Efficiency Measure in Insurance Industry – A Network DEA Model

Operation Research Application

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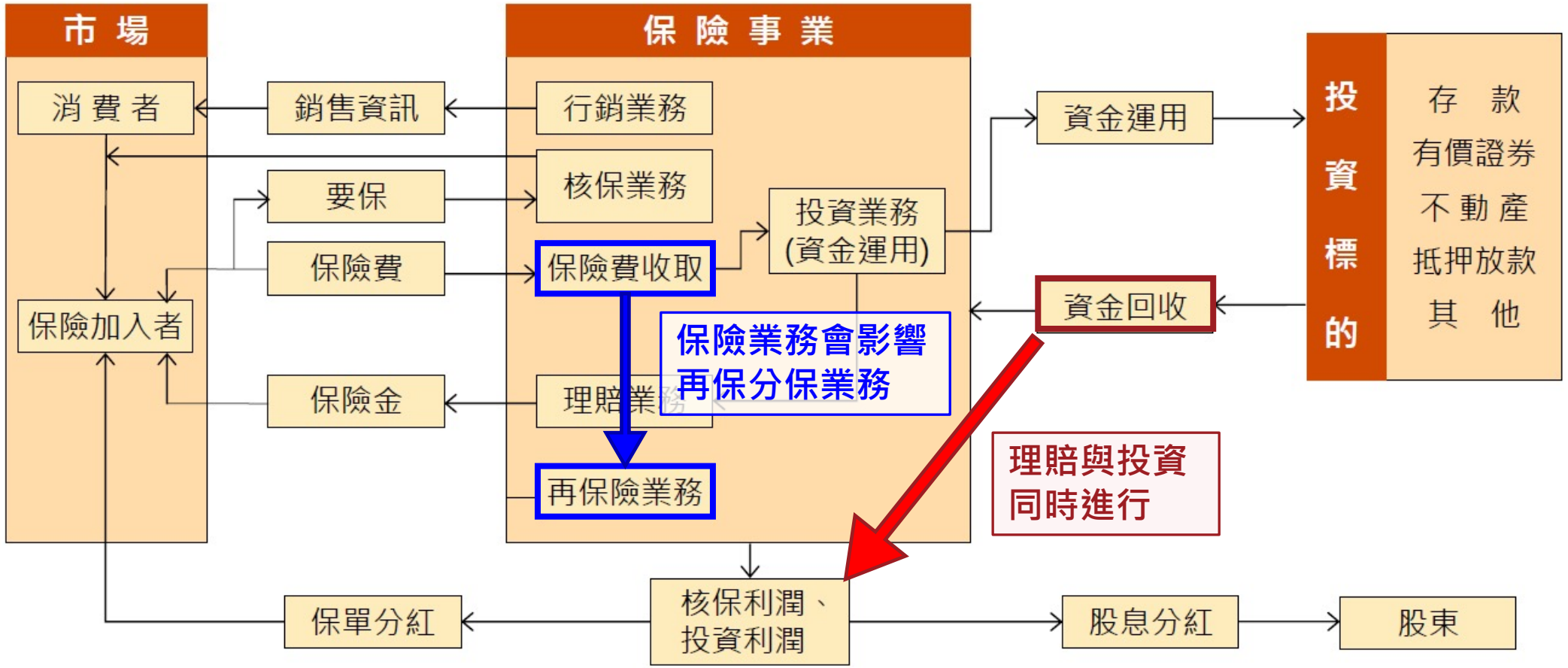
Background and Motivation

- Kao and Hwang (2008) —the **first study that applies Network-DEA to the insurance field,** which consists of 24 non-life insurance firms in Taiwan between **2001 and 2002**. Subsequently, several scholars use the same dataset in Kao and Hwang (2008) to test new Network-DEA models.

- When it comes to efficiency measure in insurance industry nowadays. It's difficult to collect similar data since **the change in financial statement** over 15 years.
- Furthermore, network framework from Kao and Hwang (2008) should be update.
- We proposed a three stage network system which contains three processes, premium acquisition, reinsurance allocation and profit generation.

Background and Motivation

保險業的經營



Methodology - network DEA model

Notations:

