Channel Allocation in Non-Cooperative Multi-Radio Multi-Channel Wireless Networks

Dejun Yang, Xi Fang, Guoliang Xue
Arizona State University

***Abstract*—While tremendous efforts have been made on channel allocation problems in wireless networks, most of them are on cooperative networks with few exceptions [6, 8, 31, 32]. Among those works on non-cooperative networks, none of them considers the network with *multiple collision domains*. Instead, they all assume the *single collision domain*, where all transmissions interfere with each other if they are on the same channel. In this paper, we fill this void and generalize the channel allocation problem to non-cooperative multi-radio multi-channel wireless networks with multiple collision domains. We formulate the problem as a strategic game, called *ChAlloc*. We show that the ChAlloc game may result in an oscillation when there are no exogenous factors to influence players' strategies. To avoid this possible oscillation, we design a charging scheme to induce players to converge to a Nash Equilibrium (NE). We bound the convergence speed and prove that the system performance in an NE is at least $(1 - \frac{\bar{r}}{h})$ of the system performance in an optimal solution, where $\bar{r}$ is the maximum number of radios equipped on wireless devices and h is the number of available channels. In addition, we develop a localized algorithm for players to find an NE strategy. Finally, we evaluate our design through extensive experiments. The results validate our analysis of the possible oscillation in the ChAlloc game lacking the charging scheme, confirm the convergence of the ChAlloc game with the charging scheme, and verify our proof on the system performance compared to the upper bounds returned by an LP-based algorithm.**

I. Introduction

The development of the IEEE 802.11a/b/g standards has spurred the emergence of broadband wireless networks. Due to the common transmission media shared by communication devices, interference arises if communication devices are operating on the same frequency. It has been shown that interference severely limits the network capacity [10]. Frequency Division Multiple Access (FDMA) is a widely used technique to enable multiple devices to share a communication medium. In FDMA, the available bandwidth is divided into multiple sub-bands, named channels. Using multiple channels in multi-radio wireless networks can greatly alleviate the interference and improve the network throughput [26]. Ideally, if there are a sufficient number of channels and each device assigns different channels to its radios, there would be no interference in the network at all. However, since the spectrum is a scarce resource, we are only allowed to divide the available bandwidth into a limited number of channels. For example, there are 3 and 12 non-overlapping channels for the IEEE 802.11b/g standards in 2.4 GHz and the IEEE 802.11a standard in 5 GHz, respectively. A fundamental problem in multi-radio multi-channel (MR-MC) wireless networks is how to allocate channels to radios, which is commonly referred to as the *channel allocation* problem (also known as the channel assignment problem).

While tremendous efforts have been made on the channel allocation problem, most of them are on cooperative networks where devices are assumed to be cooperative and unselfish; however this assumption may not hold in practice. Usually, a wireless device is owned by an independent individual, who is only interested in selfishly maximizing its own profit without respecting the system performance. There are a few works considering non-cooperative networks [6, 8, 31, 32]. However, all of these works only consider the problem in a single collision domain, which means all the transmissions will interfere with each other if they are on the same channel.

In this paper, we study the channel allocation problem in non-cooperative MR-MC networks with *multiple collision domains*. To characterize the network with multiple collision domains, we introduce interference models into the network. The results in this paper are independent of the interference model adopted as long as the model is defined on pairs of communications, for example, the *protocol interference model* is used in this paper. We model the channel assignment problem in non-cooperative MR-MC wireless networks as a strategic game. We show that the game may oscillate indefinitely when there are no exogenous factors to influence players' behavior. This possible oscillation can result in significant communication overhead and the degradation of the system performance. To avoid this undesirable outcome, we develop a charging scheme to induce players' behavior. The design of the charging scheme ensures the convergence to a Nash Equilibrium (NE). Players are in an NE if no player can improve its utility by changing its strategy unilaterally. Although NE is usually not social optimal, we can prove that the system performance in an NE is guaranteed to be at least a factor of the system performance in the optimal solution.

We summarize our main contributions as follows:

- To the best of our knowledge, we are the *first* to study the channel allocation problem in non-cooperative MR-MC wireless networks with *multiple collision domains*. We model the problem as a strategic game, called *ChAlloc*.
- We show that the ChAlloc game can result in an oscillation, where players keep changing their strategies back and force trying to improve their utilities.
- To avoid the possible oscillation, we design a charging scheme to influence players' behavior. We prove that, under the charging scheme, the ChAlloc game converges

The authors are affiliated with Arizona State University, Tempe, AZ 85287. E-mail: {dejun.yang, xi.fang, xue}@asu.edu. This research was supported in part by NSF grants 0905603 and 0901451. The information reported here does not reflect the position or the policy of the federal government.

to an NE. We also prove that the system performance in an NE is guaranteed to be at least $(1 - \frac{\bar{r}}{h})$ of the system performance in the optimal solution, where $\bar{r}$ is the maximum number of radios equipped on wireless devices and h is the number of available channels.
- We design a localized algorithm for players to find an NE and prove that it takes $O(\bar{r}hn^3(n + \log h))$ time for the ChAlloc game to converge to an NE, where n is the number of players.
- In order to verify our proof of the system performance in an NE, we give an LP-based algorithm to derive efficiently computable upper bounds on the optimal solution.
- Through extensive experiments, we validate our analysis of the possible oscillation in the ChAlloc game lacking the charging scheme and confirm the proof of the convergence of the ChAlloc game with the charging scheme. The results also show that the system performance in an NE is very close to the optimal solution and thus verify our proof of the system performance.

The remainder of this paper is organized as follows. In Section II, we review the current literature on the channel allocation problem. In Section III, we present the system model considered in our paper and formulate the channel allocation problem as a game, called *ChAlloc*. In Section IV, we use an example to show that it is possible for the ChAlloc game to oscillate endlessly when there are no exogenous factors to influence players' behavior. In Section V, we design a charging scheme to induce players to converge to an NE, compute the price of anarchy of the ChAlloc game, and develop a localized algorithm for players. In Section VI, we give an LP-based algorithm to find an upper bound on the optimal solution. In Section VII, we evaluate the performance of the ChAlloc game through extensive experiments. Finally, we form our conclusion in Section VIII.

