Model Based Trading: Group Project

Adaptive VWAP Strategy with Optimal Execution

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Apr. 19th, 2018

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Goal and Rationale

Goal

- Implementation of VWAP execution strategy in practice

- Trade performance improvement
  - Dynamically adapt VWAP to market information
  - Optimal order placement

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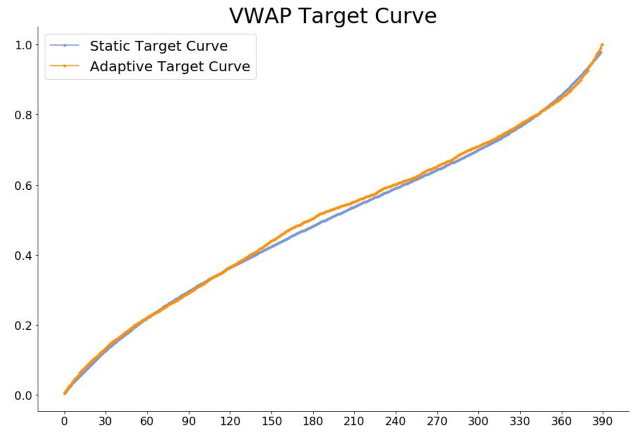
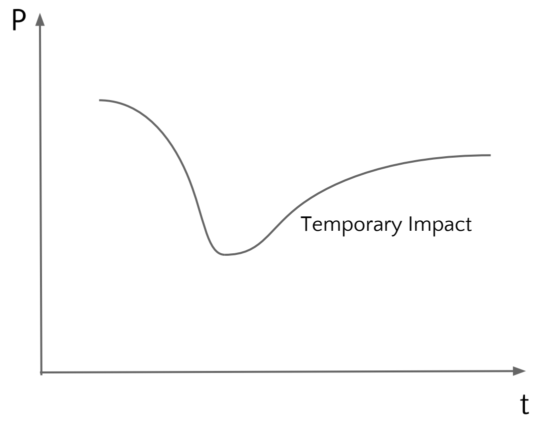


Goal

- **Implementation of VWAP execution strategy in practice**

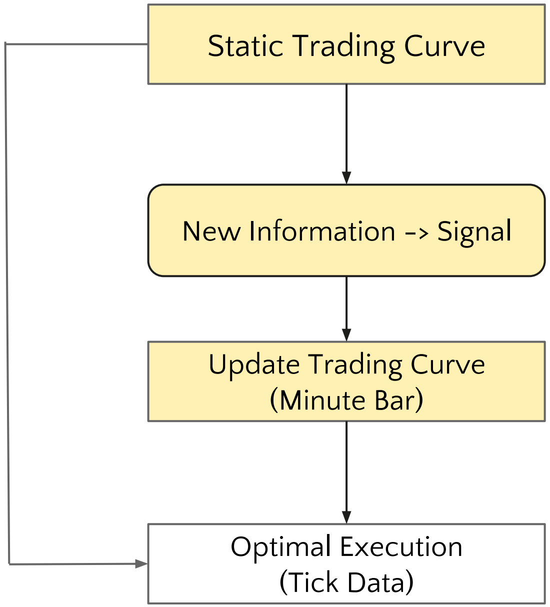
- **Trade performance improvement**
  - Dynamically adapt VWAP to market information
  - Optimal order placement



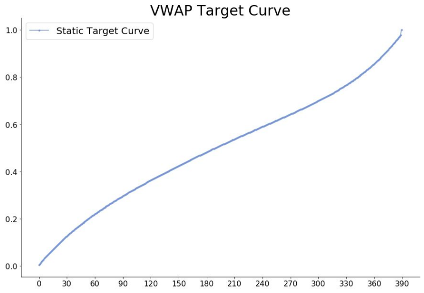
Methodology: Framework

Two basic ideas

Step 1

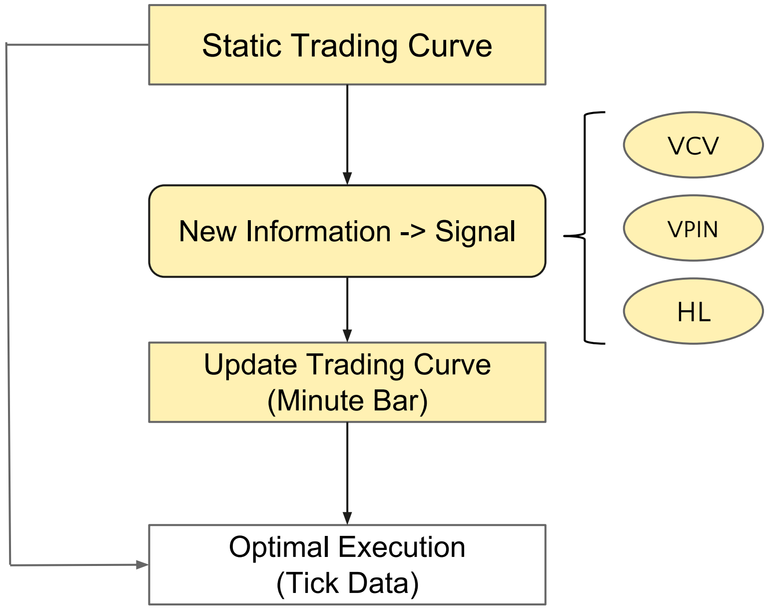


Static Trading Curve
New Information -> Signal
Update Trading Curve
(Minute Bar)
Optimal Execution
(Tick Data)



VWAP Target Curve
Static Target Curve
1.0
0.8
0.6
0.4
0.2
0.0
0
30
60
90
120
150
180
210
240
270
300
330
360
390

Step 1



Static Trading Curve
New Information -> Signal
VCV
VPIN
HL
Update Trading Curve
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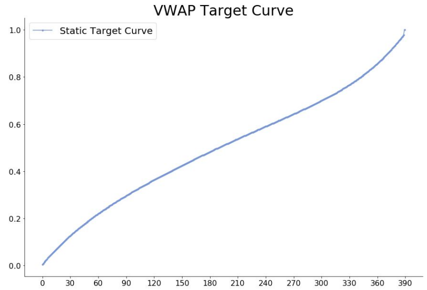
Optimal Execution
(Tick Data)



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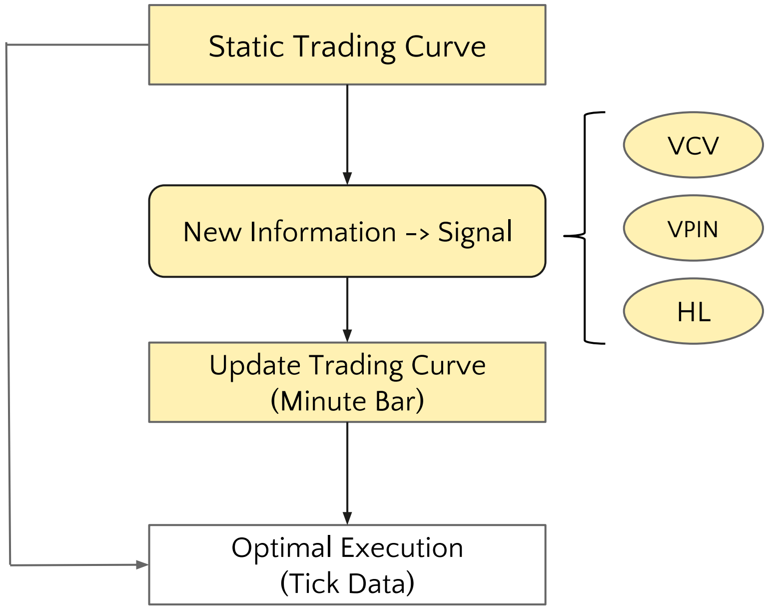
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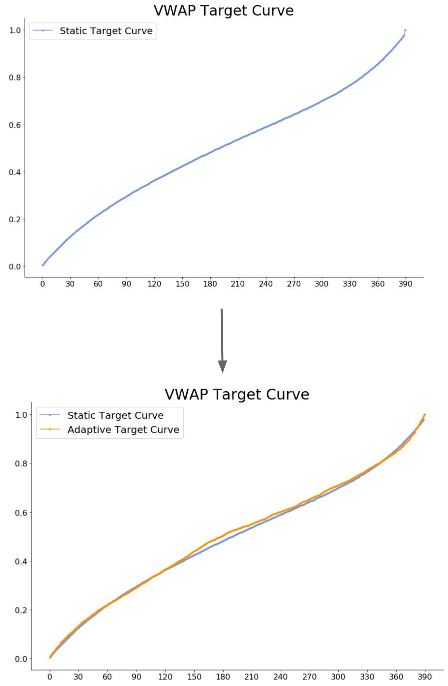
VWAP Target Curve
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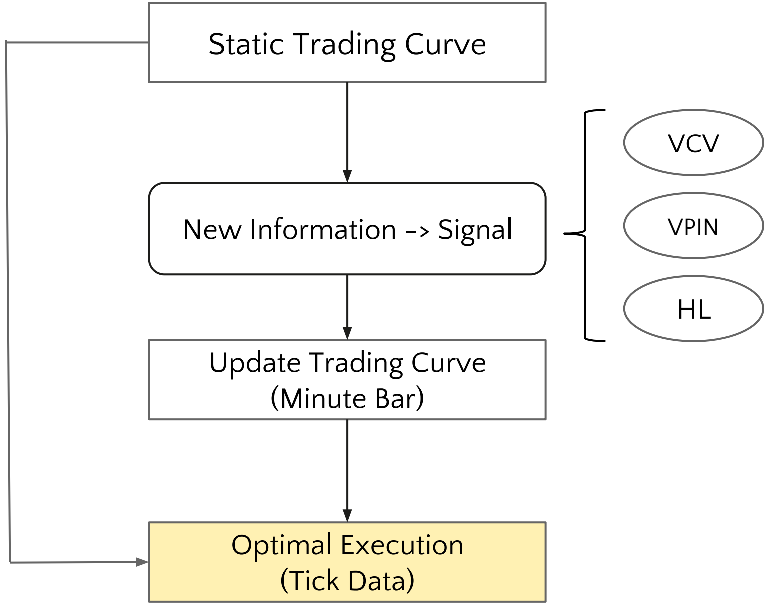
VWAP Target Curve
Static Target Curve
Adaptive Target Curve

