Columbia University
IEOR4703 – Monte Carlo Simulation (Hirsa)
Assignment 3 – Due 17:40 on Tuesday March 6th, 2018

Problem 1 (Conditional Monte Carlo): Consider the following probability

$$\theta = P(X_1 + X_2 > 8)$$

where $X_1 \sim \text{Exp}(2)$ and $X_2 \sim \text{Exp}(1)$. For $X \sim \text{Exp}(\lambda)$, the corresponding CDF is given by $F(x) = 1 - e^{-\lambda x}$.

(a) Use standard simulation to estimate it.

(b) Use Conditional Monte Carlo to estimate it.

(c) Find approximate 99% confidence intervals in both cases and compare.

Use 10,000 draws/samples (You should include your codes and print out the results).

Problem 2 (Conditional Monte Carlo): Assume that the stock price follows $GBM(r-q,\sigma)$. We would like to calculate the price of an option that has the following payoff:

$$h(X) = \begin{cases} (\lambda_1 S_{\frac{T}{2}} - S_T)^+ & : \text{ if } S_{\frac{T}{2}} < H \\ (S_T - \lambda_2 S_{\frac{T}{2}})^+ & : \text{ if } S_{\frac{T}{2}} \geq H \end{cases}$$

where $X = (S_{\frac{T}{2}}, S_T)$. The price of the option can be written as

$$\begin{aligned} P(S_0, t=0; \lambda_1, \lambda_2, H, T) &= e^{-rT}\mathbb{E}\left(h(X)\right) \\ &= e^{-rT}\mathbb{E}\left(\mathbf{1}_{S_{\frac{T}{2}}<H}\{(\lambda_1 S_{\frac{T}{2}} - S_T)^+\} + \mathbf{1}_{S_{\frac{T}{2}}\geq H}\{(S_T - \lambda_2 S_{\frac{T}{2}})^+\}\right) \end{aligned}$$

For the following parameter set: spot price of $S_0 = 100$, risk-free rate $r = 0.04$, continuous divided rate of $q = 0.015$, volatility of $\sigma = 0.30$, maturity of $T = 1$ year, level $H = 110$, $\lambda_1 = 0.9$, and $\lambda_2 = 1.1$ price the option by utilizing

(a) standard simulation to estimate the option price and find approximate 99% confidence interval.

(b) conditional Monte-Carlo simulation to estimate the option price and find approximate 99% confidence interval.

Use 20,000 simulation paths in your estimations (You should include your codes and print out the results).

Problem 3 (Importance Sampling): Assume that the stock price follows $GBM(r-q,\sigma)$. We would like to calculate the price of deep-out-of-money (European) put for the following parameter set: spot price of $S_0 = \$200$, strike price of $K = 120$, risk-free rate $r = 0.05$, continuous divided rate of $q = 0.025$, volatility of $\sigma = 0.35$, and maturity of $T = 1$ year.

(a) Use standard simulation to estimate the option price and find approximate 99% confidence interval.

(b) Use importance sampling by specifying g to estimate the option price and find approximate 99% confidence interval.

Use 20,000 simulation paths in your estimations. Compare your results with the exact solution (You should include your codes and print out the results).

Problem 4 (Stratified Sampling): Assume that the stock price follows $GBM(r-q,\sigma)$. We would like to calculate the price of deep-out-of-money (European) put for the following parameter set: spot price of $S_0 = \$200$, strike price of $K = 120$, risk-free rate $r = 0.05$, continuous divided rate of $q = 0.025$, volatility of $\sigma = 0.35$, and maturity of $T = 1$ year.

(a) Use standard simulation to estimate the option price and find approximate 99% confidence interval and compare it with the exact solution.

(b) Use stratified sampling using sub-optimal choice for n_i to estimate the option price and find approximate 99% confidence interval.

(c) Use stratified sampling using optimal choice for n_i to estimate the option price and find approximate 99% confidence interval.

Use 20,000 simulation paths in your estimations. Compare your results with the exact solution (You should include your codes and print out the results).