Model description for the implementation of System Identification methodologies on the rotorcraft SH09

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List of symbols

α	Angle of attack	[deg]
	Abbreviations	
CAS	Control Augmentation System	
$CSAS$	Control and Stability Augmentation System	

1 Introduction

Bibliography: [1], [2], [3], [4]

2 Model description

2.1 F-16 model

This section aims to present the reduced equations for the dynamics of the F-16 fixed-wing aircraft which simulated in a non-linear manner using SIDPAC software.

2.1.1 Mathematical models of aircrafts

The notation for the aircraft is the one shown in Figure 1.

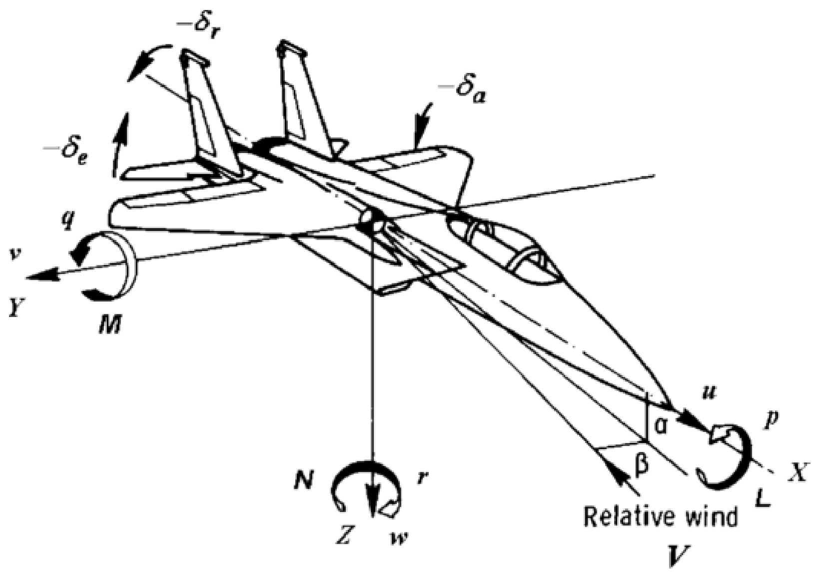

Figure 1: Airplane notation and sign conventions: u, v, w 5 body-axis components of aircraft velocity relative to Earth axes; p, q, r 5 body-axis components of aircraft angular velocity; X, Y, Z 5 body-axis components of aerodynamic force acting on the aircraft; and L, M, N 5 body-axis components of aerodynamic moment acting on the aircraft.

The components of the aerodynamic forces and moments, are the following:

Forces:

$$\text{Body axes} \qquad \text{Stability axes} \tag{1}$$

$$X = \bar{q}SC_X \qquad D = \bar{q}SC_D \tag{2}$$

$$Z = \bar{q}SC_Z \qquad L = \bar{q}SC_L \tag{3}$$

$$Y = \bar{q}SC_Y \qquad Y = \bar{q}SC_Y \tag{4}$$

Moments:

$$L = \bar{q}bSC_l \tag{5}$$

$$M = \bar{q}\bar{c}SC_m \tag{6}$$

$$N = \bar{q}bSC_n \tag{7}$$

where $\bar{q} = 1/2\rho V^2$ is the dynamic pressure, ρ is the air density, V is the airspeed, S is the wing reference area, b is the wing span and $\bar{c}$ is the mean aerodynamic chord (MAC).

The forces expressed in the wind axis systems as shown in set Equations 8.

$$\begin{aligned} C_{\mathrm{L}} &= -C_{\mathrm{Z}}\cos\alpha + C_{\mathrm{X}}\sin\alpha \\ C_{\mathrm{D}} &= -C_{\mathrm{X}}\cos\alpha - C_{\mathrm{Z}}\sin\alpha \end{aligned} \tag{8}$$

The Taylor expansion for the longitudinal motion of the aircraft are expressed in set of Equations 9.