Step 1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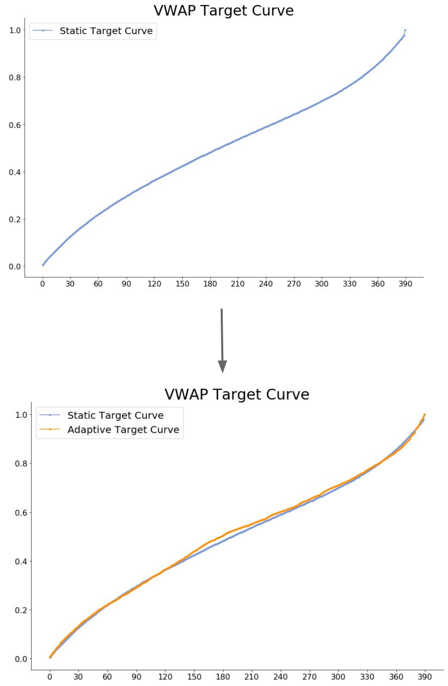
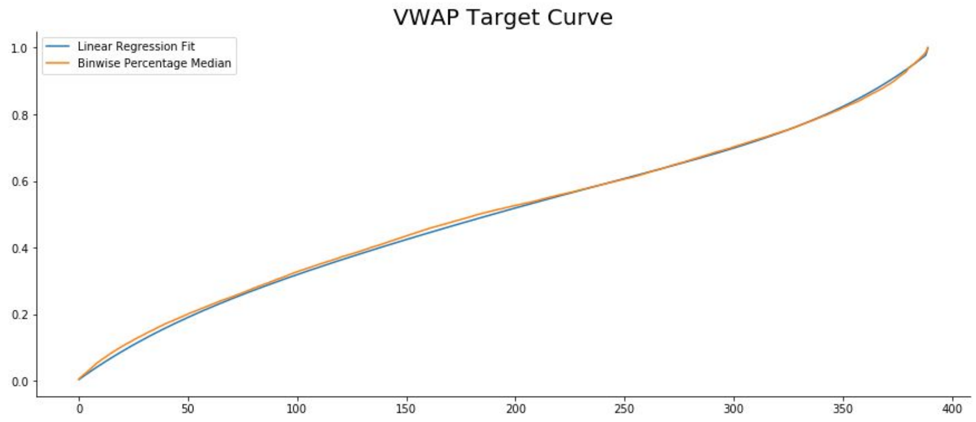

VWAP Target Curve
Static Target Curve
VWAP Target Curve
Static Target Curve
Adaptive Target Curve

VWAP Execution

Static VWAP

- Method 1: Polynomial curve by regression
- Method 2: By Historical Binwise Median



Adaptive VWAP

- Divide the trading horizon into N slices
  - In our case, each minute is a time slice, constituting 390 slices in one day

- Get a signal in each slice
  - A signal is an indicator of the price level

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- Get a signal in each slice
  - A signal is an indicator of the price level

$$s_n = \begin{cases} 1 & \textit{good price level, execute more aggressive} \\ 0 & \textit{average price level, no action} \\ -1 & \textit{bad price level, execute more conservative} \end{cases}$$

Adaptive VWAP

- Algorithm:

  - For the first iteration, use the static curve $p^{(1)} = p$.
  - For the n-th iteration where $2 \leq n \leq N-1$,

$$p_k^{(n)} = \begin{cases} p_k^{(n-1)} & \text{if} \quad k \leq n-1 \\ p_k^{(n-1)}(1+\delta s_k) & \text{if} \quad k = n \\ p_k^{(n-1)}\left(1 - \frac{\delta s_n p_n^{(n-1)}}{\sum_{l \geq n+1} p_l^{(n-1)}}\right) & \text{if} \quad k \geq n+1 \end{cases}$$

------ Previous targets unchanged

------ Current target adjusted

------ Decay effect on future targets

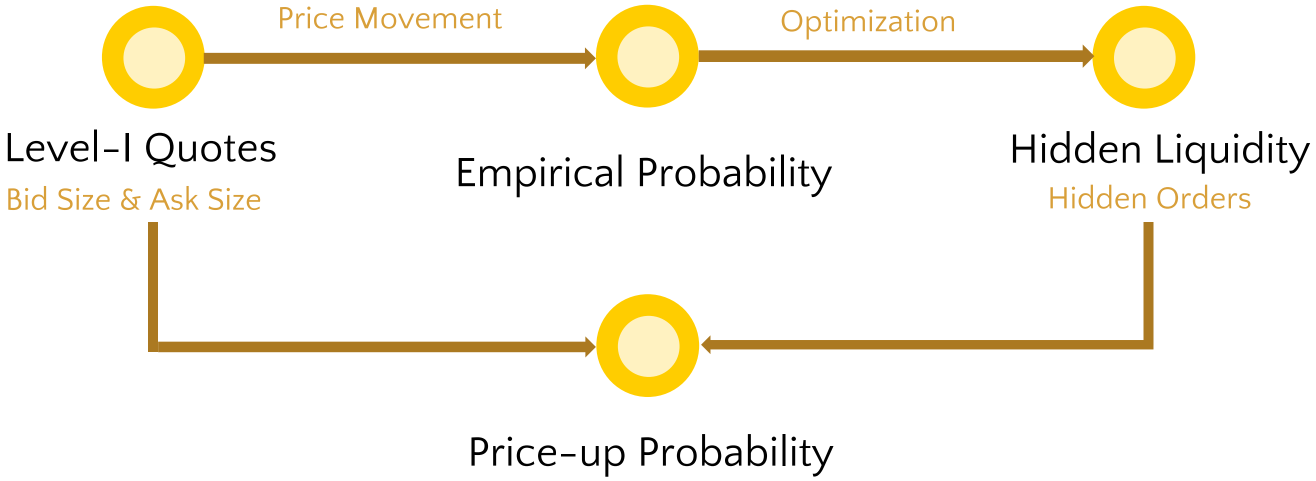
  - For the last iteration

$$p_N^{(N)} = 1 - \sum_{k=1}^{N-1} p_k^{(N-1)} .$$

Trade Signals

What does the information suggest about the price level?

Level-I Quotes Signal with Hidden Liquidity

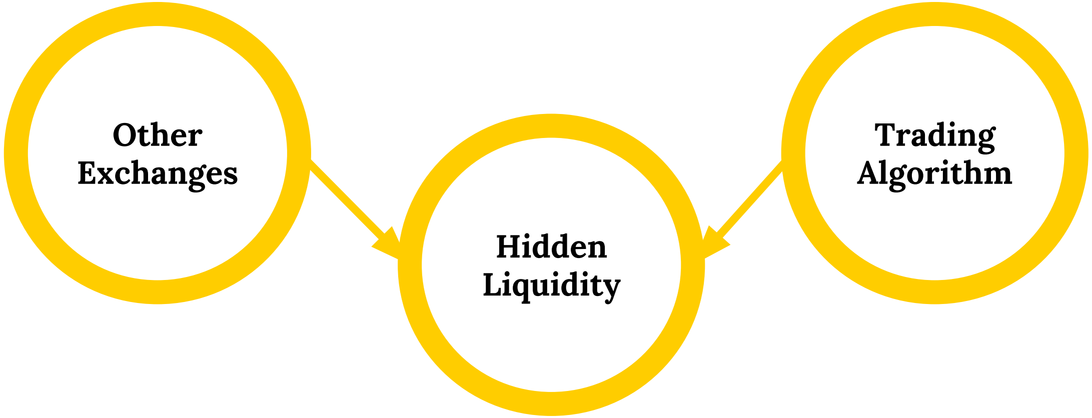


Modeling Level-I Quotes

- **Level I quotes**: the best bid/ask prices and sizes
- **“RACE TO BOTTOM”**: the queue that hits zero first causes the price to move in that direction

Hidden liquidity

- **Hidden Liquidity**: sizes that are not shown in the order book, but which may influence the probability of an upward move in the price.



The discrete Poisson model

- View ask queue and bid queue as a continuous time Markov chain (CTMC)

$$\begin{aligned}
h &= \text{minimum order size} \\
\lambda_a &= \text{arrival rate of limit orders at the ask} \\
\lambda_b &= \text{arrival rate of limit orders at the bid} \\
\mu_a &= \text{arrival rate of (buy) market orders at the ask or cancellations at the ask} \\
\mu_b &= \text{arrival rate of (sell) market orders at the bid or cancellations at the bid}
\end{aligned}$$

The discrete Poisson model

- CTMC:
  - State (X, Y), X = bid queue size, Y =ask queue size
  - Transition rates into four neighboring states:

$$\begin{aligned}\lambda_{0,1} &= \lambda_a\\ \lambda_{0,-1} &= \mu_a\\ \lambda_{-1,0} &= \mu_b\\ \lambda_{1,0} &= \lambda_b.\end{aligned}$$

