

APPENDIX

Proof of Proposition 1: Adding the constant term $(\lambda_i^{k-1})^2/2\rho$ to the right-hand side of (2a).

$$\begin{aligned} \min_{\mathbf{x}_i \in \mathcal{X}_i} L_i^k &= \frac{1}{2} \mathbf{x}_{i,\text{Re}}^T \mathbf{Q}_i \mathbf{x}_{i,\text{Re}} + \mathbf{d}_i^T \mathbf{x}_{i,\text{Re}} + \lambda_i^{k-1} (x_{i,\text{Sh}} - y_{i,\text{Sh}}^{k-1}) \\ &\quad + \frac{\rho}{2} (x_{i,\text{Sh}} - y_{i,\text{Sh}}^{k-1})^2 + \frac{(\lambda_i^{k-1})^2}{2\rho} \\ &= \frac{1}{2} \mathbf{x}_{i,\text{Re}}^T \mathbf{Q}_i \mathbf{x}_{i,\text{Re}} + \mathbf{d}_i^T \mathbf{x}_{i,\text{Re}} + \frac{\rho}{2} \left(x_{i,\text{Sh}} - y_{i,\text{Sh}}^{k-1} + \frac{\lambda_i^{k-1}}{\rho} \right)^2 \end{aligned} \quad (\text{A1})$$

Set $\mathbf{x}_{i,\text{Sh}}'^k = x_{i,\text{Sh}}^k - y_{i,\text{Sh}}^{k-1} + \lambda_i^{k-1}/\rho$, $\mathbf{x}_i' = [\mathbf{x}_{i,\text{Re}}^T, \mathbf{x}_{i,\text{Sh}}'^k]^T$, $\mathbf{x}_i' \in \mathbb{R}^{R_i+1}$, $\boldsymbol{\theta}_{i,\text{Re}} = [-\bar{x}_{i,1} \cdots -\bar{x}_{i,R_i} \quad \underline{x}_{i,1} \cdots \underline{x}_{i,R_i}]^T$, $\boldsymbol{\theta}_{i,\text{Sh}}^k = D_i - y_{i,\text{Sh}}^{k-1} + \lambda_i^{k-1}/\rho$, then the (A1) can be expressed as:

$$\min_{\mathbf{x}_i'} L_i^k(\mathbf{x}_i', \boldsymbol{\theta}_i) = \frac{1}{2} \mathbf{x}_i'^T \mathbf{Q}_i' \mathbf{x}_i' + \mathbf{d}_i'^T \mathbf{x}_i', \quad (\text{A2a})$$

$$\text{s.t. } \mathbf{A}_i \mathbf{x}_i' + \boldsymbol{\theta}_{i,\text{Re}} \leq \mathbf{0}, \mu_i \quad (\text{A2b})$$

$$\mathbf{1}^{1 \times (R_i+1)} \mathbf{x}_i' - \boldsymbol{\theta}_{i,\text{Sh}}^k = \mathbf{0}; \eta_i \quad (\text{A2c})$$

$$\mathbf{Q}_i' = \begin{bmatrix} \mathbf{Q}_i & \mathbf{0}^{R_i} \\ \mathbf{0}^{1 \times R_i} & \rho \end{bmatrix}, \mathbf{d}_i' = [\mathbf{d}_i \quad 0] \quad (\text{A2d})$$

where, (A2b) corresponds to the resource constraint (1b), and (A2c) corresponds to prosumer's power balance constraint (1c). The multipliers for (A2b) and (A2c) are denoted as μ_i and η_i .