X^1: Insurance expenses

X^2: Operation expenses

X^{21}: 50% of X_2

X^{22}: 50% of X_2

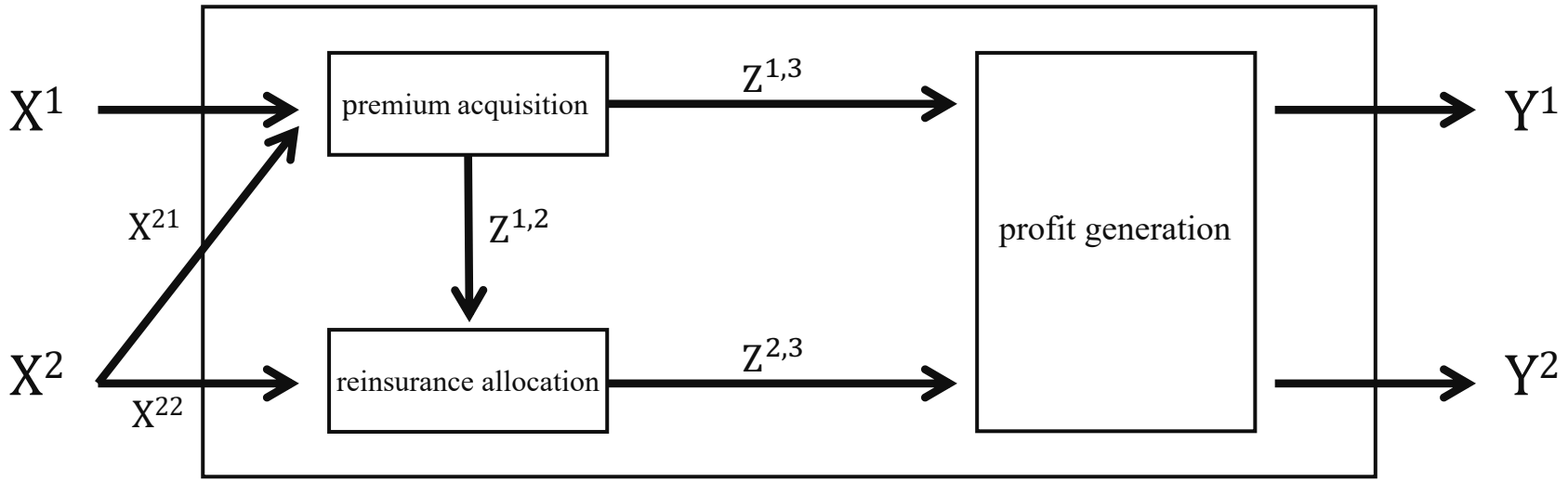
$Z^{1,2}$: Premiums ceded to reinsurers

$Z^{1,3}$: Premium income minus $Z^{1,2}$

$Z^{2,3}$: Reinsurance premiums

Y^1: Underwriting profit

Y^2: Investment profit



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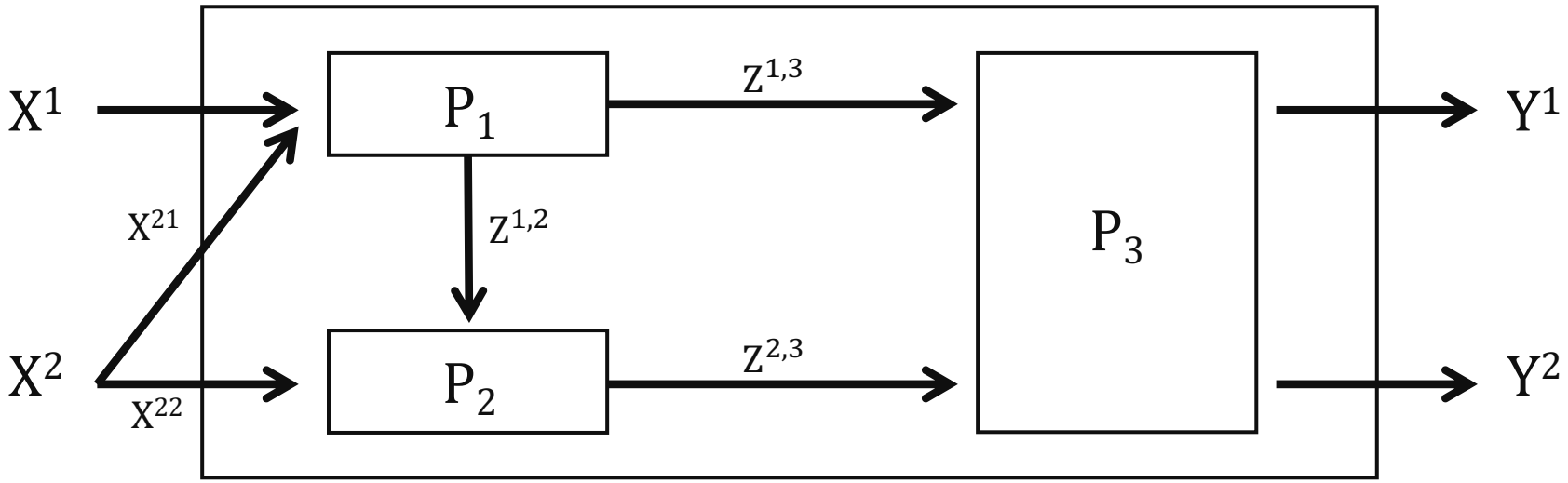


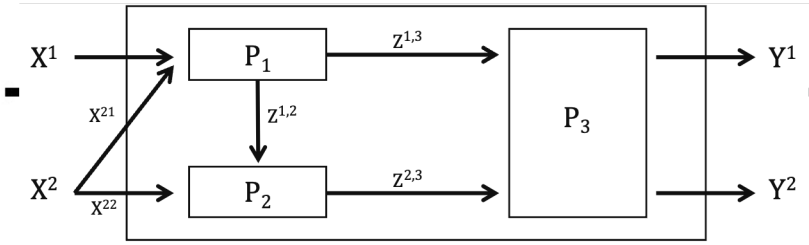
P_1: premium acquisition

P_2: reinsurance allocation

P_3: profit generation

Data Collection

https://www.tii.org.tw/tii/actuarial/actuarial1/report/result.html



產物保險公司損益表

中華民國１０７年至１０８年

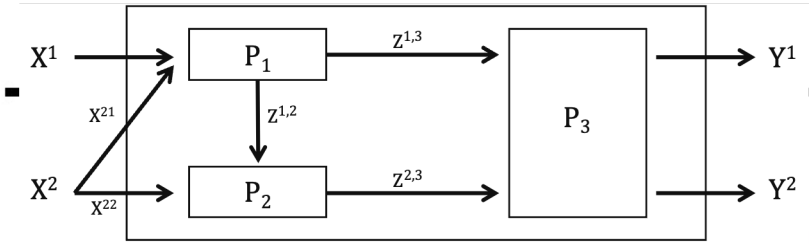
目

Items

營業收入	**Operating Revenues**
簽單保費收入	Premium income
再保費收入	Reinsurance Premium income
減：再保費支出 $Z^{1,2}$	Premiums ceded to reinsurers
減：未滿期保費準備淨變動	Provision(Recovery) for Unearned Premium Reserve
自留滿期保費收入	Retained Premiums Earned
再保佣金收入 $Z^{2,3}$	Reinsurance Commission Received
手續費收入	Service Charges
淨投資損益 Y^2	Financial Receipts (Expenses)
利息收入	Interest Income
透過損益按公允價值衡量之金融資產及負債損益	Gain (Loss) on Financial Assets and Liabilities Measured at Fair Value through Profit or Loss
透過其他綜合損益按公允價值衡量之金融資產已實現損益	Realized Gain (Loss) on Financial Assets Measured at Fair Value through Other Comprehensive Income
除列按攤銷後成本衡量之金融資產淨損益	Gain (Loss) from Derecognition of Financial Assets Measured at Amortized Cost
採用權益法認列之關聯企業及合資權益之份額	Share of the Profit or Loss of Associates and Joint Ventures Accounted Equity Method
兌換損益－投資	Realized Foreign Exchange Gains & Losses

