A Joint Real Grassmannian Quantization Strategy for MIMO Interference Alignment with Limited Feedback

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***Abstract*—Interference alignment (IA) is a scheme to approach the capacity at high signal-to-noise ratio (SNR) in multiuser multiple-input multiple-output (MIMO) interference networks. To implement the IA scheme in a frequency-division duplexing (FDD) system, transmitter channel state information (CSIT) is fed back from the receiver with finite bits. However, such CSIT is subject to quantization errors and delays of feedback channels. In this paper, we verify that interference leakage is bounded by chordal distance in the MIMO channel. Besides, a joint real Grassmannian quantization strategy is proposed to reduce chordal distance to improve CSIT quality. Meanwhile, under the noise-limited criterion, the lower bound of the codebook size of our proposed strategy is much smaller than that of the conventional complex Grassmannian quantization strategy. Simulations demonstrate that our proposed strategy provides substantial performance gains compared with the conventional strategy.**

I. Introduction

Interference alignment (IA) illustrates that the capacity of the multiuser interference network is much higher than what was previously thought. Several notable results characterize the capacity region of various channels with degrees of freedom (DOF) [1][2], and several IA algorithms are presented [3]. However, perfect transmitter channel state information (CSIT) must be guaranteed to achieve DOF in interference networks.

In a frequency-division duplexing (FDD) system, CSIT is obtained at the receiver and fed back to the transmitter. However, delays of feedback channel render CSIT outdated and quantization errors contribute to the CSIT distortion [4]. Besides, high resolution CSIT incurs a huge codebook size which is impractical. In this paper, only the quantization error problem will be analyzed.

An IA scheme is applied to eliminate interference in interference networks. However, with a finite codebook size, inaccurate CSIT results in interference leakage. Even though with the finite codebook, if interference leakage is weak enough, quality of service (QoS) of interference networks can still be satisfied. So we raise the question: How large codebook size shall be to guarantee the QoS?

Many previous literatures have addressed the issue of IA with limited feedback. In the single-input single-output (SISO) frequency-selective channel, interference leakage is analyzed and the lower bound of feedback bits to achieve DOF is presented in [5]. An extended work is developed in the multiple-input multiple-output (MIMO) frequency-selective channel in [6]. An additional filter at the receiver is applied to reduce chordal distance in the MIMO channel in [7]. Results in [8] show that the mean loss of sum rate is bounded by average interference leakage and propose an analog CSIT feedback strategy to reduce overhead. Exploiting the temporary channel correlation, authors in [9] propose a differential Grassmannian quantization strategy to reduce feedback bits in the SISO frequency-selective channel. However, the relationship between interference leakage and chordal distance has not been presented in the MIMO channel. Besides, conventional strategies usually apply the complex Grassmannian quantization which achieves performance at the cost of the codebook size.

Firstly, in this paper, the noise-limited criterion is proposed to guarantee the QoS. Furthermore, we verify that interference leakage is bounded by chordal distance. A joint real Grassmannian quantization strategy is proposed to reduce chordal distance so that the codebook size decreases. Secondly, two conventional quantization strategies and the proposed strategy are analyzed in chordal distance, and our proposed strategy outperforms others under the same QoS. Finally, simulations show a narrow performance gap between IA using our proposed strategy and IA with perfect CSIT. Besides, compared with conventional strategies, the proposed strategy achieves a significant sum rate gain at moderate and high SNR.

Notation: Capital and small bold letters represent matrices and vectors; $\mathbf{A}^H$ and $\mathbf{A}^T$ stand for conjugate transpose and transpose of $\mathbf{A}$ respectively; $\mathbf{a}^*$ represents the conjugate of vector $\mathbf{a}$; $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$; $\lfloor x \rfloor$ denotes maximum integer that is smaller than x; $\mathrm{E}\left[x\right]$ stands for the expectation of x; $\mathbb{C}^{M\times N}$ and $\mathbb{R}^{M\times N}$ are the set of complex and real matrices with M rows and N columns respectively; $G_{n,p}\left(\mathbb{C}\right)$ and $G_{n,p}\left(\mathbb{R}\right)$ denote complex and real Grassmann manifold of dimensions (n, p) respectively; $G_{n,p}^{m}\left(\mathbb{C}\right)$ represents the composite complex Grassmann manifold; $\mathrm{vec}\left(\mathbf{H}\right)$ is the aggregated vector of all column vectors in matrix $\mathbf{H}$; $\mathbf{I}_N$ is the $N \times N$ identity matrix.

The rest of the paper is organized as follows. In Section II, we present the system model and analyze the performance of IA with perfect CSIT. In Section III, we state the limited feedback IA problem. In Section IV, we quantify three quan-

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tization strategies performance. Section V presents numerical simulations. In Section VII, concluding remarks are given.