  - Additional diagonal transitions:

$$\lambda_{-1,+1} = \lambda_{+1,-1} = \eta > 0$$

The discrete Poisson model

- Drifts and variances of the queue sizes:

$$m_X = h\left(\lambda_b - \mu_b\right)$$

$$m_Y = h\left(\lambda_a - \mu_a\right)$$

$$\sigma_X^2 = h^2\left(\lambda_b + \mu_b + 2\eta\right)$$

$$\sigma_Y^2 = h^2\left(\lambda_a + \mu_a + 2\eta\right).$$

- Assume that there is symmetry between bid and offer sizes, and the drifts vanish

$$\sigma_X^2 = \sigma_Y^2 = 2h^2\left(\lambda + \eta\right)$$

- Correlation between the bid and the ask queues

$$\rho = \frac{-\eta}{\lambda + \eta}$$

Diffusion approximation

- Define $x = X/ < X >, \quad y = Y/ < Y >$
- < X > and < Y > denote, respectively, the average (or median) size of the queues X_t , Y_t
- The process (x_t , y_t) can be approximated by the diffusion

$$\sigma^2 = \frac{2h^2 (\lambda + \eta)}{< X >^2}$$

$$\begin{aligned} dx_t &= \sigma dW_t^{(1)} \\ dy_t &= \sigma dW_t^{(2)} \\ E\left(dW^{(1)} dW^{(2)}\right) &= \rho dt, \end{aligned}$$

Diffusion approximation

- **u(x, y)**: the probability that the next price move is up, given that (standardized) bid/ask sizes (x, y)
- By Ito's lemma,

$$\sigma^2 (u_{xx} + 2\rho u_{xy} + u_{yy}) = 0, \quad x > 0,\ y > 0 \qquad \begin{aligned} u(0, y) &= 0, \quad \text{for} \quad y > 0 \\ u(x, 0) &= 1, \quad \text{for} \quad x > 0 \end{aligned} \qquad (*)$$

- However, the "true" size of the queues are x + H and y + H, where H is the hidden liquidity
- **p(x, y; H)**: the probability of an upward price move conditional on the observed queue sizes (x, y) and the hidden liquidity H

$$p(x, y; H) = u(x + H, y + H)$$

Diffusion approximation

- Assume a Perfectly negatively correlated queues

$$\rho = -1$$

- Solving the PDE (*)

$$p(x, y; H) = \frac{x + H}{x + y + 2H}$$

- Obtain H by minimizing least squares

$$\min_H \sum_{i,j} \left[\left(u_{ij} - \frac{i + H}{i + j + 2H} \right)^2 d_{ij} \right]$$

Model Probability of Price-Up Example

AAPL

Hidden Liquidity = 1.3491

Bucket	0.2	0.4	0.6	0.8	1.0
0.2	0.5	0.4696	0.4428	0.4188	0.3973
0.4	0.5303	0.5	0.4729	0.4486	0.4267
0.6	0.5571	0.5270	0.5	0.4755	0.4534
0.8	0.5811	0.5513	0.5244	0.5	0.4777
1.0	0.6026	0.5732	0.5465	0.5222	0.5

IEF

Hidden Liquidity = 0.2223

Bucket	0.2	0.4	0.6	0.8	1.0
0.2	0.5	0.4042	0.3393	0.2923	0.2567
0.4	0.5957	0.5	0.4307	0.4486	0.3783
0.6	0.6606	0.5692	0.5	0.4457	0.4021
0.8	0.7076	0.6216	0.5542	0.5	0.4554
1.0	0.7432	0.6626	0.5978	0.5445	0.5

Signals - VCV Measure

VCV Measure

Information Asymmetry

Volume Coefficient of Variation

Intuition: the distribution of transaction amount depends on the correlation of individual orders

Informed Order ↑

More Skewed and dispersed Distribution

Risk aversion, informational advantage, competition with other informed traders

VCV Calculation

Compare information asymmetries over time

Time series of intraday transaction amount

$$VCV = \frac{\hat{\sigma}_v}{\hat{\mu}_v}$$

a consistent estimator of $\frac{\sigma_v}{\mu_v}$

As $M \rightarrow \infty$

$$\lim_{M \rightarrow \infty} \frac{\sigma_v}{\mu_v} = \sqrt{2\pi - 4}\,\frac{\eta}{\eta+1}$$

VCV $\uparrow$

Proportion of informed trade $\uparrow$

M: number of traders

Eta: Proportion of informed trade

VCV Measure

Positively Correlated	Negatively Correlated
Return volatility	Size
Illiquidity	Turnover
Bid-ask spread	Analyst coverage
Return autocorrelations	Return reversals

From VCV Measure to Signal

Price Changes due to informed trading are likely to be predictive of future price changes –> Push up return autocorrelations

First Step

- Use min bar data to calculate sample VCV as a benchmark

Second Step

- Execution
- Derive current VCV, average price within one minute, and running vwap

Third Step

- Convert VCV to signal
- Assume buy
- Signal = $\begin{cases} \text{1, if cur vcv > benchmark \& avg price > running vwap} \\ \text{0, if cur vcv < benchmark} \\ \text{-1,if if cur vcv > benchmark \& avg price < running vwap} \end{cases}$

Limitation

- After the earnings announcement, the proportion of informed trading goes down as information asymmetries decreases and the market becomes more attractive for uninformed traders. Thereafter, VCV goes down. According to the return autocorrelation characteristic, it's unlikely for us to predict the price movements.

Signals - VPIN Measure

VPIN Signal

- **Rationale:** What does VPIN measure?
  - Buy-sell imbalance.

Informed Trading

Volatility Spike

Liquidation Events

VPIN Signal

- **Key Ideas:** How do we generate VPIN?

1. Time Bar — (Obtaining price change and cumulative volume)
2. Trade Direction — (Deciding based on every time bar)
3. Volume Bar — (Aggregating to create constant size blocks)
4. Buy-sell Imbalance — (Calculating buy/sell volume difference)

VPIN Signal

- **Measure to Signal:** The intuition behind the conversion.

1. Price continuation or price reversion?
2. How to determine if it has predicting power?
3. How did we test it?

VPIN Signal

- **Further Applications:** Relating VPIN to econometrics.

VPIN pattern near the spikes

Possible Causes

Qualitative backtest

Predict appearances

VPIN Signal

- **Limitations:**
  - Not a good predictor for big-cap stocks
    - Infrequent observations of significant imbalances
  - Need a longer horizon to realize impact
    - A small backtest set may contain too much randomness
  - Possibly fail upon market macro changes
    - Bear market/bull market
  - Solutions
    - Time-series perspective/ Reparametrization

Optimal Execution

Optimal Execution

Goals:

- Improve execution price
  - Split orders
  - reduce market impact, therefore improving profit.
- Follow given trading curve

Optimal Execution (Theory)

Assumptions:

- There is no new information flow.
- Regard a market order as a limit order at bid price.
- Probability of the limit order being filled follows an exponential distribution.

Optimal Execution (Theory)

$$\max_{\delta_t^a} \mathbb{E}\left[-exp\left(-\gamma(X_T + q_T(S_T - b))\right)\right]$$

Where:

T: Time horizon (1 minute)

δ_t^a: The difference between quote price and mid-price

γ: Trader's risk aversion

X_T: Terminal Wealth at T

q_T: Shares lag behind target trading curve at T

b: Penalty for not achieving target trading curve at T

Optimal Execution (Solution)

- **Optimal quote**

$$S_t^a(q) = S_t + \frac{1}{k}\ln\left(1 + \frac{f(q)\times \boxed{(T-t)^q}}{\sum_{j=0}^{q-1} g(j)\times \boxed{(T-t)^j}}\right) - b$$

Where $f(q)$ and $g(q)$ are functions of remaining units of shares during $[0,T]$

- **Model Calibration:** parameters of exponential distribution, risk aversion, b.

Optimal Execution (Details)

$$\max_{\delta_t^a} \mathbb{E}\left[-exp\left(-\gamma(X_T + q_T(S_T - b))\right)\right]$$

Given a time horizon T, the goal is to optimize the expected utility of P&L at time T. γ is traders' absolute risk aversion, X_T is the wealth at T, q_T is the remaining units of shares that lag behind trading curve at time T, and b is the penalty term.

Define reference price (mid-price or bid price) of a stock follows an arithmetic Brownian motion.

$$dS_t = \mu dt + \sigma dW_t$$

Without losing generality, consider a liquidation problem and ask quotes respectively. Assume offer quotes follows:

$$dS_t^a = S_t + \delta_t^a$$

Remaining quantity $q_t = q_0 - N_t^a$, where N_t^a is a time-dependent jump process with intensity $\lambda = Aexp(-k\delta_t^a)$. Wealth process dynamics is therefore:

$$dX_t = (S_t + \delta_t^a)dN_t^a$$

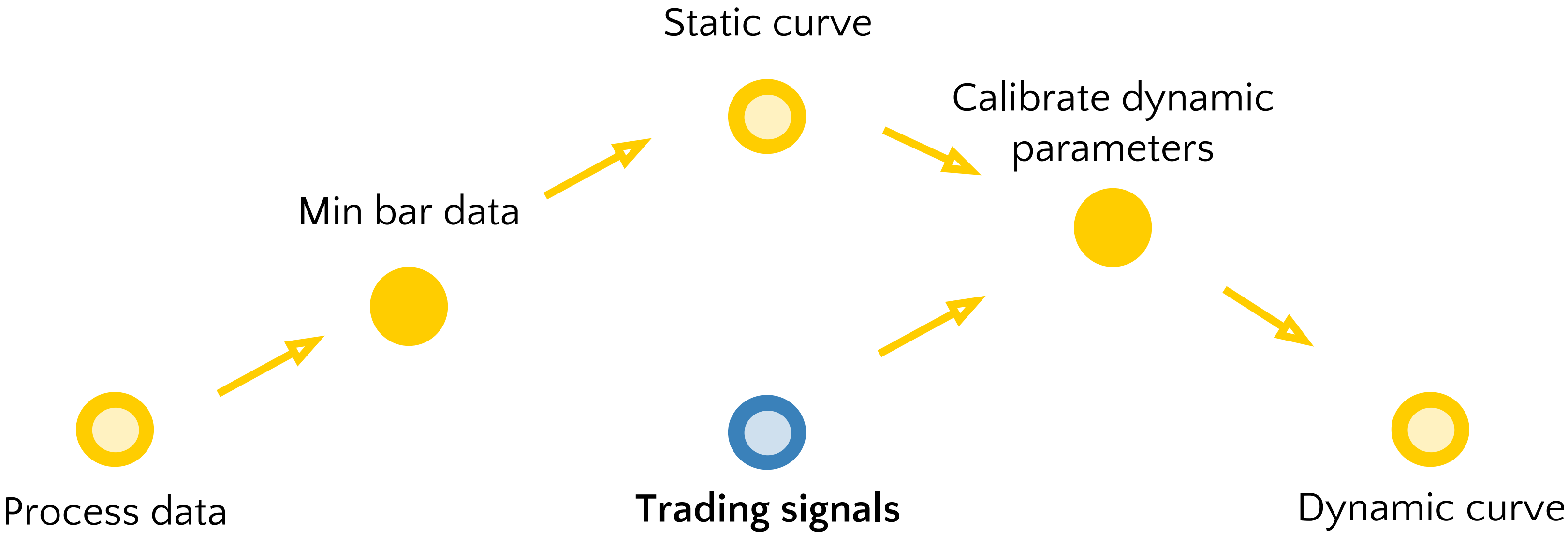
Write the stochastic control problem in Hamilton-Jacobi-Bellman equation, we obtain:

