On-Manifold Preintegration for Real-Time Visual-Inertial Odometry

Paper Reading by 高翔

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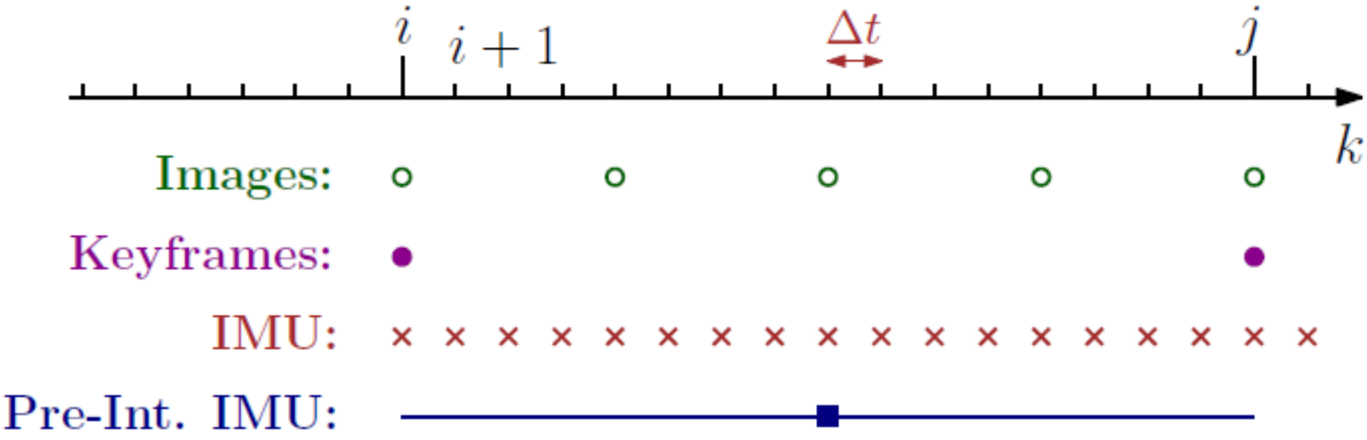
引言

- VIO or VI-SLAM 视觉与惯性融合
- IMU　测量角速度、加速度
- Vision　测量图像



VI传感器实物演示

DUO-3D 双目+IMU

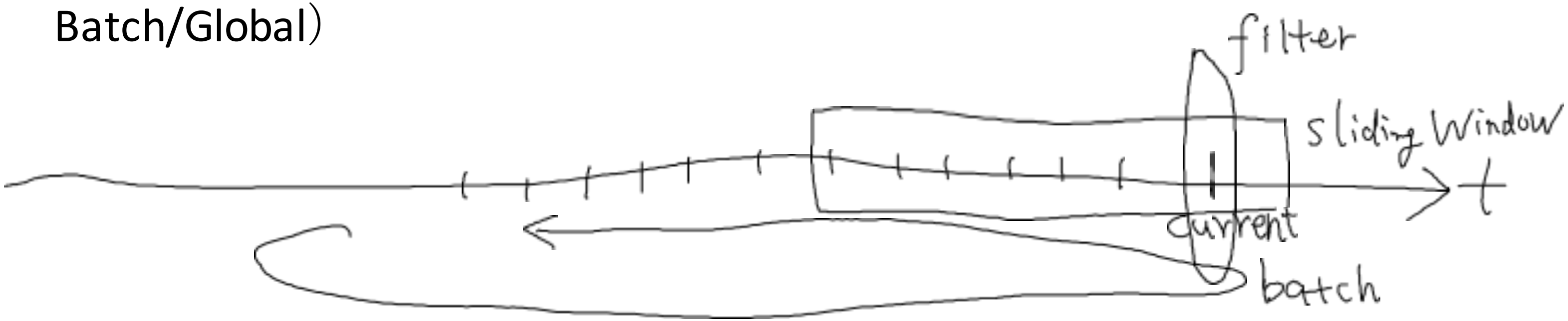
引言

- IMU：快、依赖少、只测量角速度和加速度、会漂移
- 视觉：图像信息丰富、不漂移、易受干扰（光照、遮挡、模糊、快速运动）
- VIO的处理方式：松耦合/紧耦合

