Q2.

a) absolute error $= \sin(x+h) - \sin(x)$

$= \sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)$

$= (\cos(h) - 1)\sin(x) + \cos(x)\sin(h)$

b) relative error $= \dfrac{\sin(x+h) - \sin(x)}{\sin(x)}$

$= \dfrac{(\cos(h) - 1)\sin(x) + \cos(x)\sin(h)}{\sin(x)}$

c) Condition Number

$\approx \left| \dfrac{x \sin'(x)}{\sin(x)} \right| = \left| \dfrac{x \cdot \cos(x)}{\sin(x)} \right|$

d) $\sin(x)$ is highly sensitive for x near any integer multiple of π, since $\lim_{x \to i\pi, \forall i \in \mathbb{Z}} \sin(x) = 0$ and $\lim_{x \to i\pi, \forall i \in \mathbb{Z}} \cos(x) = 1$. The value becomes infinite.

e) relative error: $\frac{x-\sin(x)}{\sin(x)} \leq \delta$

$$\frac{x}{\sin(x)} - 1 \leq \delta$$

$$\frac{x}{\sin(x)} \leq 1+\delta$$

By Taylor's series:

$$\sin(x) = x - \frac{x^3}{3!}$$

$$\frac{\sin(x)}{x} = 1 - \frac{x^2}{6}$$

$$1 - \frac{x^2}{6} = 1 \div \frac{x}{\sin(x)}$$

$$1 - \frac{x^2}{6} = \frac{1}{1+\delta}$$

$$\frac{x^2}{6} = 1 - \frac{1}{1+\delta}$$

$$x^2 = \frac{6\delta}{1+\delta}$$

$$x = \pm\sqrt{\frac{6\delta}{1+\delta}}$$

Therefore, the range for x should be between $-\sqrt{\frac{6\delta}{1+\delta}}$ and $\sqrt{\frac{6\delta}{1+\delta}}$

3. X2 is correct since with 24 bytes, the smallest non-zero number that can be represented is $\frac{1}{2^{24}}$ (only rightmost digit in binary representation is 1, which corresponds to 2^{-24}) The X_2 is smaller than this number, so if the assumption is correct, python can not store X_2. However, the truth is X_2 can be stoded in python, thus X_2 is correct.