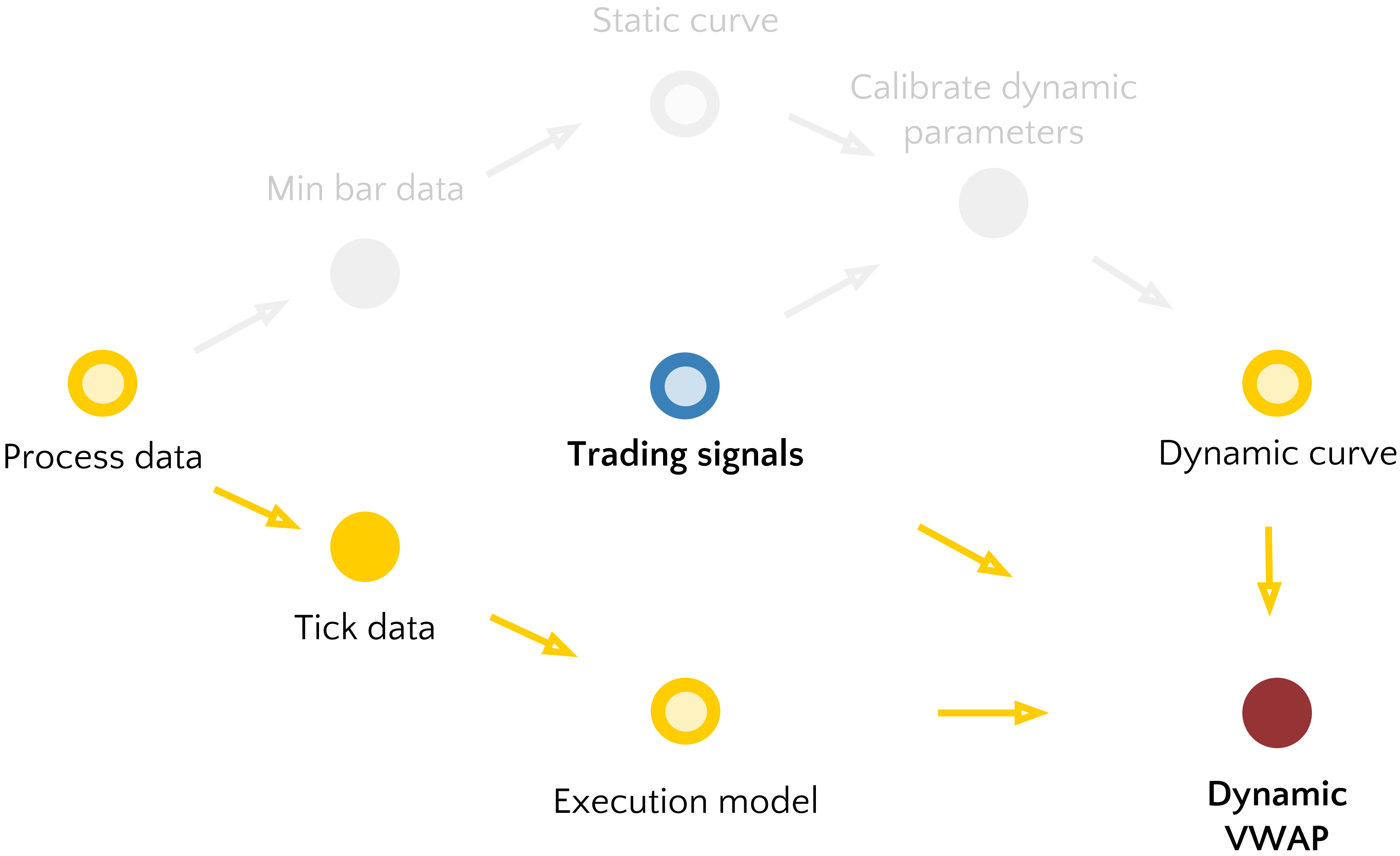
$$\delta_t^{a^*} = \frac{1}{k} log\left(\frac{w_{q(t)}}{w_{q-1(t)}}\right) - b$$

Where $w_{q(t)}$ is solution of ODEs derived from HJB equation.

We can solve $w_{q(t)}$ explicitly when we assume $\mu = \sigma = 0$, in case of no price risk.

Implementation


Static curve
Calibrate dynamic parameters
Min bar data
Process data
Trading signals
Dynamic curve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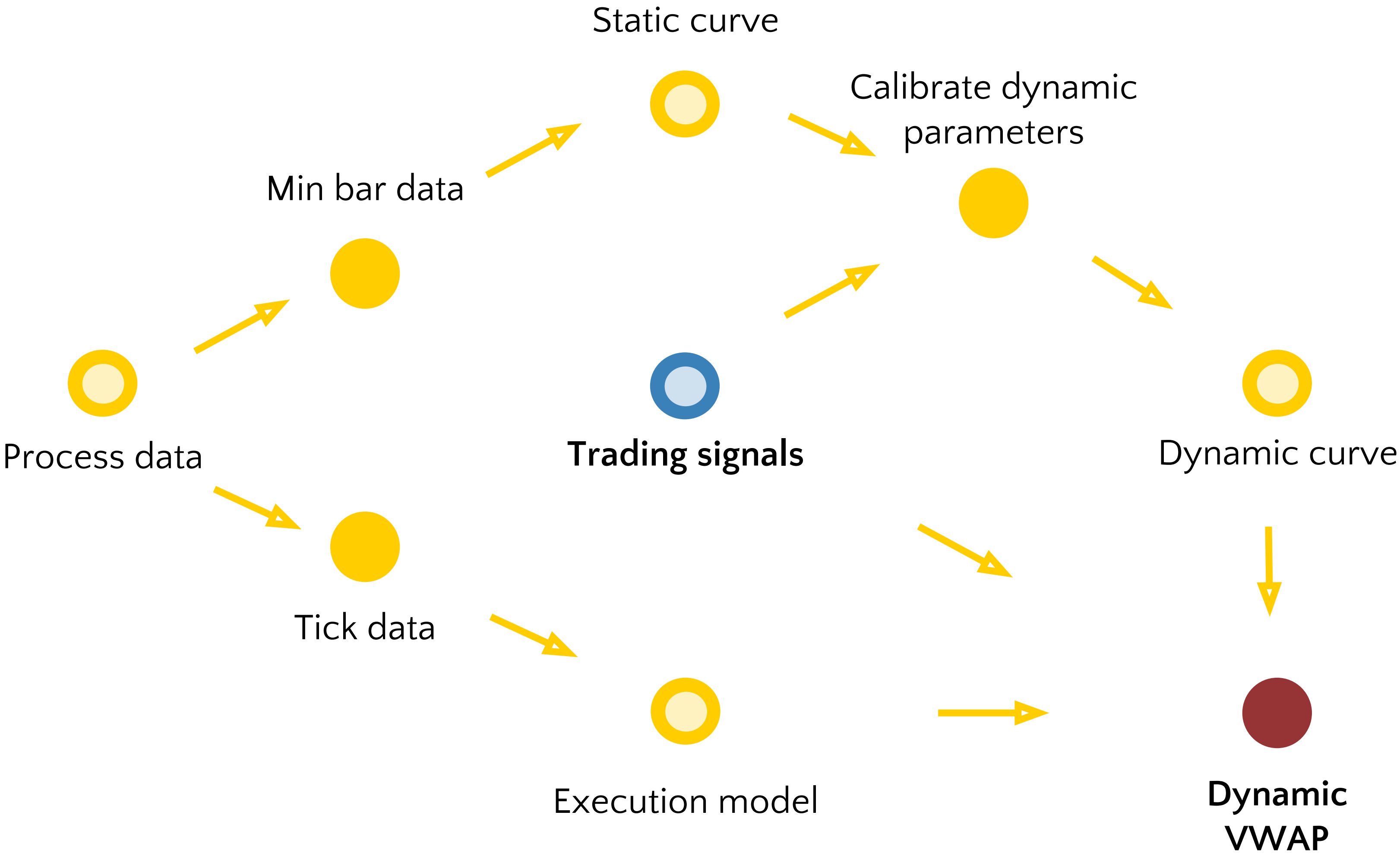
Tick data
Execution model
Dynamic VWAP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Notebook Demo

- Read and process data
- Get the static trading curve using 2 methods
- Trading signals functions: VCV, VPIN, Hidden Liquidity
- Calibrate delta parameter in generating adaptive trading curve
- Generate dynamic trading curve
- Backtest on min bar data
- Calibrate parameters in excution model
- Test on tick data
- Result analysis & Visualization

https://github.com/wuyouk/Model-Based-Trading/blob/master/adaptive_vwap_main.ipynb

Notebook Usage

Usage

Please download the following notebooks and data files and put the data files in the path */Data/*.