Loosely coupled: 视觉和IMU分开估计位置，最后融合

Tightly coupled: 视觉和IMU共同估计一个状态量

- 紧耦合是标准的做法
- 处理方式：滤波（Filtering）、局部平滑/滤波（Sliding Window）、全局优化（Batch/Global）



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预备知识

- 旋转和平移的表示

$$R \in SO(3), t \in R^3$$

- 旋转位于流形上，原点正切空间为李代数。李代数到李群有指数和对数关系：

大写的Exp带着∧

$$\begin{array}{lccccccc} \mathrm{Exp}: & \mathbb{R}^3 & \to & \mathrm{SO}(3) & ; & \phi & \mapsto & \exp(\phi^\wedge) \\ \mathrm{Log}: & \mathrm{SO}(3) & \to & \mathbb{R}^3 & ; & \mathrm{R} & \mapsto & \log(\mathrm{R})^\vee, \end{array}$$

- 本文用右乘的SO(3)，雅可比为：

右乘用R_{BW}, t_{BW}, 在B系处理更方便.

$$\mathrm{J}_r(\phi) = \mathrm{I} - \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}\phi^\wedge + \frac{\|\phi\|-\sin(\|\phi\|)}{\|\phi^3\|}(\phi^\wedge)^2.$$

- SLAM中的非线性优化&图优化（略）

SO(3)的伴随性质

$$\mathrm{R}\ \mathrm{Exp}(\phi)\ \mathrm{R}^\mathsf{T} = \exp(\mathrm{R}\phi^\wedge\mathrm{R}^\mathsf{T}) = \mathrm{Exp}(\mathrm{R}\phi)$$

$$\Leftrightarrow \quad \mathrm{Exp}(\phi)\ \mathrm{R} = \mathrm{R}\ \mathrm{Exp}(\mathrm{R}^\mathsf{T}\phi).$$

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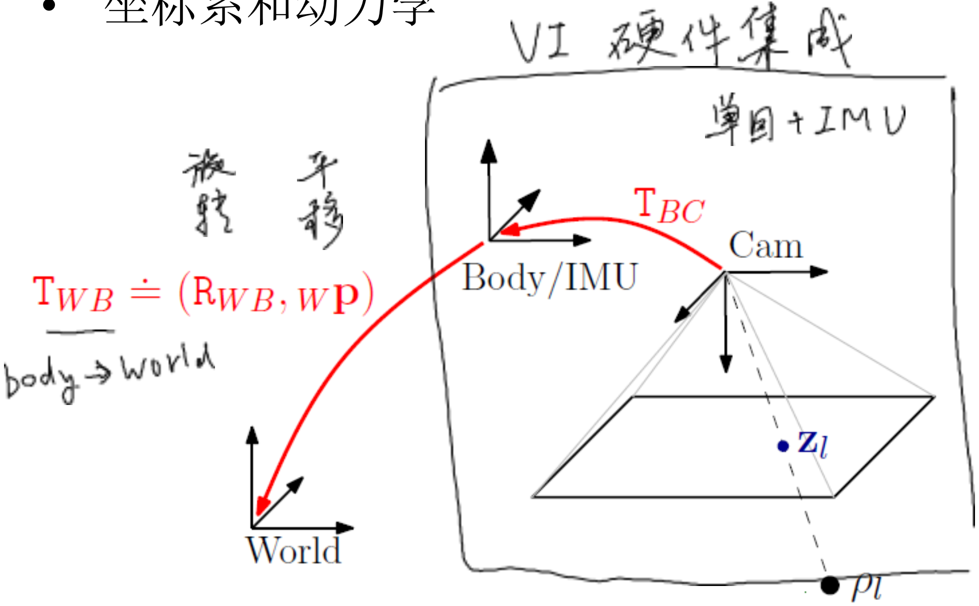
预备知识

- 坐标系和动力学



世界：固定不动.

纯视觉：仅估计Twc或Tcw

世界坐标系下动力学.

$$\dot{p}_W = v_W$$
$$\dot{v}_W = a_W$$
$$\dot{R}_{WB} = R_{WB}\omega_{WB}^{\wedge}$$

*在Body系下会更复杂

预备知识

- IMU测量什么？

$$
{}_{\mathrm{B}}\tilde{\boldsymbol{\omega}}_{\mathrm{WB}}(t) = {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(t) + \overset{\text{bias}}{\mathbf{b}^g(t)} + \overset{\text{noise}}{\boldsymbol{\eta}^g(t)}
$$

Body系下角速度

$$
{}_{\mathrm{B}}\tilde{\mathbf{a}}(t) = \mathrm{R}_{\mathrm{WB}}^{\mathsf{T}}(t)\left({}_{\mathrm{W}}\mathbf{a}(t) - \underset{\text{世界坐标系下重力}}{{}_{\mathrm{W}}\mathbf{g}}\right) + \underset{\text{bias}}{\mathbf{b}^a(t)} + \underset{\text{noise}}{\boldsymbol{\eta}^a(t)},
$$

加速度.