Defining the optimal solution of (A2) as $\mathbf{x}_i^* = [\mathbf{x}_{i,\text{Re}}^{*T}, \mathbf{x}_{i,\text{Sh}}^{*T}]^T$, the KKT conditions can be written as:

$$\frac{\partial L_i(\mathbf{x}_i^*, \boldsymbol{\theta}_i)}{\partial \mathbf{x}_i^*} = \mathbf{Q}_i' \mathbf{x}_i^* + \mathbf{d}_i'^T + \mathbf{A}_i^T \mu_i + \mathbf{1}^{R_i+1} \eta_i = \mathbf{0}, \quad (\text{A3a})$$

$$(\mathbf{A}_i)_{\text{A},s} \mathbf{x}_i^* + (\boldsymbol{\theta}_{i,\text{Re}})_{\text{A},s} = \mathbf{0}, \quad (\text{A3b})$$

$$(\mu_i)_{\text{A}} \geq \mathbf{0}, \quad (\text{A3c})$$

$$(\mathbf{A}_i)_{\text{I},s} \mathbf{x}_i^* + (\boldsymbol{\theta}_{i,\text{Re}})_{\text{I},s} \leq \mathbf{0}, \quad (\text{A3d})$$

$$(\mu_i)_{\text{I}} = \mathbf{0}, \quad (\text{A3e})$$

$$\mathbf{1}^{1 \times (R_i+1)} \mathbf{x}_i^* - \boldsymbol{\theta}_{i,\text{Sh}}^k = \mathbf{0} \quad (\text{A3f})$$

where, $(\cdot)_{\text{A},s}$ and $(\cdot)_{\text{I},s}$ are matrix reorganization operators, which that combine the rows corresponding to active and inactive constraints within the s segments into new matrices.

Support p_{A} and p_{I} denote the number of active and inactive constraints. Combining (A3a), (A3b), and (A3f), we have

$$\mathbf{Q}_i' \mathbf{x}_i^* + \mathbf{d}_i'^T + \begin{bmatrix} (\mathbf{A}_i)_{\text{A},s}^T & \mathbf{1}^{R_i} \\ \mathbf{0}^{1 \times p_{\text{A}}} & 1 \end{bmatrix} \begin{bmatrix} (\mu_i)_{\text{A}} \\ \eta_i \end{bmatrix} = \mathbf{0}, \quad (\text{A4a})$$

$$\begin{bmatrix} (\mathbf{A}_i)_{\text{A},s} & \mathbf{0}^{p_{\text{A}}} \\ \mathbf{1}^{1 \times R_i} & 1 \end{bmatrix} \mathbf{x}_i^* + \begin{bmatrix} (\boldsymbol{\theta}_{i,\text{Re}})_{\text{A},s} \\ -\boldsymbol{\theta}_{i,\text{Sh}}^k \end{bmatrix} = \mathbf{0}, \quad (\text{A4b})$$

Simplifying (A3c)~(A3d) and (A4a)~(A4b), then:

$$\mathbf{H}_{1,i,s} \boldsymbol{\Gamma}_{i,s} \geq \mathbf{0}, \mathbf{H}_{1,i,s} = \begin{bmatrix} (\mathbf{E}^{2R_i})_{\text{A},s} & \mathbf{0}^{p_{\text{A}}} \\ \mathbf{0}^{1 \times 2R_i} & 0 \end{bmatrix}, \boldsymbol{\Gamma}_{i,s} = \begin{bmatrix} (\mu_i)_{\text{A},s} \\ \eta_i \end{bmatrix} \quad (\text{A5a})$$

$$(\mathbf{A}_i)_{\text{I},s} \mathbf{x}_i^* + \mathbf{H}_{2,i,s} \boldsymbol{\theta}_i^k \leq \mathbf{0}, \mathbf{H}_{2,i,s} = \begin{bmatrix} (\mathbf{E}^{2R_i})_{\text{I},s} & \mathbf{0}^{p_{\text{I}}} \\ \mathbf{0}^{1 \times 2R_i} & 0 \end{bmatrix} \quad (\text{A5b})$$

$$\mathbf{Q}_i' \mathbf{x}_i^* + \mathbf{d}_i'^T + \mathbf{G}_{i,s}^T \boldsymbol{\Gamma}_{i,s} = \mathbf{0}, \mathbf{G}_{i,s} = \begin{bmatrix} (\mathbf{A}_i)_{\text{A},s} & \mathbf{0}^{p_{\text{A}}} \\ \mathbf{1}^{1 \times R_i} & 1 \end{bmatrix} \quad (\text{A5c})$$

$$\mathbf{G}_i \mathbf{x}_i^* + \mathbf{H}_{3,i,s} \boldsymbol{\theta}_i^k = \mathbf{0}, \mathbf{H}_{3,i,s} = \begin{bmatrix} (\mathbf{E}^{2R_i})_{\text{A},s} & \mathbf{0}^{p_{\text{A}}} \\ \mathbf{0}^{1 \times 2R_i} & -1 \end{bmatrix} \quad (\text{A5d})$$

