homework-3

```r
library(bis557)
```

Question 1

```r
#the linear Hessian is well-conditioned
set.seed(2020)
X <- matrix(rnorm(10), ncol=2)
#calculate condition number
svals<- svd(X)$d
max(svals)/ min(svals)
#> [1] 1.610345
```

```r
#the logistic variation is not
X <- cbind(rep(1,5), X)
beta <- c(0.1, 100,1)
mu <- 1 / (1+exp(-X %*% beta))
D <- diag(x=as.vector(mu), nrow=5, ncol=5)
H <- t(X) %*% D %*% X
#calculate condition number
svals<- svd(H)$d
max(svals)/ min(svals)
#> [1] 6.688635e+16
```

Question 2

1st-order solution for the GLM maximum likelihood problem: constant step size

```r
set.seed(20)
n = 1000; p = 5
X <- cbind(1, matrix(rnorm(n*(p-1)), ncol=p-1))
Y = sample(c(0,1), replace=TRUE, size=n)
glm_grad(X, Y, update=FALSE)
#> [[1]]
#>              [,1]
#> [1,] -0.70550039
#> [2,] -0.00184793
#> [3,] -0.09250006
#> [4,]  0.04454898
#> [5,]  0.06017863
```

1st-order solution for the GLM maximum likelihood problem: adaptive step size

```r
glm_grad(X, Y, update=TRUE)
```

```
#> [[1]]
#>               [,1]
#> [1,] -0.2020982895
#> [2,]  0.0008404792
#> [3,] -0.0134230734
#> [4,]  0.0051157479
#> [5,]  0.0133863702
library(MASS)
df <- as.data.frame(cbind(X,Y))
glm(Y ~ .-1, data=df, family = poisson(link="log"))
#>
#> Call:  glm(formula = Y ~ . - 1, family = poisson(link = "log"), data = df)
#>
#> Coefficients:
#>        V1         V2         V3         V4         V5
#> -0.710432  -0.001723  -0.095592   0.046264   0.061304
#>
#> Degrees of Freedom: 1000 Total (i.e. Null);  995 Residual
#> Null Deviance:       1006
#> Residual Deviance: 687.7     AIC: 1692
```

Here we can see the fitting of constant step size is a little better.