- IMU的Bias是随机游走

$$
\dot{\mathbf{b}}^g(t) = \boldsymbol{\eta}^{bg}, \qquad \dot{\mathbf{b}}^a(t) = \boldsymbol{\eta}^{ba}.
$$

$\eta^g, \eta^a, \eta^{bg}, \eta^{ba} \sim N$

高斯噪声

预备知识

- 状态变量

$$\mathbf{x}_i \doteq [\mathrm{R}_i, \mathbf{p}_i, \mathbf{v}_i, \mathbf{b}_i]. \in \mathbb{R}^{15}$$

$(\mathrm{R}_i, \mathbf{p}_i)$ belongs to SE(3)

旋转+平移

$\mathbf{v}_i \in \mathbb{R}^3$ 速度

$\mathbf{b}_i^g, \mathbf{b}_i^a \in \mathbb{R}^3$ 零偏

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预积分

- 积分形式动力学

$$\dot{p}_W = v_W$$
$$\dot{v}_W = a_W$$
$$\dot{R}_{WB} = R_{WB}\omega_{WB}^{\wedge}$$

$$\mathrm{R}_{\mathrm{WB}}(t+\Delta t) = \mathrm{R}_{\mathrm{WB}}(t)\ \mathrm{Exp}\left(\int_t^{t+\Delta t} {}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(\tau)d\tau\right)$$
$${}_{\mathrm{W}}\mathbf{v}(t+\Delta t) = {}_{\mathrm{W}}\mathbf{v}(t) + \int_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{a}(\tau)d\tau$$
$${}_{\mathrm{W}}\mathbf{p}(t+\Delta t) = {}_{\mathrm{W}}\mathbf{p}(t) + \int_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{v}(\tau)d\tau + \iint_t^{t+\Delta t} {}_{\mathrm{W}}\mathbf{a}(\tau)d\tau^2.$$

- 离散形式

$$\mathrm{R}_{\mathrm{WB}}(t+\Delta t) = \mathrm{R}_{\mathrm{WB}}(t)\ \mathrm{Exp}\left({}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(t)\Delta t\right)$$
$${}_{\mathrm{W}}\mathbf{v}(t+\Delta t) = {}_{\mathrm{W}}\mathbf{v}(t) + {}_{\mathrm{W}}\mathbf{a}(t)\Delta t$$
$${}_{\mathrm{W}}\mathbf{p}(t+\Delta t) = {}_{\mathrm{W}}\mathbf{p}(t) + {}_{\mathrm{W}}\mathbf{v}(t)\Delta t + \frac{1}{2}{}_{\mathrm{W}}\mathbf{a}(t)\Delta t^2.$$

- 用IMU观测量表达

$$\begin{aligned}
\mathrm{R}(t+\Delta t) &= \mathrm{R}(t)\ \mathrm{Exp}\left(\left(\tilde{\boldsymbol{\omega}}(t) - \mathbf{b}^g(t) - \boldsymbol{\eta}^{gd}(t)\right)\Delta t\right)\\
\mathbf{v}(t+\Delta t) &= \mathbf{v}(t) + \mathbf{g}\Delta t + \mathrm{R}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right)\Delta t\\
\mathbf{p}(t+\Delta t) &= \mathbf{p}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2}\mathbf{g}\Delta t^2\\
&\quad + \frac{1}{2}\mathrm{R}(t)\left(\tilde{\mathbf{a}}(t) - \mathbf{b}^a(t) - \boldsymbol{\eta}^{ad}(t)\right)\Delta t^2, \qquad (31)
\end{aligned}$$