The \mathbf{E}^{2R_i} denotes an identity diagonal matrix of size $2R_i \times 2R_i$. (A5a), (A5b), (A5c) and (A5d) correspond to (A3c), (A3d), (A5a) and (A5b), respectively.

Considering the mutual exclusivity of the upper and lower resource constraints, $\text{rank}(\mathbf{G}_{i,s}) = p_{\text{A}} + 1$ and $\mathbf{G}_{i,s} \mathbf{Q}_i'^{-1} \mathbf{G}_{i,s}^T$ is invertible. Hence, combining (A5c) and (A5d), we have

$$\boldsymbol{\Gamma}_{i,s} = \mathbf{M}_{1,i,s} \boldsymbol{\theta}_i^k + \mathbf{M}_{2,i,s} \quad (\text{A6a})$$

$$\mathbf{M}_{1,i,s} = (\mathbf{G}_{i,s} \mathbf{Q}_i'^{-1} \mathbf{G}_{i,s}^T)^{-1} \mathbf{H}_{3,i,s} \quad (\text{A6b})$$

$$\mathbf{M}_{2,i,s} = -(\mathbf{G}_{i,s} \mathbf{Q}_i'^{-1} \mathbf{G}_{i,s}^T)^{-1} \mathbf{G}_{i,s} \mathbf{Q}_i'^{-1} \mathbf{d}_i'^T \quad (\text{A6c})$$

By substituting (A6a) into (A5c), we obtain

$$\mathbf{x}_i^* = \mathbf{W}_{1,i,s} \boldsymbol{\theta}_i^k + \mathbf{W}_{2,i,s} \quad (\text{A7a})$$

$$\mathbf{W}_{1,i,s} = -\mathbf{Q}_i'^{-1} \mathbf{G}_{i,s}^T \mathbf{M}_{1,i,s} \quad (\text{A7b})$$

$$\mathbf{W}_{2,i,s} = -\mathbf{Q}_i'^{-1} \mathbf{d}_i'^T - \mathbf{Q}_i'^{-1} \mathbf{G}_{i,s}^T \mathbf{M}_{2,i,s} \quad (\text{A7c})$$

Combining (A7a) with the definitions of \mathbf{x}_i^* , we have

$$x_{i,\text{Sh}}^{k*} = \mathbf{w}_{1,i,s}' \boldsymbol{\theta}_i^k + \mathbf{w}_{2,i,s}' + y_{i,\text{Sh}}^{k-1} - \frac{\lambda_i^{k-1}}{\rho} \quad (\text{A8c})$$

where, $\mathbf{w}_{1,i,s}' = \mathbf{W}_{1,i,s,R_i+1}$ and $\mathbf{w}_{2,i,s}' = \mathbf{W}_{2,i,s,R_i+1}$ represent the vectors corresponding to the (R_i+1) -th row of $\mathbf{W}_{1,i,s}$ and $\mathbf{W}_{2,i,s}$, respectively. To ensure the complementary slackness condition holds, (A8c) is further substituted into (A5a)~(A5b) (A5b), yielding the following domain:

$$\Theta_i^k := \left\{ \boldsymbol{\theta}_i^k \left| \begin{aligned} &\mathbf{H}_{1,i,s} (\mathbf{M}_{1,i,s} \boldsymbol{\theta}_i^k + \mathbf{M}_{2,i,s}) \geq \mathbf{0}, \\ &(\mathbf{A}_i)_{\text{I},s} (\mathbf{W}_{1,i,s} \boldsymbol{\theta}_i^k + \mathbf{W}_{2,i,s}) + \mathbf{H}_{2,i,s} \boldsymbol{\theta}_i^k \leq \mathbf{0} \end{aligned} \right. \right\} \quad (\text{A9})$$

According to (A8c)~(A9), we can derive **Proposition 1**. The prosumer's best response can be characterized by piecewise linear function. The domain of each segment depends on the cost coefficients and the active constraints.

