2. a)

n	rel err	cond(H)
2	4.44e-16	2.70e+01
3	9.55e-15	7.48e+02
4	6.55e-14	2.84e+04
5	2.78e-12	9.44e+05
6	2.19e-10	2.91e+07
7	1.19e-08	9.85e+08

b) From the table above, we can observe that as n increases, the relative error and the conditional number is also increasing. Therefore, more accurate input data digits are required to get a correct digit in the solution as n increases. For example, 1 accurate input data digit is require to get a correct digit in solution when $n=1$, while 8 is needed when $n=7$. The exact relation should be:

The matrix is ill conditioned $\frac{\|\hat{x}-x\|}{\|x\|} \lesssim \text{cond}(A) \cdot \varepsilon_{mach}$

3. (a)

$$\begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+\epsilon \\ 2 \end{bmatrix}$$

for $k = 1$ to $2-1$:

for $i = k+1$ to 2:

$a_{11} \neq 0$,

$a_{21} = a_{21} / a_{11}$, then: $\begin{bmatrix} \epsilon & 1 \\ \frac{1}{\epsilon} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+\epsilon \\ 2 \end{bmatrix}$

for $j = k+1$ to 2:

$a_{22} = a_{22} - a_{21} \cdot a_{12}$, then: $\begin{bmatrix} \epsilon & 1 \\ \frac{1}{\epsilon} & 1-\frac{1}{\epsilon} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+\epsilon \\ 2 \end{bmatrix}$

$b_2 = b_2 - a_{21} \cdot b_1$, then:

$2 - \frac{1}{\epsilon} - 1$

$$\begin{bmatrix} \epsilon & 1 \\ \frac{1}{\epsilon} & 1-\frac{1}{\epsilon} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+\epsilon \\ 1-\frac{1}{\epsilon} \end{bmatrix}$$

b) Back Substitution:

Upper triangle:

① $\epsilon \cdot x_1 + x_2 = 1+\epsilon$

② $(1-\frac{1}{\epsilon}) \cdot x_2 = 1-\frac{1}{\epsilon}$

$x_2 = 1$

substitute x_2:

$\epsilon \cdot x_1 + 1 = 1+\epsilon$

$\epsilon \cdot x_1 = \epsilon$

$x_1 = 1$

thus, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d)

eps	rel err
1.0e-01	8.8818e-16
1.0e-02	8.8818e-16
1.0e-03	8.8818e-16
1.0e-04	1.1013e-13
1.0e-05	6.5512e-12
1.0e-06	2.8756e-11
1.0e-07	5.8387e-10
1.0e-08	6.0775e-09
1.0e-09	1.3930e-07
1.0e-10	8.2740e-08
1.0e-11	8.2740e-08
1.0e-12	1.3314e-04

The computed solution will be more inaccurate as the value of ϵ decreases. Since from the table, we can see the relatively error is getting larger as ϵ decreases.