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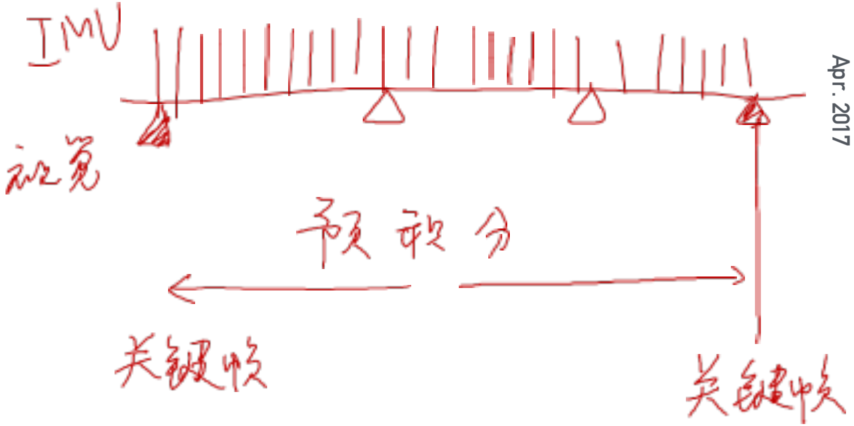
预积分

- 差分方程给出了两个IMU数据之间的关系

$$
\begin{aligned}
\mathrm{R}(t+\Delta t) &= \mathrm{R}(t)\ \mathrm{Exp}\left(\left(\tilde{\boldsymbol{\omega}}(t)-\mathbf{b}^{g}(t)-\boldsymbol{\eta}^{gd}(t)\right)\Delta t\right)\\
\mathbf{v}(t+\Delta t) &= \mathbf{v}(t)+\mathbf{g}\Delta t+\mathrm{R}(t)\left(\tilde{\mathbf{a}}(t)-\mathbf{b}^{a}(t)-\boldsymbol{\eta}^{ad}(t)\right)\Delta t\\
\mathbf{p}(t+\Delta t) &= \mathbf{p}(t)+\mathbf{v}(t)\Delta t+\frac{1}{2}\mathbf{g}\Delta t^{2}\\
&\quad+\frac{1}{2}\mathrm{R}(t)\left(\tilde{\mathbf{a}}(t)-\mathbf{b}^{a}(t)-\boldsymbol{\eta}^{ad}(t)\right)\Delta t^{2},
\end{aligned}
\tag{31}
$$



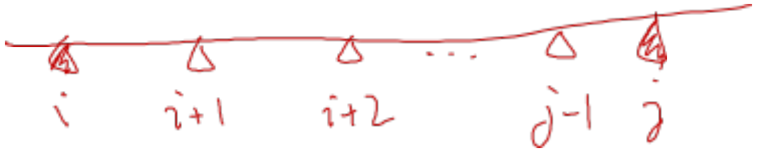
- 然而仅凭IMU无法计算Bias，所以：
- 将两个视觉帧之间的IMU积分在一起——预积分（Preintegration）

预积分

- 两个Keyframe之间有多个IMU数据

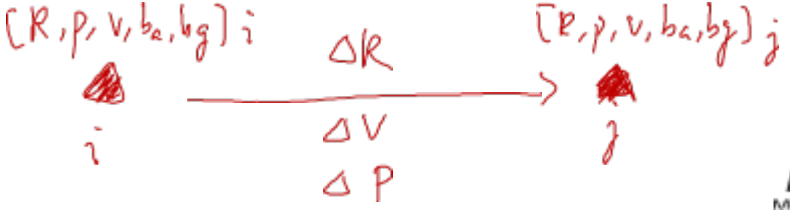
$$
\begin{aligned}
\mathrm{R}_j &= \mathrm{R}_i \prod_{k=i}^{j-1} \operatorname{Exp}\left(\left(\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_k^g - \boldsymbol{\eta}_k^{gd}\right)\Delta t\right), \\
\mathbf{v}_j &= \mathbf{v}_i + \mathbf{g}\Delta t_{ij} + \sum_{k=i}^{j-1} \mathrm{R}_k\left(\tilde{\mathbf{a}}_k - \mathbf{b}_k^a - \boldsymbol{\eta}_k^{ad}\right)\Delta t \qquad (32)\\
\mathbf{p}_j &= \mathbf{p}_i + \sum_{k=i}^{j-1}\left[\mathbf{v}_k\Delta t + \frac{1}{2}\mathbf{g}\Delta t^2 + \frac{1}{2}\mathrm{R}_k\left(\tilde{\mathbf{a}}_k - \mathbf{b}_k^a - \boldsymbol{\eta}_k^{ad}\right)\Delta t^2\right]
\end{aligned}
$$