Question 3

The classification model to generalize logistic regression to accommodate 3 classes

```
#test the algorithm
X <- matrix(rnorm(500), ncol=5)
y <- sample(c(1,2,3) , size=100, replace = T)
class_log(X,y, class=3)
#> $fit
#>             [,1]       [,2]       [,3]
#>   [1,] 0.3402711 0.15697081 0.50275809
#>   [2,] 0.3669822 0.41921569 0.21380209
#>   [3,] 0.3811645 0.49334120 0.12549434
#>   [4,] 0.2077950 0.11521606 0.67698889
#>   [5,] 0.3586300 0.47640691 0.16496314
#>   [6,] 0.2934174 0.62406504 0.08251759
#>   [7,] 0.5211966 0.18901958 0.28978381
#>   [8,] 0.1900963 0.11559882 0.69430493
#>   [9,] 0.1525582 0.45018620 0.39725560
#>  [10,] 0.3483038 0.37310135 0.27859485
#>  [11,] 0.3217075 0.09655137 0.58174112
#>  [12,] 0.5197914 0.08092050 0.39928813
#>  [13,] 0.1916419 0.41819740 0.39016070
#>  [14,] 0.3186558 0.56062751 0.12071672
#>  [15,] 0.3349993 0.42653629 0.23846446
#>  [16,] 0.2139189 0.34733949 0.43874158
#>  [17,] 0.1882419 0.21104434 0.60071378
#>  [18,] 0.2545151 0.08030394 0.66518096
```

```
#>  [19,] 0.2609638 0.47886575 0.26017049
#>  [20,] 0.3031191 0.11336138 0.58351953
#>  [21,] 0.1830860 0.33865637 0.47825762
#>  [22,] 0.2943140 0.35349787 0.35218816
#>  [23,] 0.3144615 0.18856581 0.49697265
#>  [24,] 0.3965600 0.48272439 0.12071558
#>  [25,] 0.3812611 0.31821679 0.30052211
#>  [26,] 0.3473388 0.22132010 0.43134113
#>  [27,] 0.3884189 0.40955219 0.20202890
#>  [28,] 0.2476446 0.53624815 0.21610728
#>  [29,] 0.4297631 0.19606051 0.37417637
#>  [30,] 0.3209738 0.19907556 0.47995067
#>  [31,] 0.3320733 0.10108551 0.56684114
#>  [32,] 0.3089869 0.47886716 0.21214597
#>  [33,] 0.1432741 0.08766623 0.76905962
#>  [34,] 0.2551965 0.30487422 0.43992927
#>  [35,] 0.2419120 0.55294585 0.20514215
#>  [36,] 0.3510438 0.37839768 0.27055856
#>  [37,] 0.3032531 0.50852274 0.18822419
#>  [38,] 0.3757522 0.19104987 0.43319792
#>  [39,] 0.1629021 0.18637772 0.65072022
#>  [40,] 0.5604136 0.22995364 0.20963272
#>  [41,] 0.2971970 0.36545866 0.33734438
#>  [42,] 0.3044571 0.27777479 0.41776811
#>  [43,] 0.1335030 0.72368486 0.14281217
#>  [44,] 0.3817573 0.53481330 0.08342936
#>  [45,] 0.3239478 0.19706690 0.47898525
#>  [46,] 0.2162065 0.33854833 0.44524521
#>  [47,] 0.3343163 0.36384447 0.30183918
#>  [48,] 0.3373925 0.26000441 0.40260306
#>  [49,] 0.3199763 0.12203428 0.55798939
#>  [50,] 0.3667437 0.43202362 0.20123270
#>  [51,] 0.1434877 0.34677200 0.50974033
#>  [52,] 0.1495312 0.48064474 0.36982408
#>  [53,] 0.5270187 0.35945678 0.11352451
#>  [54,] 0.3932309 0.35801768 0.24875143
#>  [55,] 0.1525723 0.35469702 0.49273068
#>  [56,] 0.5005026 0.35798409 0.14151333
#>  [57,] 0.2987871 0.53401337 0.16719950
#>  [58,] 0.2479142 0.06421850 0.68786727
#>  [59,] 0.3436198 0.57577107 0.08060914
#>  [60,] 0.2291937 0.42744073 0.34336555
#>  [61,] 0.3769664 0.29094102 0.33209258
#>  [62,] 0.2160133 0.66322185 0.12076483
#>  [63,] 0.5618954 0.28674310 0.15136147
#>  [64,] 0.5301192 0.23807516 0.23180563
#>  [65,] 0.3988250 0.34142789 0.25974707
#>  [66,] 0.2281942 0.36102795 0.41077789
#>  [67,] 0.3817828 0.35942202 0.25879519
#>  [68,] 0.2600672 0.14774866 0.59218418
```

```
#>  [69,] 0.5248993 0.20094401 0.27415669
#>  [70,] 0.2282856 0.36047731 0.41123713
#>  [71,] 0.4729243 0.21493482 0.31214086
#>  [72,] 0.2206409 0.32648108 0.45287804
#>  [73,] 0.4460325 0.22541033 0.32855713
#>  [74,] 0.6055364 0.28235392 0.11210970
#>  [75,] 0.5403254 0.20075971 0.25891491
#>  [76,] 0.2864603 0.65687466 0.05666504
#>  [77,] 0.3183391 0.26889113 0.41276975
#>  [78,] 0.3526573 0.45152268 0.19581999
#>  [79,] 0.2073068 0.15029409 0.64239909
#>  [80,] 0.4849239 0.43618274 0.07889340
#>  [81,] 0.4307104 0.11083520 0.45845438
#>  [82,] 0.2666911 0.15947311 0.57383577
#>  [83,] 0.2165699 0.13176906 0.65166101
#>  [84,] 0.3277685 0.13131127 0.54092027
#>  [85,] 0.2568678 0.22781318 0.51531899
#>  [86,] 0.4684926 0.31745088 0.21405650
#>  [87,] 0.2716617 0.53336131 0.19497697
#>  [88,] 0.1680514 0.23433290 0.59761572
#>  [89,] 0.2727501 0.60199633 0.12525362
#>  [90,] 0.1494016 0.23637658 0.61422184
#>  [91,] 0.4047775 0.11748520 0.47773726
#>  [92,] 0.3051090 0.19743003 0.49746096
#>  [93,] 0.3170900 0.21737170 0.46553825
#>  [94,] 0.2594366 0.13760319 0.60296016
#>  [95,] 0.2620679 0.36785193 0.37008016
#>  [96,] 0.2630407 0.24974410 0.48721516
#>  [97,] 0.3623842 0.28143140 0.35618444
#>  [98,] 0.1352594 0.06297348 0.80176713
#>  [99,] 0.3079810 0.24056267 0.45145631
#> [100,] 0.4030091 0.26566420 0.33132673
#>
#> $misclassification
#> [1] 0.51
```