CSC336 Assignment 1 report

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Q1:

(a) Write the Taylor expansion for f(x +h) and f(x - h), and we keep the h3 remainder term.

$f(x+h) = f(x) + f'(x)h + f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6}$

$f(x-h) = f(x) - f'(x)h + f''(x)\frac{h^2}{2} - f'''(x)\frac{h^3}{6}$

$$\begin{aligned}
f(x+h) - f(x-h) &= f(x) + f'(x)h + f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6} - f(x) + f'(x)h - f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6} \\
&= f(x) - f(x) + f'(x)h + f'(x)h + f''(x)\frac{h^2}{2} - f''(x)\frac{h^2}{2} + f'''(x)\frac{h^3}{6} + f'''(x)\frac{h^3}{6} \\
&= 2f'(x) + 2f'''(x)\frac{h^3}{6} \\
\frac{f(x+h)-f(x-h)}{2h} &= f'(x) + f'''(x)\frac{h^2}{6}
\end{aligned}$$

Truncation error: $f'''(x)\frac{h^2}{6}$, assume M is a bound on $|f'''(t)|$ for t near x, then truncation error: $\frac{Mh^2}{6}$

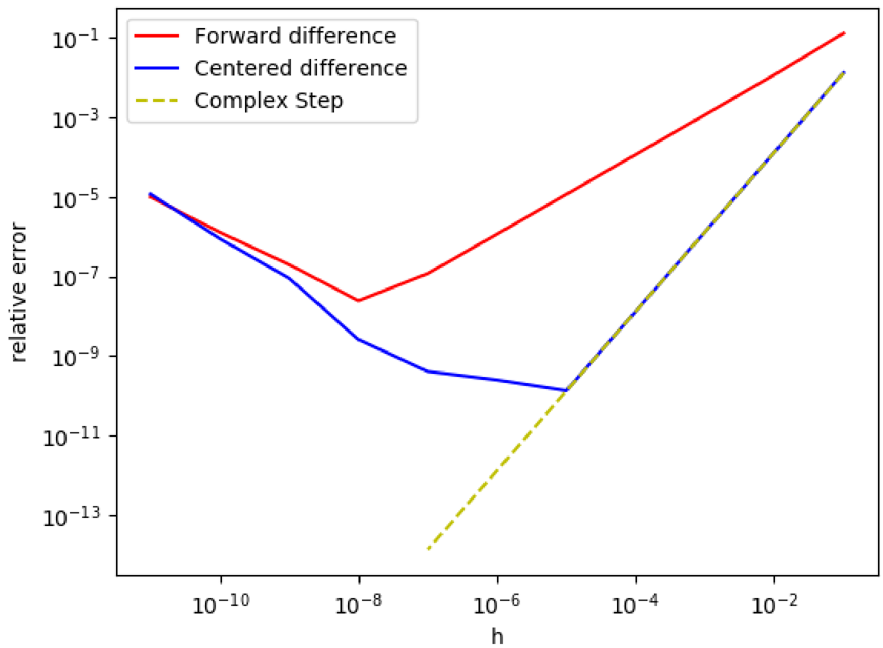
Given rounding error is $\frac{\epsilon}{h}$, then the total error $= \frac{Mh^2}{6} + \frac{\epsilon}{h}$

The derivative (h) of total error: $\frac{Mh}{3} - \frac{\epsilon}{h^2}$

$$\begin{aligned}
\frac{Mh}{3} - \frac{\epsilon}{h^2} &= 0 \\
\frac{Mh}{3} &= \frac{\epsilon}{h^2} \\
Mh^3 &= 3\epsilon \\
h^3 &= \frac{3\epsilon}{M} \\
h &= \sqrt[3]{\frac{3\epsilon}{M}} = \text{optimal value for h}
\end{aligned}$$

(b) see "a1-report.py"

(c)



(d) From the results of numerical experiment, I observed that the performances of forward difference and centred difference are similar, which are: the relative errors of both approximations decrease when step size "h" is approaching to the optimal value that we computed in the worksheet, from both left and right sides. It reaches the local ([0-1]) minimum when "h" equals to its optimal value. Therefore, the results we found for forward difference and centered difference are consistent with the theoretical analysis we derived. More specifically, the optimal h value for centered difference is larger than the optimal h value for forward difference when h is between 0 and 1. This is also consistent with the theoretical result since cubic root is involved in centered difference's optimal h value, while square root is involved in forward difference's optimal h value. As for complex step method, theoretically there should also be an optimal h, which can minimize the relative error. However, the result is inconsistent with the theory, since the graph suggests that the relative error for complex step method is decreasing as the step size "h" decreases. One potential explanation is that the optimal h for complex step method is extremely small so that we did not tested.