II. System Model

Throughout the paper, we make such four assumptions:

- There exists an error-free feedback link between each receiver and transmitter.
- Perfect channel estimation at the receiver.
- Perfect time synchronization at each time slot.
- Channel matrix is independent at each block.

Fig. 1 illustrates a MIMO interference network with K source-destination pairs, denoted by $\{S_j, D_j\}, j = 1, ..., K$. Each transmitter and receiver are equipped with M and N antennas respectively.

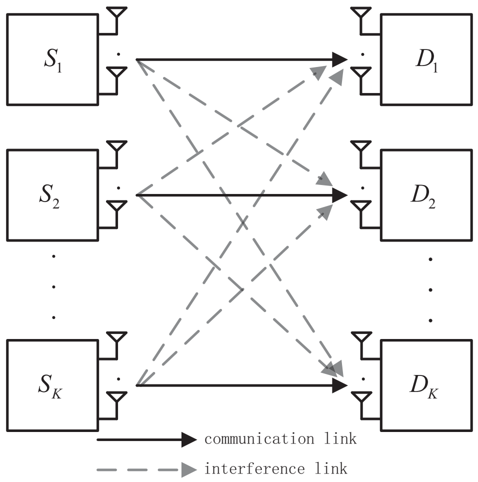

Fig. 1. K-user MIMO interference network.

$\mathbf{H}_{k,j} = \left[\mathbf{h}_{k,j}^1, \mathbf{h}_{k,j}^2, ..., \mathbf{h}_{k,j}^M\right] \in \mathbb{C}^{N\times M}$ denotes the channel matrix between S_j and D_k and $\mathbf{h}_{k,j} = \mathrm{vec}\left(\mathbf{H}_{k,j}\right) = \left[\left(\mathbf{h}_{k,j}^1\right)^T, \left(\mathbf{h}_{k,j}^2\right)^T, ..., \left(\mathbf{h}_{k,j}^M\right)^T\right]^T \in \mathbb{C}^{NM\times 1}$ denotes the channel vector whose normalized version is $\mathbf{w}_{k,j} = \mathbf{h}_{k,j}/\left\|\mathbf{h}_{k,j}\right\| \in \mathbb{C}^{NM\times 1}$. Specifically, the receive interference suppression filter matrix and transmit pre-coding filter matrix are

$$\begin{array}{l} \mathbf{U}_k = [\begin{array}{cccc} \mathbf{u}_k^1 & \mathbf{u}_k^2 & ... & \mathbf{u}_k^{d_k} \end{array}] \in \mathbb{C}^{N\times d_k}, \mathbf{U}_k^H \mathbf{U}_k = \mathbf{I}_{d_k} \\ \mathbf{V}_k = [\begin{array}{cccc} \mathbf{v}_k^1 & \mathbf{v}_k^2 & ... & \mathbf{v}_k^{d_k} \end{array}] \in \mathbb{C}^{M\times d_k}, \mathbf{V}_k^H \mathbf{V}_k = \mathbf{I}_{d_k} \end{array}$$

where d_k denotes the number of data symbols.

Assuming a perfect time synchronization [6], the m-th received symbol is

$$\begin{aligned} \left(\mathbf{u}_k^m\right)^H \mathbf{y}_k = & \left(\mathbf{u}_k^m\right)^H \mathbf{H}_{k,k} \mathbf{v}_k^m x_k^m \\ & + \underbrace{\sum_{j\neq k} \left(\mathbf{u}_k^m\right)^H \mathbf{H}_{k,j} \mathbf{v}_j^m x_j^m}_{\text{interference}} + \left(\mathbf{u}_k^m\right)^H \mathbf{z}_k \end{aligned} \tag{1}$$

where $m \in \{1, ..., d_k\}$. $x_k \in \mathbb{C}^{1\times 1}$ denotes the transmit symbol with power constraint $E\left[\left|x_k\right|^2\right] = \frac{P}{d_k}$ and P denotes the transmit power at each node. $\mathbf{z}_k \in \mathbb{C}^{N\times 1}$ denotes the $CN\left(0, N_o \mathbf{I}_N\right)$ thermal noise vector [3]. In the SISO and MIMO frequency-selective channel, exploiting diagonal channel matrix, interference leakage can be represented by the Hadamard product [5]. However, in the MIMO channel, such solution can not be applied because the channel matrix is non-diagonal. In this paper, the Kronecker product is introduced to represent interference leakage so that