keyframe i → j



- 由此推出两个Keyframe之间的Measurement model

$$
\begin{aligned}
\Delta\mathrm{R}_{ij} &\doteq \mathrm{R}_i^{\mathsf{T}}\mathrm{R}_j = \prod_{k=i}^{j-1} \operatorname{Exp}\left(\left(\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_k^g - \boldsymbol{\eta}_k^{gd}\right)\Delta t\right) \\
\Delta\mathbf{v}_{ij} &\doteq \mathrm{R}_i^{\mathsf{T}}\left(\mathbf{v}_j - \mathbf{v}_i - \mathbf{g}\Delta t_{ij}\right) = \sum_{k=i}^{j-1} \Delta\mathrm{R}_{ik}\left(\tilde{\mathbf{a}}_k - \mathbf{b}_k^a - \boldsymbol{\eta}_k^{ad}\right)\Delta t \\
\Delta\mathbf{p}_{ij} &\doteq \mathrm{R}_i^{\mathsf{T}}\left(\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i\Delta t_{ij} - \tfrac{1}{2}\textstyle\sum_{k=i}^{j-1}\mathbf{g}\Delta t^2\right) \\
&= \sum_{k=i}^{j-1}\left[\Delta\mathbf{v}_{ik}\Delta t + \frac{1}{2}\Delta\mathrm{R}_{ik}\left(\tilde{\mathbf{a}}_k - \mathbf{b}_k^a - \boldsymbol{\eta}_k^{ad}\right)\Delta t^2\right] \qquad (33)
\end{aligned}
$$



预积分

- Measurement和IMU的bias、noise都有关，且关系复杂
- 先假设Bias不动，仅讨论噪声，然后再讨论bias

$$
\begin{aligned}
\Delta \mathrm{R}_{ij} &\overset{\text{eq.}\boxed{7}}{\simeq} \prod_{k=i}^{j-1} \left[\mathrm{Exp}\left((\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_i^g)\Delta t \right) \mathrm{Exp}\left(-\mathrm{J}_r^k \, \boldsymbol{\eta}_k^{gd} \, \Delta t \right) \right] \\
&\overset{\text{eq.}\boxed{11}}{=} \Delta \tilde{\mathrm{R}}_{ij} \prod_{k=i}^{j-1} \mathrm{Exp}\left(-\Delta \tilde{\mathrm{R}}_{k+1j}^{\mathsf{T}} \, \mathrm{J}_r^k \, \boldsymbol{\eta}_k^{gd} \, \Delta t \right) \\
&\doteq \Delta \tilde{\mathrm{R}}_{ij} \mathrm{Exp}\left(-\delta \boldsymbol{\phi}_{ij} \right)
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
\Delta \mathbf{v}_{ij} &\overset{\text{eq.}\boxed{4}}{\simeq} \sum_{k=i}^{j-1} \Delta \tilde{\mathrm{R}}_{ik} (\mathbf{I} - \delta \boldsymbol{\phi}_{ik}^{\wedge}) (\tilde{\mathbf{a}}_k - \mathbf{b}_i^a) \Delta t - \Delta \tilde{\mathrm{R}}_{ik} \boldsymbol{\eta}_k^{ad} \Delta t \\
&\overset{\text{eq.}\boxed{2}}{=} \Delta \tilde{\mathbf{v}}_{ij} + \sum_{k=i}^{j-1} \left[\Delta \tilde{\mathrm{R}}_{ik} (\tilde{\mathbf{a}}_k - \mathbf{b}_i^a)^{\wedge} \delta \boldsymbol{\phi}_{ik} \Delta t - \Delta \tilde{\mathrm{R}}_{ik} \boldsymbol{\eta}_k^{ad} \Delta t \right] \\
&\doteq \Delta \tilde{\mathbf{v}}_{ij} - \delta \mathbf{v}_{ij}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
\Delta \mathbf{p}_{ij} &\overset{\text{eq.}\boxed{4}}{\simeq} \sum_{k=i}^{j-1} \Big[(\Delta \tilde{\mathbf{v}}_{ik} - \delta \mathbf{v}_{ik}) \Delta t + \frac{1}{2} \Delta \tilde{\mathrm{R}}_{ik} (\mathbf{I} - \delta \boldsymbol{\phi}_{ik}^{\wedge}) (\tilde{\mathbf{a}}_k - \mathbf{b}_i^a) \Delta t^2 \\
&\qquad - \frac{1}{2} \Delta \tilde{\mathrm{R}}_{ik} \boldsymbol{\eta}_k^{ad} \Delta t^2 \Big] \\
&\overset{\text{eq.}\boxed{2}}{=} \Delta \tilde{\mathbf{p}}_{ij} + \sum_{k=i}^{j-1} \Big[- \delta \mathbf{v}_{ik} \Delta t + \frac{1}{2} \Delta \tilde{\mathrm{R}}_{ik} (\tilde{\mathbf{a}}_k - \mathbf{b}_i^a)^{\wedge} \delta \boldsymbol{\phi}_{ik} \Delta t^2 \\
&\qquad - \frac{1}{2} \Delta \tilde{\mathrm{R}}_{ik} \boldsymbol{\eta}_k^{ad} \Delta t^2 \Big] \\
&\doteq \Delta \tilde{\mathbf{p}}_{ij} - \delta \mathbf{p}_{ij},
\end{aligned} \tag{37}
$$