Data Collection

https://www.tii.org.tw/tii/actuarial/actuarial1/report/result.html



產物保險公司損益表

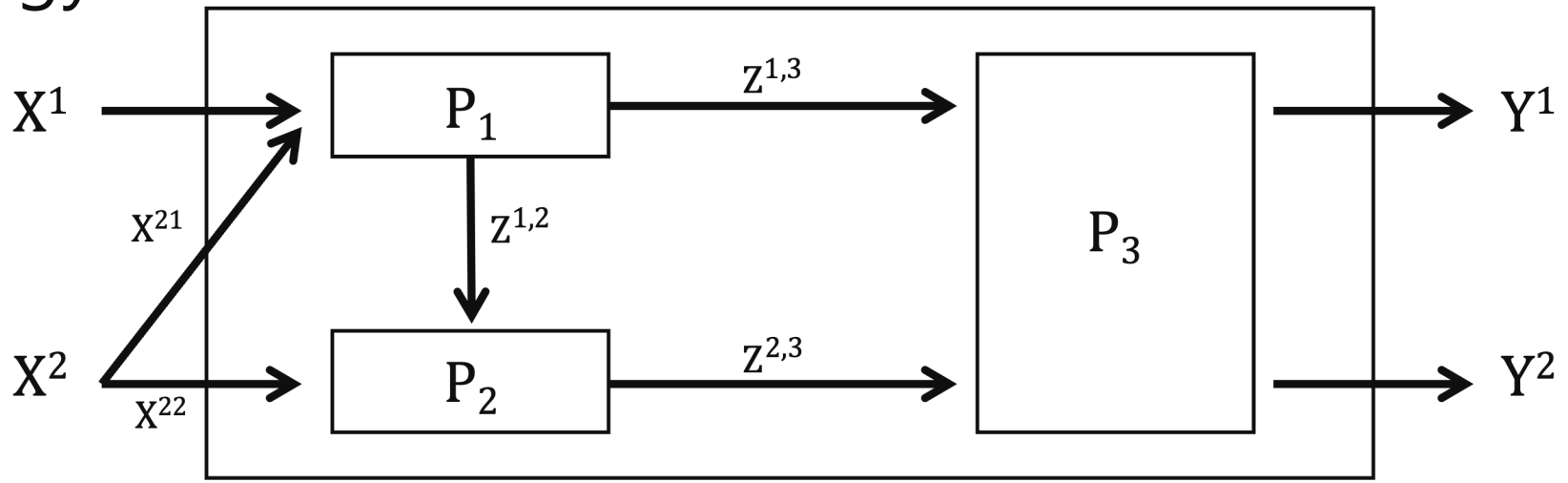
中華民國１０７年至１０８年

目

Items

承保費用支出	X^1	Acquisition Expenses
佣金費用		Commission Expenses
其他營業成本	X^2	Other Operating Expenses
財務成本		Finance Costs
分離帳戶保險商品費用		Insurance Product Expenses - Separate Account
營業費用		**Operating Expenses**
營業外收入及支出		**Non-Operating Income and Expenses**
繼續營業單位稅前純益（純損）		**Continuing Operating Income before Tax**
所得稅（費用）利益		**Tax Expense (Benefit)**
繼續營業單位本期淨利（淨損）		**Net Profit (Net Loss) from Continuing Operations**
停業單位損益		**Profit (Loss) from Discontinued Operations**
本期淨利(淨損)		**Net Profit (Loss)**
本期其他綜合損益（稅後淨額）		**Other Comprehensive Income(Net of Tax)**
本期綜合損益總額		**Total Comprehensive Income**

Methodology - network DEA model CRS



$$\text{Max} \quad u^1 Y_k^1 + u^2 Y_k^2 Y_{2k}$$

$$\text{s.t.} \quad v^1 X_k^1 + v^2 X_k^2 = 1$$

$$P_1 \quad \left(w^{1,3} Z_j^{1,3} + w^{1,2} Z_j^{1,2}\right) - \left(v^1 X_j^1 + v^2 X_j^{21}\right) \le 0 \;\; j = 1, \dots, n$$

$$P_2 \quad \left(w^{2,3} Z_j^{2,3}\right) - \left(v^2 X_j^{22} + w^{1,2} Z_j^{1,2}\right) \le 0 \;\; j = 1, \dots, n$$

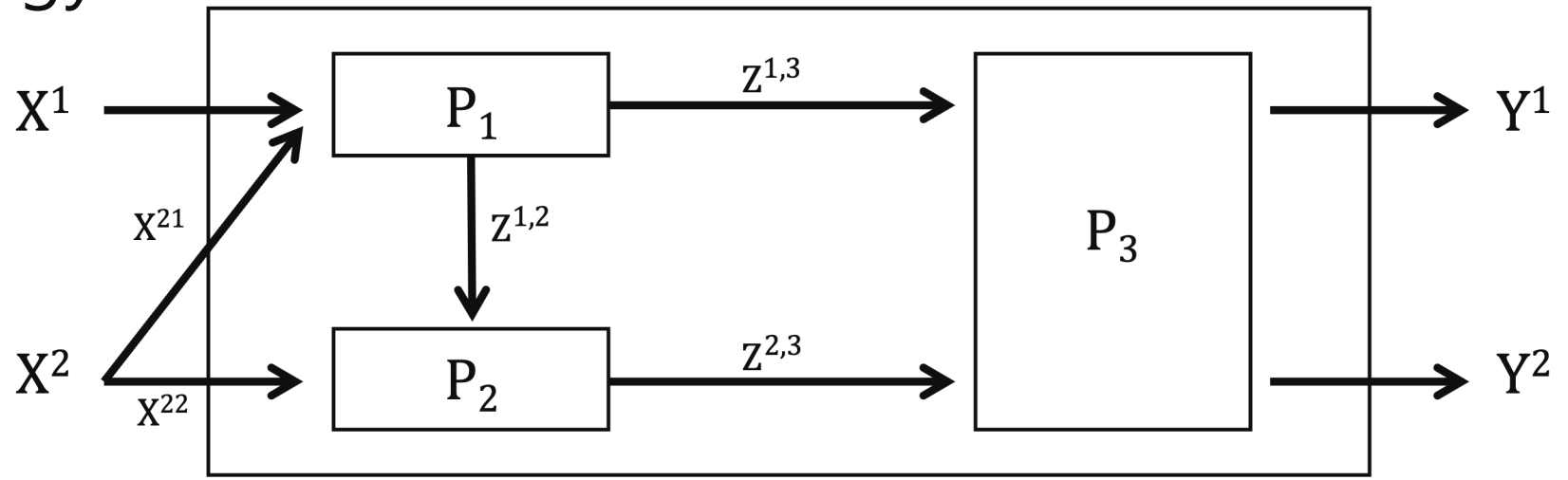
$$P_3 \quad \left(u^1 Y_j^1 + u^2 Y_j^2\right) - \left(w^{1,3} Z_j^{1,3} + w^{2,3} Z_j^{2,3}\right) \le 0 \;\; j = 1, \dots, n$$