II. Related Work

Most previous works on channel allocation can be categorized into two categories, *channel allocation in cooperative networks* [5, 15, 21, 24, 25, 27–29] and *channel allocation in non-cooperative networks* [6, 8, 31, 32]. We summarize the related works in Table I.

TABLE I
Related work

	Single Collision Domain	Multiple Collision Domains
Cooperative	none	[5, 15, 21, 24, 25, 27–29]
Non-cooperative	[6, 8, 31, 32]	our work

A. Channel Allocation in Cooperative Networks

There is a considerable amount of works on the channel allocation problem in Wireless Mesh Networks (WMNs). In [5], Das *et al.* presented two mixed integer linear programming (ILP) models to solve the channel allocation problem in WMNs with the objective to maximize the number of simultaneously transmitting links. However, it is known that solving an ILP is NP-hard. In [24], Ramachandran *et al.* proposed a centralized channel allocation algorithm utilizing a novel interference estimation technique in conjunction with an extension to the conflict graph model, called the multi-radio conflict graph. In [28], Sridhar *et al.* proposed a localized channel allocation algorithm called LOCA, which is a heuristic algorithm. Subramanian *et al.* [29] and Marina *et al.* [21] studied the channel allocation problem where each link is assigned a channel with the constraint that the number of different channels assigned to the links incident on any node is at most the number of radios on that node. Subramanian *et al.* [29] developed a centralized algorithm based on Tabu search and a distributed algorithm based on the Max-K-cut problem. Marina *et al.* [21] proposed a greedy heuristic channel allocation algorithm, termed CLICA. In [15], Ko *et al.* studied the channel allocation problem with a different objective function and proposed a distributed algorithm without any performance guarantee. In [27], Shin *et al.* considered the channel allocation problem to maximize the throughput or minimize the delay, and presented the an allocation scheme, called SAFE, which is a distributed heuristic.

All the above related works are based on the assumption that wireless devices in the network cooperate to achieve a high system performance. However, this assumption might not hold in practice. Usually, a wireless device is owned by an independent individual, who is only interested in selfishly maximizing its own profit without respecting the system performance or considering others' profits.

B. Channel Allocation in Non-cooperative Networks

Game theory has been widely used to solve problems in non-cooperative wireless networks, for instance, Aloha networks [19] and CSMA/CA networks [3, 16]. Based on a graph coloring game model, Halldórsson *et al.* [12] provided bounds on the *price of anarchy* of the channel allocation game. However, their model does not apply to multi-radio networks.

In an earlier work, Félegyházi *et al.* [6] formulated the channel allocation problem in non-cooperative MR-MC wireless networks as a game, analyzed the existence of Nash Equilibria and presented two algorithms to achieve an NE. Along this line, Wu *et al.* [32] introduced a payment formula to ensure the existence of a *strongly dominant strategy equilibrium* (SDSE). Furthermore, when the system converges to an SDSE, it also achieves global optimality in terms of system throughput. In [8], Gao *et al.* extended the problem to multi-hop networks and also addressed coalition issues. Most recently, Wu *et al.* [31] studied the problem of adaptive-width channel allocation in non-cooperative MR-MC wireless networks, where contiguous channels may be combined to provide a better utilization of the available channels. However, all the above results can only be applied to a single collision domain, without considering multiple collision domains. In this paper, we fill this void and study the channel allocation problem in non-cooperative MR-MC networks with multiple collision domains.

III. System Model and Game Formulation

A. Network Model

The network model in this paper closely follows the models in [6, 8, 31, 32]. We consider a static wireless network consisting of a set $\mathcal{L} = \{L_1, L_2, \ldots, L_n\}$ of n communication links. Each link L_i is modeled as an undirected link between two nodes v_i and u_i, where v_i and u_i denote two wireless devices communicating with each other. The use of the undirected link model reflects the fact that the IEEE 802.11 DCF requires the sender to be able to receive the acknowledgement message from the receiver for every transmitted packet. Since links are undirected, two nodes are able to coordinate to select the same channels for communication. As in [6, 8, 31, 32], we assume that the links are backlogged and always have packets to transmit. Each wireless device is equipped with multiple radio interfaces. We further assume that each transmission must be between two radios, of which one functions as a transmitter and the other as a receiver. Thus, it is reasonable to assume that both nodes of L_i have the same number of radios, denoted by r_i. We assume the wireless devices have the same maximum transmission power, but each of the devices adjusts its actual data transmission power according to the length of the transmission link, denoted by l_i. Let R denote the transmission range under the maximum transmission power. Furthermore, there are $h > 1$ orthogonal channels available in the network, e.g. 12 orthogonal channels in the IEEE 802.11a protocol. We denote the set of channels by $\mathcal{C} = \{c_1, c_2, \ldots, c_h\}$.

To communicate, two nodes of a link have to tune at least one of their radios to the same channel(s). Parallel communications are allowed between two nodes if they share multiple channels on radios. In order to avoid the co-radios interference in a device [8], we assume that different radios on a node should be tuned to different channels. Therefore, it is reasonable to assume that $r_i < h$ for all $L_i \in \mathcal{L}$ as it would be straightforward to allocate channels otherwise. Allocating each channel to at most one radio has also been proved to be a necessary condition to maximize the device's transmission data rate [6, 8, 31].

B. Interference Model

Due to the common transmission medium, wireless transmission along a communication link may interfere with the transmissions along other communication links, especially those within its vicinity. While existing works [6, 8, 31, 32] have studied the channel allocation problem in non-cooperative networks for both single-hop and multi-hop models, all the results can only be applied to a single collision domain. In other words, they assume that all transmissions interfere with each other if they share at least one channel. However, the strength of a wireless transmission signal decays exponentially with respect to the distance it travels from the transmitter. Therefore the signal from a distant transmission is, if not negligible, not destructive enough to prevent another transmission from succeeding. In this paper, we generalize the channel allocation problem to networks with multiple collision domains.