预积分

- 从两个Keyframe状态定义出来的Motion：

$$\Delta\tilde{\mathrm{R}}_{ij} = \mathrm{R}_i^{\mathsf{T}}\mathrm{R}_j \mathrm{Exp}\left(\delta\phi_{ij}\right)$$

$$\Delta\tilde{\mathbf{v}}_{ij} = \mathrm{R}_i^{\mathsf{T}}\left(\mathbf{v}_j - \mathbf{v}_i - \mathbf{g}\Delta t_{ij}\right) + \delta\mathbf{v}_{ij}$$

$$\Delta\tilde{\mathbf{p}}_{ij} = \mathrm{R}_i^{\mathsf{T}}\left(\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i\Delta t_{ij} - \frac{1}{2}\mathbf{g}\Delta t_{ij}^2\right) + \delta\mathbf{p}_{ij}$$

它和观测模型相差噪声

$$[\delta\boldsymbol{\phi}_{ij}^{\mathsf{T}}, \delta\mathbf{v}_{ij}^{\mathsf{T}}, \delta\mathbf{p}_{ij}^{\mathsf{T}}]^{\mathsf{T}}.$$

和η成非线性关系，可近似.

- 这些噪声项满足什么分布？

$$\delta\boldsymbol{\phi}_{ij} = -\mathrm{Log}\left(\prod_{k=i}^{j-1}\mathrm{Exp}\left(-\Delta\tilde{\mathrm{R}}_{k+1j}^{\mathsf{T}}\,\mathrm{J}_r^k\,\boldsymbol{\eta}_k^{gd}\,\Delta t\right)\right).$$

$$\delta\mathbf{v}_{ij} \simeq \sum_{k=i}^{j-1}\left[-\Delta\tilde{\mathrm{R}}_{ik}\left(\tilde{\mathbf{a}}_k - \mathbf{b}_i^a\right)^{\wedge}\delta\boldsymbol{\phi}_{ik}\Delta t + \Delta\tilde{\mathrm{R}}_{ik}\boldsymbol{\eta}_k^{ad}\Delta t\right] \qquad (43)$$

$$\delta\mathbf{p}_{ij} \simeq \sum_{k=i}^{j-1}\left[\delta\mathbf{v}_{ik}\Delta t - \frac{1}{2}\Delta\tilde{\mathrm{R}}_{ik}\left(\tilde{\mathbf{a}}_k - \mathbf{b}_i^a\right)^{\wedge}\delta\boldsymbol{\phi}_{ik}\Delta t^2 + \frac{1}{2}\Delta\tilde{\mathrm{R}}_{ik}\boldsymbol{\eta}_k^{ad}\Delta t^2\right]$$

在一阶近似下，可以认为高斯分布
这在非线性优化的information matrix中用到

预积分

- 处理Bias
- 观测量对bias的导数（附录B）

$$\Delta\tilde{\mathrm{R}}_{ij}(\mathrm{b}_i^g) \simeq \Delta\tilde{\mathrm{R}}_{ij}(\bar{\mathrm{b}}_i^g)\ \mathrm{Exp}\left(\frac{\partial\Delta\bar{\mathrm{R}}_{ij}}{\partial\mathrm{b}^g}\delta\mathrm{b}^g\right) \tag{44}$$

