Operations Research Applications

Assignment 1

Due Date: Mar. 9, 2021, 5pm

Please solve the following questions and justify your answer by using Python. Show all your analysis result in your report. **Upload your "zip" file including MS Word report and Python code with file name: ORA_Assignment1_ID_Name.zip" to *NTU COOL*** by due. The late submission is not allowed.

- This assignment is just for reviewing the linear programming (LP) and Markov chain (MC) in your undergraduate course "Operations Research". If you are familiar with the LP and MC, it's great. If you know nothing about them, don't worry and it's fine, just google it and learn!
- In particular, you may read https://github.com/PO-LAB/Python-Gurobi-Pulp for Python + Pulp.

Questions (100%)

Please answer following questions and justify your answer. Show all your works in details.

1. Problem 3.4-17. in Hillier and Lieberman (2010)

3.4-17. Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches. They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content of each food choice and its cost are given in the table below.

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (¢)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp)	75	100	0	4	4
Strawberry jelly (1 tbsp)	0	50	3	0	7
Graham cracker (1 cracker)	20	60	0	1	8
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice).

Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements.

i. Define the set, index, parameters, and decision variables.
ii. Formulate a linear programming (LP) model with objective function and constraints.
iii. Solve this model by the **Python+Pulp** solver. You may describe the code in the report.
iv. Show the solution results. What's the managerial implication or insight?

2. Problem 16.5-10. in Hillier and Lieberman (2010)

16.5-10. An important unit consists of two components placed in parallel. The unit performs satisfactorily if one of the two components is operating. Therefore, only one component is operated at a time, but both components are kept operational (capable of being operated) as often as possible by repairing them as needed. An operating component breaks down in a given period with probability 0.2. When this occurs, the parallel component takes over, if it is operational, at the beginning of the next period. Only one component can be repaired at a time. The repair of a component starts at the beginning of the first available period and is completed at the end of the next period. Let X_t be a vector consisting of two elements U and V, where U represents the number of components that are operational at the end of period t and V represents the number of periods of repair that have been completed on components that are not yet operational. Thus, $V = 0$ if $U = 2$ or if $U = 1$ and the repair of the nonoperational component is just getting under way. Because a repair takes two periods, $V = 1$ if $U = 0$ (since then one nonoperational component is waiting to begin repair while the other one is entering its second period of repair) or if $U = 1$ and the nonoperational component is entering its second period of repair. Therefore, the state space consists of the four states (2, 0), (1, 0), (0, 1), and (1, 1). Denote these four states by 0, 1, 2, 3, respectively. $\{X_t\}$ $(t = 0, 1, \ldots)$ is a Markov chain (assume that $X_0 = 0$) with the (one-step) transition matrix

$$\mathbf{P} = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0 & 1 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 \end{bmatrix} \end{array}.$$

i. What is the Markovian property?

ii. What is the probability that the unit will be inoperable (because both components are down) after n periods, for n = 2, 5, 10, 20?

iii. What are the steady-state probabilities of the state of this Markov chain?

iv. If it costs $30,000 per period when the unit is inoperable (both components down) and zero otherwise, what is the (long-run) expected average cost per period?

Note

1. Show all your work in detail. <u>**Innovative** idea is encouraged</u>.
2. If your answer refers to any external source, please “must” give an academic citation. Any “plagiarism” is not allowed.