Supplemental Material

Lemma 1. *For adjacency matrices $\mathcal{A}$ and $\mathcal{A}(t)$ defined in our model, let $\Lambda_{max}(\mathcal{A})$ and $\Lambda_{max}(\mathcal{A}(t))$ be the largest real eigenvalues of $\mathcal{A}$ and $\mathcal{A}(t)$, respectively, we have $\Lambda_{max}(\mathcal{A}) \geq \Lambda_{max}(\mathcal{A}(t))$.*

Proof. Assuming that the number of all nodes is N in the network. Without node sleep scheduling, the network topology is fixed with an adjacency matrix $\mathcal{A}$. Due to the employment of node sleep scheduling, there will be some sleeping nodes at each time step disconnecting from their neighbors, and thus the network topology is changing with time with a temporal adjacency matrix $\mathcal{A}(t)$. Both $\mathcal{A}$ and $\mathcal{A}(t)$ are Hermitian matrices. Let the number of sleeping nodes at time t be n. The temporal adjacency matrix $\mathcal{A}(t)$ can be expressed in terms of submatrices,

$$\mathcal{A}(t) = \begin{bmatrix} B_{(N-n)\times(N-n)} & 0_{(N-n)\times n} \\ 0_{n\times(N-n)} & 0_{n\times n} \end{bmatrix},$$

where B is the principal submatrix representing the connections among the $N-n$ active nodes. Then, all eigenvalues of matrix $\mathcal{A}(t)$ are comprised of all eigenvalues of matrix B and n zero eigenvalues. So, we have

$$\Lambda_{\text{max}}(\mathcal{A}(t)) = \Lambda_{\text{max}}(B), \tag{1}$$

where $\Lambda_{\text{max}}(B)$ is the largest real eigenvalue of matrix B. Similarly, matrix $\mathcal{A}$ can be partitioned into the following form,

$$\mathcal{A} = \begin{bmatrix} B_{(N-n)\times(N-n)} & C_{(N-n)\times n} \\ C^{*} & D_{n\times n} \end{bmatrix}.$$

Let us rank all the eigenvalues of A and B as $\lambda_1(\mathcal{A}) \leq \lambda_2(\mathcal{A}) \leq \cdots \leq \lambda_N(\mathcal{A})$ and $\lambda_1(B) \leq \lambda_2(B) \leq \cdots \leq \lambda_{N-n}(B)$, respectively. According to Cauchy's theorem (Theorem 4.3.28 in [1]),

$$\lambda_i(\mathcal{A}) \leq \lambda_i(B) \leq \lambda_{i+n}(\mathcal{A}), \quad i = 1, 2, \ldots, N-n.$$

When $i = N-n$, we have $\lambda_N(\mathcal{A}) \geq \lambda_{N-n}(B)$, i.e.,

$$\Lambda_{\text{max}}(\mathcal{A}) \geq \Lambda_{\text{max}}(B). \tag{2}$$

Then, according to Eqs. (1) and (2), we have

$$\Lambda_{\text{max}}(\mathcal{A}) \geq \Lambda_{\text{max}}(\mathcal{A}(t)).$$

■

Theorem 1. *The epidemic threshold β_c of our model satisfies $\beta_c \geq (1+\frac{u}{v})\frac{\gamma}{\Lambda_{max}(\mathcal{A})}$.*

Proof. As shown in Eq. (16) of the main paper, we have

$$\left((1+\frac{u}{v})\frac{\gamma}{\beta}E - \mathcal{A}(t)\right)\epsilon = 0,$$

where $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_N)^T$. When $\epsilon \neq 0$, $(1+\frac{u}{v})\frac{\gamma}{\beta}$ is one of the eigenvalue of $\mathcal{A}(t)$. Thus, we have

$$\Lambda_{\text{max}}(\mathcal{A}(t)) \geq (1+\frac{u}{v})\frac{\gamma}{\beta}.$$

According to **Lemma 1**, we obtain

$$\Lambda_{\text{max}}(\mathcal{A}) \geq (1+\frac{u}{v})\frac{\gamma}{\beta},$$

i.e.,

$$\beta_c \geq (1+\frac{u}{v})\frac{\gamma}{\Lambda_{max}(\mathcal{A})}.$$

■

REFERENCES

[1] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge university press, 2012.