$$\begin{aligned} I_k^m &= \sum_{j\neq k} \frac{P}{d_j} \left|\left(\mathbf{u}_k^m\right)^H \mathbf{H}_{k,j} \mathbf{v}_j^m\right|^2 \\ &= \sum_{j\neq k} \frac{P}{d_j} \left|\left(\left(\mathbf{v}_j^m\right)^T \otimes \left(\mathbf{u}_k^m\right)^H\right) \mathrm{vec}\left(\mathbf{H}_{k,j}\right)\right|^2 \\ &= \sum_{j\neq k} \frac{P}{d_j} \left|\mathbf{b}_{k,j}^H \mathbf{h}_{k,j}\right|^2 \end{aligned} \tag{2}$$

where $\mathbf{b}_{k,j} = \left(\left(\mathbf{v}_j^m\right)^T \otimes \left(\mathbf{u}_k^m\right)^H\right)^H \in \mathbb{C}^{NM\times 1}$. The feasibility conditions for IA with perfect CSIT are

$$\begin{array}{l} \mathbf{b}_{k,j}^H \mathbf{h}_{k,j} = 0 \quad \forall j \neq k, m \\ \mathbf{b}_{k,k}^H \mathbf{h}_{k,k} \geq c > 0 \quad \forall k, m. \end{array} \tag{3}$$

If channel coefficients are i.i.d, the second condition can be automatically satisfied almost surely [3]. However, in the frequency-selective channel, channel taps must scale proportionately fast with the auxiliary parameter of symbol extensions so that the second condition can be satisfied [6]. Such requirement is very hard to meet, so the MIMO channel has an advantage for the feasibility condition.

When $K > R$, the achievable DOF is

$$\lim_{P\to\infty} \frac{R_{sum}}{\log P} = \frac{RK \min\left(N, M\right)}{R+1} \tag{4}$$

where $R = \left\lfloor \frac{\max(M,N)}{\min(M,N)} \right\rfloor$ [2]. For simplicity, we just analyze the case $K > R$ in this paper.

III. IA with limited feedback

In the FDD system, CSIT will be quantized with limited bits before fed back to the transmitter. The performance of the limited feedback system depends on the quantization error. In this section, we will investigate the relationship between interference leakage and chordal distance.

A. *Interference Alignment with Limited Feedback*

$\hat{\mathbf{H}}_{k,j} = \left[\hat{\mathbf{h}}_{k,j}^1, \hat{\mathbf{h}}_{k,j}^2, ..., \hat{\mathbf{h}}_{k,j}^M\right] \in \mathbb{C}^{N\times M}$ and $\hat{\mathbf{h}}_{k,j} = \mathrm{vec}\left(\hat{\mathbf{H}}_{k,j}\right)$ denote the quantized version of $\mathbf{H}_{k,j}$ and $\mathbf{h}_{k,j}$ respectively. $\hat{\mathbf{w}}_{k,j} \in \mathbb{C}^{NM\times 1}$ is the quantized version of $\mathbf{w}_{k,j}$. Using the quantized channel matrix, the corresponding transmitter pre-coding filter $\hat{\mathbf{V}}_k$ and receiver interference suppression filter $\hat{\mathbf{U}}_k$ can be calculated. (1) can be rewritten as

$$\begin{aligned} \left(\hat{\mathbf{u}}_k^m\right)^H \mathbf{y}_k = & \left(\hat{\mathbf{u}}_k^m\right)^H \mathbf{H}_{k,k} \hat{\mathbf{v}}_k^m x_k^m \\ & + \underbrace{\sum_{j\neq k} \left(\hat{\mathbf{u}}_k^m\right)^H \mathbf{H}_{k,j} \hat{\mathbf{v}}_j^m x_j^m}_{\text{interference}} + \left(\hat{\mathbf{u}}_k^m\right)^H \mathbf{z}_k. \end{aligned} \tag{5}$$

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The interference leakage is

$$\hat{I}_k^m = \sum_{j \neq k} \frac{P}{d_j} \left| \hat{\mathbf{b}}_{k,j}^H \mathbf{h}_{k,j} \right|^2. \quad (6)$$

As the solution in [5], a set of normalized vectors $\{\mathbf{q}_1, ..., \mathbf{q}_{NM-2}\} \in \mathbb{C}^{NM \times 1}$ can be found to form an orthonormal basis in $\mathbb{C}^{NM}$ which is denoted as $\left[\hat{\mathbf{b}}_{k,j} / \left\| \hat{\mathbf{b}}_{k,j} \right\|, \hat{\mathbf{h}}_{k,j} / \left\| \hat{\mathbf{h}}_{k,j} \right\|, \mathbf{q}_1, ..., \mathbf{q}_{NM-2}\right]$. Then we expand $\mathbf{h}_{k,j}$ and derive

$$\left\| \mathbf{h}_{k,j} \right\|^2 \geq \left| \frac{\hat{\mathbf{b}}_{k,j}^H}{\left\| \hat{\mathbf{b}}_{k,j} \right\|} \mathbf{h}_{k,j} \right|^2 + \left| \frac{\left(\hat{\mathbf{h}}_{k,j}\right)^H}{\left\| \hat{\mathbf{h}}_{k,j} \right\|} \mathbf{h}_{k,j} \right|^2. \quad (7)$$