To characterize networks with multiple collision domains, an appropriate interference model is necessary. Various interference models have been proposed in the literature, for example, the primary interference model [11], the protocol interference model [10, 14], and the physical interference model (a.k.a SINR interference model) [10, 14]. The results in

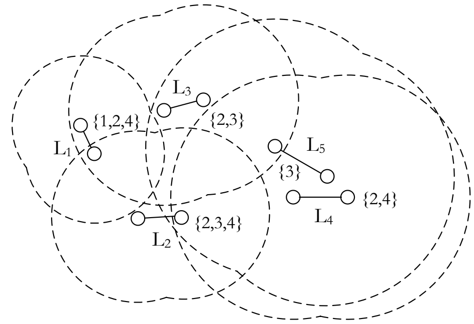
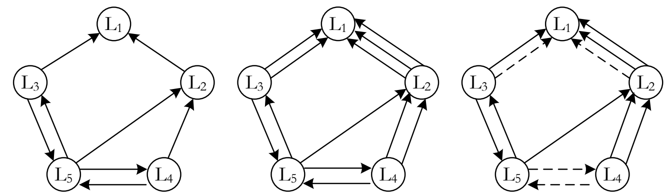


Fig. 1. A 5-link network, where $\mathcal{L} = \{L_1, L_2, L_3, L_4, L_5\}$, $r_1 = 3, r_2 = 3, r_3 = 2, r_4 = 2, r_5 = 1$ and $\mathcal{C} = \{1, 2, 3, 4\}$.



(a) PING (b) MPING (c) MING

Fig. 2. The PING, MPING and MING for the example in Fig. 1

this paper are independent of the specific interference model used *as long as the interference model is defined on pairs of communication links*. For the sake of presentation, the *protocol interference model* is adopted throughout this paper. This model has been used by most of the works on channel allocation problems [1, 2, 21, 26, 29, 30]. In this model, each node has an interference range γl_i, which is at least as large as the transmission range (equal to l_i), i.e., $\gamma \geq 1$. We assume that $l_i \leq \frac{R}{\gamma}$, for any L_i. Any node u will be interfered by node v if u is within v's interference range. We can imagine that, associated with each L_i, there is an interference disk D_{u_i} centered at u_i, and an interference disk D_{v_i} centered at v_i. The union of D_{u_i} and D_{v_i}, denoted by $D_{u_i} \cup D_{v_i}$, constitutes the *interference area* of L_i. Link L_i *interferes* with link L_j if and only if either of v_j and u_j is in $D_{u_i} \cup D_{v_i}$, and two links share at least one common channel. Before the channels on the radios are known, we can only say that L_i *potentially interferes* with L_j. As an illustrating example, Fig. 1 shows a 5-link network, where dashed peanut-shaped curves represent the boundaries of interference areas (numbers in parentheses will be explained in Section III-D). In this example, L_3 potentially interferes with L_1 while L_1 cannot interfere with L_3.

Conflict graphs are widely used to facilitate the design of channel allocation algorithms [14, 21, 22, 24, 29]. We use a similar concept, called *potential interference graph* (PING), to characterize the interfering relationships among the links. Different from the conflict graph, the edges in the PING are directed due to the heterogeneity of the interference range. In a PING, $G_p = (V_p, A_p)$, nodes correspond to communication links. Hereafter, we also use L_i to denote the corresponding node in G_p. There is an arc from L_i to L_j if L_i potentially interferes with L_j. Fig. 2(a) shows a PING of the example in Fig. 1. Unfortunately, the above defined PING does not accurately model the devices with multiple radios. For example, if L_i potentially interferes with L_j and both links have two radio pairs, there should be two interference arcs. Therefore, we extend the PING to model multi-radio networks and call the new model *multi-radio potential interference graph* (MPING). An MPING is a directed multigraph, $G_m = (V_m, A_m)$, where nodes still represent transmission links, arcs represent potential interference between links, and parallel directed arcs may exist between two vertices. There are $\min\{r_i, r_j\}$ arcs from L_i to L_j if $(L_i, L_j) \in A_p$. Let $A_m^-(L_i)$ and $A_m^+(L_i)$ be the set of in-arcs and the set of out-arcs, respectively. The in-arc set $A_m^-(L_i)$ of L_i is the set of arcs going into L_i and the out-arc set $A_m^+(L_i)$ of L_i is the set of arcs going from L_i. The in-arcs in $A_m^-(L_i)$ are called the *potential interference arcs* of L_i. Let $N_m^-(L_i)$ be the set of in-neighbors and $N_m^+(L_i)$ be the set of out-neighbors in the MPING G_m. The in-neighbor set $N_m^-(L_i)$ of L_i is the set of vertices, which are the tails of in-arcs and the out-neighbor set $N_m^+(L_i)$ of L_i is the set of vertices, which are the heads of out-arcs. The MPING corresponding to Fig. 1 is shown in Fig. 2(b).

C. Game Theory Concepts in a Nutshell

Game theory [7] is a discipline aimed at modeling scenarios where individual decision-makers have to choose specific actions that have mutual or possibly conflicting consequences. A game consists of a set $\mathcal{P} = \{P_1, P_2, \ldots, P_n\}$ of players. Each player $P_i \in \mathcal{P}$ has a non-empty strategy set Π_i. Let s_i denote the selected strategy by P_i. A strategy profile s consists of all the players' strategies, i.e., $s = (s_1, s_2, \ldots, s_n)$. Obviously, we have $s \in \Pi = \times_{P_i \in \mathcal{P}} \Pi_i$. Let s_{-i} denote the strategy profile excluding s_i. As a notational convention, we then have $s = (s_i, s_{-i})$. The utility (or payoff) function $u_i(s)$ of P_i measures P_i's valuation on strategy profile s. We say that P_i prefers s_i to s_i' if $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.

When other players' strategies are fixed, P_i can select a strategy, denoted by $b_i(s_{-i})$, which maximizes its utility function. Such a strategy is called a *best response* [7] of P_i, which is formally defined as follows.