1. mbt_g1_strategy
2. mbt_g1_signal_liquidity
3. mbt_g1_signal_vpin
4. tick data AAPL_20180117
5. bar data AAPL_170101_180413

- Note: For part of the visualization, you need to have a plotly account and set credentials in the first block `tls.set_credentials_file(username=' ', api_key=' ')`

Results Analysis & Visualization

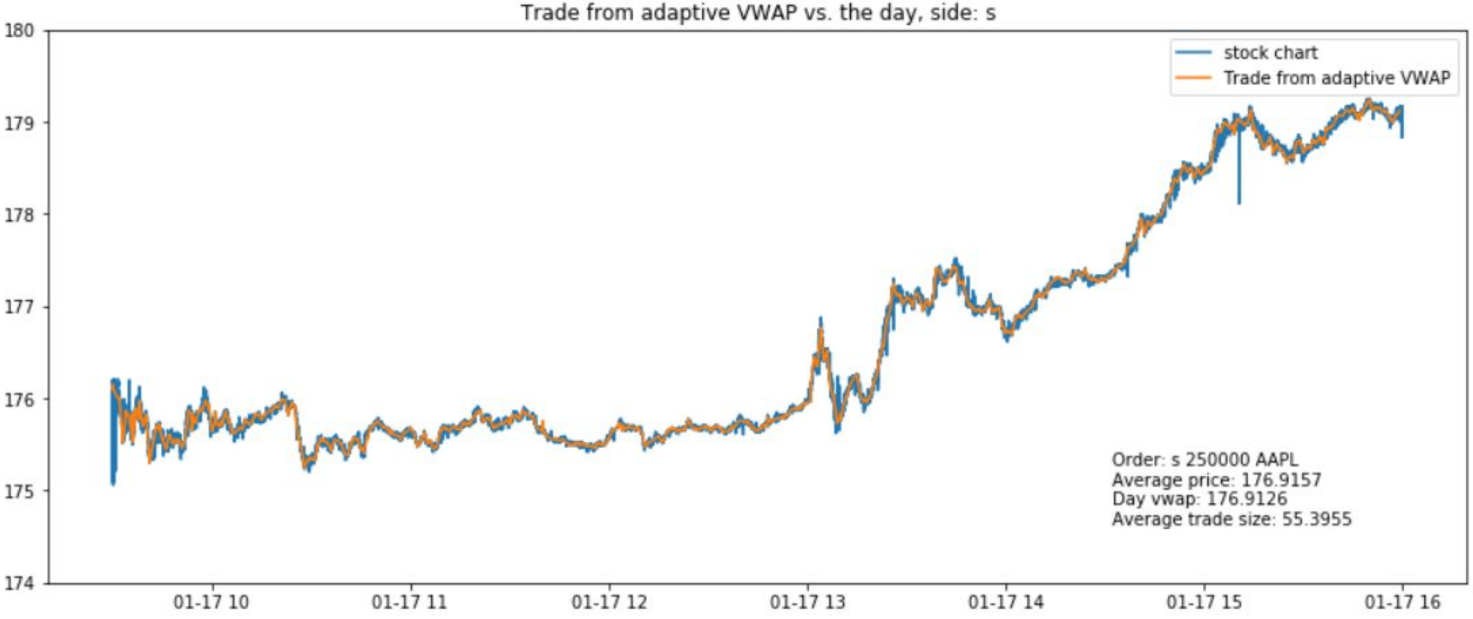
VWAP	176.912
Execution	176.791

Result (Static Trading Curve)

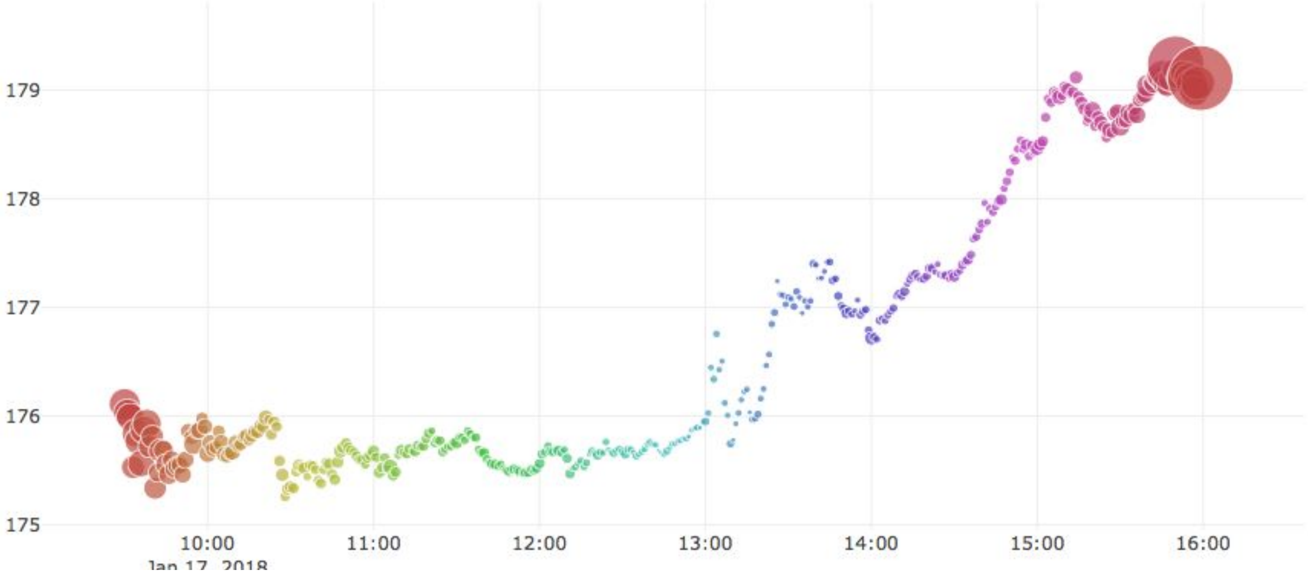
Results of
Hidden Liquidity

VWAP	176.912
Execution	176.916

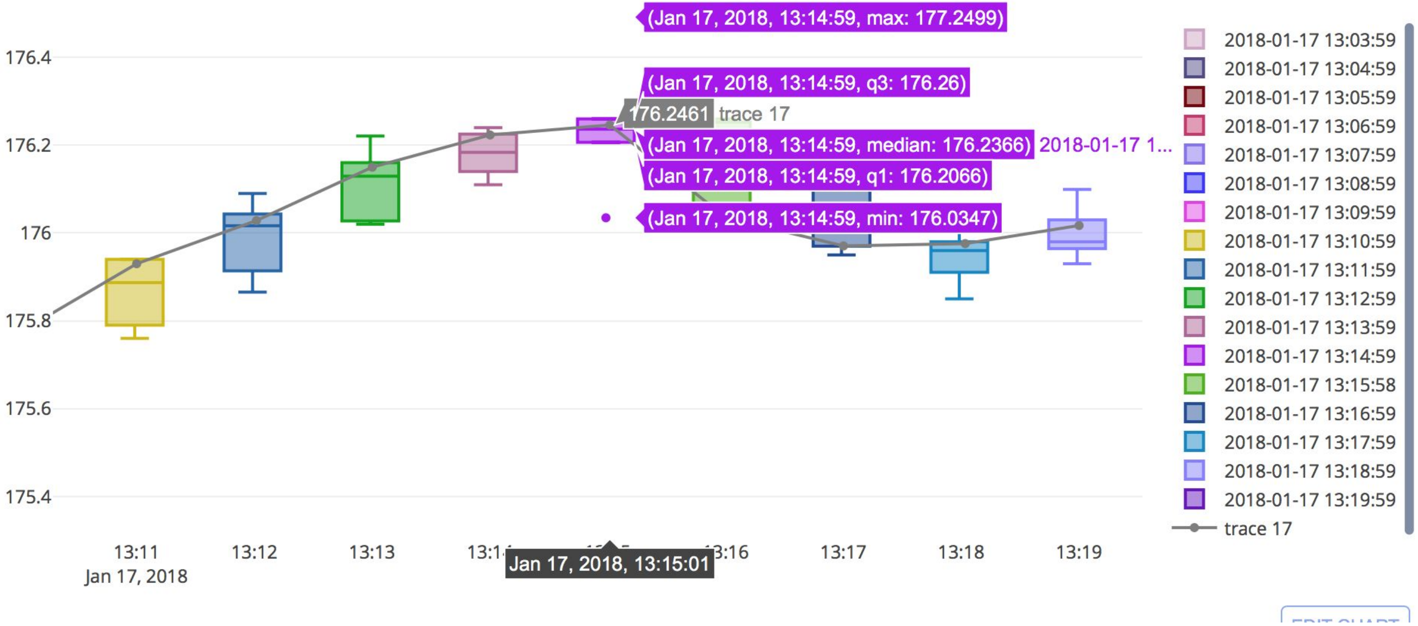
Result (Hidden Liquidity)



