

扩展卡尔曼滤波（EKF）仿真演示

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一、 问题描述

如图 1 所示，从空中水平抛射出的物体，初始水平速度 $v_x(0)$ ，初始位置坐标 $(x(0), y(0))$ ；受重力 g 和阻尼力影响，阻尼力与速度平方成正比，水平和垂直阻尼系数分别为 k_x, k_y ；还存在不确定的零均值白噪声干扰力 δa_x 和 δa_y 。在坐标原点处有一观测设备（不妨想象成雷达），可测得距离 r （零均值白噪声误差 δr ）、角度 α （零均值白噪声误差 $\delta \alpha$ ）。

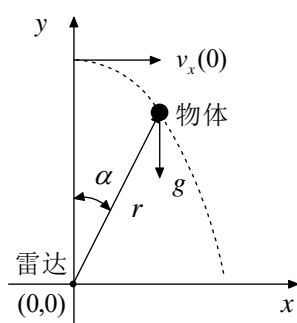


图 1 雷达观测示意图

二、 建模

$$\text{系统方程: } f: \begin{cases} \dot{x} = v_x \\ \dot{v}_x = -k_x v_x^2 + \delta a_x \\ \dot{y} = v_y \\ \dot{v}_y = k_y v_y^2 - g + \delta a_y \end{cases}$$

$$\text{量测方程: } h: \begin{cases} r = \sqrt{x^2 + y^2} + \delta r \\ \alpha = \arctan(x/y) + \delta \alpha \end{cases}$$

选状态向量 $\mathbf{x} = [x \ v_x \ y \ v_y]^T$ ，量测向量 $\mathbf{z} = [r \ \alpha]^T$

$$\text{系统 Jacobian 矩阵 } \frac{\partial f}{\partial \mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2k_x v_x & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2k_y v_y \end{bmatrix}$$
$$\text{量测 Jacobian 矩阵 } \frac{\partial h}{\partial \mathbf{x}} = \begin{bmatrix} \frac{1}{\sqrt{x^2 + y^2}} & 0 & \frac{1}{\sqrt{x^2 + y^2}} & 0 \\ \frac{1/y}{1 + (x/y)^2} & 0 & \frac{-x/y^2}{1 + (x/y)^2} & 0 \end{bmatrix}$$

三、 Matlab 仿真

```
function test_ekf
    kx = .01; ky = .05; % 阻尼系数
    g = 9.8; % 重力
    t = 10; % 仿真时间
    Ts = 0.1; % 采样周期
    len = fix(t/Ts); % 仿真步数
```

```

% 真实轨迹模拟
dax = 1.5; day = 1.5; % 系统噪声
X = zeros(len,4); X(1,:) = [0, 50, 500, 0]; % 状态模拟的初值
for k=2:len
    x = X(k-1,1); vx = X(k-1,2); y = X(k-1,3); vy = X(k-1,4);
    x = x + vx*Ts;
    vx = vx + (-kx*vx^2+dax*randn(1,1))*Ts;
    y = y + vy*Ts;
    vy = vy + (ky*vy^2-g+day*randn(1,1))*Ts;
    X(k,:) = [x, vx, y, vy];
end
figure(1), hold off, plot(X(:,1),X(:,3),'-b'), grid on
% 构造量测量
mrad = 0.001;
dr = 10; dafa = 10*mrad; % 量测噪声
for k=1:len
    r = sqrt(X(k,1)^2+X(k,3)^2) + dr*randn(1,1);
    a = atan(X(k,1)/X(k,3)) + dafa*randn(1,1);
    Z(k,:) = [r, a];
end
figure(1), hold on, plot(Z(:,1).*sin(Z(:,2)), Z(:,1).*cos(Z(:,2)),'*')
% ekf 滤波
Qk = diag([0; dax; 0; day])^2;
Rk = diag([dr; dafa])^2;
Xk = zeros(4,1);
Pk = 100*eye(4);
X_est = X;
for k=1:len
    Ft = JacobianF(X(k,:), kx, ky, g);
    Hk = JacobianH(X(k,:));
    fX = fff(X(k,:), kx, ky, g, Ts);
    hfX = hhh(fX, Ts);
    [Xk, Pk, Kk] = ekf(eye(4)+Ft*Ts, Qk, fX, Pk, Hk, Rk, Z(k,:)-hfX);
    X_est(k,:) = Xk';
end
figure(1), plot(X_est(:,1),X_est(:,3), '+r')
xlabel('X'); ylabel('Y'); title('ekf simulation');
legend('real', 'measurement', 'ekf estimated');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function F = JacobianF(X, kx, ky, g) % 系统状态雅可比函数
    vx = X(2); vy = X(4);
    F = zeros(4,4);
    F(1,2) = 1;
    F(2,2) = -2*kx*vx;
    F(3,4) = 1;
    F(4,4) = 2*ky*vy;

function H = JacobianH(X) % 量测雅可比函数
    x = X(1); y = X(3);
    H = zeros(2,4);
    r = sqrt(x^2+y^2);
    H(1,1) = 1/r; H(1,3) = 1/r;
    xy2 = 1+(x/y)^2;
    H(2,1) = 1/xy2*1/y; H(2,3) = 1/xy2*x*(-1/y^2);