$$\begin{aligned} C_D &= C_{D_0} + C_{D_V}\frac{\Delta V}{V_0} + C_{D_\alpha}\Delta\alpha + C_{D_q}\frac{q\bar{c}}{2V_0} + C_{D_{\delta_\mathrm{e}}}\Delta\delta_\mathrm{e} \\ C_L &= C_{L_0} + C_{L_V}\frac{\Delta V}{V_0} + C_{L_\alpha}\Delta\alpha + C_{L_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{L_q}\frac{q\bar{c}}{2V_0} + C_{L_{\delta_\mathrm{e}}}\delta_\mathrm{e} \\ C_m &= C_{m_0} + C_{m_V}\frac{\Delta V}{V_0} + C_{m_\alpha}\Delta\alpha + C_{m_{\dot\alpha}}\frac{\dot\alpha\bar{c}}{2V_0} + C_{m_q}\frac{q\bar{c}}{2V_0} + C_{m_{\delta_\mathrm{e}}}\delta_\mathrm{e} \end{aligned} \tag{9}$$

The set of equations that describe the motion of the aircraft, obtained from the Taylor series expansion is the one represented in Equation 10.

$$\begin{aligned} C_Y &= C_{Y_0} + C_{Y_\beta}\Delta\beta + C_{Y_p}\frac{pb}{2V_0} + C_{Y_r}\frac{rb}{2V_0} + C_{Y_{\delta_\mathrm{a}}}\Delta\delta_\mathrm{a} + C_{Y_{\delta_\mathrm{r}}}\Delta\delta_\mathrm{r} \\ C_l &= C_{l_0} + C_{l_\beta}\Delta\beta + C_{l_p}\frac{pb}{2V_0} + C_{l_r}\frac{rb}{2V_0} + C_{l_{\delta_\mathrm{a}}}\Delta\delta_\mathrm{a} \qquad (C_{l_{\delta_\mathrm{r}}} \ll 1) \\ C_n &= C_{n_0} + C_{n_\beta}\Delta\beta + C_{n_p}\frac{pb}{2V_0} + C_{n_r}\frac{rb}{2V_0} + C_{n_{\delta_\mathrm{r}}}\Delta\delta_\mathrm{r} \qquad (C_{l_{\delta_\mathrm{a}}} \ll 1) \end{aligned} \tag{10}$$

2.2 Helicopter model

2.2.1 Model forces and moments coefficients

Forces:

$$X_u, X_v, X_w, X_p, X_q, X_r Y_u, Y_v, Y_w, Y_p, Y_q, Y_r Z_u, Z_v, Z_w, Z_p, Z_q, Z_r$$

Moments:

$$L_u, L_v, L_w, L_p, L_q, L_r M_u, M_v, M_w, M_p, M_q, M_r N_u, N_v, N_w, N_p, N_q, N_r$$

Controllability, **G** matrix: Forces

$$X_{\theta_{\text{lon}}}, X_{\theta_{\text{lat}}}, X_{\theta_{\text{ped}}}, X_{\theta_{\text{col}}} Y_{\theta_{\text{lon}}}, Y_{\theta_{\text{lat}}}, Y_{\theta_{\text{ped}}}, Y_{\theta_{\text{col}}} Z_{\theta_{\text{lon}}}, Z_{\theta_{\text{lat}}}, Z_{\theta_{\text{ped}}}, Z_{\theta_{\text{col}}}$$

Moments

$$M_{\theta_{\text{lon}}}, M_{\theta_{\text{lat}}}, M_{\theta_{\text{ped}}}, M_{\theta_{\text{col}}} N_{\theta_{\text{lon}}}, N_{\theta_{\text{lat}}}, N_{\theta_{\text{ped}}}, N_{\theta_{\text{col}}} N_{\theta_{\text{lon}}}, N_{\theta_{\text{lat}}}, N_{\theta_{\text{ped}}}, N_{\theta_{\text{col}}}$$