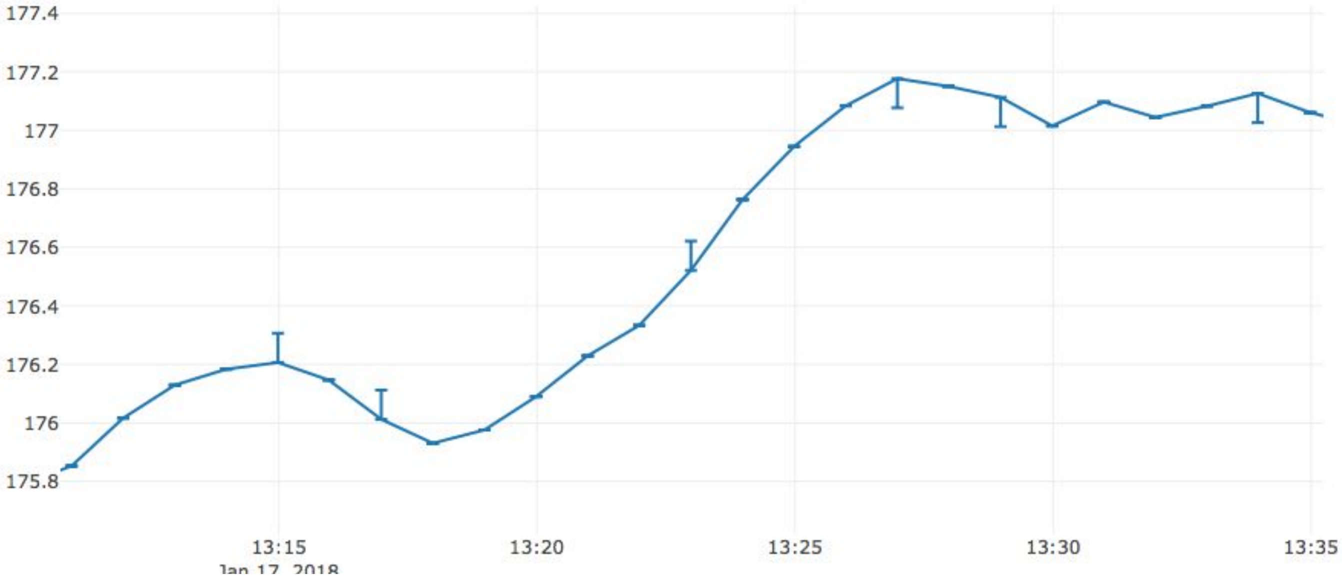
Bubble Chart (Hidden Liquidity)



Box Chart (Hidden Liquidity)



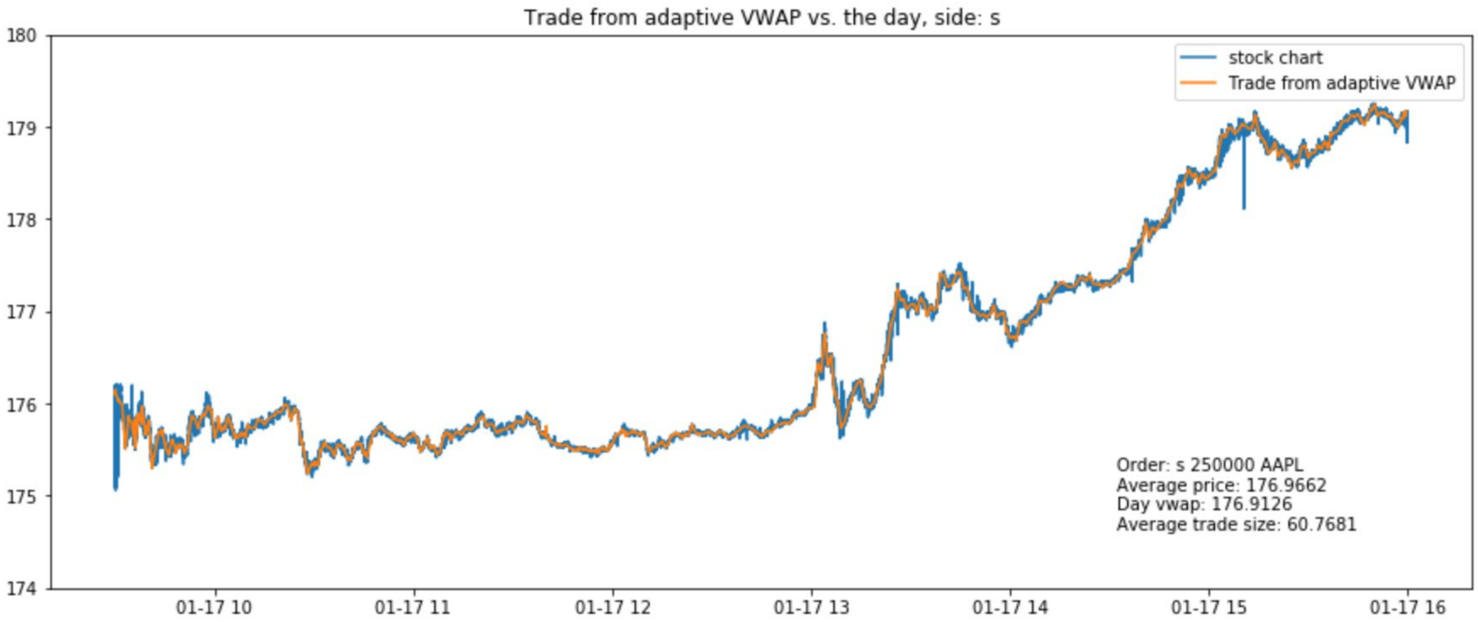
Signal Plot (Hidden Liquidity)



Results of VCV Measure

VWAP	176.912
Execution	176.966

Result (VCV Measure)



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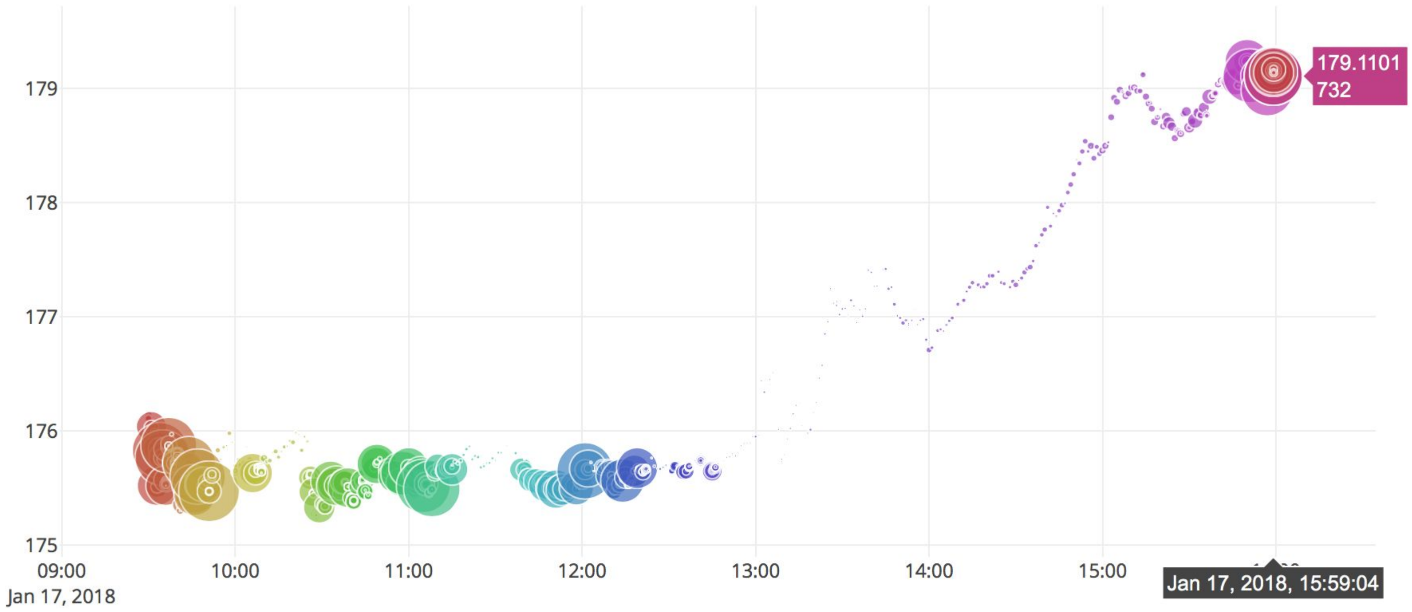

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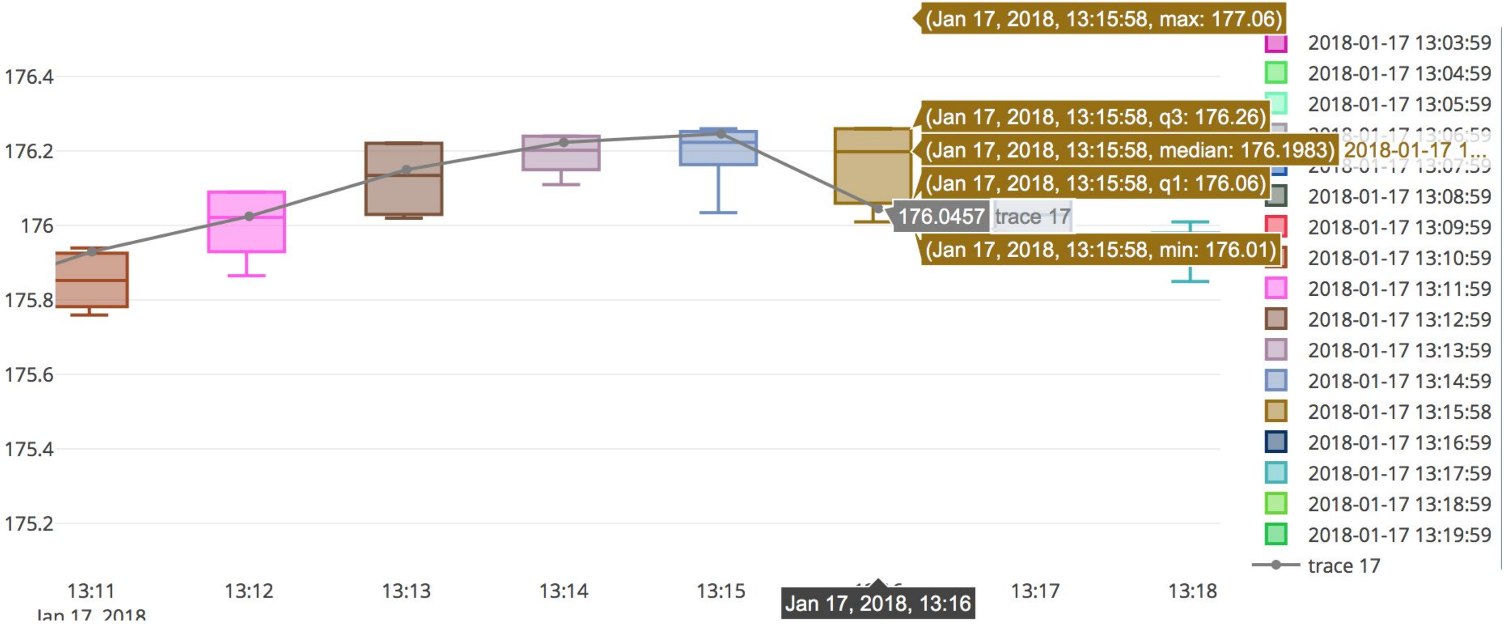