The interference leakage is bounded by

$$\begin{aligned} \hat{I}_k^m &\leq \sum_{j \neq k} \frac{P}{d_j} \left\| \hat{\mathbf{b}}_{k,j} \right\|^2 \left(\left\| \mathbf{h}_{k,j} \right\|^2 - \left| \frac{\left(\hat{\mathbf{h}}_{k,j}\right)^H}{\left\| \hat{\mathbf{h}}_{k,j} \right\|} \mathbf{h}_{k,j} \right|^2 \right) \\ &= \sum_{j \neq k} \frac{P}{d_j} \left\| \hat{\mathbf{b}}_{k,j} \right\|^2 \left\| \mathbf{h}_{k,j} \right\|^2 d^2(\mathbf{w}_{k,j}, \hat{\mathbf{w}}_{k,j}) \end{aligned} \quad (8)$$

where $d^2(\mathbf{w}_{k,j}, \hat{\mathbf{w}}_{k,j}) = 1 - \left| \mathbf{w}_{k,j}^H \hat{\mathbf{w}}_{k,j} \right|^2$ is the chordal distance between the channel vector and its quantized version. An important result is that leakage interference is bounded by chordal distance. The result is similar to that in the SISO and MIMO frequency-selective channel in [5][6].

The mean loss of sum rate is bounded by

$$\Delta R_{sum} \leq \sum_{k=1}^{K} \sum_{m=1}^{d_k} \log_2 \left(1 + \frac{\mathrm{E}\left[\hat{I}_k^m\right]}{N_o} \right)$$

which illustrates performance loss depending on average interference leakage [8].

B. Power Strength of Average Interference Leakage

In this subsection, we will simplify average interference leakage.

Firstly, we consider a Rayleigh fading channel where each element of channel matrix is distributed as $CN(0, 1)$. The squared magnitude $\left\| \mathbf{h}_{k,j} \right\|^2$ is distributed as χ_{2NM}^2 with a mean NM so that

$$\mathrm{E}\left[\left\| \mathbf{h}_{k,j} \right\|^2 \right] = NM. \quad (9)$$

Secondly, $\hat{\mathbf{v}}_j^m \in \mathbb{C}^{M \times 1}$ is an unit vector with $\sum_{l=0}^{M-1} \left| \hat{v}_j^m(l) \right|^2 = 1$. A similar expression for $\hat{\mathbf{u}}_k^m \in \mathbb{C}^{N \times 1}$, i.e., $\sum_{i=0}^{N-1} \left| \hat{u}_k^m(i) \right|^2 = 1$. So we derive that

$$\left\| \hat{\mathbf{b}}_{k,j} \right\|^2 = \sum_{l=0}^{M-1} \sum_{i=0}^{N-1} \left| \hat{u}_k^m(i)^* \right|^2 \left| \hat{v}_j^m(l) \right|^2 = 1. \quad (10)$$

Thirdly, each user can achieve $\min(M, N) \frac{R}{R+1}$ DOF through the IA scheme so that

$$\mathrm{E}[d_j] = \min(M, N) \frac{R}{R+1}. \quad (11)$$

We define the signal-to-noise ratio (SNR) as P/N_o. With results in (9), (10) and (11), average interference leakage is given by

$$\mathrm{E}\left[\hat{I}_k^m\right] \leq c\mathrm{SNR} N_o \mathrm{E}\left[d^2(\mathbf{w}_{k,j}, \hat{\mathbf{w}}_{k,j})\right] \quad (12)$$

where

$$c = \frac{(R+1)(K-1)\max(M, N)}{R}$$

is a constant. This result indicates that average interference leakage is decided by the number of users, the number of antennas, the transmit power and chordal distance. A good quantization strategy can reduce chordal distance to improve quantization quality and mitigate average interference leakage.

C. The Noise-limited Criterion

If interference leakage is mitigated to a low level with the IA scheme, the QoS still can be satisfied. The noise-limited criterion is defined as

$$\mathrm{E}\left[I_k^m\right] \leq N_o \quad (13)$$

which means that average interference leakage is no more than that of the thermal noise [10]. In this paper, this criterion is used to derive the lower bound of the codebook size.