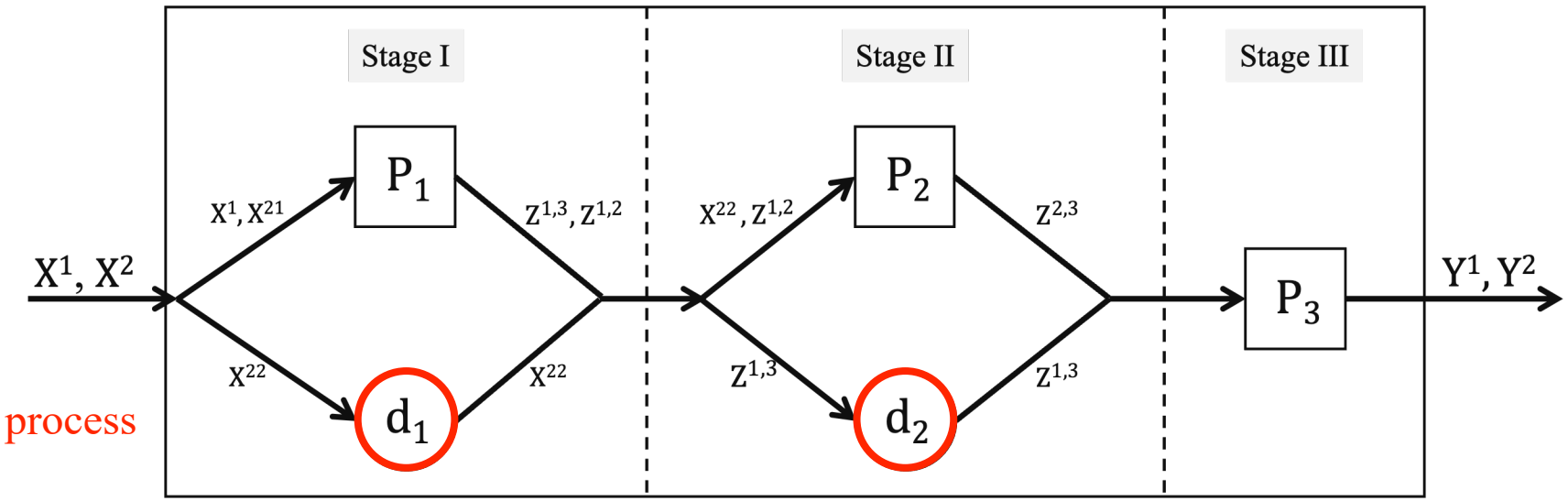
$$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2 \ge \varepsilon$$

$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2$

$X^1, X^2, Z^{1,2}, Z^{1,3}, Z^{2,3}, Y^1, Y^2$ 的權重

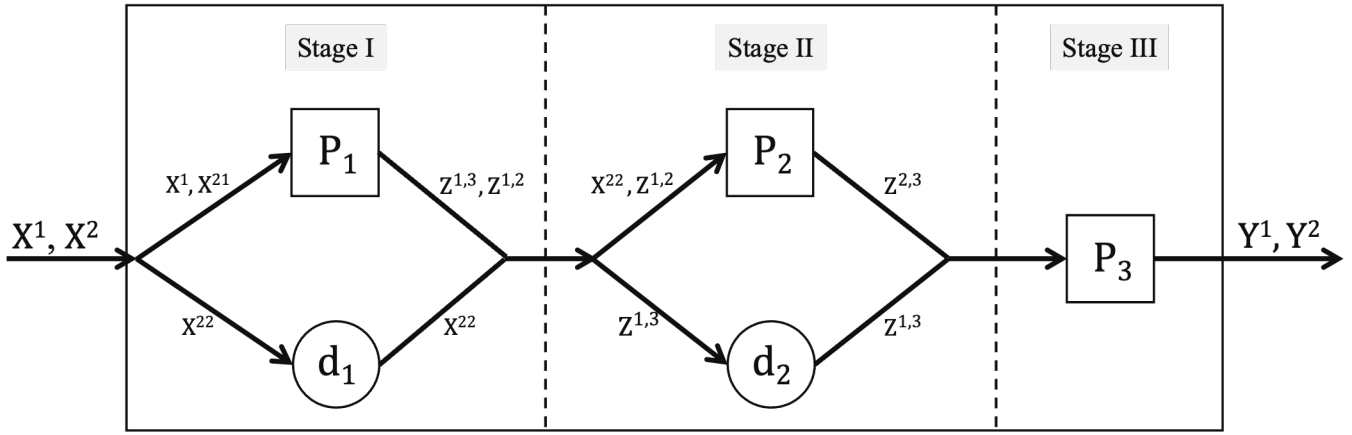
Methodology - network DEA model CRS





dummy process

Methodology - network DEA model CRS



$$E_k^{(1)} = (w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3})/(v^{1*}X_k^1 + v^{2*}X_k^{21})$$

$$E_k^{(2)} = (w^{2,3*}Z_k^{2,3})/(v^{2*}X_k^{22} + w^{1,2*}Z_k^{1,2})$$

$$E_k^{(3)} = (u^{1*}Y_k^1 + u^{2*}Y_k^2)/(w^{2,3*}Z_k^{2,3} + w^{1,3*}Z_k^{1,3})$$