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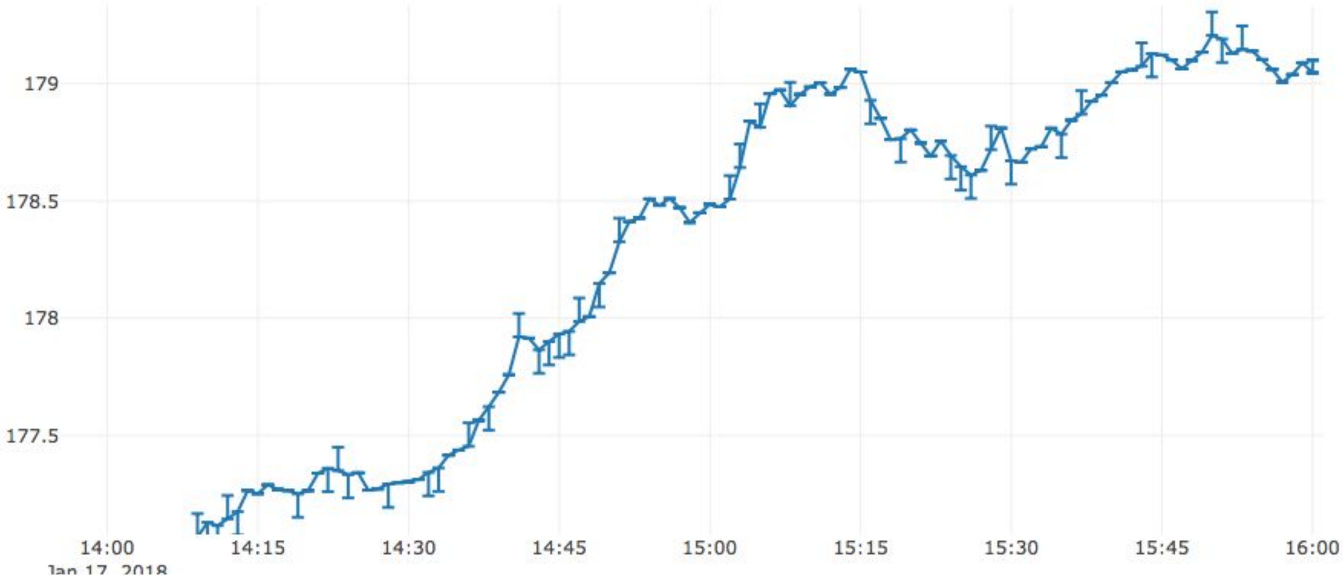
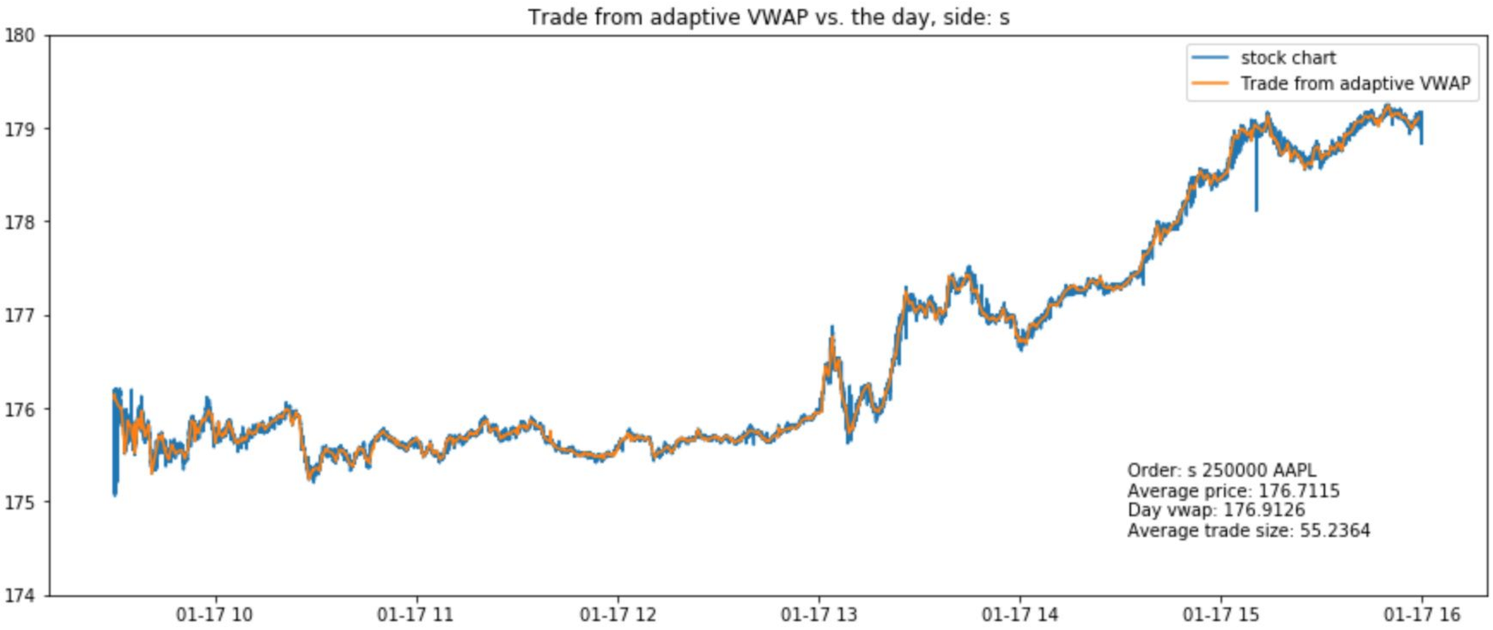


Results of VPIN Measure

VWAP	176.912
Execution	176.712

Result (VPIN Measure)



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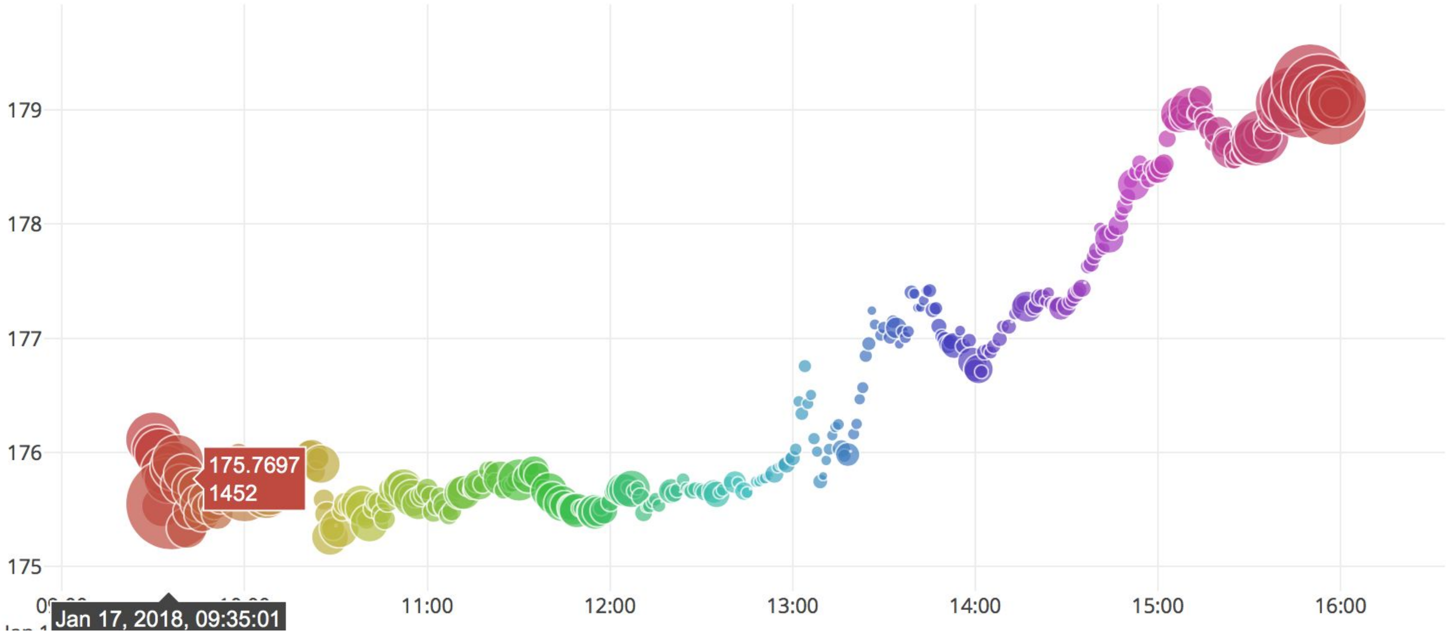