Time delays

$$\tau_{\text{lon}}, \tau_{\text{lat}}, \tau_{\text{ped}}, \tau_{\text{col}}$$

2.2.2 Equations of motion for a linearised model

The linearisation of forces and moments by means of the small perturbation theory about the body axes centre leads to:

$$X = m\{\dot{u} + qW_e - d_x(q^2 + r^2) + d_y(pq - \dot{r}) + d_z(pr + \dot{q})\} \tag{11}$$

$$Y = m\{\dot{v} - pW_e + rU_e + d_x(pq + \dot{r}) - d_y(p^2 + r^2) + d_z(qr - \dot{p})\} \tag{12}$$

$$Z = m\{\dot{w} - qU_e + d_x(pr - \dot{q}) + d_y(qr + \dot{p}) + d_z(p^2 + q^2)\} \tag{13}$$

$$L = I_{xx}\dot{p} - I_{xz}\dot{r} - Yd_z + Zd_y \tag{14}$$

$$M = I_{yy}\dot{q} - Zd_x + Xd_z \tag{15}$$

$$N = I_{zz}\dot{r} - I_{xz}\dot{p} - Xd_y + Yd_x \tag{16}$$

And, considering the forces and moments arise from aerodynamic, gravitational and control sources, it can be written that:

$$X = u\mathring{X}_u + w\mathring{X}_w + q\mathring{X}_q - \theta mg\cos\theta_e + \theta_{\text{lon}}\mathring{X}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{X}_{\theta_{\text{col}}} \tag{17}$$

$$Y = v\mathring{Y}_v + p\mathring{Y}_p + r\mathring{Y}_r + \phi mg\sin\theta_e + \psi mg\cos\theta_e + \theta_{\text{lat}}\mathring{Y}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{Y}_{\theta_{\text{ped}}} \tag{18}$$

$$Z = u\mathring{Z}_u + w\mathring{Z}_w + q\mathring{Z}_q - \theta mg\sin\theta_e + \theta_{\text{lon}}\mathring{Z}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{Z}_{\theta_{\text{col}}} \tag{19}$$

$$L = v\mathring{L}_v + p\mathring{L}_p + r\mathring{L}_r + \theta_{\text{lat}}\mathring{L}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{L}_{\theta_{\text{ped}}} + \theta_{\text{lon}}\mathring{L}_{\theta_{\text{lon}}} \tag{20}$$

$$M = u\mathring{M}_u + w\mathring{M}_w + q\mathring{M}_q + \theta_{\text{lon}}\mathring{M}_{\theta_{\text{lon}}} + \theta_{\text{col}}\mathring{M}_{\theta_{\text{col}}} + \theta_{\text{ped}}\mathring{M}_{\theta_{\text{ped}}} + \theta_{\text{lat}}\mathring{M}_{\theta_{\text{lat}}} \tag{21}$$

$$N = v\mathring{N}_v + p\mathring{N}_p + r\mathring{N}_r + \theta_{\text{lat}}\mathring{N}_{\theta_{\text{lat}}} + \theta_{\text{ped}}\mathring{N}_{\theta_{\text{ped}}} \tag{22}$$

2.2.3 Longitudinal and lateral dynamics from the linearized model

The linearized model for longitudinal dynamics results on:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \mathbf{A}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{B}\begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{23}$$

$$\mathbf{y} = \mathbf{C}\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \mathbf{D}\begin{bmatrix} \theta_{\text{col}} \\ \theta_{\text{lon}} \end{bmatrix} \tag{24}$$

Ultimately, this will result in the following characteristic equation:

$$(T_1 s + 1)(T_2 s + 1)/(s^2 + 2\zeta\omega_n s + \omega_n^2) = 0$$

And for the lateral, results on:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \mathbf{A}\begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{B}\begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{25}$$

$$\mathbf{y} = \mathbf{C}\begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \mathbf{D}\begin{bmatrix} \theta_{\text{lat}} \\ \theta_{\text{ped}} \end{bmatrix} \tag{26}$$

Ultimately, this will result in the following characteristic equation for the lateral/directional motion:

$$(T_1 s + 1)(T_2 s + 1)/(s^2 + 2\zeta\omega_n s + \omega_n^2)s = 0$$

3 Theory of Helicopters

3.1 Equations of Motion for Rigid Airframe

As shown in [5]...