■ End of proof of **Proposition 1**.

Proof of Proposition 3:

Since prosumers are not coupled in (4), it is equivalent to show that the $\hat{\mathbf{x}}_i^k$ of each prosumer satisfies the (15).

1) For prosumers in \mathcal{I}^k . According to the KKT conditions, (15) is satisfied with $\hat{\mathbf{x}}_i^k = \mathbf{x}_i^{k*}$ and $\hat{\boldsymbol{\alpha}}_i^k = \boldsymbol{\alpha}_i^{k*}$.

2) For prosumers in $\mathcal{I}/\mathcal{I}^k$. Combining (1c) and (13) and A1

$$\sum_{r \in \mathcal{R}_i} (\underline{x}_{i,r} - x_{i,r}^*) \leq x_{i,\text{Sh}}^{k*} - \tilde{x}_{i,\text{Sh}}^k \leq \sum_{r \in \mathcal{R}_i} (\bar{x}_{i,r} - x_{i,r}^*) \quad (\text{A10a})$$

$$-\xi \leq x_{i,\text{Sh}}^{k*} - \tilde{x}_{i,\text{Sh}}^k \leq \xi \quad (\text{A10b})$$

According to (13) and (1c) existing δ_i have

$$\max\{(\underline{x}_{i,r} - x_{i,r}^{k*}), -\xi\} \leq \delta_{i,r} \leq \min\{(\bar{x}_{i,r} - x_{i,r}^{k*}), \xi\}, \quad (\text{A11a})$$

$$\sum_{r \in \mathcal{R}_i} \delta_{i,r} = x_{i,\text{Sh}}^{k*} - \hat{x}_{i,\text{Sh}}^k \quad (\text{A11b})$$

Assume the $\hat{x}_{i,r}^k = x_{i,r}^{k*} + \delta_{i,r}$, then we have

$$\hat{x}_{i,\text{Sh}}^k + \sum_{r \in \mathcal{R}_i} \hat{x}_{i,r}^k = x_{i,\text{Sh}}^{k*} + \sum_{r \in \mathcal{R}_i} x_{i,r}^{k*} = D_i \quad (\text{A12a})$$

$$\underline{x}_{i,r}^k \leq \hat{x}_{i,r}^k \leq \bar{x}_{i,r}^k, -\xi \leq \hat{x}_i^k - x_i^{k*} \leq \xi \quad (\text{A12b})$$

Under \hat{x}_i^k , the multipliers of the inactive constraints in (4) are set to 0, and that of active constraints are consistent with $\alpha_{i,r}^{k*}$. The complementary slackness conditions are satisfied.

■ End of proof of **Proposition 3**.

Proof of Proposition 5:

According to (3c), (4) and the definition of d_p^k , we have

$$d_p^k = \frac{\partial f(x^k)}{\partial x} + A_C^T \lambda^k + \rho A_C^T B_C (-y^{k-1} + y^k) + A_C^T \alpha^k \quad (\text{A13})$$

Combining (5c) and the KKT conditions of (5a)~(5b), then

$$P_G - B_C^T \lambda^k + \mathbf{1}^{I+1} \gamma^k = \mathbf{0} \quad (\text{A14})$$

Support the auxiliary variable u^k , where

$$u^k := d_p^k - A_C^T \lambda^k - \rho A_C^T B_C (-y^{k-1} + y^k) - A_P^T \alpha^k = \frac{\partial f(x^k)}{\partial x} \quad (\text{A15})$$