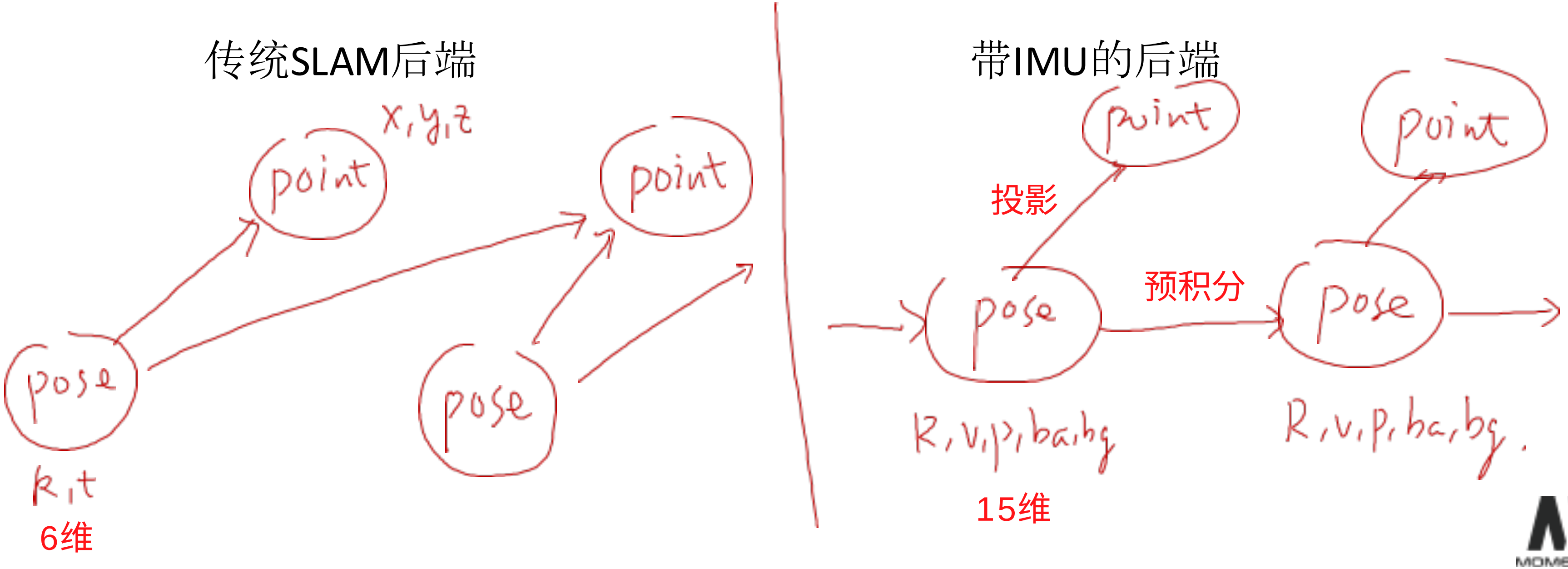
$$\Delta\tilde{\mathbf{v}}_{ij}(\mathrm{b}_i^g,\mathrm{b}_i^a) \simeq \Delta\tilde{\mathbf{v}}_{ij}(\bar{\mathrm{b}}_i^g,\bar{\mathrm{b}}_i^a) + \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathrm{b}^g}\delta\mathrm{b}_i^g + \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathrm{b}^a}\delta\mathrm{b}_i^a$$

$$\Delta\tilde{\mathbf{p}}_{ij}(\mathrm{b}_i^g,\mathrm{b}_i^a) \simeq \Delta\tilde{\mathbf{p}}_{ij}(\bar{\mathrm{b}}_i^g,\bar{\mathrm{b}}_i^a) + \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathrm{b}^g}\delta\mathrm{b}_i^g + \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathrm{b}^a}\delta\mathrm{b}_i^a$$