The axes to be used are the helicopter body axes (O, x, y, z) fixed in the helicopter and with its origin at the body axes centre. The components of velocity and force along the Ox, Oy and Oz axes are U, V, W and X, Y, Z, respectively. The components of the rates of rotation about the same axes are p, q and r and the moments L, M and N.

Considering the position that the position of the centre of gravity CG is given by the co-ordinates d_x, d_y and d_z relative to the body axes centre, the absolute velocity of CG is given by u', v' and w':

$$u' = U - rd_y + qd_z \qquad v' = V - pd_z + rd_x \qquad w' = W - qd_x + pd_y \tag{27}$$

and similarly, for the accelerations of the CG:

$$a'_x = \dot{U}' - rv' + qw' \qquad a'_y = \dot{v}' - pw' + ru' \qquad a'_z = \dot{w}' - qu' + pv' \tag{28}$$

3.2 Relationship between feathering law and flapping law for hovering

The flapping equation for hover is given by:

$$M^{a,E}_{b,y_{A1}} + k_\beta \beta + I_\beta \left(\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} + \Omega^2 \beta \right) + x_{\mathrm{GB}} M_P \Omega^2 e \beta = 0,$$

where the aerodynamic moment is given by:

$$M^{a,E}_{b,y_{A1}} = \rho a c \Omega^2 R^4 \left\{ \left[-\frac{1}{6} + \frac{1}{4}\frac{e}{R} - \frac{1}{12}\left(\frac{e}{R}\right)^3 \right] \lambda_i + \left[\frac{1}{8} - \frac{1}{3}\frac{e}{R} ... \right] \right\}, \tag{29}$$

then, substituting $\psi = \Omega t$:

$$\frac{\mathrm{d}^2\beta}{\mathrm{d}\psi^2} + \eta_\beta \frac{\mathrm{d}\beta}{\mathrm{d}\psi} + \lambda_\beta^2 \beta + \delta_\beta \lambda_i - \alpha_\beta \theta = 0, \tag{30}$$

where η_β, δ_β and α_β are function of the Lock number $\gamma = \rho a c R^4 / I_\beta$, the blade eccentricity e and the rotor radius R:

$$\eta_\beta = \frac{\gamma}{8}\left[1 - \frac{8}{3}\frac{e}{R} + 2\left(\frac{e}{R}\right)^2 - \frac{1}{3}\left(\frac{e}{R}\right)^4\right], \tag{31}$$

$$\delta_\beta = \frac{\gamma}{8}\left[-\frac{4}{3} + 2\frac{e}{R} - \frac{2}{3}\left(\frac{e}{R}\right)^3\right], \tag{32}$$

$$\alpha_\beta = \frac{\gamma}{8}\left[1 - \frac{4}{3}\frac{e}{R} + \frac{1}{3}\left(\frac{e}{R}\right)^4\right]. \tag{33}$$

The blade natural damped natural frequency λ_β is given by:

$$\lambda_\beta = \sqrt{1 + \frac{3}{2(1-e/R)}\frac{e}{R} + 3\frac{k_\beta}{R^3 m_P \Omega^2}\frac{1}{(1-e/R)^3}}.$$

The feathering control law is given by:

$$\theta(\phi) = \theta_0 + \theta_{1C}\cos\phi + \theta_{1S}\sin\phi$$

Then, it can be assumed that the flapping angle can be reduced to its first harmonic:

$$\beta(\phi) = \beta_0 + \beta_{1C}\cos\phi + \beta_{1S}\sin\phi.$$

Then the constant part β_0 is given by:

$$\beta_0 = \frac{\alpha_\beta\theta_0 - \delta_\beta\lambda_{i0}}{\lambda_\beta^2},$$

and β_{1C} and β_{1C} are given by:

$$\beta_{1C} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2}[(\lambda_\beta^2 - 1)\theta_{1C} - \eta_\beta\theta_{1S}], \tag{34}$$

$$\beta_{1S} = \frac{\alpha_\beta}{(\lambda_\beta^2 - 1)^2 + \eta_\beta^2}[\eta_\beta\theta_{1C} + (\lambda_\beta^2 - 1)\theta_{1S}]. \tag{35}$$

4 Hydraulic actuator

4.1 Introduction

The hydraulic actuators are used to the amplification of the pilot forces. They consist on an actuator body, a input lever, a lever arm, a piston and a piston rod.