Redundant in LP formulation

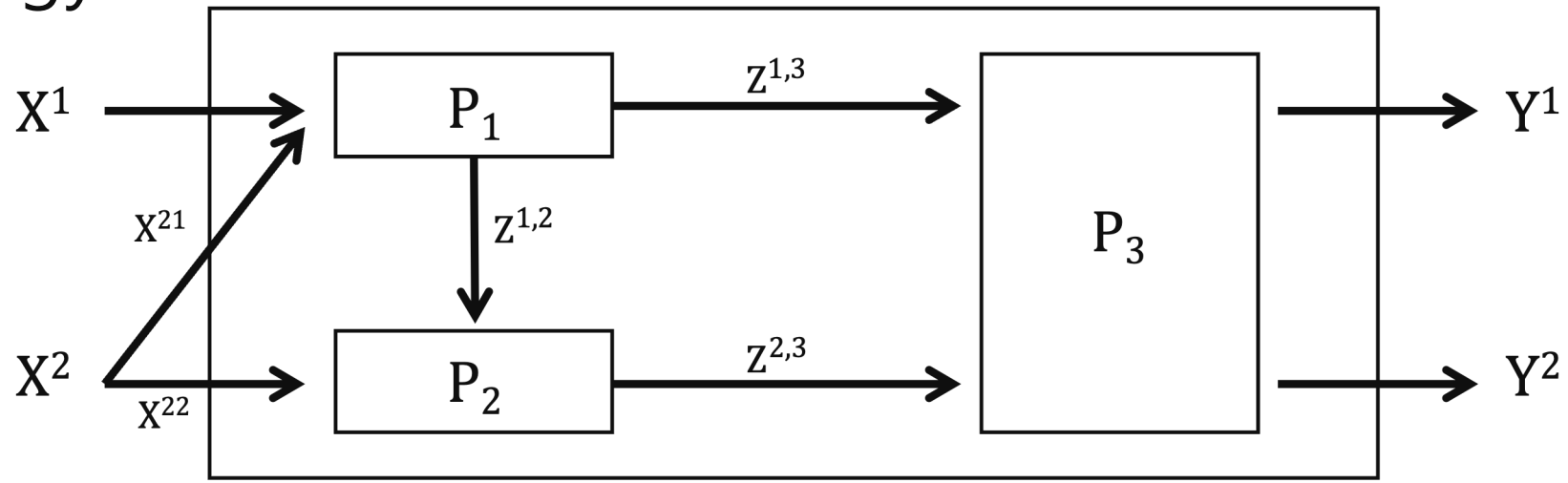
$$E_k^{I} = (w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3} + v^{2*}X_k^{22})/(v^{1*}X_k^1 + v^{2*}X_k^{21} + v^{2*}X_k^{22})$$

$$E_k^{II} = (w^{2,3*}Z_k^{2,3} + w^{1,3*}Z_k^{1,3})/(w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3} + v^{2*}X_k^{22})$$

$$E_k^{III} = E_k^{(3)}$$

$$E_k = E_k^{I} * E_k^{II} * E_k^{III}$$

Methodology - network DEA model CRS



$$\text{Max} \quad u^1 Y_k^1 + u^2 Y_k^2 Y_{2k}$$

$$\text{s.t.} \quad v^1 X_k^1 + v^2 X_k^2 = 1$$

$$\text{P}_1 \quad \left(w^{1,3} Z_j^{1,3} + w^{1,2} Z_j^{1,2} + v^2 X_k^{22}\right) - \left(v^1 X_j^1 + v^2 X_j^{21} + v^2 X_k^{22}\right) \le 0 \quad j = 1, \dots,$$

$$\text{P}_2 \quad \left(w^{2,3} Z_j^{2,3} + w^{1,3} Z_k^{1,3}\right) - \left(v^2 X_j^{22} + w^{1,2} Z_j^{1,2} + w^{1,3} Z_k^{1,3}\right) \le 0 \quad j = 1, \dots, n$$

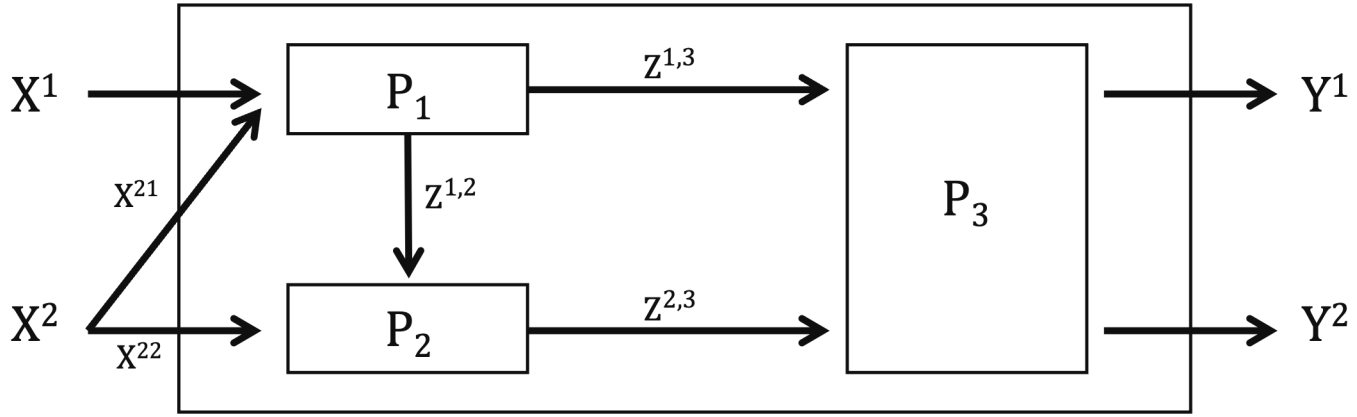
$$\text{P}_3 \quad \left(u^1 Y_j^1 + u^2 Y_j^2\right) - \left(w^{1,3} Z_j^{1,3} + w^{2,3} Z_j^{2,3}\right) \le 0 \quad j = 1, \dots, n$$