Since $f(x)$ is a convex function, we have

$$\langle u^k + A_C^T \lambda^* + A_P^T \alpha^*, x^k - x^* \rangle \geq 0 \quad (\text{A16a})$$

$$\langle B_C^T \lambda^k - \mathbf{1}^{I+1} \gamma^k - B_C^T \lambda^* + \mathbf{1}^{I+1} \gamma^*, y^k - y^* \rangle = 0 \quad (\text{A16b})$$

Bring (5c) and (A15) to (A16), then we obtain

$$\begin{aligned} 0 &\leq \langle d_p^k, x^k - x^* \rangle - \langle \lambda^k - \lambda^*, A_C(x^k - x^*) \rangle \\ &\quad - \langle \rho B_C(-y^{k-1} + y^k), A_C(x^k - x^*) \rangle \\ &\quad - \langle (\lambda^k - \lambda^*), -B_C(y^k - y^*) \rangle \\ &\quad + \langle -(\alpha^k - \alpha^*), A_P(x^k - x^*) \rangle \\ &\quad + \langle -(\gamma^k - \gamma^*), \mathbf{1}^{I+1}(y^k - y^*) \rangle \end{aligned} \quad (\text{A17})$$

According to **Proposition 3**, we have that

$$\langle -(\alpha^k - \alpha^*), A_P(x^k - x^*) \rangle \leq 0 \quad (\text{A18a})$$

$$\langle -(\gamma^k - \gamma^*), \mathbf{1}^{I+1}(y^k - y^*) \rangle = 0 \quad (\text{A18b})$$

Combining (A17) and (A18), we can get

$$\begin{aligned} &\langle d_p^k, x^k - x^* \rangle - \langle \rho B_C(-y^{k-1} + y^k), A_C(x^k - x^*) \rangle \\ &\quad - \langle \lambda^k - \lambda^*, -B_C(y^k - y^*) + A_C(x^k - x^*) \rangle \geq 0 \end{aligned} \quad (\text{A19})$$

Bringing (5c) into the third term on (A19), we have

$$\begin{aligned} &\langle d_p^k, x^k - x^* \rangle - \left\langle \lambda^k - \lambda^*, \frac{1}{\rho}(\lambda^k - \lambda^{k-1}) \right\rangle \\ &\quad \geq \langle \rho B_C(-y^{k-1} + y^k), A_C(x^k - x^*) \rangle \end{aligned} \quad (\text{A20})$$

With multiplying both sides of (A20) by 2ρ , (19) is obtained

■ End of proof of **Proposition 5**.

Proof of Proposition 6:

According to the definition of inner product, we have

$$\frac{2}{\rho} \langle \omega^{k-1} - x^k, d_p^k \rangle \leq \frac{2}{\rho} \langle \omega^{k-1} + |x^k|, |d_p^k| \rangle \quad (\text{A21})$$

According to (16) and the definition of d_p^k , there are

$$\left| \frac{\partial L'_p(x^k, \alpha^k)}{\partial x} - \frac{\partial L'_p(x^{k*}, \alpha^{k*})}{\partial x} \right| = |d_p^k| \leq \xi' \quad (\text{A22})$$

Combining (17b)~(17c) and (A22), we can get

$$\frac{2}{\rho} \langle \omega^{k-1} + |x^k|, |d_p^k| \rangle \leq \frac{2}{\rho} \langle \omega^{k-1} + \bar{x}_{\max}^k, \xi' \rangle \quad (\text{A23a})$$

$$|\omega^{k-1}| \leq \bar{\omega}^{k-1} \quad (\text{A23b})$$

Then, we can obtain

$$\langle \omega^{k-1} + |x^k|, |d_p^k| \rangle \leq \langle \bar{\omega}^{k-1} + \bar{x}_{\max}^k, \xi' \rangle \quad (\text{A24})$$

According to (17a), (A22) and (A24), the (20) is satisfied.

■ End of proof of **Proposition 6**.