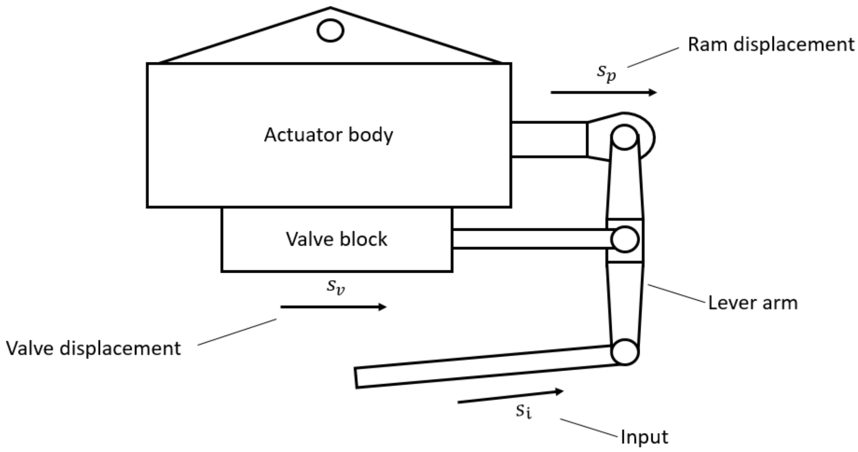

Figure 2: Actuator schematic view.

4.2 Modeling

The model of a hydraulic actuator is as shown in Figure 3. In this model, the internal dynamics of the actuator are represented in terms of force-producer system plus which is mixed with the inertial forces due to the upper controls displacement and the external forces introduced to the aerodynamic surfaces.

On the left side of the diagram, the pilot input position x_i is mixed to the piston position x_p within the Kinematic lever model. From there, a spool position is extracted and the consequent pressure gain in each of the chambers of the actuator are calculated. The time delay for the spool operation is accounted for with 1st order system block with T_v as time constant. The pressure of the hydraulic fluid acting in each of the piston area A_{piston} produces a consequent force due to hydraulic pressure F_{hyd}. At this point, the total force acting on the piston rod is calculated adding F_{hyd} the force due to the Coulomb friction force F_{f}, the force due to the hydraulic damping which opposes the movement of the piston F_{d} and the force F_{up} which comes from the inertial forces due to the acceleration of the upper controls components F_{inertia} and external forces excitation F_{ext}.

4.2.1 Pressure gain

The name pressure gain is assigned to the relationship between spool position and pressure in each of the four chambers of the hydraulic actuator. This is a nonlinear relationship and it may be different for each system.

Once the pressure gain curve is obtained, it can be convinient to multiply the obtained pressure by the effective piston area A_{piston} to arrive to the force gain curve, which is shown in