Definition 1: [Best Response] Given other player's strategies s_{-i}, a *best response* strategy of P_i is a strategy $s_i \in \Pi_i$ such that $b_i(s_{-i}) = \arg\max_{s_i \in \Pi_i} u_i(s_i, s_{-i})$, where Π_i is the strategy space of P_i. □

In order to study the interaction of players, we adopt the concept of *Nash Equilibrium* (NE) [7].

Definition 2: [Nash Equilibrium] A strategy profile $s^{ne} = (s_1^{ne}, s_2^{ne}, \ldots, s_n^{ne})$ constitutes a *Nash Equilibrium* if, for each P_i, we have $u_i(s_i^{ne}, s_{-i}^{ne}) \geq u_i(s_i, s_{-i}^{ne})$ for all $s_i \in \Pi_i$. □

In other words, none of the players can improve its utility by unilaterally deviating from its current strategy in an NE. Mathematically, it means $b_i(s_{-i}^{ne}) = s_i^{ne}$ for all $P_i \in \mathcal{P}$. To characterize and quantify the inefficiency of the system performance due to the lack of cooperation among the players, we use the concept of *price of anarchy* (POA) [17].

Definition 3: [Price of Anarchy] The *price of anarchy* of the game is the ratio of the system performance in the worst Nash Equilibrium to the system performance in the social optimal solution, that is

$$POA = \frac{\min_{s^{ne} \in \Pi^{ne}} U(s^{ne})}{\max_{s \in \Pi} U(s)},$$

where $\Pi^{ne} \subseteq \Pi$ is the set of all NEs and U is a function of the strategy profile measuring the system performance. □

Note that $U(s)$ is not necessarily the sum of the utilities of all players as player's utility may not always reflect the system performance. The POA in game theory is an analogue of the approximation ratio in combinatorial optimization. *If a game has a POA lower bounded by $\alpha \leq 1$, it means that for any instance of the game, the system performance in* **any** *NE is at least α times the system performance in the optimal solution.*

D. ChAlloc Game Formulation

We formulate the channel allocation problem in non-cooperative MR-MC wireless networks as a game, called *ChAlloc*. In this game, each transmission link is a player, whose strategy space is the set of all the possible channel allocations on its radios. We assume that players are selfish, rational and honest. We leave the case where players can cheat for our future work. Throughout the rest of this paper, we will use link and player interchangeably. The channel allocation of L_i is defined to be a vector $s_i = (s_{i1}, s_{i2}, \ldots, s_{ih})$, where $s_{ik} = 1$ if L_i assigns channel c_k to one radio pair and $s_{ik} = 0$ otherwise. To sufficiently utilize the channel resource, we require that $\sum_{k=1}^{h} s_{ik} = r_i$, which is also proved to be optimal for each player for the single-collision domain case [6]. The strategy profile s is then an $n \times h$ matrix defined by all the players' strategies, $s = (s_1, s_2, \ldots, s_n)^T$.

Although previous works in the literature [6, 8, 31, 32] have used achievable data rate as the utility function, they assume that all the links are in a single collision domain. Because of the hidden terminal problem, it is unlikely to have a closed-form expression to calculate the achievable data rate for each player in the network with multiple collision domains. This difficulty has also been discussed in [6, 32]. An alternative is to use the interference as a performance metric. As shown in [34, Eq.(4)], the data rate is approximately a linear function of the interference that the link can overhear. The use of the interference as a performance metric can also be found in [24, 29, 30].

Given a strategy profile s, we say a communication radio pair *interferes* with L_i if this radio pair belongs to a link interfering with L_i and has been tuned to a channel that is also allocated by L_i. We define the *interference number* of L_i, denoted by $I_i(s)$, to be the number of communication radio pairs interfering with L_i. Mathematically, we have

$$I_i(s) = \sum_{L_j \in N_m^-(L_i)} s_i \cdot s_j,$$

where the symbol $\cdot$ is the dot product between two vectors. Note that $I_i(s) \leq |A_m^-(L_i)|$ for all $L_i \in \mathcal{L}$. When s is given, we can construct the *multi-radio interference graph* (MING), $G_m(s) = (V_m(s), A_m(s))$, from the MPING by removing corresponding potential interference arcs. The number of arcs from L_i to L_j is equal to $s_i \cdot s_j$.

In this paper, we define the utility function of a player to be a function of its interference number. More specifically, the utility function $u_i(s)$ of player L_i is defined as

$$u_i(s) = |A_m^-(L_i)| - I_i(s). \quad (1)$$

In other words, the objective of L_i is to remove as many of the potential interference arcs as possible from $N_m^-(L_i)$ by allocating channels to its radios. When the network is given, $|A_m^-(L_i)|$ is a constant. Hence maximizing (1) can achieve the goal of minimizing $I_i(s)$, which is the interference suffered by L_i under the strategy profile s.

Intuitively, the system performance function is defined as

$$U(s) = |A_m| - \sum_{L_i \in \mathcal{L}} I_i(s), \qquad (2)$$

which is the total potential interference removed from the MPING under allocation profile s. Likewise, $|A_m|$ is a constant, hence maximizing (2) can achieve the goal of minimizing $\sum_{L_i \in \mathcal{L}} I_i(s)$, which is the overall network interference.

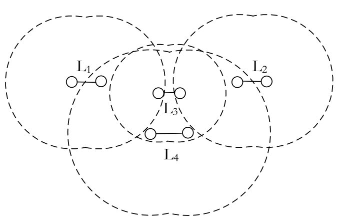
Use the example in Fig. 1 for illustration. The numbers in the parentheses associated to each link represent the allocated channels. The corresponding channel allocation vectors are $s_1 = (1,1,0,1)$, $s_2 = (0,1,1,1)$, $s_3 = (0,1,1,0)$, $s_4 = (0,1,0,1)$, and $s_5 = (0,0,1,0)$. The interference graph under s is shown in Fig. 2(c). Under this strategy profile, we have $u_1(s) = 2$, $u_2(s) = 0$, $u_3(s) = 0$, $u_4(s) = 1$ and $u_5(s) = 1$. The system performance is $U(s) = 4$.