Proof of Proposition 7:

According to (17c) and **Proposition 6**, we can get

$$\begin{aligned} &\|\omega^{k-1} - x^*\|_2^2 - \|\omega^k - x^*\|_2^2 \\ &= -\|\omega^k - \omega^{k-1}\|_2^2 - 2\langle \omega^{k-1} - x^*, \omega^k - \omega^{k-1} \rangle \\ &= -\rho^2 \|d_p^k\|_2^2 + 2\rho \langle \omega^{k-1} - x^k, d_p^k \rangle + 2\rho \langle x^k - x^*, d_p^k \rangle \quad (\text{A25}) \\ &\geq -\rho^2 \|d_p^k\|_2^2 + 2\rho \langle x^k - x^*, d_p^k \rangle - 2\rho \langle \omega^k - x^*, d_p^k \rangle \\ &\geq 2\rho \langle x^k - x^*, d_p^k \rangle - \rho^2 \sigma \|A_C x^k - B_C y^{k-1}\|_2^2 \end{aligned}$$

Similarly, we have

$$\begin{aligned} &\|\lambda^{k-1} - \lambda^*\|_2^2 - \|\lambda^k - \lambda^*\|_2^2 \\ &= \|\lambda^{k-1} - \lambda^k\|_2^2 + 2\langle \lambda^{k-1} - \lambda^k, \lambda^k - \lambda^* \rangle \end{aligned} \quad (\text{A26})$$

Furthermore, according to **Proposition 5**, (A25)~(A26) and (5c), we have

$$\begin{aligned} &\Phi^{k-1} - \Phi^k \\ &\geq \rho^2 \|A_C x^k - B_C y^k\|_2^2 - \rho^2 \sigma \|A_C x^k - B_C y^{k-1}\|_2^2 \\ &\quad - \rho^2 \|B_C(-y^k + y^*)\|_2^2 + \rho^2 \|B_C(-y^{k-1} + y^*)\|_2^2 \\ &\quad - 2\rho^2 \langle B_C(-y^k + y^{k-1}), A_C(x^k - x^*) \rangle \end{aligned} \quad (\text{A27})$$

Due to $A_C x^* - B_C y^* = 0$ and Cauchy-Schwarz Inequality, the fifth term on the right-hand side of (A27) can be written as

$$\begin{aligned} &2\rho^2 \langle B_C(-y^k + y^{k-1}), A_C(x^k - x^*) \rangle \\ &= \rho^2 \left(\|B_C(-y^{k-1} + y^k)\|_2^2 + \|A_C x^k - B_C y^k\|_2^2 - \|A_C x^k - B_C y^{k-1}\|_2^2 \right) \quad (\text{A28}) \\ &\quad - \rho^2 \left(\|B_C(-y^{k-1} + y^k)\|_2^2 + \|B_C(-y^k + y^*)\|_2^2 - \|B_C(-y^{k-1} + y^*)\|_2^2 \right) \end{aligned}$$

Bring (A28) to (A27), we obtain

$$\Phi^{k-1} - \Phi^k \geq \rho^2 (1 - \sigma) \|A_C x^k - B_C y^{k-1}\|_2^2 \quad (\text{A29})$$

Obviously, is a non-increasing and positive sequence. Then,

$\{y^k\}$, $\{\lambda^k\}$, and $\{\omega^k\}$ are bounded sequences. And we can get

$$\lim_{k \rightarrow \infty} A_C x^k - B_C y^{k-1} = 0 \quad (A30)$$

Besides, based on **Proposition 3**, $\{x^k\}$ is bounded sequence. Since $\{x^k\}$, $\{y^k\}$, $\{\lambda^k\}$ is bounded, $\{\alpha^k\}$ and $\{\gamma^k\}$ are bounded, according to KKT conditions of (4) and (5).

■ End of proof of **Proposition 7**.

