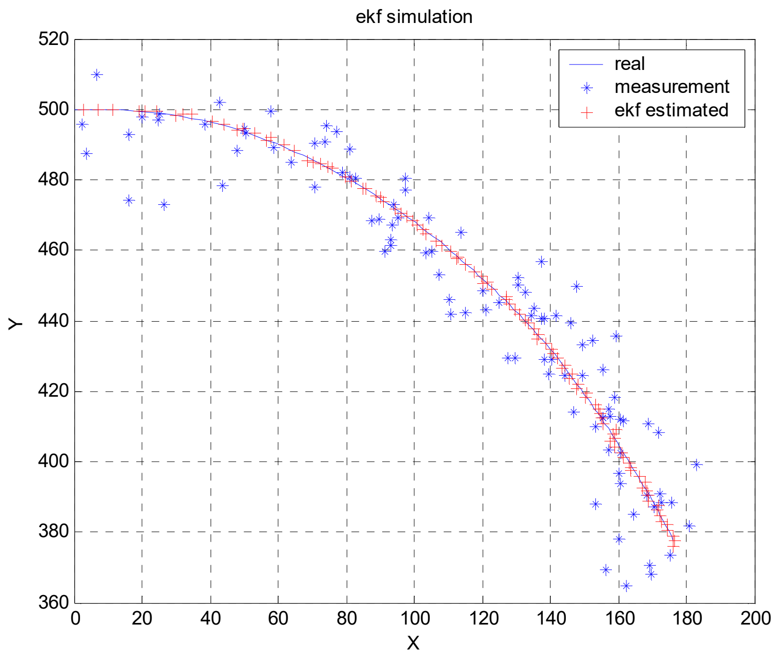

图 2　仿真结果