The Unsteady Navier-Stokes Flow Using Adaptive Mixed Finite Elements

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Abstract

1 Accuracy Solution

The Navier-Stokes equations:

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} - \nu\nabla^2\vec{u} + \vec{u}\cdot\nabla\vec{u} + \nabla p &= \vec{0}, \\ \nabla\cdot\vec{u} &= 0, \end{aligned} \quad (1)$$

The accuracy solutions of euqations (1) is defined as follows:

$$\begin{aligned} u(x,y,t) &= -cos(x)sin(y)e^{-2\nu t}, \\ v(x,y,t) &= sin(x)sin(y)e^{-2\nu t}, \\ p(x,y,t) &= \frac{1}{2}(sin^2(x) + sin^2(y))e^{-4\nu t} + \text{constant} \end{aligned} \quad (2)$$

with the computing domain $[0,2\pi]\times[0,2\pi]$.

2 Numerical result

2.1 Uniform mesh

2.1.1 Error analysis

From Table [1], we can conclude L_2 error of velocity is $O(h^2)$, H_1 semierror of velocity is $O(h)$.

2.1.2 Mesh 20 * 20

Steamline of velocity and contour of pressure is shown in Figure (1).

Mesh (P)	$\|\|u-u_h\|\|_{L^2}$	$\|\|u-u_h\|\|_{H^1}$	$\|\|v-v_h\|\|_{L^2}$	$\|\|v-v_h\|\|_{H^1}$	$\|\|p-p_h\|\|_{L^2}$	$\|\|p-p_h\|\|_{H^1}$
20×20	$6.734e{-3}$	$2.319e{-1}$	$6.734e{-3}$	$2.311e{-1}$	$1.547e{-2}$	$4.098e{-1}$
40×40	$1.545e{-3}$	$1.131e{-1}$	$1.535e{-3}$	$1.126e{-1}$	$3.917e{-3}$	$2.041e{-1}$
80×80	$3.816e{-4}$	$5.642e{-2}$	$3.789e{-4}$	$5.615e{-2}$	$6.444e{-4}$	$1.019e{-1}$

Table 1: Error for Accuracy test using uniform mesh, $\nu = 0.05, t = 1s$.

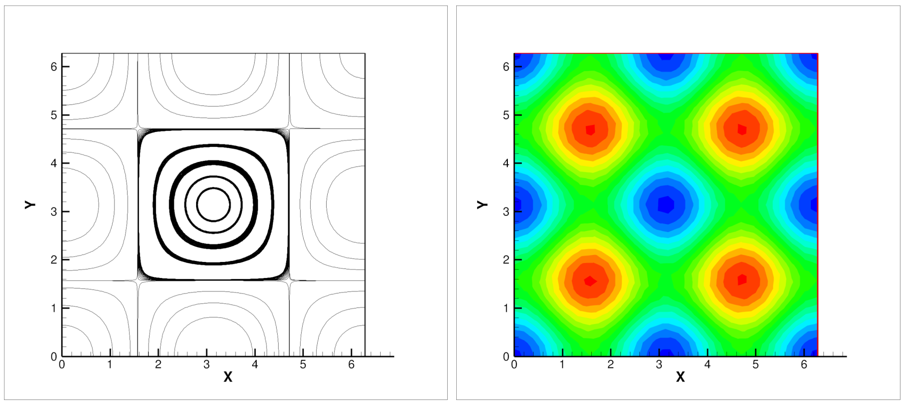

Figure 1: Uniform mesh 20×20 Left: streamline, right: pressure contour. $t = 1s, \nu = 0.05$.

2.1.3 Mesh 40 * 40

In Figure (4), velocity streamline and pressure contour is present.

2.2 Moving Mesh

We choose

$$G_1 = \sqrt{1.0 + \alpha|\nabla\vec{u}|^{\beta}} \tag{3}$$

as monitor, here α and β are two positive constants. In follow experiments, we set $\alpha = 5, \beta = 2$.

2.2.1 Error analysis

Mesh (P)	$\|\|u-u_h\|\|_{L^2}$	$\|\|u-u_h\|\|_{H^1}$	$\|\|v-v_h\|\|_{L^2}$	$\|\|v-v_h\|\|_{H^1}$	$\|\|p-p_h\|\|_{L^2}$	$\|\|p-p_h\|\|_{H^1}$
20×20	$1.113e{-2}$	$2.960e{-1}$	$1.124e{-2}$	$2.961e{-1}$	$4.117e{-2}$	$4.666e{-1}$
40×40	$2.713e{-3}$	$1.517e{-1}$	$2.693e{-3}$	$1.511e{-1}$	$1.613e{-2}$	$2.432e{-1}$

Table 2: Error for Accuracy test using moving mesh, $\nu = 0.05, t = 1s$.