IV. Oscillation in the ChAlloc Game

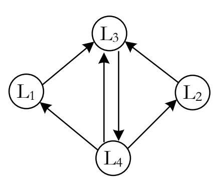
In this section, we show that players might not converge to any stable status, i.e. NE, according to the current defined utility function. Consider the network illustrated in Fig.3(a). Obviously, we have the MPING as shown in Fig.3(b). Assume that each link is equipped only one radio pair and there are two channels $\{c_1, c_2\}$ available. Due to the special topology and the dependency relation, we have the following conclusions.

- If L_4 uses c_1, both L_1 and L_2 will use c_2.
- If both L_1 and L_2 use c_2, L_3 will use c_1.
- If L_3 uses c_1, L_4 will use c_2.
- If L_4 uses c_2, both L_1 and L_2 will use c_1.
- ...

This process turns into an infinite loop.



(a) A four-link example



(b) MPING

Fig. 3. An example where players oscillate forever

This possible oscillation is definitely undesirable for two reasons: 1) The channel switching delays can be in the order of milliseconds [4], an order of magnitude higher than typical packet transmission time (in microseconds). 2) It can introduce a significant amount of communication overhead, as two devices need to coordinate to switch channels.

V. Nash Equilibria

As we have discussed in Section IV, the ChAlloc game can run into an oscillation problem, which is undesirable from the system's perspective. In order to induce players to converge to an NE, we design a charging scheme to influence the players in this section. We then prove that, based on the newly defined utility function considering the charge, the ChAlloc game must converge to an NE. In addition, we prove that even at the worst NE, the system performance is at least $(1-\frac{\bar{r}}{h})$ times the system performance in the optimal channel allocation, where $\bar{r}$ is the maximum number of radios equipped on the nodes and h is the number of channels available in the network. Finally, we present a localized algorithm for players to converge to an NE.

A. Charging Scheme Design

Similar approaches have also been used in [31, 32]. Their charging functions are designed based on the globally optimal channel allocation. Essentially, players who deviate from the optimal channel allocation will be punished according to the charging function. Unfortunately, it has been proved that the optimization problem of maximizing the system performance function (2) is NP-hard [29]. Different variations of the channel allocation problem have also been shown to be NP-hard [1, 21, 24, 26]. Therefore, we focus on designing a charging scheme, which can make the ChAlloc game converge to an NE and achieve guaranteed system performance.

As in [31–33], we assume that there exists a virtual currency in the system. Each player needs to pay certain amount of virtual money to the system administrator based on the strategy profile s. We define the charge p_i of player L_i as

$$p_i(s) = \sum_{L_j \in N_m^+(L_i)} s_i \cdot s_j, \qquad (3)$$

which is the total interference player L_i imposes on the others. The charge can be considered to be the fee for accessing the channels. We then redefine the utility function for each player $L_i \in \mathcal{L}$ as

$$u_i(s) = \left|A_m^-(L_i)\right| - I_i(s) - p_i(s), \qquad (4)$$

which is equal to the original utility minus its payment to the system administrator.

B. Existence of Nash Equilibria

Having defined a new utility function for the player, we next prove the existence of Nash Equilibria with the help of the concept of *potential game* [23].

Definition 4: [Potential Game] A function $\Phi : \Pi \mapsto Z^*$ is an *exact potential function* for a game if the change of any player's utility can be exactly expressed in the function. Formally, Φ should satisfy

$$\Phi(s_i, s_{-i}) - \Phi(s_i', s_{-i}) = u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}),$$

for all s_{-i} and $s_i, s_i' \in \Pi_i$. A game is called a *potential game* if it admits an exact potential function. □

A nice property of being a potential game is that if Φ is bounded, we can prove that the game possesses an NE and any *improvement path* leads to an NE. An improvement path is a sequence of strategy profiles, each of which (except the first one) is formed from the previous one by changing a unique player's strategy to improve the player's utility. Therefore we first prove that the ChAlloc game is a potential game and then prove that its corresponding potential function is bounded.

Lemma 1: The ChAlloc game is a potential game. □

Proof: We prove this lemma by constructing an exact potential function Φ. Define Φ as

$$\Phi(s) = \frac{1}{2}\sum_{L_i\in\mathcal{L}} u_i(s).$$

We next prove that $\Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) = u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i})$ for all s_{-i} and $s_i, s'_i \in \Pi_i$. First, we have

$$\begin{aligned}\Phi(s) &= \frac{1}{2}\sum_{L_i\in\mathcal{L}}\left(\left|A_m^-(L_i)\right| - \sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j - \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j\right)\\ &= \frac{|A_m|}{2} - \frac{1}{2}\sum_{L_i\in\mathcal{L}}\left(\sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j\right), \end{aligned}\quad (5)$$

where the second equality follows from the fact that $|A_m| = \sum_{L_i\in\mathcal{L}}|A_m^-(L_i)|$. We then have

$$\begin{aligned}&\Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i})\\ =&\frac{1}{2}\left(\sum_{L_j\in N_m^-(L_i)} s_j\cdot s'_i - \sum_{L_j\in N_m^-(L_i)} s_j\cdot s_i\right.\\ &+\sum_{L_j\in N_m^+(L_i)} s_j\cdot s'_i - \sum_{L_j\in N_m^+(L_i)} s_j\cdot s_i\\ &+\sum_{L_j\in N_m^-(L_i)} s'_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s'_i\cdot s_j\\ &\left.-\left(\sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j\right)\right)\\ =&\left(\sum_{L_j\in N_m^-(L_i)} s'_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s'_i\cdot s_j\right.\\ &\left.-\left(\sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j\right)\right)\\ =&u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}),\end{aligned}$$

where the first equality follows from the fact that the change of player L_i's strategy only affects players in $N_m^-(L_i)$ and $N_m^+(L_i)$, and the last equality follows from $u_i(s_i, s_{-i}) - u_i(s'_i, s_{-i}) = \sum_{L_j\in N_m^-(L_i)} s'_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s'_i\cdot s_j - \left(\sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j + \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j\right)$.

We have proved that $\Phi(s)$ is an exact potential function (Definition 4) of the ChAlloc game. Hence the ChAlloc game is a potential game. ■

The bound of $\Phi(s)$ is given in the following lemma.