Proof of Proposition 8:

With Bolzano-Weierstrass Theorem, there are \mathcal{K} satisfying

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}} \lambda^k = \bar{\lambda}, \lim_{k \rightarrow \infty, k \in \mathcal{K}} \alpha^k = \bar{\alpha}, \lim_{k \rightarrow \infty, k \in \mathcal{K}} \gamma^k = \bar{\gamma} \quad (A31)$$

According to **Proposition 6**, (A30) and the definition of ω^k , we have

$$\langle \omega^{k-1} - x^k, d_p^k \rangle = \|d_p^k\|^2 = 0 \quad (A32)$$

Since $\{\omega^k\}$ and $\{x^k\}$ are bounded, we obtain

$$\langle x^k, d_p^k \rangle = \langle \omega^k, d_p^k \rangle = \langle A_C x^k - B_C y^{k-1}, A x^k \rangle = 0 \quad (A33)$$

Support the Fenchel conjugate function of f is f^* . According to Legendre Transform Definition, there are

$$u^k = \partial f(x^k) \Leftrightarrow x^k = \partial f^*(u^k) \quad (A34)$$

Due to convexity, we have

$$\begin{aligned} f^*(-A_C^T \bar{\lambda}^* - A_p^T \alpha^*) \\ \geq f^*(u^k) + \langle \partial f^*(u^k), -A_C^T \bar{\lambda}^* - A_p^T \alpha^* - u^k \rangle \end{aligned} \quad (A35)$$

Bring (A15) into (A35), we can get

$$\begin{aligned} f^*(-A_C^T \bar{\lambda}^* - A_p^T \alpha^*) \\ \geq f^*(u^k) + \langle x^k, A_C^T (\lambda^{k-1} - \lambda^*) \rangle \\ + \langle A_C x^k, \rho(A_C x^k - B_C y^{k-1}) \rangle \\ - \langle x^k, d_p^k \rangle + \langle x^k, A_p^T (\alpha^k - \alpha^*) \rangle \end{aligned} \quad (A36)$$

According to (A15) and (A30)~(A33), we have

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}} u^k = -A_C^T \bar{\lambda} - A_p^T \bar{\alpha} \quad (A37)$$

Besides, γ^k and γ^* satisfy the KKT conditions of problem (1) and (5a)~(5b). Hence, there are

$$\mathbf{1}^{1 \times (I+1)} y^k = \mathbf{1}^{1 \times (I+1)} y^* = 0 \quad (A38a)$$

$$P_G - B_C^T \lambda^k + \mathbf{1}^{I+1} \gamma^k = 0, P_G - B_C^T \lambda^* + \mathbf{1}^{I+1} \gamma^* = 0 \quad (A38b)$$

Based on (A31)~(A33) and (A38b), the (A36) can be written as following when $k \rightarrow \infty, k \in \mathcal{K}$.

$$\begin{aligned} f^*(-A_C^T \bar{\lambda}^* - A_p^T \alpha^*) \\ \geq f^*(-A_C^T \bar{\lambda} - A_p^T \bar{\alpha}) + \langle A_C x^k, \bar{\lambda} - \lambda^* \rangle \\ + \langle A_p x^k, \alpha^k - \alpha^* \rangle - \langle B_C y^{k-1}, \bar{\lambda} - \lambda^* \rangle \\ + \langle y^{k-1}, \mathbf{1}^{I+1} (\gamma^{k-1} - \gamma^*) \rangle \end{aligned} \quad (A39)$$

Bringing (A30), (A38b) and (15a) into (A39) yields

$$\begin{aligned} f^*(-A_C^T \bar{\lambda}^* - A_p^T \alpha^*) \\ \geq f^*(-A_C^T \bar{\lambda} - A_p^T \bar{\alpha}) - \langle b_p, \bar{\alpha} \rangle \\ - \langle A_p x^k + b_p, \alpha^* \rangle + \langle b_p, \alpha^* \rangle \\ \geq f^*(-A_C^T \bar{\lambda} - A_p^T \bar{\alpha}) - \langle b_p, \bar{\alpha} \rangle + \langle b_p, \alpha^* \rangle \end{aligned} \quad (A40)$$