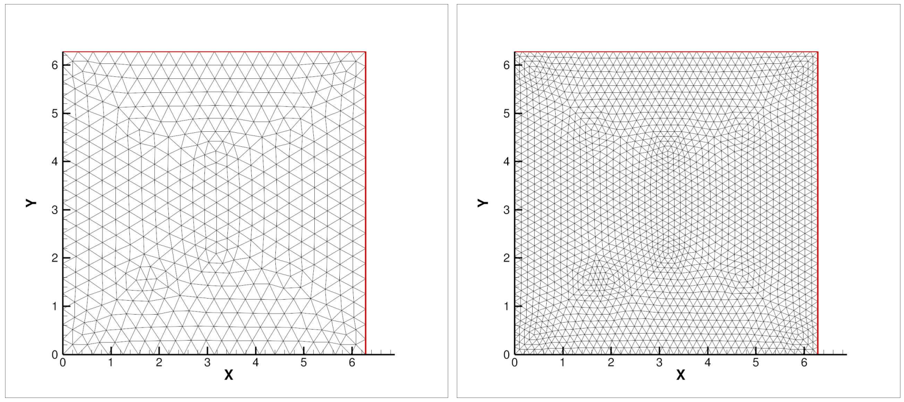

Figure 2: Uniform mesh 20×20, Left: mesh P, right: mesh V, $t = 1s, \nu = 0.05$.

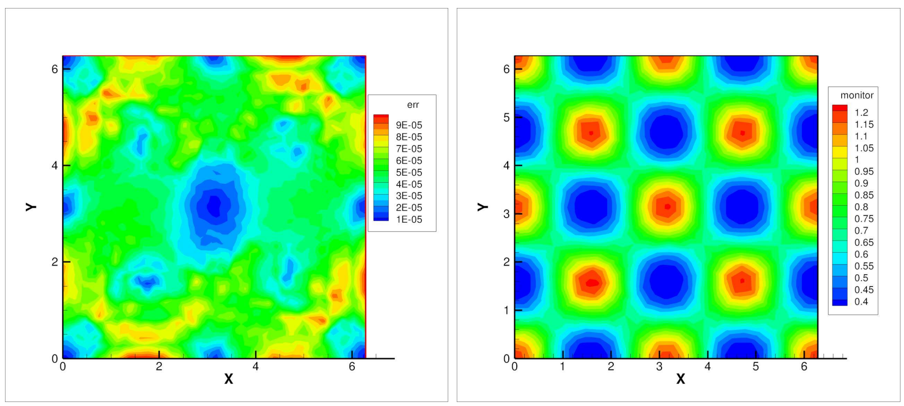

Figure 3: Uniform mesh 20×20 Left: distribution of $||\vec{u} - \vec{u}_h||_{L^2}$ error, right: monitor G_1 in 3.

2.2.2 Mesh 20 * 20

Numerical solution and moving mesh are show in FIgure(6) and Figure(7).

From Figure(8), mesh is moving from the place where value of monitor is big to that which value is small.

2.2.3 Mesh 40 * 40

From Figure(11), mesh is moving from the place where value of monitor is big to that which value is small.

3 Remarks

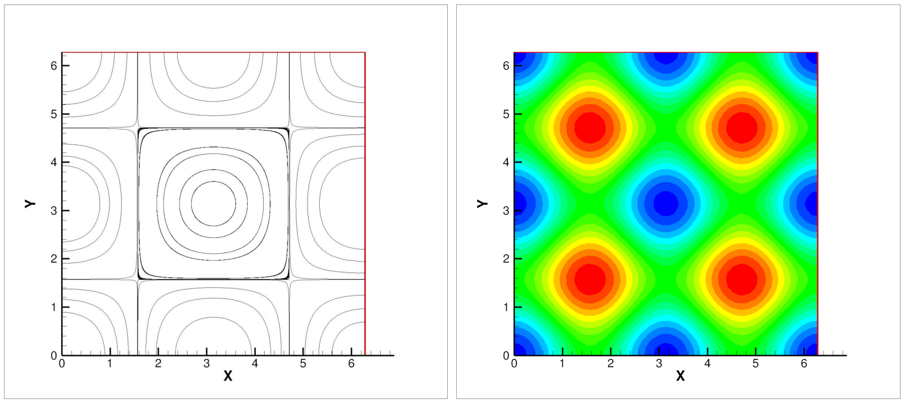

Figure 4: Uniform mesh 40×40, Left: streamline, right: pressure contour. $t = 1s, \nu = 0.05$.