Lemma 2: For any $s\in\Pi$, $\Phi(s)$ is bounded by $O(\bar{r}n^2)$. □

Proof: By (5), we have

$$\Phi(s) \le \frac{|A_m|}{2} \le \frac{\sum_{L_i\in\mathcal{L}} r_i(n-1)}{2} \le \frac{1}{2}\bar{r}n^2.$$

This completes the proof. ■

Now we give the main theorem in this section.

Theorem 1: The ChAlloc game possesses an NE. □

Proof: Combining Lemma 1 and Lemma 2, this theorem directly follows from Corollary 2.2 in [23], which states that every finite potential game possesses a Nash Equilibrium. ■

C. Price of Anarchy

Although we have proved that there exist Nash Equilibria in the ChAlloc game, we know that NE is usually not socially efficient in the sense that the system performance in an NE is not optimized. Nevertheless, we prove in this section that the POA of the ChAlloc game is independent of the number of players involved in the game and is lower bounded by a constant when the number of channels and the number of radios equipped on devices are fixed.

Theorem 2: In the ChAlloc game, $POA \ge \left(1 - \frac{\bar{r}}{h}\right)$. Recall that $\bar{r}$ is the maximum number of radios equipped on wireless devices and h is the number of available channels. □

Proof: Before proving the POA of the ChAlloc game, we first find a lower bound of the utility of any player in an NE. Let $s^{ne} = (s_1^{ne}, s_2^{ne}, \ldots, s_n^{ne})^T$ be any NE of the ChAlloc game. Let $s^{opt} = (s_1^{opt}, s_2^{opt}, \ldots, s_n^{opt})^T$ be a social optimum. We have

$$\begin{aligned}&u_i(s^{ne})\\ =&\left|A_m^-(L_i)\right| - \sum_{L_j\in N_m^-(L_i)} s_i^{ne}\cdot s_j^{ne} - \sum_{L_j\in N_m^+(L_i)} s_i^{ne}\cdot s_j^{ne}\\ \ge&\sum_{s_i\in\Pi_i}\left(\left|A_m^-(L_i)\right| - \sum_{L_j\in N_m^-(L_i)} s_i\cdot s_j^{ne} - \sum_{L_j\in N_m^+(L_i)} s_i\cdot s_j^{ne}\right)/|\Pi_i| \quad (6)\\ =&\left|A_m^-(L_i)\right| - \frac{r_i}{h}\left(\left|A_m^-(L_i)\right| + \left|A_m^+(L_i)\right|\right) \quad (7)\\ \ge&\left|A_m^-(L_i)\right| - \frac{\bar{r}}{h}\left(\left|A_m^-(L_i)\right| + \left|A_m^+(L_i)\right|\right), \quad (8)\end{aligned}$$

where (6) follows from the definition of NE, (7) follows from the fact that each arc is counted $\binom{h-1}{r_i-1}$ times and $|\Pi_i| = \binom{h}{r_i}$, next (8) follows from $\bar{r} = \max_{L_i\in\mathcal{L}} r_i$.

Then the system performance is

$$\begin{aligned}U(s^{ne}) &= \sum_{L_i\in\mathcal{L}}\left(u_i(s^{ne}) + \sum_{L_j\in N_m^+(L_i)} s_i^{ne}\cdot s_j^{ne}\right) \quad (9)\\ &\ge \sum_{L_i\in\mathcal{L}}\left(\left|A_m^-(L_i)\right| - \frac{\bar{r}}{h}\left(\left|A_m^-(L_i)\right| + \left|A_m^+(L_i)\right|\right)\right)\\ &\quad+\sum_{L_i\in\mathcal{L}}\sum_{L_j\in N_m^+(L_i)} s_i^{ne}\cdot s_j^{ne} \quad (10)\\ &=|A_m| - \frac{2\bar{r}}{h}|A_m| + \sum_{L_i\in\mathcal{L}}\sum_{L_j\in N_m^+(L_i)} s_i^{ne}\cdot s_j^{ne} \quad (11)\\ &=|A_m| - \frac{2\bar{r}}{h}|A_m| + |A_m| - U(s^{ne}), \quad (12)\end{aligned}$$

where (9) follows from (2) and (4), (10) follows from (8), and (12) follows from the fact that $|A_m| = U(s^{ne}) + \sum_{L_i\in\mathcal{L}}\sum_{L_j\in N_m^+(L_i)} s_i^{ne}\cdot s_j^{ne}$.

Considering the obvious fact that $U(s^{opt}) \le |A_m|$, we have

$$U(s^{ne}) \ge \left(1 - \frac{\bar{r}}{h}\right)U(s^{opt}). \quad (13)$$

Since (13) holds for any NE of the ChAlloc game, it is straightforward to prove that $POA \geq \left(1 - \frac{\bar{r}}{h}\right)$. ■

Note that we were very conservative when we derived (10), making the bound of POA very loose. We will leave the derivation of a tighter bound for the future study.

D. A Localized Algorithm for the ChAlloc Game

Since the ChAlloc game is a potential game (Lemma 1), any improvement path leads to an NE [23]. To form an improvement path, we need to require the sequential action of the players. Each player takes its best response strategy upon its turn. Note that $b_i(s_{-i})$ can be computed just based on the strategies of the players in $N_m^-(L_i)$ and $N_m^+(L_i)$. Therefore we can design a localized algorithm for players to find an NE. A localized algorithm needs no information to propagate through the whole network. Thus it is scalable to the network size and robust to the topology change.

