Operations Research Applications

Assignment 2

Due Date: March 30, 2021, 5pm

Please zip your files, including MS Word, Excel, Python code files or others, with the file name: **ORA_Assignment2_ID_NAME.zip**, and submit your homework to **NTU COOL** by due.

Questions (100%)

1. (**45%**) Stochastic Programming

Consider the following problem and solve it using stochastic programming via Python+Gurobi (or pulp). You grow wheat, corn, and sugar beet on 500 acres of land. Planting an acre costs $150, $230, and $260, respectively. Need 200t of wheat and 240t of corn for cattle feed. Excess production of wheat and corn is sold at $170/t and $150/t; however, if less is produced, it is bought at $238/t and 210/t, respectively. Sugar beet sells at $36/t below 6000t, and sells at $10/t above 6000t (laws courtesy of the European Commission)

The average yields of crop are: 2.5t/acre for wheat, 3t/acre for corn, and 20t/acre for beet. Depending on how good the weather is, these yields may decrease or increase by 20%. That is, there are three scenarios with equal probability: high yield, average yield, and low yield (i.e. discrete scenario).

How to solve this problem and maximize total profit? You should decide how many acres to use for each crop.

(a) (5%) Define decision variables and give linear programming (LP) formulation with respect to average yield scenario (i.e. EV solution).
(b) (5%) Solve the solution of Scenario Analysis and the EV solution?
(c) (5%) Define new decision variables and give deterministic equivalent problem (DEP) formulation of two-stage recourse problem (RP).
(d) (5%) Solve the RP model.
(e) (5%) What's the Expected Value of Perfect Information (EVPI)? What's the value of the stochastic solution (VSS)?
(f) (5%) Do you think RP providing a good solution in this study? Why?
(g) (15%) For continuous scenarios, if the weather results in a continuous probability distribution of the yield **rate**, which follows the normal distribution $\mathrm{N}(1, 0.1)$. Calculate the objective function (i.e. expected value of the resource function) with the confidence intervals of the upper bound and lower bound, respectively, by Monte Carlo sampling average approximation (SAA). In the Monte Carlo simulation, we use the sample size $N = \bar{N} = 30$ and the replications (batches) $M = T = 15$.

2. (**55%**) Decision Analysis and Value of Information

NTU is planning her production strategy for NTU mug in the souvenir shop. Three alternatives are being considered for production: 10,000, 6000, and 3,000 mugs per year. NTU's president decides to

categorize the demand for mugs for the next year as either High (H) or Low (L). The payoffs (i.e. profit) measured in NTD and probabilities of states are presented in the table below.

Decision Alternatives	High Demand ($\theta = 1$)	Low Demand ($\theta = 0$)
Strategy A: 10,000	1,000,000	-400,000
Strategy B: 6,000	600,000	300,000
Strategy C: 3,000	100,000	400,000
(Prior) Probability	0.41	0.59

How to solve this problem by drawing decision tree in decision analysis?

(a) (5%) How many decision nodes are required? How many branches come out of each decision node?
(b) (5%) How many chance nodes are required? How many branches come out of each chance node?
(c) (5%) Draw the decision tree. Label each branch completely including probabilities and payoffs.
(d) (5%) Solve the decision tree and find the best production strategy.

Based on the above mug production problem, if president would like to hire a marketing research firm to help estimate the demand more accurately. Consider the reliabilities of the marketing research firm with <u>two marketing results</u>: Encouraging ($X = 1$) or Discouraging ($X = 0$). The conditional probability of Encouraging given High Demand (i.e. $P(X=1|\theta=1)$) is 0.8; on the other side, the conditional probability of Discouraging given Low Demand (i.e. $P(X=0|\theta=0)$) is 0.7.

(e) (5%) Calculate the joint probabilities, i.e. $P(X=0, \theta=0)$, $P(X=0, \theta=1)$, $P(X=1, \theta=0)$, and $P(X=1,\theta=1)$.
(f) (5%) Calculate the marginal probabilities, i.e. $P(X=1)$ and $P(X=0)$.
(g) (5%) Calculate the posterior probabilities, i.e. $P(\theta=0|X=0)$, $P(\theta=1|X=0)$, $P(\theta=0|X=1)$, and $P(\theta=1|X=1)$.
(h) (5%) Redraw the revised decision tree with considering <u>two alternatives</u>: Hire or Non-Hire marketing research firm.
(i) (5%) Calculate the EVPI.
(j) (5%) Calculate the EVE when we directly use the marketing results, i.e. if X=1 then Strategy A; otherwise X=0 then Strategy C.
(k) (5%) If hiring marketing research firm costs NTD 50,000, do you suggest NTU President to hire?

Note

1. Show all your work in detail. **<u>Innovative</u>** <u>idea is encouraged</u>.
2. If your answer refers to any external source, please "must" give an academic citation. Any "plagiarism" is not allowed.