Columbia University

IEOR4703 – Monte Carlo Simulation (Hirsa)

Assignment 2 – Due 17:40 on Tuesday Feb 20th, 2018

Problem 1 (CI based on Chebyshev's Inequality): Chebyshev's Inequality states that for a random variable X with expectation $\mathbb{E}(X) = \theta$ and for any $\alpha > 0$,

$$P(|X - \theta| \geq \alpha) \leq \frac{\text{Var}(X)}{\alpha^2}$$

In our case we would write it as

$$P(|\hat{\theta}_n - \theta| \geq \alpha) \leq \frac{\text{Var}(\hat{\theta}_n)}{\alpha^2}$$

where $\hat{\theta}_n = \frac{1}{n}\sum_{i=1}^{n} h(X_i)$ is the sample mean. Use the inequality to construct confidence intervals for θ. Show that it generally yields to conservative confidence intervals.

Problem 2 (CLT vs. Bootstrap): Consider a European call option under Black-Merton-Scholes model with risk-free rate $r = 0.05$, continuous divided rate of $q = 0.01$, volatility of $\sigma = 0.30$, maturity of $T = 1$ year, spot price of $S_0 = \$100$, and strike price of $K = \$90$.

(a) Use standard simulation to estimate the option price and compare it with the exact solution (use 10,000 simulation paths

(b) Find approximate 95% confidence interval based on Central limit theorem.

(c) Find approximate 95% confidence interval based on bootstrap approach.

(You should include your codes and print out the results)

Problem 3 (Antithetic Variates): Consider the following average price call

$$\theta = e^{-rT}\mathbb{E}[(\overline{S} - K)^+]$$

with $\overline{S} = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}$ for $t_i = i\Delta t$ where $\Delta t = T/m$. Assume S_t follows a risk-neutral probability measure $GBM(r - q, \sigma)$ with risk-free rate $r = 0.04$, continuous divided rate of $q = 0.015$, and volatility of $\sigma = 0.30$. Assume maturity of $T = 1$ year, monitoring times of $m = 12$, spot price of $S_0 = \$1,000$, and strike price of $K = \$1,100$.

(a) Use standard simulation to estimate the option price and find approximate 99% confidence interval.

(b) Use simulation w/ antithetic variates to estimate the option price and find approximate 99% confidence interval.

Use 50,000 simulation paths in your estimations (You should include your codes and print out the results).

Problem 4 (Control Variates): Consider the following average price call

$$\theta = e^{-rT}\mathbb{E}[(\overline{S} - K)^+]$$

with $\overline{S} = \frac{1}{m}\sum_{i=1}^{m} S_{t_i}$ for $t_i = i\Delta t$ where $\Delta t = T/m$. Assume S_t follows a risk-neutral probability measure $GBM(r - q, \sigma)$ with risk-free rate $r = 0.04$, continuous divided rate of $q = 0.015$, and volatility of $\sigma = 0.30$. Assume maturity of $T = 1$ year, monitoring times of $m = 12$, spot price of $S_0 = \$1,000$, and strike price of $K = \$1,100$.

(a) Use standard simulation to estimate the option price and find approximate 99% confidence interval.

(b) Set $Z = S_T$ as a control variate to estimate the option price and find approximate 99% confidence interval.

(c) Set $Z = \exp(-rT)\max(S_T - K, 0)$ as a control variate to estimate the option price and find approximate 99% confidence interval.

(d) Set $Z = \exp(-rT)\max(\sqrt[m]{\prod_{i=1}^{m} S_{t_i}} - K, 0)$ as a control variate to to estimate the option price and find approximate 99% confidence interval. Note that for the geometric average price the expectation is known (i.e. $\mathbb{E}(Z)$

Use 50,000 simulation paths in your estimations. From your results conclude whine one results in a substantial variance reduction (You should include your codes and print out the results).