That is

$$f^*(-A_C^T \bar{\lambda}^* - A_p^T \alpha^*) - \langle b_p, \alpha^* \rangle \geq f^*(-A_C^T \bar{\lambda} - A_p^T \bar{\alpha}) - \langle b_p, \bar{\alpha} \rangle \quad (A41)$$

Moreover, solving the primary problem (1) is equivalent to solving its dual problem:

$$\min_{\lambda, \alpha, \gamma} f^*(-A_C^T \lambda - A_p^T \alpha) - P_G - \langle b_p, \alpha \rangle - \langle 0, \lambda \rangle - \langle 0, \gamma \rangle \quad (A42)$$

Since (1) is a convex programming, $\bar{\lambda}, \bar{\alpha}, \bar{\gamma}$ is the global optimal solution $\lambda^*, \alpha^*, \gamma^*$. Similarly, there are $\mathcal{K}' \subseteq \mathcal{K}$ satisfying

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}'} x^k = \bar{x}, \lim_{k \rightarrow \infty, k \in \mathcal{K}'} y^k = \bar{y} \quad (A43)$$

According to (A15) and (A31)~(A33), we have

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}'} u^k = \bar{u} = -A_C^T \bar{\lambda} - A_p^T \bar{\alpha} = \frac{\partial f(\bar{x})}{\partial x} \quad (A43)$$

According to $\bar{\lambda} = \lambda^*, \bar{\alpha} = \alpha^*, \bar{\gamma} = \gamma^*$, (15), (A30), (A38) and (A43), the KKT conditions of (1) are satisfied by $\{\bar{\lambda}, \bar{\alpha}, \bar{\gamma}, \bar{x}, \bar{y}\}$.

Therefore, it is the optimal solution.

Based on (A32), $\{\omega^k\}$ is a global convergence sequence that converges to ω^∞ . Similarly, there is $\mathcal{K}'' \subseteq \mathcal{K}'$ satisfying

$$\lim_{k \rightarrow \infty} \omega^k = \omega^\infty \quad (A44a)$$

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}''} \|\omega^k - y^*\|_2^2 = \liminf_{k \rightarrow \infty} \|\omega^k - y^*\|_2^2 \quad (A44b)$$

Introduce an auxiliary variable which satisfies

$$\phi^k = \|\omega^k - x^*\|_2^2 + \rho^2 \|\omega^k - y^*\|_2^2 \quad (A45)$$

According to (A44), we have

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}''} \phi^k = \liminf_{k \rightarrow \infty} \phi^k = \|\omega^\infty - x^*\|_2^2 + \rho^2 \liminf_{k \rightarrow \infty} \|\omega^k - y^*\|_2^2 \quad (A46)$$

From (A21a), Φ^k is global convergence. Then, we can get

$$\limsup_{k \rightarrow \infty} \Phi^k = \lim_{k \rightarrow \infty} \Phi^k = \lim_{k \rightarrow \infty, k \in \mathcal{K}''} \Phi^k \quad (A47)$$

According to $\mathcal{K}'' \subseteq \mathcal{K}'$, (21b), (A31), and $\bar{\lambda} = \lambda^*$, we have

$$\lim_{k \rightarrow \infty, k \in \mathcal{K}''} \Phi^k = \lim_{k \rightarrow \infty, k \in \mathcal{K}''} \phi^k = \liminf_{k \rightarrow \infty} \phi^k \quad (A48)$$

Combining (21b) and (A45), we obtain

$$\begin{aligned} \limsup_{k \rightarrow \infty} \|\lambda^k - \lambda^*\|^2 &= \limsup_{k \rightarrow \infty} (\Phi^k - \phi^k) \\ &\geq \limsup_{k \rightarrow \infty} \Phi^k - \liminf_{k \rightarrow \infty} \phi^k = 0 \end{aligned} \quad (A49)$$

That is $\lim_{k \rightarrow \infty} \lambda^k = \lambda^*$. Since $\Phi^k, \lambda^k, \omega^k$ is global convergence, y^k is also global convergence. So

$$\lim_{k \rightarrow \infty} y^k = y^* \quad (A50)$$

■ End of proof of **Proposition 8**.