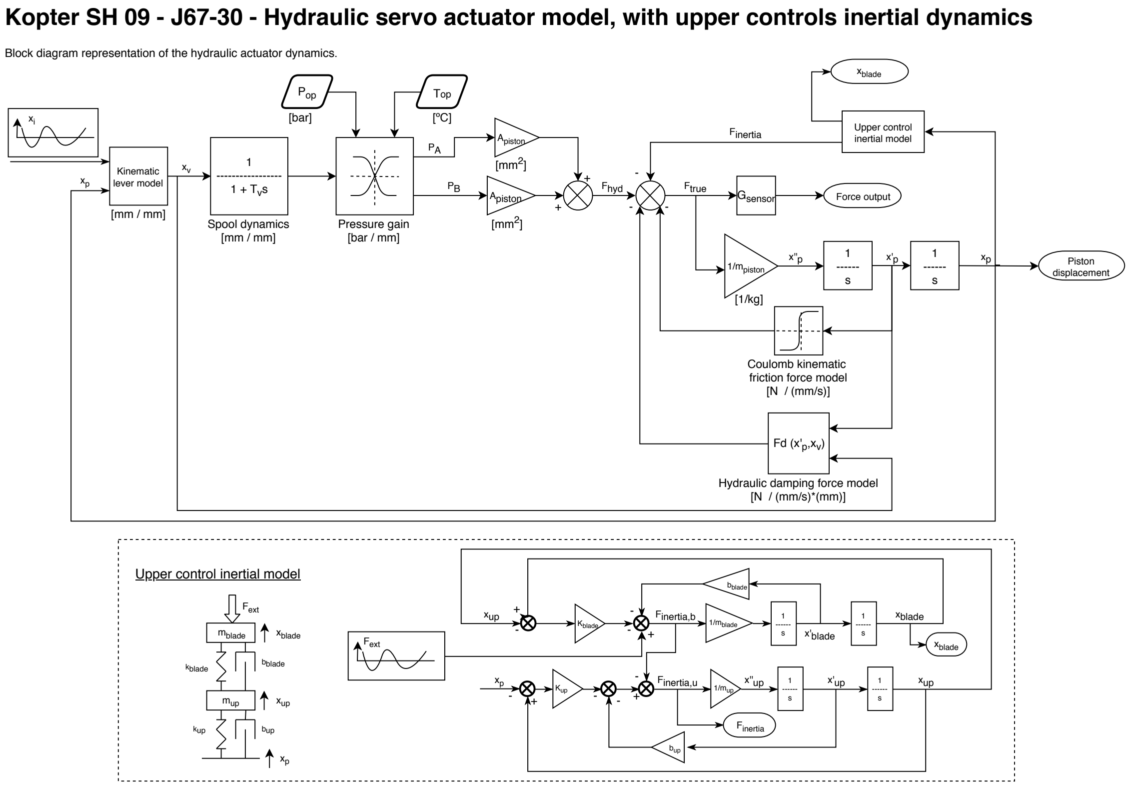

Figure 3: Kopter SH09 Hydraulic servo actuator dynamic block model, with upper controls inertial dynamics and external excitation.

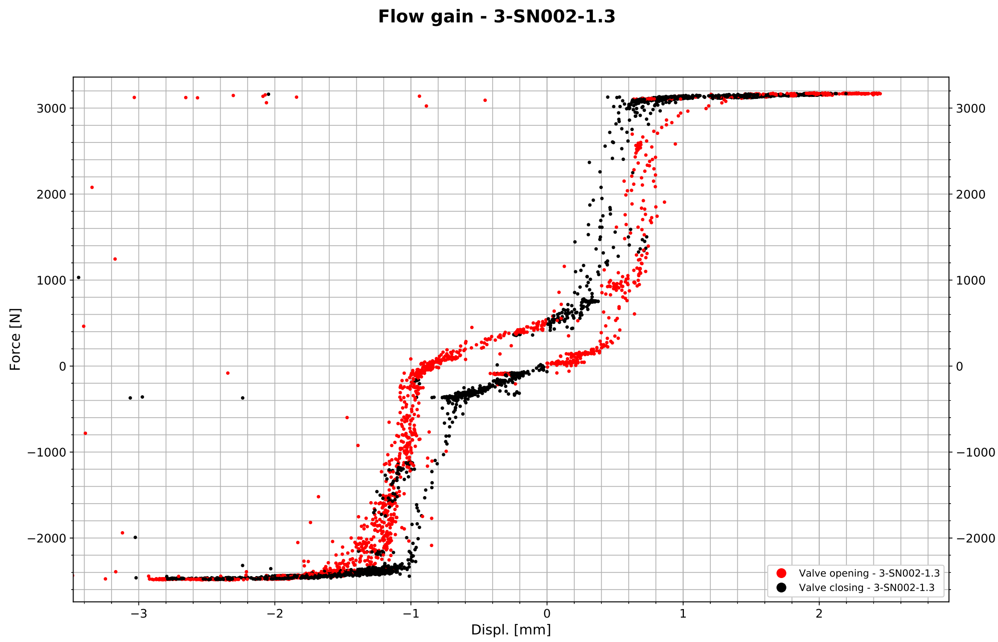

Figure 4: Force gain curve example. The data correspond to the performed P3 actuators qualification tests.