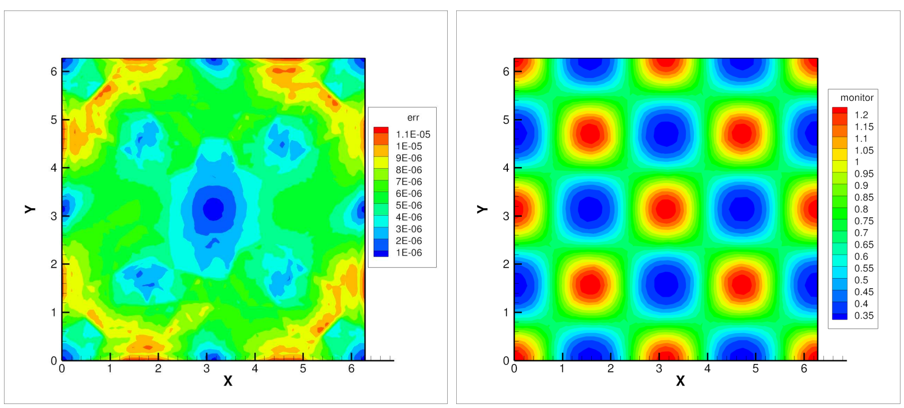

Figure 5: Uniform mesh 40×40 Left: distribution of $||\vec{u} - \vec{u}_h||_{L^2}$ error, right: monitor G_1 in 3.

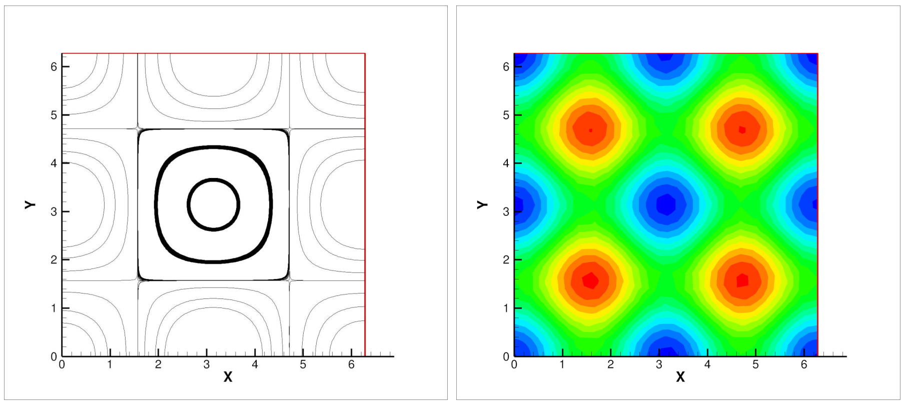

Figure 6: Moving mesh 20×20, Left: streamline, right: pressure contour. $t = 1s, \nu = 0.05$.

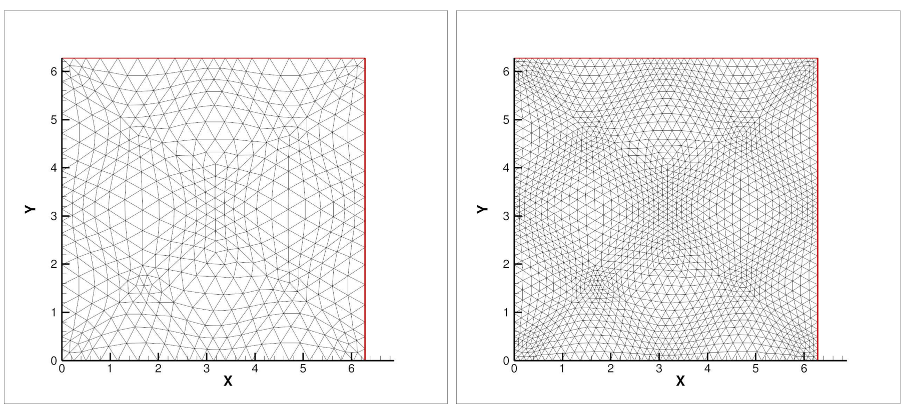

Figure 7: Moving mesh 20×20, Left: mesh P, right: mesh V, using monitor G_1 in (3). $t = 1s, \nu = 0.05$.