$$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2 \ge \varepsilon$$

$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2$

$X^1, X^2, Z^{1,2}, Z^{1,3}, Z^{2,3}, Y^1, Y^2$ 的權重

Methodology - network DEA model VRS input-oriented



$$\text{Max} \quad u^1 Y_k^1 + u^2 Y_k^2 Y_{2k} - u_0$$

$$\text{s.t.} \quad v^1 X_k^1 + v^2 X_k^2 = 1$$

$$\left(w^{1,3} Z_j^{1,3} + w^{1,2} Z_j^{1,2} - w_0^1\right) - \left(v^1 X_j^1 + v^2 X_j^{21}\right) \le 0 \quad j = 1, \dots, n$$

$$\left(w^{2,3} Z_j^{2,3} - w_0^2\right) - \left(v^2 X_j^{22} + w^{1,2} Z_j^{1,2} - w_0^1\right) \le 0 \quad j = 1, \dots, n$$

$$\left(u^1 Y_j^1 + u^2 Y_j^2 - u_0\right) - \left(w^{1,3} Z_j^{1,3} - w_0^1 + w^{2,3} Z_j^{2,3} - w_0^2\right) \le 0 \quad j = 1, \dots, n$$

$$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2 \ge \varepsilon$$

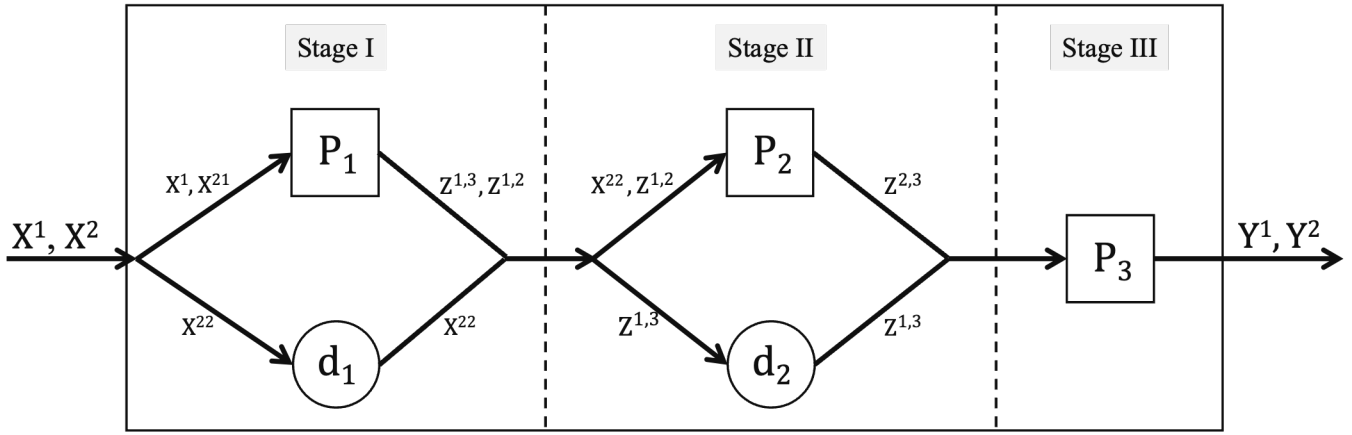
$v^1, v^2, w^{1,2}, w^{1,3}, w^{2,3}, u^1, u^2$

$X^1, X^2, Z^{1,2}, Z^{1,3}, Z^{2,3}, Y^1, Y^2$ 的權重

w_0^1, w_0^2, u_0

Multiplier variables

Methodology - network DEA model VRS input-oriented



$$T_k^{(1)} = (w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3} - w_0^{1*})/(v^{1*}X_k^1 + v^{2*}X_k^{21})$$

$$T_k^{(2)} = (w^{2,3*}Z_k^{2,3} - w_0^{2*})/(v^{2*}X_k^{22} + w^{1,2*}Z_k^{1,2} - w_0^{1*})$$

$$T_k^{(3)} = (u^{1*}Y_k^1 + u^{2*}Y_k^2 - u_0^*)/(w^{2,3*}Z_k^{2,3} - w_0^{2*} + w^{1,3*}Z_k^{1,3} - w_0^{1*})$$

$$T_k^I = (w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3} - w_0^{1*} + v^{2*}X_k^{22})/(v^{1*}X_k^1 + v^{2*}X_k^2)$$

$$T_k^{II} = (w^{2,3*}Z_k^{2,3} - w_0^{2*} + w^{1,3*}Z_k^{1,3} - w_0^{1*})/(w^{1,2*}Z_k^{1,2} + w^{1,3*}Z_k^{1,3} - w_0^{1*} + v^{2*}X_k^{22})$$

$$T_k^{III} = T_k^{(3)}$$

$$T_k = T_k^I * T_k^{II} * T_k^{III}$$

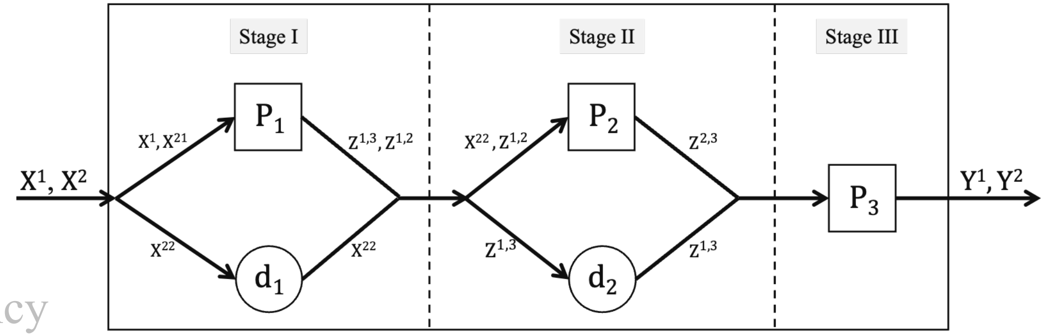
Efficiency Measure in Insurance Industry – A Network DEA Model