4.2.2 Kinematic lever model

The kinematic relationship between the is governed by the .

These four points and its positioning variables are identified as shown in Figure XX.

- 1: End of the piston rod, center of the piston lug P_1
- 2: Center of middle circular hole of the input lever P_2
- 3: Center of circular hole of the input lever at its connection to the pilot input rod P_3
- 4: Center of circular hole of the input lever at its connection to the piston rod lug P_4

Additionally, these parameters are constant for a considered geometry of the involved components:

- l_3: Distance between P_2 and P_4
- l_1: Distance between P_2 and P_3
- l_2: Distance between P_2 and P_1
- l_4: Distance between P_1 and P_4
- ψ_1: Angle between the line that connects P_2 and P_3, and the line that connects P_2 and P_1

The value of l_1, l_2, l_3 and ψ_1 are obtained from the geometry of the input lever and the pivoting small lever. Finally, the value of l_4 can be obtained as a function of the previously introduced parameters as shown in Equation 36:

$$l_4 = \sqrt{(l_1 \sin \psi_1)^2 + (l_2 + l_1 \cos \psi_1)^2}. \tag{36}$$

The points movements could be assumed to be in-plane motion, so two coordinates are needed to define the location of each of the previously defined points. However, P_4 and P_1 are constrained to an unidirectional movement. Finally, for each of the points, its location is defined as follows a function of the unitary vectors $\vec{i}$ and $\vec{j}$ along the x and y axes, respectively:

$$\begin{aligned}
\vec{p_1} &= x_1\vec{i} \\
\vec{p_2} &= x_2\vec{i} + y_2\vec{j} \\
\vec{p_3} &= x_3\vec{i} + y_3\vec{j} \\
\vec{p_4} &= x_4\vec{i}
\end{aligned} \tag{37}$$

where $x_{1,2,3,4}$ and $y_{3,4}$ are expressed in the absolute reference system located in the initial position of P_4. The initial conditions of these parameters are the following:

$$\begin{aligned}
x_1 &= l_3 \\
x_2 &= l_3 \\
x_3 &= l_3 + l_1 \sin \psi_1 \\
x_4 &= 0 \\
y_2 &= 0 \\
y_3 &= -l_1 \cos \psi_1
\end{aligned} \tag{38}$$

The target is now to obtain the position of those points that are unknown. To begin with, the position $\vec{p}_2$ of P_2 is known when $\vec{p}_1$ and $\vec{p}_4$ are known. The equations which allow the calculation of $\vec{p}_2$ are the following pair of implicit equations:

$$\begin{aligned} 0 &= (x_1 - x_2)^2 + (l_2 - y_2)^2 - l_2^2 \\ 0 &= (x_4 - x_2)^2 + y_2^2 - l_3^2, \end{aligned} \tag{39}$$

where x_2 and y_2 are the unknowns. Once $\vec{p}_4$ is calculated, the position $\vec{p}_3$ of the point P_3 can be calculated using $\vec{p}_1$ and $\vec{p}_4$ as follows:

$$\begin{aligned} 0 &= (x_3 - x_2)^2 + (y_3 - y_2)^2 - l_1^2 \\ 0 &= (x_3 - (l_3 + x_1))^2 + (y_3 - l_2)^2 - l_4^2, \end{aligned} \tag{40}$$

where x_3 and y_3 are the unknowns. The Equations 40 and 39 allow the calculation of the points.

Now, considering the problem of a known input, then the position $\vec{p}_3$ of the point P_3 is known. Considering the position $\vec{p}_1$ from the previous operation, it is possible to obtain $\vec{p}_2$ using Equation 40. With this information, it is possible to obtain $\vec{p}_4$ using 39. This defines the opening of the valve which is the input to the pneumatic model of the actuator. The outcome of the model is the piston rod displacement expressed as $\Delta\vec{p}_1$ which is then used in the next iteration.

References

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