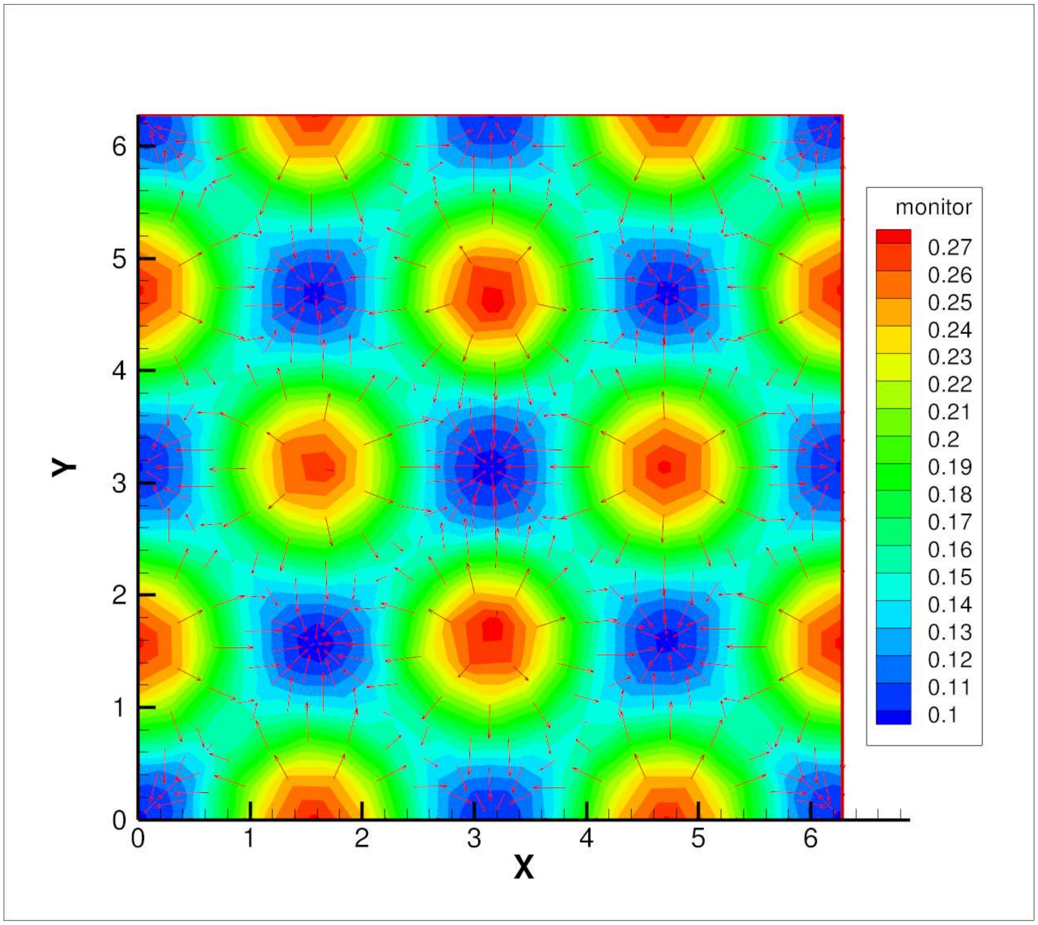

Figure 8: Moving mesh 20×20, contour of monitor $m = \frac{1}{G_1}$ in (3) and mesh move direction. $t = 1s, \nu = 0.05$.

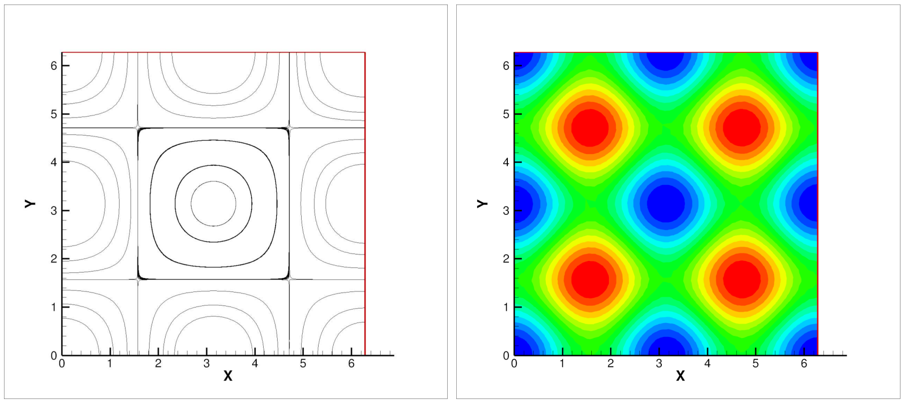

Figure 9: Moving mesh 40×40, Left: streamline, right: pressure contour. $t = 1s, \nu = 0.05$.