IV. GRASSMANNIAN QUANTIZATION

There is a tradeoff between the accuracy of CSIT and the codebook size. In this section, three Grassmannian quantization strategies are analyzed, and the lower bounds of the codebook size are derived respectively. Under the noise-limited criterion, the joint real Grassmannian quantization strategy outperforms other conventional strategies with a smaller codebook size.

A. Complex Grassmannian Quantization to Achieve DOF

In this subsection, we will quantize the channel vector so that the DOF of interference networks will be achieved. Previous literatures prefer to apply a complex Grassmannian codebook to quantize the channel vector which is a *Grassmannian line packing* problem. The composite complex Grassmann manifold $G_{n,p}^m(\mathbb{C})$ which is defined as the Cartesian product of m copies of $G_{n,p}(\mathbb{C})$ [11], is introduced to quantize the aggregated channel matrix in [6]. Such strategy can be considered to quantize each channel vector respectively as the number of feedback bits increases [7]. It is also the Grassmannian line packing problem.

The Grassmannian line packing problem is the problem of finding the set of lines in $G_{n,1}(\mathbb{C})$ that has the maximum minimum distance between any two lines [10]. In this paper, we try to quantize $\mathbf{w}_{k,j}$ with $B = \log(F)$ bits which means finding a code in the codebook $\mathbf{W} = [\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2, ..., \hat{\mathbf{w}}_{2^B}]^T \in \mathbb{C}^{F \times NM}$ in $G_{NM,1}(\mathbb{C})$, where F denotes the codebook size.

The minimum chordal distance of any two codes in the codebook is defined as $\delta(\mathbf{W})$. With results in [10], chordal distance must be shorter than $\delta(\mathbf{W})$, so the upper bound of chordal distance is

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$$d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)\leq\delta^2\left(\mathbf{W}\right)\leq 4\left(\frac{1}{2^{\frac{B_d}{NM-1}}}\right). \tag{14}$$

(8) can be simplified as

$$\hat{I}_k^m\leq\sum_{j\neq k}\frac{P}{d_j}\left\|\hat{\mathbf{b}}_{k,j}^{m,m}\right\|^2\left\|\mathbf{h}_{k,j}\right\|^2 4\left(\frac{1}{2^{\frac{B_d}{NM-1}}}\right). \tag{15}$$

Using previous results in [5], the number of feedback bits is

$$B_d\geq\left(NM-1\right)\log\left(KP\right) \tag{16}$$

so that interference leakage is bounded by a constant. Next, as the $P\rightarrow\infty$, the codebook size tends to infinity so that the maximum quantization error tends to zero. Finally, DOF is achievable as interference leakage tends to zero.

The corresponding codebook size is

$$F_d\geq 2^{(NM-1)\log(KP)}. \tag{17}$$

Similar results are also given in the SISO and MIMO frequency-selective channel [5][6]. However, an infinity codebook is impractical which makes such lower bound of codebook size non-realistic to some extent.

B. Complex Grassmannian Quantization

In this subsection, we will quantize the channel vector with the complex Grassmannian codebook. Besides, we try to find a tighter bound of chordal distance to ensure a tighter lower bound of the codebook size. When the codebook size is huge enough, average chordal distance is bounded by [13] (Th. 2)

$$\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)\right]\leq\frac{\Gamma\left(\frac{1}{NM-1}\right)}{NM-1}\left(p_{NM,2}F_C\right)^{\frac{-1}{NM-1}}$$

where $\ p_{NM,2}=1\quad NM\geq 2\ $ is a constant. Such bound is tighter than that based on $\delta\left(\mathbf{W}\right)$ in (14). Under the noise-limited criterion, the codebook size is bounded by

$$F_C\geq\left(c_2\mathrm{SNR}\right)^{NM-1} \tag{18}$$

where

$$c_2=\frac{\Gamma\left(\frac{1}{NM-1}\right)\left(R+1\right)\left(K-1\right)\max\left(M,N\right)}{\left(NM-1\right)R}$$

is a constant which is determined by the number of users and the number of antennas in the network. It's obviously that the codebook size grows exponentially with $NM-1$. At high SNR, the codebook size becomes huge which is impractical to implement.

The corresponding number of feedback bits is

$$B_C=\log_2\left(F_C\right)\geq\left(NM-1\right)\log_2\left(c_2\mathrm{SNR}\right). \tag{19}$$

C. Joint Real Grassmannian Quantization

In this subsection, a joint quantization strategy is proposed to reduce chordal distance in $G_{NM,1}\left(\mathbb{C}\right)$. More concretely, the real part and imaginary part of the channel vector will be quantized with the same real Grassmannian codebook respectively. Using such strategy, a smaller codebook can be applied to achieve the same QoS.