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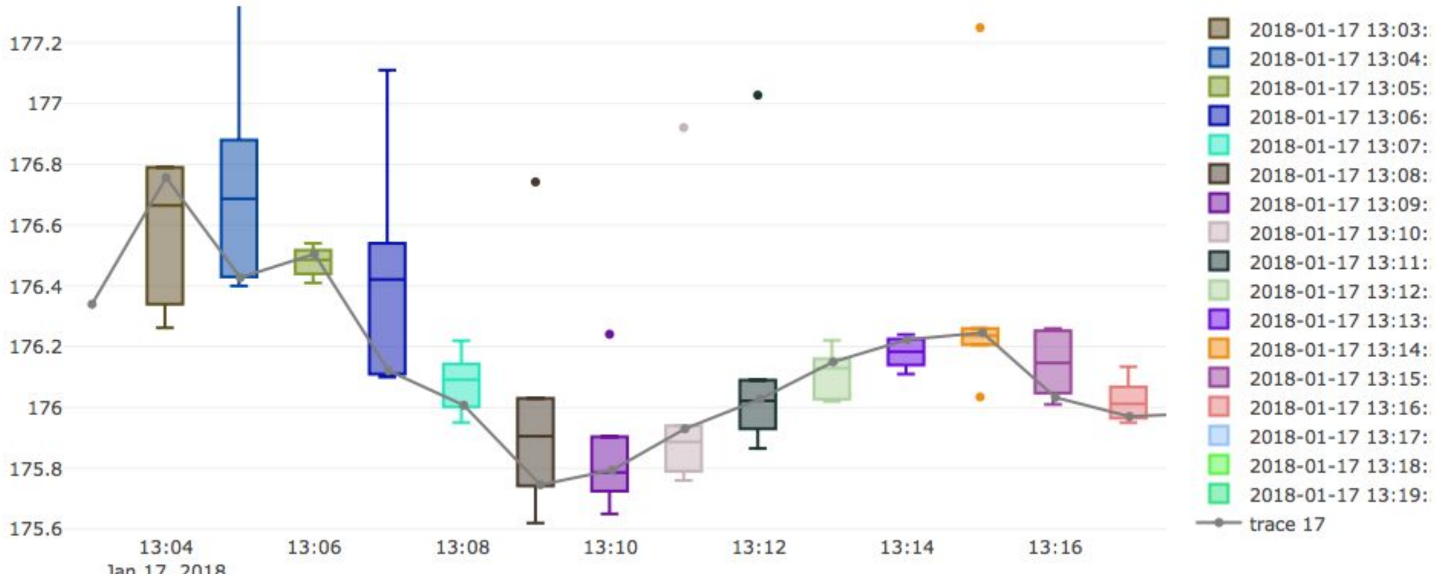

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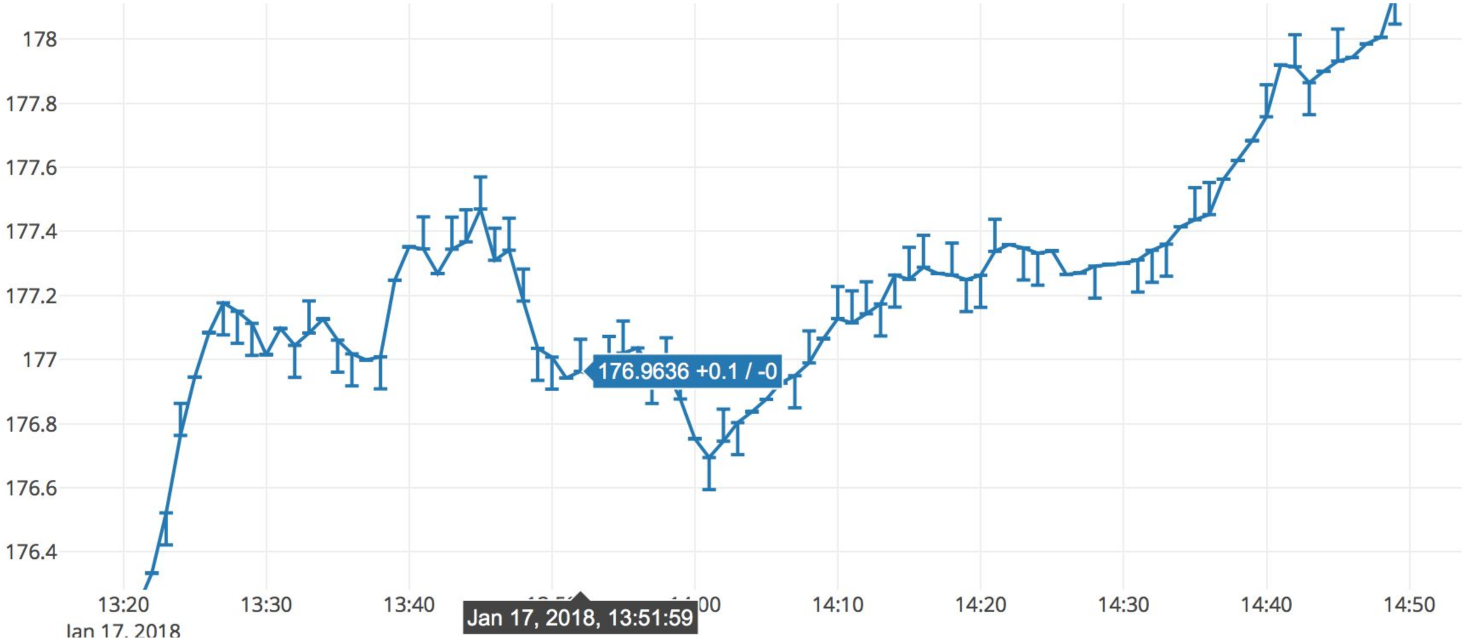


Transaction Cost Analysis

Trading Strategy	Arrival Price	Market VWAP	Static Curve	Adaptive (Hidden Liquidity)	Adaptive (VCV Measure)	Adaptive (VPIN Measure)
Average Price	176.15	176.9126	176.7910	176.9157	176.9662	176.7115

Transaction Cost Analysis

Price	VWAP	Adaptive VWAP	Slippage
Mean	173.7762	173.5857	-0.1905
Variance	18.2991	18.7856	1.0151

Limitations & Improvements

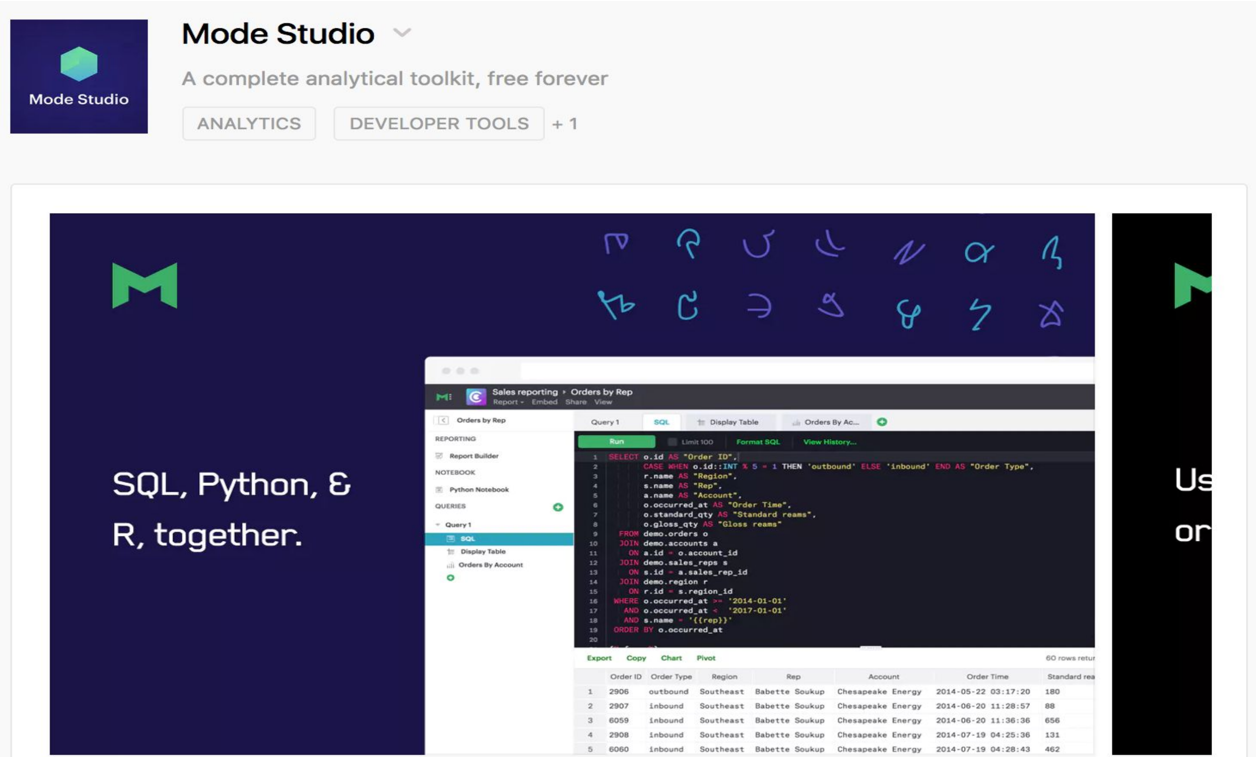
Limitations

- Limited training data to estimate parameters.
- Lack of tick data and testing samples.

- Time consuming: more than 30 minutes to test.

Improvements

Improvements



https://modeanalytics.com/

Improvements

- More types of signals
  - Venue liquidity, order imbalance, momentum, volume clustering
  - Trade autocorrelation, relative value, news / events

Conclusion

Conclusion

- Adaptive performs better than static
- Use more data
- Consider more signals

References

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3. Cartea, Álvaro, and Sebastian Jaimungal. "Optimal execution with limit and market orders." *Quantitative Finance* 15.8 (2015): 1279-1291.
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5. Avellaneda, Marco, Josh Reed, and Sasha Stoikov. "Forecasting Prices in the Presence of Hidden Liquidity." Preprint (2010).
6. Lof, Matthijs and van Bommel, Jos, Asymmetric Information and the Distribution of Trading Volume (April 19, 2018). Available at SSRN: https://ssrn.com/abstract=2726187 or http://dx.doi.org/10.2139/ssrn.2726187

Data source

1. Dukascopy: www.dukascopy.com
2. NxCore: http://nxcoreapi.com/doc/
3. TickData: https://www.tickdata.com/equity-data/
4. Finam: www.finam.ru
5. Google Finance: https://www.google.com/finance/getprices?i=[PERIOD]&p=[DAYS]d&f=d,o,h,l,c,v&df=cpct&q=[TICKER]

Thanks!

Any questions ?