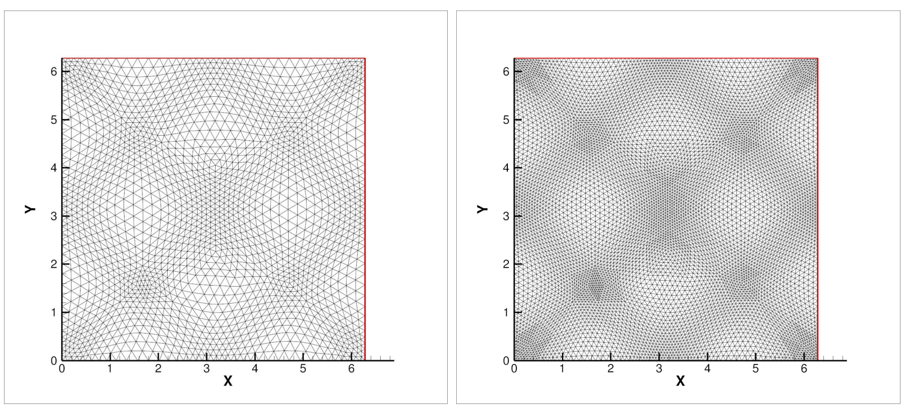

Figure 10: Moving mesh 40×40, Left: mesh P, right: mesh V, using monitor G_1 in (3). $t = 1s, \nu = 0.05$.

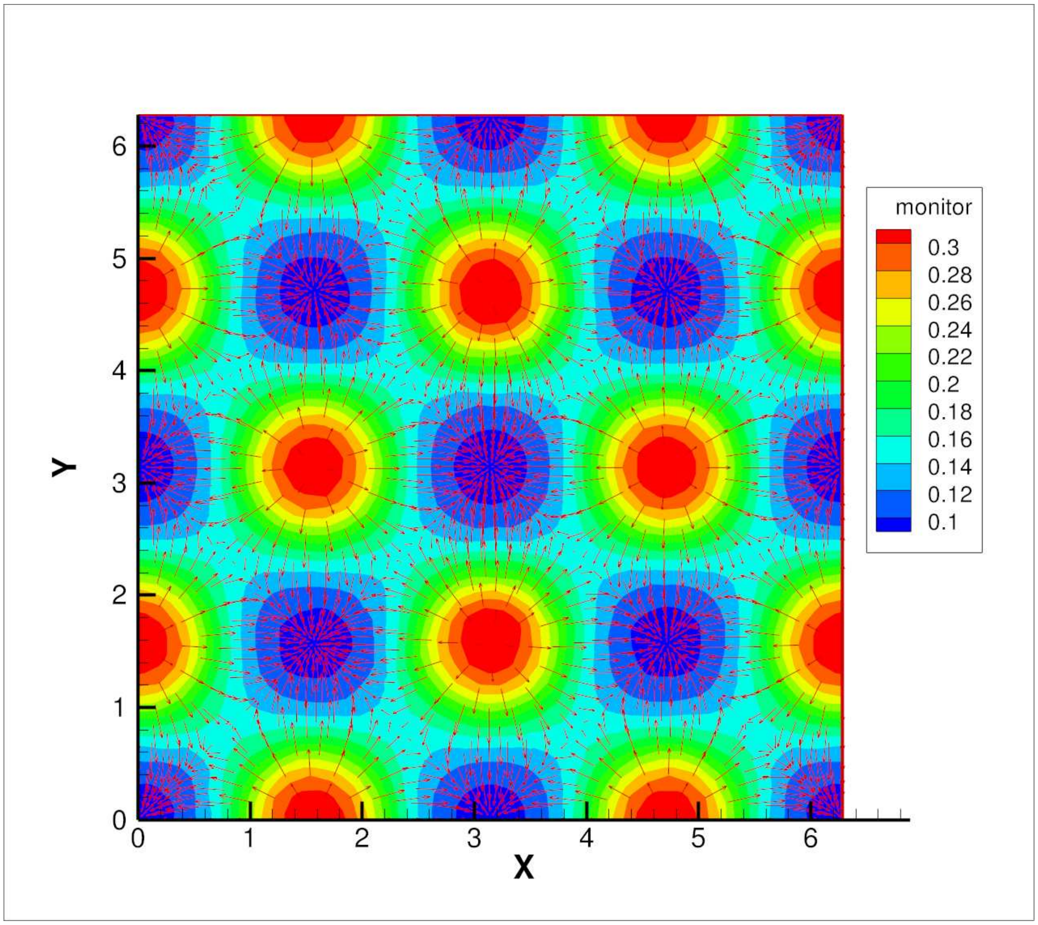

Figure 11: Moving mesh 40×40, contour of monitor $m = \frac{1}{G_1}$ in (3). and mesh move direction. $t = 1s, \nu = 0.05$.