$$\begin{aligned}
\frac{\partial\Delta\bar{\mathrm{R}}_{ij}}{\partial\mathrm{b}^g} &= -\sum_{k=i}^{j-1}\left[\Delta\tilde{\mathrm{R}}_{k+1j}(\bar{\mathrm{b}}_i)^{\mathsf{T}}\ \mathrm{J}_r^k\ \Delta t\right] \\
\frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathrm{b}^a} &= -\sum_{k=i}^{j-1}\Delta\bar{\mathrm{R}}_{ik}\Delta t \\
\frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathrm{b}^g} &= -\sum_{k=i}^{j-1}\Delta\bar{\mathrm{R}}_{ik}\left(\tilde{\mathbf{a}}_k-\bar{\mathrm{b}}_i^a\right)^\wedge\frac{\partial\Delta\bar{\mathrm{R}}_{ik}}{\partial\mathrm{b}^g}\Delta t \\
\frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathrm{b}^a} &= \sum_{k=i}^{j-1}\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathrm{b}^a}\Delta t-\frac{1}{2}\Delta\bar{\mathrm{R}}_{ik}\Delta t^2 \\
\frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathrm{b}^g} &= \sum_{k=i}^{j-1}\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathrm{b}^g}\Delta t-\frac{1}{2}\Delta\bar{\mathrm{R}}_{ik}\left(\tilde{\mathbf{a}}_k-\bar{\mathrm{b}}_i^a\right)^\wedge\frac{\partial\Delta\bar{\mathrm{R}}_{ik}}{\partial\mathrm{b}^g}\Delta t^2
\end{aligned}$$

应用

- 怎么在VIO里用预积分？
- 原作：SVO+gtsam，不过也完全可以用g2o和ceres实现

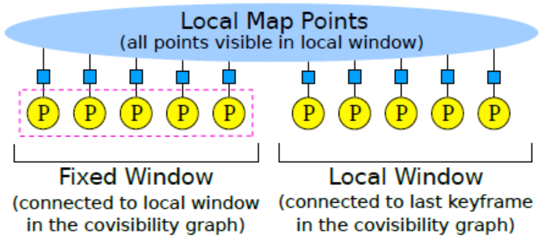


应用

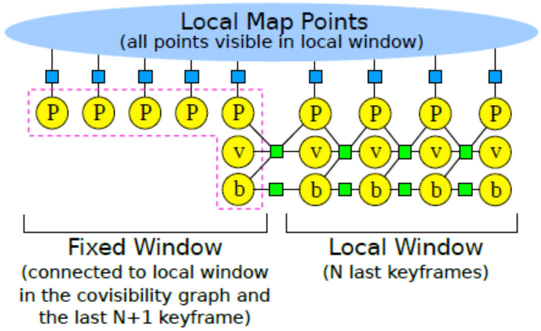
大同小异

用gtsam实现，改写factor.

- In ORB+IMU

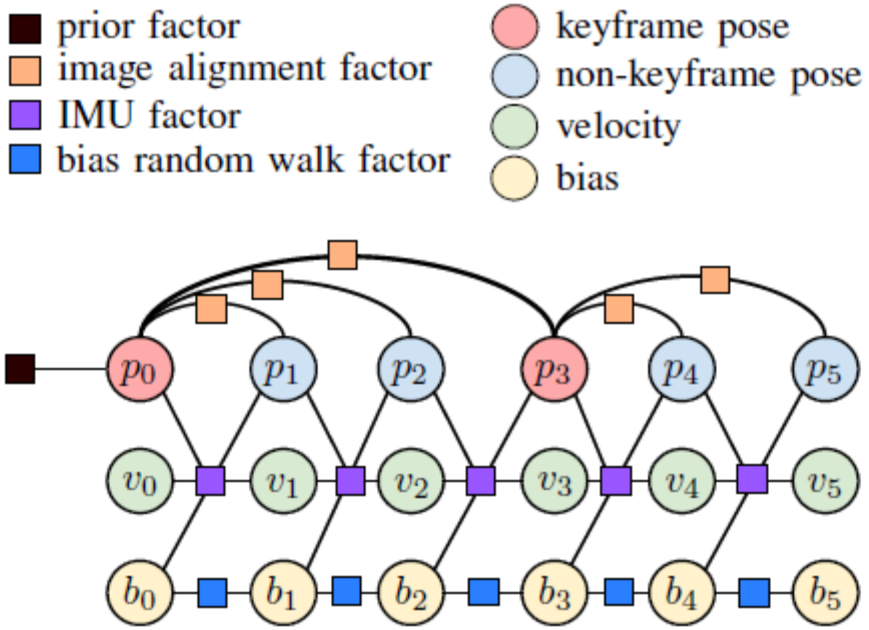


ORB-SLAM's Local BA



Visual-Inertial ORB-SLAM's Local BA

- LSD+IMU



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THANK YOU

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