The channel vector can be represented by $\mathbf{h}_{k,j}=\mathbf{h}_{k,j}^R+j\mathbf{h}_{k,j}^I\in\mathbb{C}^{NM\times 1}$ where $\mathbf{h}_{k,j}^R$ and $\mathbf{h}_{k,j}^I$ denote the corresponding real part and imaginary part respectively. The quantized version of the channel vector is $\hat{\mathbf{h}}_{k,j}=\hat{\mathbf{h}}_{k,j}^R+j\hat{\mathbf{h}}_{k,j}^I$ whose real part and imaginary part are denoted as $\hat{\mathbf{h}}_{k,j}^R$ and $\hat{\mathbf{h}}_{k,j}^I$. As a float can be used to represent the magnitude of the channel vector, we assume that the magnitude quantization is perfect which means $\left\|\mathbf{h}_{k,j}\right\|=\left\|\hat{\mathbf{h}}_{k,j}\right\|$. The quantized versions of $\mathbf{w}_{k,j}^R$ and $\mathbf{w}_{k,j}^I$ are $\hat{\mathbf{w}}_{k,j}^R$ and $\hat{\mathbf{w}}_{k,j}^I$ respectively.

Firstly, we assume that $d^2\left(\mathbf{w}_{k,j}^R,\hat{\mathbf{w}}_{k,j}^R\right)\geq d^2\left(\mathbf{w}_{k,j}^I,\hat{\mathbf{w}}_{k,j}^I\right)$. The chordal distance of real part is

$$d^2\left(\mathbf{w}_{k,j}^R,\hat{\mathbf{w}}_{k,j}^R\right)=1-\frac{\left|\left(\mathbf{h}_{k,j}^R\right)^T\hat{\mathbf{h}}_{k,j}^R\right|^2}{\left\|\mathbf{h}_{k,j}^R\right\|^4}=\Delta^2$$

where Δ^2 is a constant. As the same codebook is applied to quantize the real part and imaginary part, we have

$$\begin{aligned}\left(\left(\mathbf{h}_{k,j}^R\right)^T\hat{\mathbf{h}}_{k,j}^R\right)^2&=\left(1-\Delta^2\right)\left\|\mathbf{h}_{k,j}^R\right\|^4\\ \left(\left(\mathbf{h}_{k,j}^I\right)^T\hat{\mathbf{h}}_{k,j}^I\right)^2&\geq\left(1-\Delta^2\right)\left\|\mathbf{h}_{k,j}^I\right\|^4.\end{aligned} \tag{20}$$

The chordal distance $d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)=d$ is bounded by

$$\begin{aligned}d&=1-\frac{\left|\left(\mathbf{h}_{k,j}^R+j\mathbf{h}_{k,j}^I\right)^H\left(\hat{\mathbf{h}}_{k,j}^R+j\hat{\mathbf{h}}_{k,j}^I\right)\right|^2}{\left\|\mathbf{h}_{k,j}\right\|^4}\\ &\leq 1-\frac{\left(\left(\mathbf{h}_{k,j}^R\right)^T\hat{\mathbf{h}}_{k,j}^R+\left(\mathbf{h}_{k,j}^I\right)^T\hat{\mathbf{h}}_{k,j}^I\right)^2}{\left\|\mathbf{h}_{k,j}\right\|^4}\\ &\leq 1-\frac{\left(1-\Delta^2\right)\left(\left\|\mathbf{h}_{k,j}^R\right\|^4+\left\|\mathbf{h}_{k,j}^I\right\|^4+2\left\|\mathbf{h}_{k,j}^R\right\|^2\left\|\mathbf{h}_{k,j}^I\right\|^2\right)}{\left(\left\|\mathbf{h}_{k,j}^R\right\|^2+\left\|\mathbf{h}_{k,j}^I\right\|^2\right)^2}\\ &=\Delta^2.\end{aligned} \tag{21}$$

So $d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)\leq d^2\left(\mathbf{w}_{k,j}^R,\hat{\mathbf{w}}_{k,j}^R\right)$. The first inequality above follows from the loss of the imaginary part, and the second inequality follows from (20). It is obviously that $\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)\right]\leq\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^R,\hat{\mathbf{w}}_{k,j}^R\right)\right]$.

Secondly, we assume that $d^2\left(\mathbf{w}_{k,j}^I,\hat{\mathbf{w}}_{k,j}^I\right)\geq d^2\left(\mathbf{w}_{k,j}^R,\hat{\mathbf{w}}_{k,j}^R\right)$, and a similar result is $\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j},\hat{\mathbf{w}}_{k,j}\right)\right]\leq\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^I,\hat{\mathbf{w}}_{k,j}^I\right)\right]$.