Algorithm 1: A Localized Algorithm for L_i

```
Randomly allocate r_i channels as s_i;
W_i ← i, ctr ← 0;
while true do
    if W_i = 0 then
        Get the current channel allocation;
        s'_i ← b_i(s_{-i});
        if s'_i = s_i then
            if ctr = n then break; else ctr ← ctr + 1;
        else s_i ← s'_i, ctr ← 0;
        W_i ← n;
    else W_i ← W_i − 1;
end
```

The localized algorithm is illustrated in Algorithm 1. The implementation issue will be discussed later. To avoid the simultaneous change in channel allocations of different players, we let each player L_i have a counter W_i, which is initially set to i. At the beginning of the algorithm, each player L_i randomly picks r_i channels as its initial strategy. In every iteration, L_i checks the value of W_i. If W_i is equal 0, L_i gets the current channel allocations and calculates its best response strategy $b_i(s_{-i})$. If the best response strategy is the same with its current strategy, it increases another counter *ctr* by 1. Otherwise, it updates its strategy and resets *ctr* to 0. The value of *ctr* indicates how many times its current strategy has been the best response strategy consecutively. The use of *ctr* is to avoid early termination before the ChAlloc game converges to an NE. If the counter W_i has not reached 0, L_i decreases its value by 1.

Lemma 3: The best response strategy $b_i(s_{-i})$ can be computed in $O(h(n + \log h))$ time. □

Proof: We prove this lemma by giving an algorithm to compute $b_i(s_{-i})$. For each channel $c_k \in \mathcal{C}$, we compute the value of $\sum_{L_j \in N_m^-(L_i)} e_k \cdot s_j + \sum_{L_j \in N_m^+(L_i)} e_k \cdot s_j$, where e_k denotes the vector with a 1 in the kth coordinate and 0's elsewhere. This can be finished in $O(hn)$ time. Sort the channels in a nondecreasing order, which can be finished in $O(h \log h)$ time. Since channels are independent, L_i selects the first r_i channels as $b_i(s_{-i})$. Hence the above algorithm can be finished in time bounded by $O(h(n + \log h))$. ■

Theorem 3: For any instance of the ChAlloc game, if all the players follow Algorithm 1, it takes $O(\bar{r}hn^3(n + \log h))$ time to converge to an NE. □

Proof: According to Lemma 1 and Lemma 2, every time a player changes its strategy (to one introducing better utility), the potential function $\Phi(s)$ will be increased accordingly and the value of $\Phi(s)$ is bounded by $O(\bar{r}n^2)$. Therefore the number of strategy updates is bounded by $O(\bar{r}n^2)$. Since there will be at least one strategy update in each round, the number of rounds is also bounded by $O(\bar{r}n^2)$. Using Lemma 3 and the fact that n players take actions sequentially in each round, we can prove that it takes $O(\bar{r}hn^3(n + \log h))$ time for the ChAlloc game to converge to an NE. ■

Implementation Issue: Existing works [13, 15, 27, 28] on distributed or localized channel allocation algorithms all assume that the interference sets are given, but do not discuss how to find them in a distributed manner. We assume that during the channel allocation stage, all the players are using the same channel on one of their radios, which is called the control channel, and using the maximum transmission power to send packets. Because of the assumption that $l_i \leq \frac{R}{\gamma}$, it is guaranteed that all the links in the interference range of link L_i can overhear the packets. The packet to be exchanged during the channel allocation stage is of form (i, s_i). For each player L_i, upon its turn, it sends out the packet. During other time periods, it listens to the control channel and receives packets from others. For the packet received from L_j, L_i computes its distance from L_j according to the received signal strength[1], puts L_j in $N_m^+(L_i)$ if L_j is within its interference range, and puts L_j in $N_m^-(L_i)$ if it is within L_j's interference range.

VI. Upper Bounds on Optimal Channel Allocation

In this section, we derive efficiently computable upper bounds on the channel allocation problem, which will be used in Section VII to evaluate the system performance of the ChAlloc game. We first formulate the channel allocation problem as an integer linear program (ILP) and then relax the constraints to achieve an upper bound on the optimal solution.

Let $s_{ik} \in \{0, 1\}$ denote L_i's allocation on c_k, where $s_{ik} = 1$ if L_i allocates c_k to one of its radios and $s_{ik} = 0$ otherwise. Let $x_{ijk} \in \{0, 1\}$ denote the interference from L_i to L_j via c_k. Our ILP can be formulated as follows,

$$\max \quad |A_m| - \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \sum_{k=1}^{h} x_{ijk}$$

$$\text{s.t.} \quad \sum_{k=1}^{h} s_{ik} = r_i \quad (L_i \in \mathcal{L}) \tag{14}$$

$$x_{ijk} \geq s_{ik} + s_{jk} - 1 \quad ((L_i, L_j) \in A_m, c_k \in \mathcal{C}) \tag{15}$$

$$s_{ik} \in \{0, 1\} \quad (L_i \in \mathcal{L}, c_k \in \mathcal{C})$$

$$x_{ijk} \in \{0, 1\} \quad (L_i, L_j \in \mathcal{L}, L_i \neq L_j, c_k \in \mathcal{C}),$$

[1]Other distance measurement techniques can also be used [20].

where Constraints (14) follow our system model, and Constraints (15) guarantee that $x_{ijk} = 1$ if and only if both L_i and L_j have one radio tuned to channel c_k.

Unfortunately, solving an ILP is in general NP-hard [9], which means it may take exponential time to find the optimal solution. Hence we relax the above ILP to an LP by allowing s_{ik} and x_{ijk} to be real values between 0 and 1. The LP has been shown to be solvable in polynomial time [18]. As the constraints are relaxed, the LP only gives an *upper bound* on the ILP's optimal solution.

VII. Evaluations

A. Experiment Setup

In the simulations, links were randomly distributed in a $1000m \times 1000m$ square. The length of each link was uniformly distributed over $[1, 30]$. The interference range of the node was set to 2 times of the link length. The number of available channels was varied from 5 to 12 with increment of 1. The number of links was varied from 10 to 100 with increment of 10. The number of radio pairs on each link was uniformly selected over $[1, r]$, where $r \in \{2, 3, 4, 5\}$. Note that $\bar{r} = r$ in most cases. For every setting, we randomly generated 100 instances and averaged the results.

1) Channel Allocation Algorithms: To evaluate the system performance of the ChAlloc game, we compare the ChAlloc game with other two algorithms listed as below.