```

```
function fX = fff(X, kx, ky, g, Ts) % 系统状态非线性函数
```

```
    x = X(1); vx = X(2); y = X(3); vy = X(4);
```

```
    x1 = x + vx*Ts;
```

```
    vx1 = vx + (-kx*vx^2)*Ts;
```

```
    y1 = y + vy*Ts;
```

```
    vy1 = vy + (ky*vy^2-g)*Ts;
```

```
    fX = [x1; vx1; y1; vy1];
```

```
function hfX = hhh(fX, Ts) % 量测非线性函数
```

```
    x = fX(1); y = fX(3);
```

```
    r = sqrt(x^2+y^2);
```

```
    a = atan(x/y);
```

```
    hfX = [r; a];
```

```
function [Xk, Pk, Kk] = ekf(Phikk_1, Qk, fXk_1, Pk_1, Hk, Rk, Zk_hfX) % ekf 滤波函数
```

```
    Pkk_1 = Phikk_1*Pk_1*Phikk_1' + Qk;
```

```
    Pxz = Pkk_1*Hk';    Pzz = Hk*Pxz + Rk;    Kk = Pxz*Pzz^-1;
```

```
    Xk = fXk_1 + Kk*Zk_hfX;
```

```
    Pk = Pkk_1 - Kk*Pzz*Kk';
```

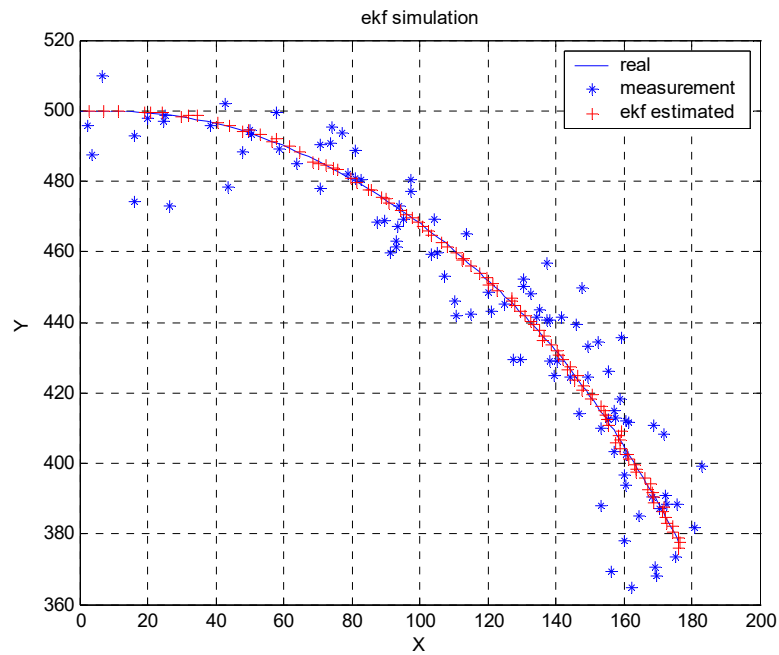


图2 仿真结果