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Basing on two results above, we have

$$\begin{aligned} & \mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}, \hat{\mathbf{w}}_{k,j}\right)\right] \\ & \leq \max\left\{\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^R, \hat{\mathbf{w}}_{k,j}^R\right)\right], \mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^I, \hat{\mathbf{w}}_{k,j}^I\right)\right]\right\}. \end{aligned} \tag{22}$$

As the real part and imaginary part are independent and quantized with the same codebook, the upper bound of average chordal distance of both is the same. The upper bound is given by [12] (Th. 2)

$$\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^R, \hat{\mathbf{w}}_{k,j}^R\right)\right] = \mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}^I, \hat{\mathbf{w}}_{k,j}^I\right)\right] \leq \Delta_1^2.$$

According to (22), average chordal distance is bounded by

$$\mathrm{E}\left[d^2\left(\mathbf{w}_{k,j}, \hat{\mathbf{w}}_{k,j}\right)\right] \leq \Delta_1^2 \tag{23}$$

where

$$\Delta_1^2 = \frac{2\Gamma\left(\frac{2}{NM-1}\right)}{NM-1}\left(p_{NM,1}F_R\right)^{\frac{-2}{NM-1}}$$

$$p_{NM,1} = \frac{\Gamma\left(\frac{NM}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{NM}{2}+\frac{1}{2}\right)} \quad NM \geq 2\ .$$

Considering the noise-limited criterion, the codebook size is bounded by

$$F_R \geq \frac{\left(c_1\mathrm{SNR}\right)^{\frac{NM-1}{2}}}{p_{NM,1}} \tag{24}$$

where

$$c_1 = \frac{2\left(R+1\right)\left(K-1\right)\max\left(N,M\right)\Gamma\left(\frac{2}{NM-1}\right)}{\left(NM-1\right)R}.$$

c_1 is a constant that is determined by the network. Ignoring trivial constants, $F_R \ll F_C$, i.e., the codebook size is much smaller than that of the complex Grassmannian quantization strategy.

The corresponding number of feedback bits is bounded by

$$\begin{aligned} B_R &= 2\log_2\left(F_R\right) \\ &\geq \left(NM-1\right)\log_2\left(c_1\mathrm{SNR}\right) - 2\log_2\left(p_{NM,1}\right). \end{aligned} \tag{25}$$

As trivial constants can be neglected, $B_R \approx B_C$, i.e., the overhead of two quantization strategies is nearly the same. So under the same QoS, the proposed strategy only requires a smaller codebook compared with the conventional strategy.

V. Simulations and Results

In this section, we present simulations to demonstrate the performance of IA using the joint real Grassmannian quantization strategy. Consider a 3 user MIMO channel where each node is equipped with multiple antennas, and the Rayleigh fading channel whose channel coefficients are distributed as $CN\left(0,1\right)$. Each user has the same transmit power, and noise power is normalized to unity at each destination.

In Fig. 2, we compare the lower bounds of the codebook size of different quantization strategies versus SNR.

1) The complex Grassmannian quantization strategy in Section-IV B.
2) The complex Grassmannian theory strategy to achieve DOF in Section-IV A.

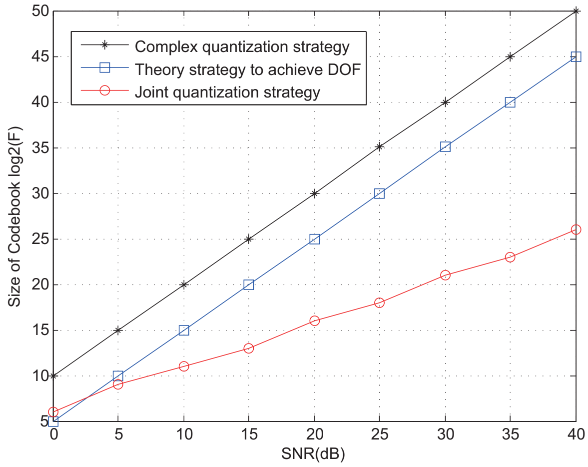

Fig. 2. Codebook size using different quantization strategies in 2×2 IA system with $K = 3$.

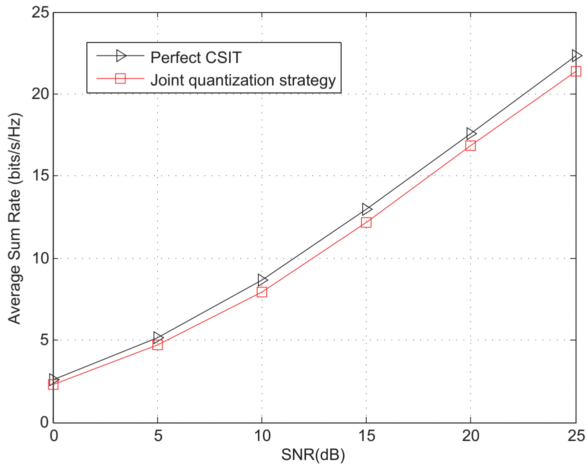

Fig. 3. Performance of IA using joint real Grassmannian quantization in 2×2 IA system with $K = 3$.