Analysis Result

Life insurance

measure by process efficiency

measure by stage efficiency



	OE_total		TE_total		SE_total		OE_p1		TE_p1		SE_p1		OE_p2		TE_p2		SE_p2		OE_p3		TE_p3		SE_p3	
臺銀人壽	0.227	6	0.227	10	1.000	1	1.000	2	1.000	1	1.000	4	0.034	12	0.034	14	1.000	10	0.418	17	0.418	18	1.000	7
台灣人壽	0.277	4	0.293	8	0.947	10	0.683	5	0.762	9	0.896	13	0.083	8	0.078	11	1.058	3	0.687	8	0.690	11	0.995	11
保誠人壽	0.102	17	0.115	19	0.891	14	0.151	17	0.211	18	0.713	16	0.000	22	0.117	8	-0.002	21	0.903	4	0.690	12	1.308	2
國泰人壽	0.164	12	1.000	1	0.164	20	0.419	13	1.000	1	0.419	18	0.023	15	1.000	1	0.023	19	0.675	10	1.000	4	0.675	20
中國人壽	0.289	3	0.316	7	0.913	12	0.886	3	1.000	1	0.886	14	0.061	10	0.057	13	1.056	5	0.572	15	0.589	16	0.970	12
南山人壽	0.180	10	0.468	4	0.384	19	0.314	15	0.853	8	0.369	19	0.010	19	1.000	1	0.010	20	0.874	5	1.000	1	0.874	15
新光人壽	0.184	9	0.194	13	0.952	8	0.455	11	0.526	13	0.866	15	0.016	17	0.013	17	1.203	2	0.865	6	0.826	9	1.047	3
富邦人壽	0.195	8	0.210	11	0.929	11	0.597	8	1.000	1	0.597	17	0.046	11	-0.450	22	-0.102	22	0.579	14	0.378	20	1.532	1
三商美邦人壽	0.203	7	0.204	12	0.991	7	0.491	9	0.540	12	0.910	12	0.079	9	0.075	12	1.057	4	0.680	9	0.669	13	1.016	4
遠雄人壽	0.157	13	0.182	14	0.863	16	0.862	4	0.862	7	1.000	4	0.011	18	0.011	19	1.000	14	0.427	16	0.495	17	0.863	17
宏泰人壽	0.341	1	0.341	5	1.000	1	0.625	6	0.625	10	1.000	4	0.006	21	0.006	21	1.000	10	1.000	1	1.000	1	1.000	7
安聯人壽	0.047	21	0.778	2	0.060	22	0.142	18	0.557	11	0.255	21	0.743	4	1.000	1	0.743	18	0.112	22	1.000	7	0.112	22
中華郵政	0.339	2	0.339	6	1.000	1	1.000	1	1.000	1	1.000	4	1.000	1	1.000	1	1.000	15	0.339	19	0.339	21	1.000	7
第一金人壽	0.171	11	0.180	15	0.950	9	0.432	12	0.432	16	1.000	4	0.114	6	0.114	9	1.000	8	0.610	13	0.642	14	0.950	13
合作金庫人壽	0.134	14	0.160	16	0.838	17	0.091	20	0.091	21	1.000	4	0.361	5	0.361	7	1.000	10	0.621	11	0.742	10	0.838	18
保德信國際人壽	0.085	18	0.085	20	1.000	1	0.182	16	0.182	19	1.000	4	0.089	7	0.089	10	1.000	9	0.613	12	0.613	15	1.000	6
康健人壽	0.043	22	0.064	22	0.676	18	0.353	14	1.000	1	0.353	20	0.030	13	0.030	15	1.017	7	0.771	7	1.000	5	0.771	19
元大人壽	0.133	15	0.146	17	0.911	13	0.620	7	0.461	15	1.346	2	0.023	14	0.012	18	1.955	1	0.380	18	0.405	19	0.937	14
全球人壽	0.245	5	0.247	9	0.995	6	0.471	10	0.500	14	0.942	11	0.021	16	0.020	16	1.044	6	0.949	3	0.943	8	1.007	5
友邦人壽	0.065	19	0.065	21	1.000	5	0.117	19	0.112	20	1.041	3	0.009	20	0.009	20	0.999	16	1.000	1	1.000	1	1.000	7
法國巴黎人壽	0.063	20	0.633	3	0.100	21	0.058	21	0.266	17	0.217	22	1.000	1	1.000	1	1.000	10	0.119	21	1.000	6	0.119	21
安達人壽	0.109	16	0.125	18	0.875	15	0.032	22	0.015	22	2.187	1	0.984	3	1.000	1	0.984	17	0.231	20	0.265	22	0.873	16
avg.	0.171		0.290		0.793		0.454		0.591		0.864		0.215		0.299		0.866		0.610		0.714		0.904	
std.	0.085		0.227		[illegible]		[illegible]		[illegible]		[illegible]		[illegible]		[illegible]		0.477		0.256		0.243		0.307	

Technical efficiency lower than 0

Scale efficiency greater than 1

Most of the results follow the common sense

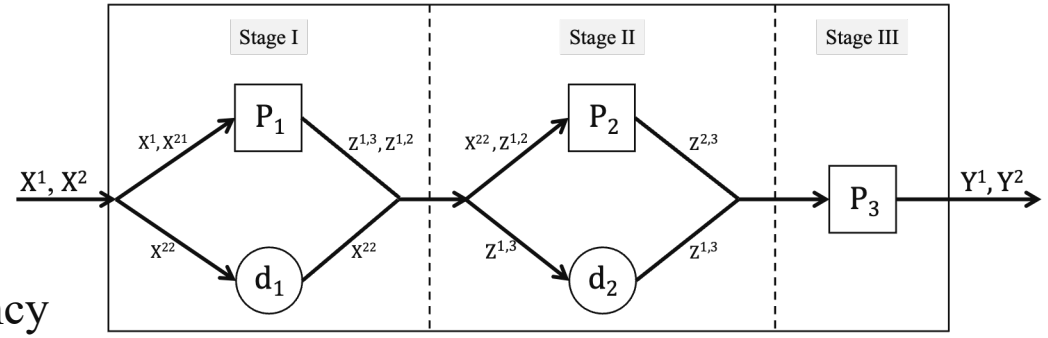
Efficiency Measure in Insurance Industry – A Network DEA Model