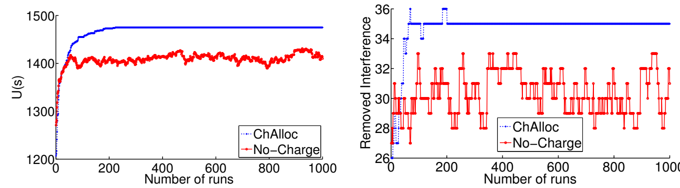
- *LP-based Algorithm (LP)*: This algorithm is based on the LP formulation in Section VI.
- *Random Allocation Algorithm (Rand)*: In this algorithm, each link L_i randomly select r_i channels out of the h channels. To certain extent, $Rand$ serves as a lower bound of the system performance in any channel allocation.

2) Performance Metric: The performance metrics include the *system performance* defined by (2), the player's *removed interference* defined by (1), and the *convergence speed* defined as the number of rounds before an NE is reached.

B. Convergence of the ChAlloc Game

We first verify that the ChAlloc game with the charging scheme, denoted by ChAlloc, does converge while the one without the charging scheme, denoted by No-Charge, may oscillate. In this set of simulations, we set n to 50, r to 3, and h to 8. The x-axis represents the number of runs, each of which is an iteration of the while-loop in Algorithm 1. Using runs can show the results in at a more granular level than using rounds. Although the algorithm will terminate when the game reaches an NE, we let it keep running for the sake of comparison. We have the results for 10000 runs, but only show the first 1000 runs due to the space limitation. Fig. 4(a) shows the system performance of these two difference game settings. As expected, ChAlloc converges to an NE after 222 runs, while No-Charge still oscillates even after 1000 runs. The factors affecting the convergence speed will be investigated in the next section.

Fig. 4(b) shows the removed interference for a random player (player 44). We observe that the removed interference of the player stays the same after about 200 runs in ChAlloc, but oscillates even after 1000 runs in No-Charge. Note that other players have the similar results.



(a) System performance (b) Player's performance

Fig. 4. Convergence of the ChAlloc game

C. Convergence Speed

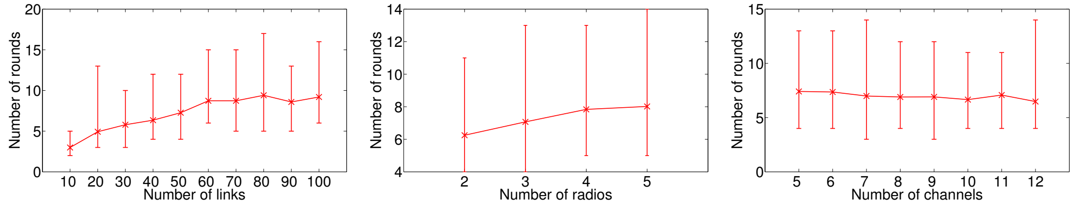
We next verify our analysis of the convergence speed, measured by the number of rounds. According to Theorem 3, the theoretical value is $O(\bar{r}n^2)$. We set h to 8 and r to 3 in Fig. 5(a). We set n to 50 and r to 3 in Fig. 5(b). We set n to 50 and h to 8 in Fig. 5(c). Fig. 5(a) and Fig. 5(b) show the impact of n and the impact of r on the convergence speed, respectively. We observe that the convergence speed is much faster than the theoretical speed, and that all the instances can converge within 10 rounds on average. Fig. 5(c) shows that the convergence speed is almost independent of h, with the varying range of the average being less than 1.

D. System Performance

We now compare the system performance of the ChAlloc game with those of other algorithms in Section VII-A. Fig. 6 shows all the results. As expected, LP has the best performance while $Rand$ has the worst. The first observation is that all the results are consistent with our performance analysis in Theorem 2. In particular, Fig. 6(a), Fig. 6(b) and Fig. 6(c) show the impact of n, r and h on the system performance, respectively. The results confirm that the system performance of ChAlloc compared to LP is independent of n. The less radios or the more channels there are, the closer the performance of ChAlloc is to the performance of LP. Another observation is that the gap between the performance of $Rand$ and the performance of ChAlloc gets narrower when the number of channels increases or the number of radios decreases. The reason is that when the value of $\frac{\bar{r}}{h}$ decreases, the probability that two interfering links share the same channels decreases as well.

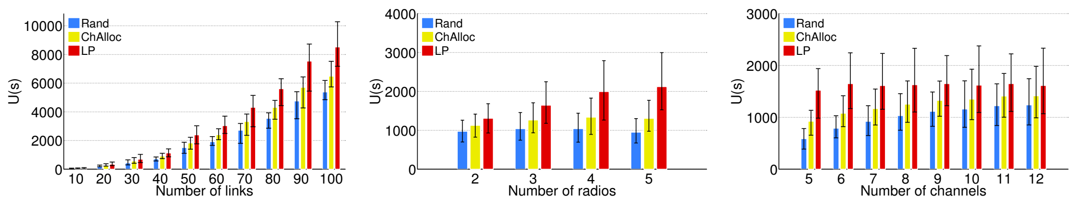
VIII. Conclusion

In this paper, we have studied the channel allocation problem in non-cooperative MR-MC networks. Compared with existing works, we removed the single collision domain assumption and considered networks with multiple collision domains. We modeled the problem as a strategic game, called ChAlloc. Via an example, we showed that ChAlloc may result in an oscillation when no exogenous factors exist. To avoid this possible oscillation, we design a charging scheme to influence players' behavior. We then proved that ChAlloc will converge to an NE in polynomial number of steps. We further proved that the system performance in any NE is guaranteed to be at least $(1 - \frac{\bar{r}}{h})$ of that in the optimal solution, where $\bar{r}$ is the maximum number of radios equipped on wireless devices



(a) The impact of the number of links (b) The impact of the number of radios (c) The impact of the number of channels

Fig. 5. Convergence speed



(a) The impact of the number of links (b) The impact of the number of radios (c) The impact of the number of channels

Fig. 6. Comparison on the system performance

and h is the number of available channels. We also developed a localized algorithm for players to find an NE strategy. Through extensive experiments, we validated our analysis of the possible oscillation and the convergence when there is and is not the charging scheme. Finally, the experiment results also confirmed our proof on the system performance compared to the upper bounds returned by an LP-based algorithm.

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