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hw7

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IEOR4703 – Monte Carlo Simulation (Hirsa)
Assignment 7 – Due Midnight on Monday April 30th, 2018

Problem 1 (Hamiltonian Monte-Carlo): As you recall **Gibbs** Sampler failed to sample from the following distribution:

$$f(x,y) = \begin{cases} \frac{1}{2} & : \quad -1 < x < 0 \ \& \ -1 < y < 0 \\ \frac{1}{2} & : \quad 0 < x < 1 \ \& \ 0 < y < 1 \end{cases}$$

Use Hamiltonian Monte Carlo to sample from $f(x,y)$.

Problem 2 (Pricing American via Simulation): The code *americanViaSimulation.m* can be used for pricing an American put option under geometric Brownian motion (GBM). One of the issues with the code is that we have to create N simulated paths for each time intervals in advance (as done in the code). And then start at maturity and marching backward as done in the dynamic programming routine in that code. That makes it slow and inefficient. Instead, we can start at maturity and then utilize Brownian bridge to get the level at each time step. Modify the code to accommodate this approach.

P1. $U(q) = -\log(\text{pdf})$

$$= \begin{cases} -\log\frac{1}{2}, & -1<x<0 \ \& \ -1<y<0 \\ -\log\frac{1}{2}, & 0<x<1 \ \& \ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

$$K(p) = \frac{p^2}{2}$$

P2. Brownian Bridge :

$$(W_t \mid W_s = x, W_v = y) \sim N\left(\frac{(v-t)x + (t-s)y}{v-s}, \frac{(v-t)(t-s)}{v-s}\right)$$

$W_T \sim N(0, T)$, $W_0 = 0$

generate $(W_{T-dt} \mid W_0, W_T)$, $(W_{T-2dt} \mid W_0, W_{T-dt})$, ...