3) The joint real Grassmannian quantization strategy in Section-IV C.

The codebook size using the proposed strategy is much smaller than that of the other two strategies. What's more, the gap between our proposed strategy and the conventional strategies becomes broader.

In Fig. 3, the sum rate of following schemes with respect to transmit power is compared.

1) The iterative IA scheme with perfect CSIT.
2) The iterative IA scheme using the proposed strategy in Section-IV C.

The performance of average sum rate is measured by Monte Carlo simulations. There exists a narrow performance gap between IA using the proposed strategy and IA with perfect CSIT at low and moderate SNR. However, as the codebook size can not be generated when SNR is larger than 25 dB, we just do simulations within 25 dB.

In Fig. 4, sum rate curves of two quantization strategies with

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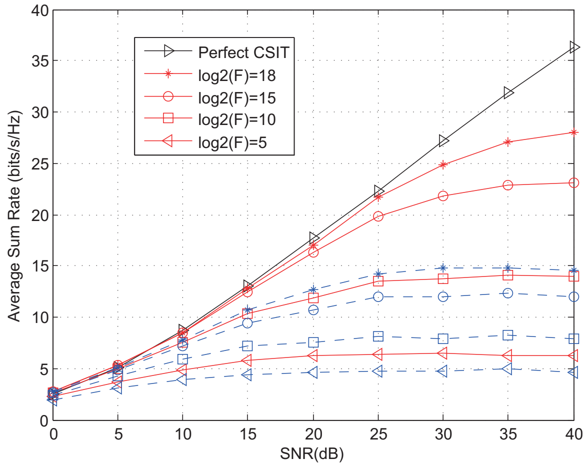

Fig. 4. Sum rate comparison in 2×2 IA system with $K = 3$.

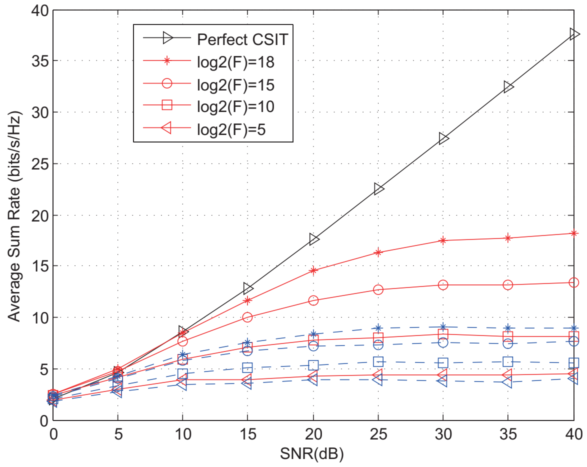

Fig. 5. Sum rate comparison in 2×3 IA system with $K = 3$.

the same codebook size are compared versus transmit power.

1) The iterative IA scheme with perfect CSIT.
2) The iterative IA scheme using the proposed strategy with various codebook sizes in Section-IV C.
3) The iterative IA scheme using the conventional strategy with the same codebook size as 2) in Section-IV B.

The sum rate curves of our proposed strategy is depicted with red lines, while the curves of the conventional strategy are plotted with blue lines. Compared with the conventional strategy, the proposed strategy exhibits significantly enhanced performance. More concretely, the proposed strategy can obtain a performance gain of 90% when the codebook size is 2^{18} at high SNR. From the plot of Fig. 4, it can be seen that a considerable performance gain can be achieved if the codebook size increases. However, our proposed strategy achieves sum rate gains at the cost of double feedback bits.

Finally, compared with a system where numbers of transmit and receive antenna are $M = 2$ and $N = 2$, we depict sum rate curves when $M = 2$ and $N = 3$ in Fig. 5. As more channel coefficients need to be quantized, quantization quality decreases so that some performance loss exists compared with the system in Fig. 4. However, our proposed strategy still obtain nearly 90% sum rate gain when the codebook size is 2^{18} at high SNR.

VI. Acknowledgment

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VII. Conclusion

In this paper, we have derived the bound of interference leakage and proposed a joint real Grassmannian quantization strategy to mitigate interference leakage in the MIMO interference network. Basing on the analysis, we have verified that the codebook size is reduced and accuracy of CSIT is improved with our proposed strategy. Simulations show that the sum rate gap between IA using our proposed strategy and IA with perfect CSIT is small. Compared with the conventional strategy, the proposed strategy with the same codebook size reduces interference leakage and contributes to higher sum rate.

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