3. a)

1st step: B^{-1} — n^3 flops (if we take advantage of particular form of the right-hand side vector e_j)

2nd step: $2A+\mathbb{1}$ — "$2A = A+A$" takes n^2 flops,

"$2A+\mathbb{1}$" also takes n^2 flops,

thus total is $2n^2$ flops

3rd step: $B^{-1}\cdot(2A+\mathbb{1})$ — B^{-1} : $n\times n$ matrix,

$(2A+\mathbb{1})$: $n\times n$ matrix,

thus, the multiplication takes n^3 flops

4th step: C^{-1} — n^3 flops (if we take advantage of particular form of the right-hand side vector e_j)

5th step: $C^{-1}+A$ — takes n^2 flops

6th step: $B^{-1}(2A+\mathbb{1})\cdot(C^{-1}+A)$ — "$B^{-1}(2A+\mathbb{1})$" is $n\times n$ matrix,

"$C^{-1}+A$" is also $n\times n$ matrix,

thus, the multiplication takes n^3 flops

7th step: $B^{-1}(2A+\mathbb{1})\cdot(C^{-1}+A)\cdot b$ — "$B^{-1}(2A+\mathbb{1})\cdot(C^{-1}+A)$" is $n\times n$ matrix, and b is a $n\times 1$ vector, thus

the muplication takes $n\times n\times 1 = n^2$ flops.

Therefore, the computation takes $n^3+2n^2+n^3+n^3+n^2+n^3+n^2$

$= 4n^3+4n^2$ flops

b) 1st Step: $2A+\mathbb{1}$ — takes $2n^2$ flops in total (same as (a))

2nd Step: $B^{-1}\cdot(2A+\mathbb{1})=\alpha$

$(2A+\mathbb{1})=B\alpha$ → solve $(2A+\mathbb{1})=B\alpha$ using LU factorization and Forward, Back substitution on each column, thus the total is $\frac{n^3}{3}$ for LU, $2\times\frac{n^2}{2}\times n$ for F/B substitution $=\frac{4n^3}{3}$ flops.

3rd step: $C^{-1}\cdot b=\beta$

$b=C\beta$ → solve $b=C\beta$ using LU factorization and forward, Back substitution, thus, the total is $\frac{n^3}{3}$ for LU, $2\times\frac{n^2}{2}\times 1$ for F/B substitution $=\frac{n^3}{3}+n^2$ flops.

4th step: $A\cdot b$ — the multiplication takes $n\times n\times 1=n^2$ flops

5th step: $C^{-1}b+Ab$ — $C^{-1}b$: $n\times 1$

Ab : $n\times 1$,

thus, it takes n flops

6th step: $B^{-1}(2A+\mathbb{1})\cdot((C^{-1}+A)b$ — $B^{-1}(2A+\mathbb{1})$: $n\times n$ matrix

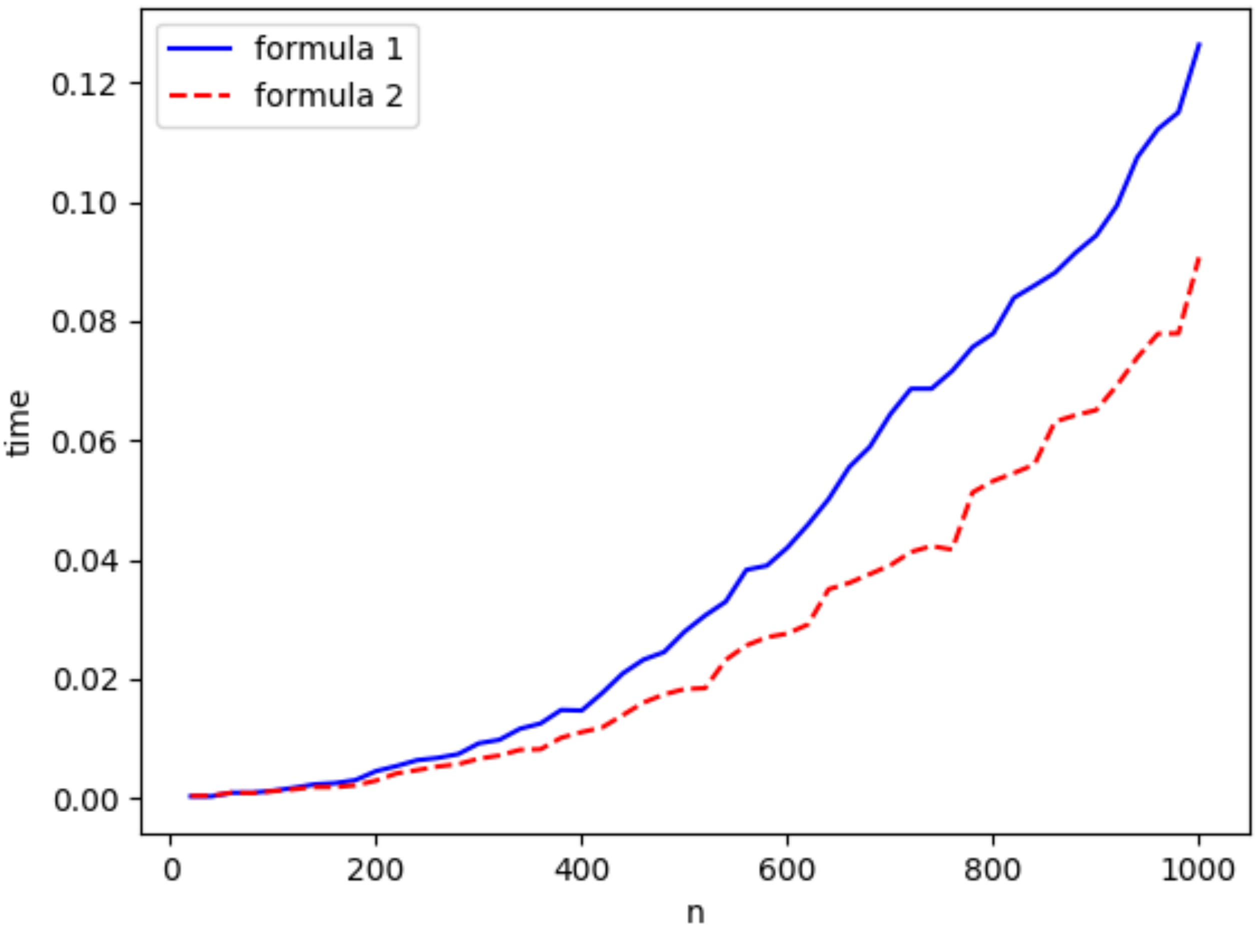
$(C^{-1}+A)b$: $n\times 1$ matrix,

thus the multiplication takes $n \times n \times 1 = n^2$ flops.

Therefore, the computation takes $2n^2 + \frac{4n^3}{3} + \frac{n^3}{3} + n^2 + n^2 + n + n^2$

$$= \frac{5}{3}n^3 + 5n^2 + n \text{ flops}$$

c)



d)

The result of the experiments exacty fits the conclusion of the analysis. When n is very small, the algorithm 1 should performs slightly better than the algorithm 2. The graph on the right proves this idea. The red line is $y_1 = 4x^3 + 4x^2$ and the blue line is $y_2 = \frac{5}{3}x^3 + 5x^2 + x$. However, as n keep increasing, the algorithm 2 becomes much more efficient, since the dominant term of formula 1 is $4n^3$, which almost dobled the dominant term of formula 2, which is $\frac{5}{3}n^3$.