Analysis Result

measure by process efficiency

Life insurance

measure by stage efficiency



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臺銀人壽	0.227	6	0.227	10	1.000	1	1.000	2	1.000	6	1.000	5	0.543	5	0.543	8	1.000	7	0.418	17	0.418	18	1.000	7
台灣人壽	0.277	4	0.293	8	0.947	10	0.830	5	0.878	8	0.945	15	0.486	7	0.483	11	1.008	4	0.687	8	0.690	11	0.995	11
保誠人壽	0.102	17	0.115	19	0.891	14	0.573	17	0.590	18	0.973	11	0.198	20	0.283	19	0.700	20	0.903	4	0.690	12	1.308	2
國泰人壽	0.164	12	1.000	1	0.164	20	0.704	12	1.000	1	0.704	20	0.344	16	1.000	1	0.344	22	0.675	10	1.000	4	0.675	20
中國人壽	0.289	3	0.316	7	0.913	12	0.939	4	1.000	1	0.939	16	0.538	6	0.537	9	1.002	5	0.572	15	0.589	16	0.970	12
南山人壽	0.180	10	0.468	4	0.384	19	0.654	15	0.853	9	0.767	19	0.314	17	0.549	7	0.572	21	0.874	5	1.000	1	0.874	15
新光人壽	0.184	9	0.194	13	0.952	8	0.712	11	0.744	14	0.957	13	0.299	18	0.315	18	0.949	16	0.865	6	0.826	9	1.047	3
富邦人壽	0.195	8	0.210	11	0.929	11	0.788	8	1.000	1	0.788	18	0.426	10	0.555	6	0.769	19	0.579	14	0.378	20	1.532	1
三商美邦人壽	0.203	7	0.204	12	0.991	7	0.730	9	0.765	12	0.954	14	0.409	12	0.399	15	1.023	2	0.680	9	0.669	13	1.016	4
遠雄人壽	0.157	13	0.182	14	0.863	16	0.942	3	0.94												0.495	17	0.863	17
宏泰人壽	0.341	1	0.341	5	1.000	1	0.797	6	0.797	10	1.000	1	0.428	9	0.428	12	1.000	1	1.000	1	1.000	1	1.000	7
安聯人壽	0.047	21	0.778	2	0.060	22	0.540	19	0.778	11	0.694	21	0.779	4	1.000	5	0.779	18	0.112	22	1.000	7	0.112	22
中華郵政	0.339	2	0.339	6	1.000	1	1.000	1	1.000	1	1.000	5	1.000	1	1.000	2	1.000	7	0.339	19	0.339	21	1.000	7
第一金人壽	0.171	11	0.180	15	0.950	9	0.702	13	0.702	16	1.000	10	0.401	13	0.401	14	1.000	6	0.610	13	0.642	14	0.950	13
合作金庫人壽	0.134	14	0.160	16	0.838	17	0.512	21	0.512	21	1.000	5	0.422	11	0.422	13	1.000	7	0.621	11	0.742	10	0.838	18
保德信國際人壽	0.085	18	0.085	20	1.000	1	0.580	16	0.580	19	1.000	9	0.238	19	0.238	20	1.000	7	0.613	12	0.613	15	1.000	6
康健人壽	0.043	22	0.064	22	0.676	18	0.666	14	1.000	1	0.666	22	0.085	22	0.064	22	1.317	1	0.771	7	1.000	5	0.771	19
元大人壽	0.133	15	0.146	17	0.911	13	0.792	7	0.730	15	1.084	1	0.443	8	0.494	10	0.896	17	0.380	18	0.405	19	0.937	14
全球人壽	0.245	5	0.247	9	0.995	6	0.721	10	0.745	13	0.967	12	0.358	15	0.351	17	1.022	3	0.949	3	0.943	8	1.007	5
友邦人壽	0.065	19	0.065	21	1.000	5	0.542	18	0.540	20	1.004	3	0.129	21	0.121	21	0.996	14	1.000	1	1.000	1	1.000	7
法國巴黎人壽	0.063	20	0.633	3	0.100	21	0.529	20	0.633	17	0.836	17	1.000	1	1.000	2	1.000	7	0.119	21	1.000	6	0.119	21
安達人壽	0.109	16	0.125	18	0.875	15	0.479	22	0.471	22	1.019	2	0.985	3	1.000	2	0.985	15	0.231	20	0.265	22	0.873	16
avg.	0.171		0.290		0.793		0.715		0.785		0.923		0.464		0.526		0.926		0.610		0.714		0.904	
std.	0.085		0.227		0.311		0.170		0.191		0.159		0.259		0.259		0.195		0.256		0.243		0.307	

Corrected technical efficiency which is lower than 0

Scale efficiency greater than 1

We can gain more reasonable results

Conclusion

Contributions:

1. Use **dummy process** to deal with complicated network system, to handle arithmetic problem in network-DEA VRS model
2. Update network framework in insurance industry
3. Update insurance data sets from 2002 to 2019

Restrictions:

1. Efficiency decomposition in scale efficiency can still greater than 1 for some arithmetic evidence
2. Some final output was lower than one according to financial statement, there would be better way instead of adding all output of DMUs uniformly
3. We cannot tell the whole efficiency simply based on financial statement

Conclusion

Future work:

1. Use penal data to evaluate two dimensional efficiency decomposition in insurance industry
2. Trade off between underwriting profit and investment profit
3. IFRSs 17 would change the financial statement in insurance industry. It would be a chance to renew model

Q&A

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